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An Application of Control Charts in Manufacturing Industry

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Abstract

The range control chart and the X bar control chart are the well known and the most popular tools for detecting out- of-control signals in the Statistical Quality Control (SQC). The control charts has shown his worth in the manufacturing industry. In this study we have applied the range and the X bar control charts to a product of Swat Pharmaceutical Company. The variables under study were weight/ml, Ph, Citrate % and the amount of fill. Besides the X bar control chart, the exponentially weighted moving average control chart and the multivariate Hotelling's T^2 control chart were applied to the same data.

Mathematics Subject Classification: 97K40, 93E14, 93E10, 26E60
Keywords: Descriptive Statistics, Data Smoothing, Estimation and Detection, Mean

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1 Introduction

Walter, A. Shewhart of the Bell telephone Industries in 1920s developed the Statistical control chart concept. This is considered as the beginning of the Statistical Quality Control (SQC).In 1924, he proposed the concept of control charts (Montgomery, 2000). A control chart shows the value of the quality characteristic of interest as a function of time or sample number. Generally, a control chart is made of a centerline which represents the mean value for the in-control process, and two horizontal lines, the upper control limit (UCL) and the lower control limit (LCL). In 1931, Shewhart published "Economic Control of Quality of Manufactured Product," a book that outlines statistical methods for use in production and control charts methods. A summary of the historical background of statistical quality control is found in "Quality Control and Industrial Statistics" Duncan (1986).

Another program of wide spread interest is the Japanese Quality Control (or Q.C circle). In Japan, the QC circle program is been extremely successful. In western industries adapted the same program by different name called the "Productivity Improvement Team".

2 Methodology

In this study we have applied the control chart approach to the data collected from Swat Pharmaceutical Company. The variables under study were

X_1 = weight/ml, X_2 = Ph, X_3 = Citrate% and X_4 = Amount of fill.

In dataset there were 21 batches of 5 observations each. The following approaches based upon the principles of Statistical Quality Control (SQC) were applied

i. The Range and the X bar control charts.

The upper and lower control limits for Range chart are

$$LCL=D_3\ \bar{R},$$

where $D_3 = 1 - 3\frac{d_3}{d_2}$

$$UCL=D_4\ \bar{R},$$

where $D_4 = 1 + 3\frac{d_3}{d_2}$

The upper and lower control limits for mean are

$$UCL = \bar{\bar{X}} + \frac{3}{d_2\sqrt{n}}\bar{R} = \bar{\bar{X}} + A_2\ \bar{R}$$

$$LCL = \bar{\bar{X}} - \frac{3}{d_2\sqrt{n}}\bar{R} = \bar{\bar{X}} - A_2\ \bar{R}$$

ii. The Exponentially Weighted Moving Average (EWMA) Control Chart.
The upper and lower control limits are

$$\overline{\overline{X}} + 3\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2t}}$$

iii. The Multivariate Quality Control Chart.
The Hotelling's T^2 statistic is

$$T^2 = m\ (\overline{X}_j - \overline{\overline{X}})^T\ S^{-1}\ (\overline{X}_j - \overline{\overline{X}})$$

Plotting the T^2 values on the time axis. The lower control limit is zero, and the upper control limit is

$$UCL = \frac{(n-1)\ (m-1)}{nm-n-p+1} F_{2,\ nm-n-p+1}(0.05)$$

3 Main Results

3.1 Range and X bar control charts for Weight/ ml

For dataset 1 of the weight /ml the range and X bar control charts are shown in Figures 3.1(a) and 3.1(b), respectively.
For R chart the LCL= 0.000 and the UCL =0.03635
For X bar chart the LCL = 1.092 and the UCL =1.131

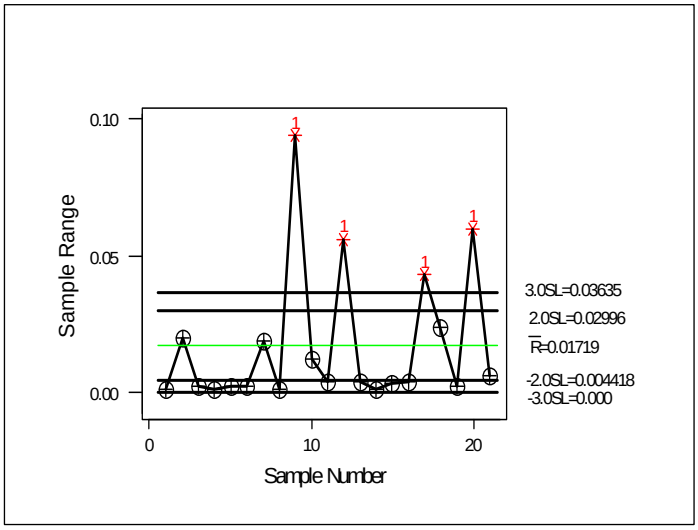

Figure 3.1(a): Range Control Chart for wt/ ml

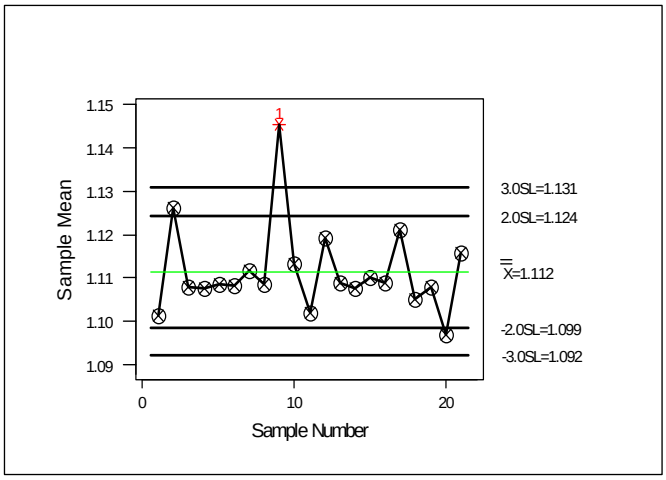

Figure 3.1(b): X bar Control, Chart for wt/ ml

Any point falling outside the control limits indicates that assignable causes had affected the process and the process is out of control. Looking into the X bar control chart in Figure 3.1b) the 9th point has crossed the upper control limit. It is also observed that fourteen points are falling below the center line. Both the range and the X bar control charts shows that the points falling between the control limits are not random. It means that the assignable causes have affected the process.

For the same dataset 1 Exponentially Weighted Moving Average Control Chart is shown in Figure3.1(c).

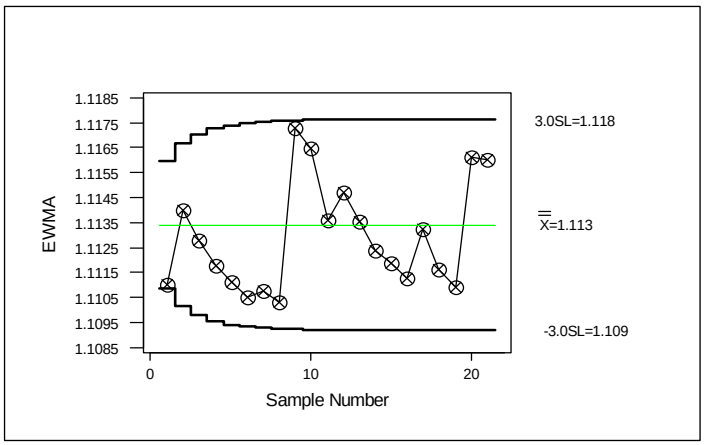

Figure 3.1(c): EWMA Control Chart for wt /ml

In the EWMA control chart, though all the points falling inside the control limits but a sharp shift is observed after the second point and the same is observed after 9th point till 19th point. These are the clear indications that the points falling with in the control limits are not showing random behavior and the process is out of control. Hence both the X bar control chart and the EWMA control charts show that the process is out of control.

3.2 Range and X bar Control Chart for Ph

For dataset 2nd of Ph the range and X bar control charts are shown in Figures 3.2(a) and 3.2(b) respectively.
For R chart the LCL= 0.000 and the UCL =0.04602
For X bar chart the LCL = 5.554 and the UCL =5.581

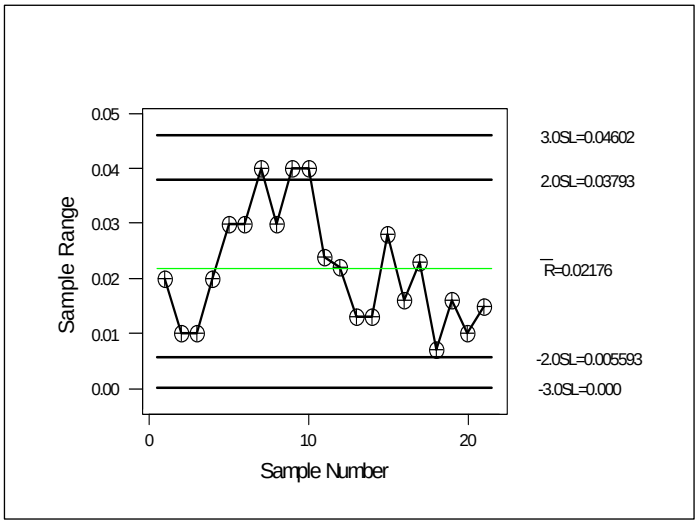

Figure 3.2(a): Range Control Chart for Ph

In X bar control chart for Ph it is observed that seven points have crossed the upper and as will as the lower control limits. These are the indications that the process is out of control.

For the same dataset 2nd the EWMA control chart is constructed in Figure 3.2(c).

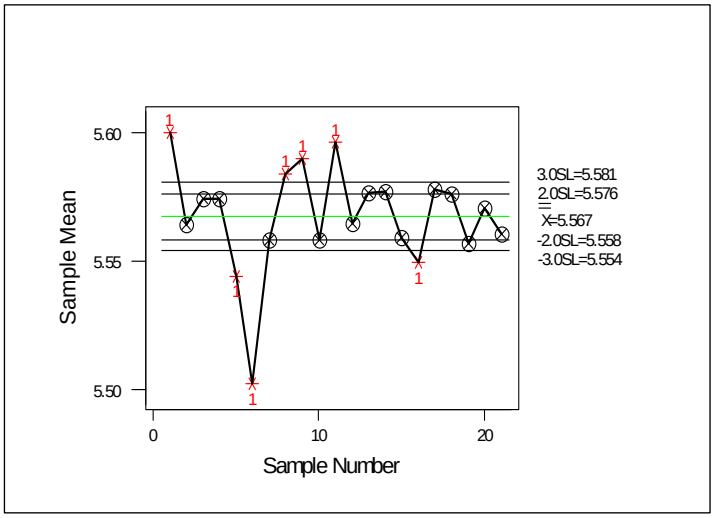

Figure 3.2(b): X bar Control Chart for Ph

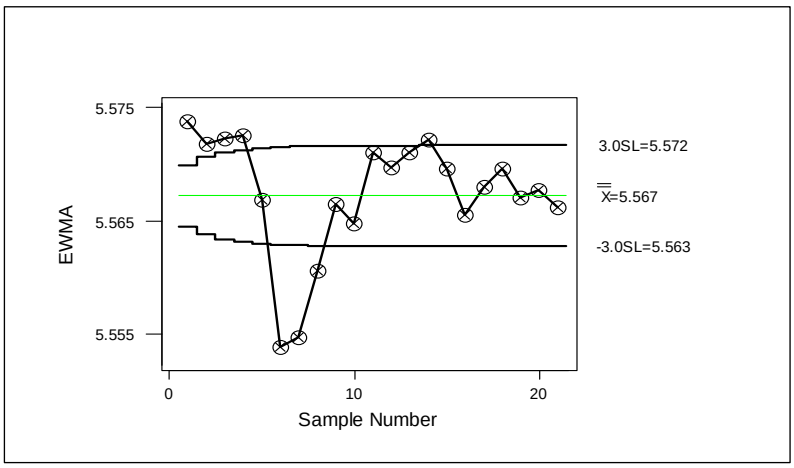

Figure 3.2(c): EWMA Control Chart for Ph.

Looking into the EWMA Control Chart of Ph it is observed that the first four points in a row, and the 6th, 7th and 8th point falling outside the control limits. It indicates that the process is out of control. Comparing the X bar and EWMA control charts, both shows that assignable causes has affected the process.

3.3 Range and X bar Control Charts for Citrate %

For the dataset 3rd of citrate % the range chart and X bar control charts are shown in Figure 3.3(a) and 3.3(b) respectively.
For R chart the LCL= 0.000 and the UCL =1.446
For X bar chart the LCL = 100.3 and the UCL =101.1
Looking into the X bar chart of citrate % all the points except one falls outside the control limits, so the process is out of control.

For the same dataset 3rd the EWMA control chart is constructed in Figure 3.3(c).

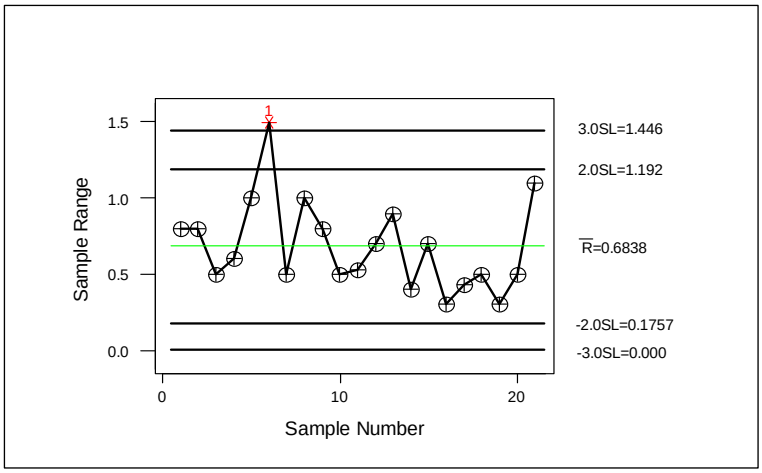

Figure 3.3(a): Range Chart for Citrate %

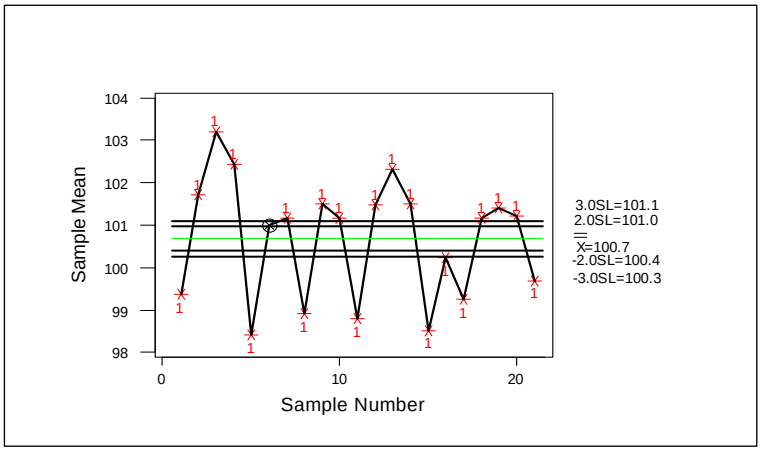

Figure 3.3(b): X bar Chart for Citrate %

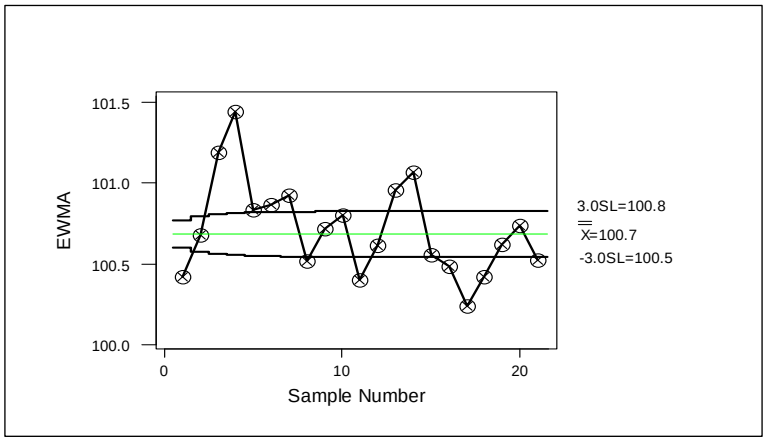

Figure 3.3(c): EWMA Control Chart for Citrate %

Like the X bar chart, the same pattern is shown in the EWMA control chart. Maximum of the points falling outside the control limits. Therefore the process is out of control.

3.4 Range and X bar Control Charts for Amount of fill

For the dataset 4th of amount of fill the range chart and X bar control charts are shown in Figure 3.4(a) and 3.4(b) respectively.
For R Chart the LCL=0.000 and the UCL =4.179
For X bar chart the LCL =28.22 and the UCL =30.47

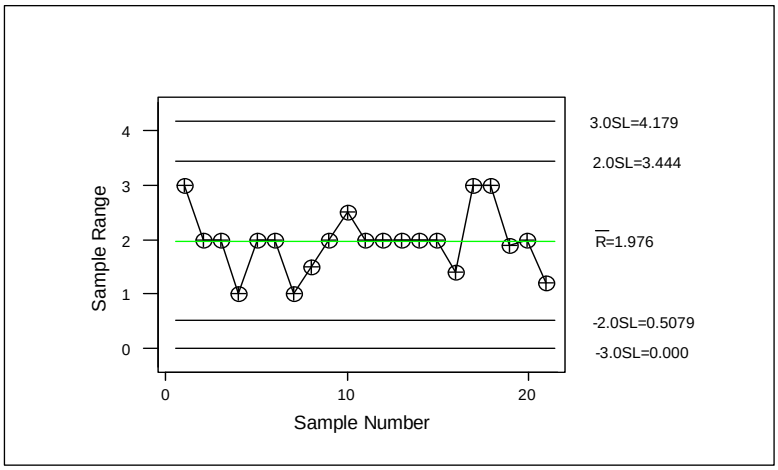

Figure 3.4(a): Range Chart for amount of fill

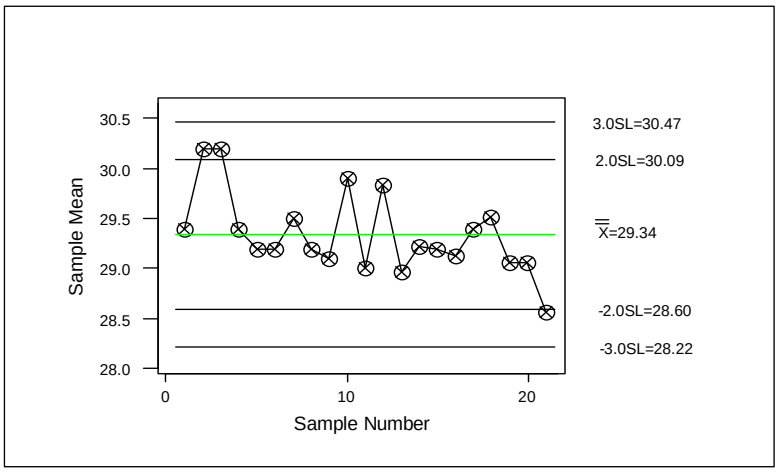

Figure 3.4(b) : X bar Chart for amount of fill

Looking into the X bar chart for the amount of fill the second and third points have crossed the upper warning limit and the last point has crossed the lower warning limit. All the other points in the process are inside the limits and at the same time showing random attitude. These are the indications that the process is very much smooth and in control.

For the same dataset 4th the EWMA control chart is constructed as follows

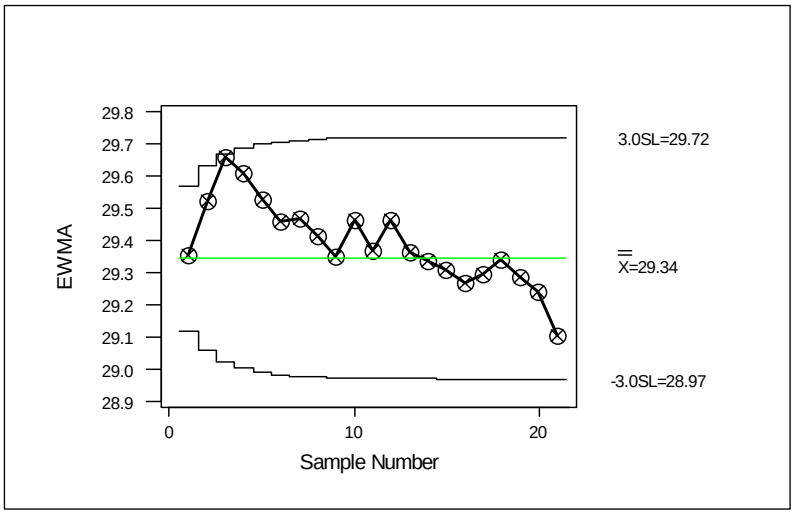

Figure 3.4(c): EWMA Control Chart for amount of fill

Like the X bar control chart, the EWMA control chart, all the points are inside the control limits. The process is in statistical control.

4 Multivariate Quality Control Chart

The Hotelling's T^2 statistic is

$$T^2 = m\ (\overline{X}_j - \overline{\overline{X}})^T\ S^{-1}\ (\overline{X}_j - \overline{\overline{X}})$$

Plotting the T^2 values on the time axis. The lower control limit is zero, and the upper control limit is

$$UCL = \frac{(n-1)\ (m-1)}{nm-n-p+1} F_{2,\ nm-n-p+1}(0.05)$$

where n = 21, m = 5 and p = 4

$$UCL = (0.98765)\ F_{2,81}(0.05) = 3.0888$$

The mean and variance-covariance matrices are as $\overline{\overline{X}} = \begin{pmatrix} 1.1134 \\ 5.5672 \\ 100.68 \\ 29.345 \end{pmatrix}$

$$S = \begin{pmatrix} 0.0004867 & 0.0000198 & 0.0030566 & 0.0016469 \\ 0.0000198 & 0.0004974 & -0.0008386 & 0.0006358 \\ 0.0030566 & -0.0008386 & 1.9379663 & 0.2446408 \\ 0.0016469 & 0.0006358 & 0.2446408 & 0.7238425 \end{pmatrix}$$

$$S^{-1} = \begin{pmatrix} 2088.57 & -83.2197 & -2.86155 & -3.71172 \\ -83.2197 & 2018.46 & 1.25826 & -2.00886 \\ -2.86155 & 1.25826 & 0.543565 & -0.178305 \\ -3.71172 & -2.00886 & -0.178305 & 1.45199 \end{pmatrix}$$

Now putting the values we get T_j^2

$T_1^2 = 3.31323$	$T_2^2 = 1.53887$	$T_3^2 = 3.82717$	$T_4^2 = 1.91318$
$T_5^2 = 3.92998$	$T_6^2 = 9.12675$	$T_7^2 = 1.66585$	$T_8^2 = 0.294206$
$T_9^2 = 2.17100$	$T_{10}^2 = 3.40705$	$T_{11}^2 = 3.64705$	$T_{12}^2 = 0.692889$
$T_{13}^2 = 2.22814$	$T_{14}^2 = 0.692763$	$T_{15}^2 = 2.86045$	$T_{16}^2 = 0.0908045$
$T_{17}^2 = 1.39555$	$T_{18}^2 = 0.494162$	$T_{19}^2 = 0.715299$	$T_{20}^2 = 0.37604$
$T_{21}^2 = 1.13555$			

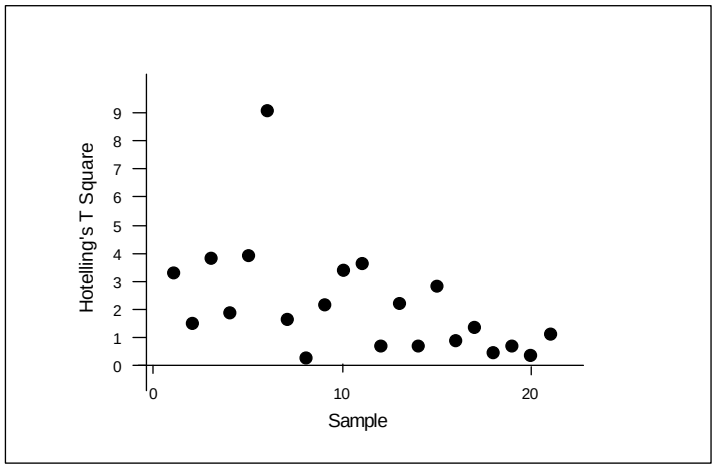

Figure 4: Hotelling's T^2 Control Chart

Looking into the Multivariate Hotelling's T^2 control chart based on subsample means, six points have crossed the upper control limit. It means that some assignable causes have affected the process and process is out of control.

5 Summary

In any manufacturing industry monitoring of their key characteristics is most important. The absence of a quality control section or the inefficient quality control section may be one cause of poor quality. The range control chart and the X bar control chart are the well known and the most popular tools for detecting out- of-control signals in the Statistical Quality Control (SQC). The control charts has shown his worth in the production process. In this study we have applied the range and the X bar control charts to a product of Swat Pharmaceutical Company. The variables under study were Weight/ml, Ph, Citrate % and the Amount of fill. Beside, X bar control chart the exponentially weighted moving average control charts (EWMA) are applied on the same data.

In Figure 3.1(a) all the points are inside the control limits except one. This indicates that the process is out of control. In Figure 3.1(c) for the same dataset the EWMA control chart is shown, all the points falling inside the control limits but a sharp shift is observed after the second point and the same is observed after 9^{th} point till 19th point. These are the clear indications that the points falling with in the control limits are not showing random behavior and the process is out of control. Hence both the X bar control chart and the EWMA control charts show that the process is out of control.

In Figure 3.2(b) seven points have crossed the upper and as will as the lower control limits. For the same dataset in Figure 3.2(c) the first four points in a row, the 6^{th}, 7^{th} and 8^{th} point falling outside the control limits. It indicates that the process is out of control. Comparing the X bar and EWMA control charts, both shows that assignable causes has affected the process and the process is out of control.

In Figure 3.3(a) except one point, all the other points falling outside the control limit. For the same dataset in figure 3.3(c) thirteen points are falling out side the control limits. The process is affected by the assignable causes and the process is out of control.

In Figure 3.4(b) all the points are falling inside the control limits and for the same dataset in Figure 3.4(c) all the points falling inside the control limits. The process is in statistical control.

In Figure 4, the Multivariate Hotelling's T^2 control chart based on subsample means is shown. In this control chart six points have crossed the upper control limit. This means that some assignable causes have affected the process and the process is out of control.

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Appendix

Table 1: Range, X bar and EWMA control limits for weight/ml

Batch#	X_1= weight/ml					$\overline{X}$	R	Z_t	LCL	UCL
1	1.101	1.101	1.101	1.102	1.102	1.1014	0.001	1.11856	1.1100201	1.1139799
2	1.12	1.12	1.12	1.13	1.14	1.126	0.02	1.11468	1.1094644	1.1145356
3	1.109	1.108	1.108	1.107	1.108	1.108	0.002	1.11325	1.1091654	1.1148346
4	1.107	1.107	1.108	1.108	1.108	1.1076	0.001	1.11142	1.1089895	1.1150104
5	1.109	1.108	1.107	1.109	1.109	1.1084	0.002	1.11142	1.1088822	1.1151177
6	1.109	1.108	1.107	1.109	1.108	1.1082	0.002	1.11082	1.1088155	1.1151845
7	1.12	1.12	1.109	1.109	1.101	1.1118	0.019	1.11225	1.1087734	1.1152266
8	1.12	1.12	1.109	1.108	1.109	1.1132	0.012	1.11244	1.1087468	1.1152532
9	1.108	1.109	1.108	1.109	1.108	1.1084	0.001	1.11635	1.1087299	1.1152701
10	1.109	1.12	1.201	1.109	1.107	1.1452	0.094	1.11831	1.1087191	1.1152809
11	1.102	1.103	1.103	1.099	1.103	1.102	0.004	0.9386	1.1087122	1.1152877
12	1.129	1.094	1.101	1.122	1.150	1.119	0.006	0.9747	1.1087078	1.1152921
13	1.111	1.107	1.110	1.108	1.109	1.109	0.004	1.00156	1.1087050	1.1152921
14	1.108	1.107	1.107	1.108	1.108	1.108	0.001	1.02280	1.1087032	1.1152968
15	1.109	1.111	1.109	1.109	1.112	1.110	0.003	1.0402	1.1087021	1.1152979
16	1.110	1.106	1.108	1.110	1.110	1.109	0.004	1.05396	1.1087015	1.1152985
17	1.098	1.123	1.131	1.141	1.112	1.121	0.043	1.0674	1.1087009	1.1152991
18	1.116	1.112	1.111	1.095	1.092	1.105	0.024	1.0749	1.1087006	1.1152994
19	1.107	1.108	1.108	1.109	1.108	1.108	0.002	1.0815	1.1087000	1.1152998
20	1.104	1.122	1.062	1.079	1.118	1.117	0.060	1.0886	1.1086995	1.1153002
21	1.113	1.119	1.113	1.116	1.118	1.116	0.006	1.09408	1.1086991	1.1153006
Average						1.112	0.01719			

Table 2: Range, X bar and EWMA Control Limits for Ph

Batch#		X_2 = Ph				$\overline{X}$	R	Zt	LCL	UCL
1	5.6	5.6	5.6	5.59	5.61	5.600	0.02	5.573600	5.564490	5.569509
2	5.57	5.56	5.56	5.56	5.57	5.564	0.01	5.5716800	5.5637856	5.570214
3	5.58	5.58	5.57	5.57	5.57	5.574	0.01	5.7214400	5.563406	5.570593
4	5.56	5.57	5.58	5.58	5.58	5.574	0.02	5.5725150	5.563183	5.570816
5	5.54	5.54	5.55	5.53	5.56	5.544	0.03	5.5668120	5.563047	5.570952
6	5.5	5.51	5.52	5.49	5.48	5.500	0.04	5.5534496	5.562963	5.571037
7	5.6	5.59	5.58	5.59	5.57	5.586	0.03	5.5599597	5.562909	5.571090
8	5.56	5.56	5.58	5.54	5.55	5.558	0.04	5.5595678	5.562876	5.571124
9	5.60	5.58	5.58	5.57	5.59	5.584	0.03	5.5644542	5.562854	5.571145
10	5.6	5.61	5.58	5.59	5.57	5.59	0.04	5.5695634	5.562841	5.571159
11	5.588	5.588	5.612	5.604	5.588	5.596	0.02	5.5748507	5.562832	5.571168
12	5.557	5.571	5.557	5.557	5.559	5.560	0.01	5.5718806	5.562827	5.571173
13	5.573	5.571	5.575	5.579	5.584	5.577	0.01	5.5729045	5.562823	5.571177
14	5.573	5.575	5.575	5.586	5.575	5.576	0.02	5.5735236	5.562821	5.571192
15	5.546	5.560	5.569	5.574	5.547	5.557	0.03	5.5702189	5.562819	5.571181
16	5.559	5.544	5.544	5.543	5.557	5.547	0.03	5.5655751	5.562818	5.571182
17	5.587	5.564	5.587	5.565	5.586	5.578	0.02	5.5680601	5.562818	5.571182
18	5.574	5.576	5.581	5.574	5.575	5.576	0.01	5.5696481	5.562817	5.571183
19	5.552	5.552	5.559	5.568	5.553	5.557	0.02	5.5671185	5.562817	5.571183
20	5.569	5.569	5.568	5.578	5.568	5.570	0.01	5.5676948	5.562817	5.571183
21	5.556	5.558	5.556	5.571	5.561	5.561	0.02	5.5663558	5.562817	5.571183
Average						5.567	0.02176			

Table 3: Range, X bar and EWMA control limits for Citrate %

Batch #		X_3 = Citrate%				$\overline{X}$	R	Z t	LCL	UCL
1	99.5	99.7	98.9	99.2	99.5	99.36	0.8	100.432	100.64121	100.75879
2	101.5	101.2	101.9	102.0	102.0	101.72	0.8	100.689	100.59898	100.80102
3	103.0	103.2	103.5	103.2	103.2	103.22	0.5	101.196	100.58707	100.81293
4	102.0	102.5	102.5	102.6	102.6	102.44	0.6	101.444	100.58007	100.81993
5	98.0	98.5	98.5	98.0	99.0	98.40	1.0	100.836	100.57579	100.82421
6	100.0	101.0	101.5	101.3	101.2	101.0	0.5	100.869	100.57313	100.82687
7	99.0	99.1	99.5	99.8	99.5	99.38	0.8	100.571	100.57146	100.82854
8	101.0	101.1	101.2	101.5	101.0	101.16	0.5	100.689	100.57040	100.82960
9	99.5	98.5	99.5	98.5	98.50	98.90	1.0	100.331	100.56972	100.83028
10	101.5	101.2	102.0	101.3	101.5	101.5	0.8	100.565	100.56929	100.83071
11	98.57	99.0	98.57	99.1	98.67	98.782	0.53	100.408	100.56902	100.83098
12	101.1	101.5	101.3	101.7	101.8	101.48	0.7	100.622	100.56884	100.83116
13	102.7	101.9	102.8	102.1	102.1	102.32	0.9	100.962	100.56873	100.83127
14	101.5	101.5	101.7	101.3	101.5	101.5	0.4	101.069	100.56866	100.83134
15	98.3	98.53	99.0	98.3	98.4	98.446	0.7	100.544	100.56862	100.83138
16	100.1	100.1	100.3	100.4	100.2	100.78	0.3	100.592	100.56859	100.831141
17	99.21	99.21	99.34	99.0	99.43	99.236	0.43	100.321	100.56857	100.83143
18	101.0	101.1	101.2	101.5	101.0	101.16	0.5	100.489	100.56856	100.83144
19	101.5	101.2	101.3	101.5	101.5	101.4	0.3	100.671	100.56855	100.83145
20	101.0	101.3	101.3	101.5	101.0	101.22	0.5	100.781	100.56855	100.83145
21	98.9	99.83	99.85	100.0	99.8	99.876	1.1	100.599	100.56855	100.83145
Average						100.7	0.6838			

Table 4: Range, X bar and EWMA control limits for the Amount of fill

Batch #	X_4 =Amount of fill					$\overline{X}$	R	Z_t	LCL	UCL
1	29	30	28	29	31	29.4	3.0	29.38	29.12349	29.546971
2	30	31	29	30	31	30.2	2.0	29.611	29.000724	29.623275
3	29	30	31	30	31	30.2	2.0	29.688	29.000736	29.625771
4	29	30	29	29	30	29.4	1.0	29.631	29.000440	29.625885
5	30	29	28	28	29	28.8	2.0	29.465	29.000381	29.656912
6	29	30	28	29	30	29.2	2.0	29.412	29.000186	29.667110
7	29	30	30	29	29.5	29.5	1.0	29.429	29.000190	29.668221
8	30	29.5	28.5	29	29.5	29.3	1.5	29.404	29.000164	29.670001
9	30	29	29	29	28	29.1	2.0	29.433	28.988641	29.676883
10	31	30	29	31	28.5	29.9	2.5	29.454	28.988589	29.681761
11	28	29	30	28	30	29.0	2.0	29.363	28.988361	29.686382
12	30	31	29	29.2	30	29.84	2.0	29.458	28.988286	29.686414
13	28.8	30	29	28	29	28.96	2.0	29.358	28.988262	29.687138
14	30	29.1	29	30	28	29.22	2.0	29.331	28.988130	29.688000
15	29	30	29	28	30	29.20	2.0	29.305	28.988100	29.688096
16	29	29	28.6	30	29	29.12	1.4	29.268	28.987514	29.688186
17	28	29	31	29	30	29.40	3.0	29.294	28.987463	29.689107
18	30	28	28.6	30	31	29.52	3.0	29.339	28.987315	29.689285
19	29	29.2	30	28.1	29	29.06	1.9	29.283	28.987136	29.700934
20	29	29.1	28	29.2	30	29.06	2.0	29.238	28.975730	29.718952
21	28.6	28	29.2	29	28	28.56	1.2	29.103	28.970014	29.718948
Average						29.34	1.976			