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Multivariate Statistical Analysis of Gongola Basin Residual Gravity Anomalies for Hydrocarbon Exploration

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Abstract

An efficient treatment of the gravitational inverse problem is possible if an optimization method is applied in the determination of the residual field. An optimal residual model should adapt to the observed gravity field data in the best possible way and take into account the geology of the area.

Three polynomial functional models of the first and second degree in one and two variables were used in the computation of the regional anomaly which was separated from the Bouguer gravity anomaly to obtain the residual vector. The polynomial fittings were applied from two options; firstly to the individual Bouguer gravity anomaly profiles and secondly to the entire network of points within the basin. The linearization of the models yielded a set of linear equations which were solved by method of least squares adjustment. Applications of multivariate statistics in analyzing the results obtained from the least squares

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adjustment in terms of multivariate confidence intervals (MCI), null hypothesis test and correlation coefficients were carried out to determine the model that is most suitable for basin analysis.
The quadratic polynomial model as applied to the individual profiles was found have the least sum of squares of residuals and va/riances. Its correlation coefficient is also higher than that obtained in the other models applied in the two options. The null hypotheses were not rejected at 5% significant level. The quadratic model is considered the most suitable for basin analysis in terms of hydrocarbon exploration.

Keywords: Bouguer anomaly, residual, polynomial, least squares, Confidence interval, null hypotheses

1 Introduction

Bouguer gravity anomaly maps always contain the superposition of disturbances of noticeable order of size. These superpositions are as a result of deep-seated structures. In Gongola basin, the presence of large scale, deep-seated structural features and density effects caused by intrabasement lithologic changes cause significant regional variations in the gravitational field. The removal of the regional gravity anomaly resulting from the deep seated structures which often distort or obscure the effects of the structures that are sought in oil exploration is of great concern to geophysicist. In achieving a good result in the separation of the regional gravity from the Bouguer gravity anomaly, polynomial functional models of the first and second degree involving one and two variables were applied for the computation of the regional gravity anomaly using the least squares technique. The first model (A) was developed for the computation of regional anomaly by utilizing the relationship between the station elevation, the weathered tertiary sediment density, the free air anomaly and Bouguer correction in the computation of Bouguer anomaly. The weathered tertiary density value of 2.2 was adopted in the computation of Bouguer gravity anomaly. This value was obtained from the application of Nettleton and Parasnis methods and validated using the density log from the basin. The second and third models (B and C) are first and second degree polynomials in two variables.

Applications of multivariate statistics for analyzing results obtained from the least squares adjustment in terms of multivariate confidence intervals (MCI) and null hypothesis testing and correlation coefficients were carried out to determine the model that is most suitable for basin analysis.

The Criteria for selecting the best regional gravity anomaly whose residuals will be used for basin analysis are based on the following:

i. The residual anomaly derived from the regional anomaly should correlate with the geology of the basin
ii. The sum of squares of the residual gravity anomaly obtained from the regional anomaly should be of minimum value $(\sum v^2 = \min imum)$.
iii. The null hypotheses should be satisfied at 5% significant level.
iv. The correlation coefficient should be maximum.

1.1 Multivariate Interval Estimation

In order to know how good the estimate of the polynomial coefficients and the variance obtained are in terms of probability, the confidence intervals were used. The confidence interval sets up the probability statement concerning the critical limits of the parameters. This in turn forms the basis for hypothesis testing. In least squares adjustment, the a-posteriori variance of unit weight is defined by:

$$\hat{\sigma}_0^2 = \frac{\hat{V}^T P \hat{V}}{df}, \tag{1}$$

where

df = degrees of freedom,

P = unit weight matrix of observation, which is defined as

$$P = \sigma_0^2 \sum\nolimits_{L^b}^{-1} , \tag{2}$$

σ_0^2 = a-priori variance of unit weight

$\sum_{L^b}$ = vector of observation

Ayeni (1981) has shown that $\hat{V}^T \sum_{L^b}^{-1} \hat{V}$ is distributed as Chi-squared (X^2) with degrees of freedom df (same as in eqn. (1)). The probabilistic statement (see Ayeni,1981)

$$X^2_{\alpha/2}\sigma_0^2 \leq \hat{V}^T P \hat{V} \leq X^2_{1-\alpha/2}\sigma_0^2 \tag{3}$$

Expresses the multivariate confidence interval (MCI) on $\hat{V}^T P \hat{V}$ at $(1-\alpha/2)$ percent. The critical region for testing the null hypothesis on $\hat{V}^T P \hat{V}$ is carried using the MCI.

Further treatment of the confidence intervals with respect to the polynomial coefficients can be obtained in Krumbien and Graybill (1965) pages 229-231 respectively.

1.2 Multivariate Hypothesis Testing

This is required to test whether $\hat{V}^T PV$ is too large or too small compared with the a-priori variance unit weight σ_0^2 assumed for the adjustment. From the confidence interval in equation (3). The three possible hypotheses used in this research are:

i. $Ho_1 : \hat{V}^T P\hat{V} = \sigma_0^2$, corresponds to testing if $\hat{V}^T P\hat{V}$ is too large or too small the criterion for rejecting Ho_1 is as follows:

Reject Ho_1, if $X^2 > X^2_{1-\alpha/2}(df)$ or $X^2 < X^2_{1-\alpha/2}(df)$

ii. $Ho_2 : \hat{V}^T P\hat{V} = \sigma_0^2$ corresponds to testing if $\hat{V}^T P\hat{V}$ is too small, which is one tailed test $Ho_2 : \hat{V}^T P\hat{V} < \sigma_0^2$

Reject Ho_2, if $X^2 < X^2_{\alpha}(df)$

iii. $Ho_3 : \hat{V}^T P\hat{V} = \sigma_0^2$ corresponds to testing if $\hat{V}^T P\hat{V}$ is too large

$H_3 : \hat{V}^T P\hat{V} > \sigma_0^2$

Reject Ho_3, if $X^2 > X^2_{1-\alpha}(df)$

1.3 Linear Correlation

The simple linear model and the multiple correlation coefficients were computed as a measure to indicate the adequacy of the variables (coordinates, elevation and distances) in the prediction of the regional gravity anomaly. The coefficient of correlation is positive for direct correlation in the case of basement complex region while it is negative for inverse correlation in the case of sedimentary region for a simple linear model in one variable. The sedimentary rocks basin has low densities and thus produces a negative gravity anomaly. The negative anomaly is high where the basin is deepest.

The estimate of the square of the multiple correlation coefficients is given by (Krumbien and Graybill, 1965)

$$\hat{R}^2 = \frac{\sum_{i=1}^{k} R_i r_i}{\sum_{i=1}^{n} (Y_i - \hat{Y})^2}, \qquad (4)$$

where $\sum_{i=1}^{k} R_i r_i$ is the sum of the products of elements in the column labeled g in the abbreviated Doolittle format (Krumbien and Graybill, 1965), and the quantity

$$\sum_{i=1}^{k} R_i r_i = \sum_{i=1}^{k} \hat{\beta}_i (\sum_{i=1}^{n} X_{ji} Y_j), \tag{5}$$

is called “the reduction due to estimating the parameters $\beta_1, \beta_2 .... \beta_k$ in the linear model.”

2 Methodology

2.1 Data Acquisition

The gravity data used in this research were secondary data obtained from Shell Nigeria Exploration and Production Company. The study area is OPL 803/806/809 within the Gongola basin. It contains 1831 observed gravity station data with station interval of 500m.

2.2 Data Quality

The gravity observations were repeated ten times at each gravity station. The standard deviation of each observed gravity value was found to be 0.013mgal. Also, the average standard error of the gravity base stations was found to be 0.015mgal (Idowu, 2006). The difference between the observed gravity anomalies and the predicted gravity values using least squares collocation technique was given as $\sum V_{col} = -8.4x10^{-6}$ while the mean square error is $\sum V_{col}^2 = 7.06x10^{-11}$. Based on the application of least squares collocation in the prediction of the gravity data by Idowu (2006), it can be inferred that the validity, reliability and the quality of the gravity data used in this research are satisfactory.

2.3 The Polynomial functional Model Computations

The polynomial functional models are expressed as follows:

$$\textit{Model}\ \text{A:}\ \ \Delta g_{reg} = \alpha + 0.2164\beta H_s s_s + e_i \tag{6}$$

$$\textit{Model}\ \text{B:}\ \ \Delta g_{reg} = \alpha x_s + \beta y_s + \gamma + e_i \tag{7}$$

$$\textit{Model}\ \text{C:}\ \ \Delta g_{reg} = \alpha x_s^2 + \beta y_s^2 + \gamma x_s y_s + \lambda x_s + \xi y_s + \kappa + e_i \tag{8}$$

where Δg_{reg} = regional gravity anomaly.

$\alpha, \beta, \gamma, \lambda, \xi, \kappa =$ the polynomial coefficients,

$H_s =$ elevation,

$\text{s}_s =$ distance with respect to the first station

$$x_s = X_s - X_0 \tag{9}$$

$$y_s = Y_s - Y_0 \tag{10}$$

where X_s, Y_s are the station point coordinates, X_0, Y_0 are coordinates of the map origin, $X_0 = 625000\text{mE}$, $Y_0 = 1096818\text{mN}$.

In the equations (6), (7) and (8), $E(e_i) = 0$, variance $(e_i) = \sigma^2$. The two options adopted in the application of the models are:

Option 1: This involves the determination of a generalized polynomial fitting and modl equations for the entire 1813 gravity observations of the basin. Fig.1 shows a model of the gravity observations with the generalized linear fitting for all the observations irrespective of the line. The coefficients were used in the determination of a generalized regional field of the basin.

Option 2: This involves the determination of the polynomial fitting and the model equation with respect to the individual line of observation. The polynomial coefficients were used in the determination of the regional field of each line. The combination of the regional field produced by individual lines was used in the determination of the entire regional field of the basin. In Fig 1, the straight lines on theY-axis crossing the observation points represent the polynomial fitting of each line.

The confidence intervals, hypotheses testing and correlation coefficients were determined by computing the following:

i. The 95% confidence interval for α with confidence coefficient $1-\gamma$ equal to $(t_{\gamma/2}(n-2)),(t_{\gamma/2}(n-3))$ and $(t_{\gamma/2}(n-6))$ respectively.

ii. The 95% confidence interval for β with confidence coefficient $1-\gamma$ equal to $(t_{\gamma/2}(n-2)),(t_{\gamma/2}(n-3))$ and $(t_{\gamma/2}(n-6))$ respectively.

iii. The 95% confidence interval for σ^2 with confidence coefficient $1-\gamma$ equal to $(X^2_{\gamma/2}(n-2))$, $(X^2_{\gamma/2}(n-3))$ and $(X^2_{\gamma/2}(n-6))$ respectively.

iv. Test the hypotheses $H_0:\alpha=\alpha_0$, where α_0 is a given constant.

v. Test the hypotheses $H_0:\beta=\beta_0$, where β_0 is a given constant.

vi. Test the hypotheses $H_0:\sigma^2=\sigma_0^2$, where σ_0^2 is a given constant.

ix. Test the hypotheses $H_0:\alpha_0=\beta_0=0$, for the two variable polynomial

x. Test the hypotheses for the second degree polynomial in two variable as :

$$H_0:\alpha_0=\beta_0=\gamma_0=\lambda_0=\xi_0=0,$$

xi. The linear correlation coefficient $\hat{p}$.

xii. The multiple linear correlation coefficient $\hat{R}^2$.

.

3 Results and Analysis

3.1 Results

The following results in Tables 1, 2, 3 and 4 were obtained for options 1 and 2 with respect to models A, B and C.

3.2 Analysis

3.2.1 Polynomial Functional Model in One and Two Variables (Models A, B and C)

Based on the results obtained, in Tables 1, 2, 3 and 4, the following are inferred:

1. Columns 2, 3 and 4 (Table 1a) and Columns 3,4 and 5 (Table 1b) show the 95% confidence interval, while column 2 (Table 2a) and column 4 (Table 2b) show the a-priori variance for model A (options 1 and 2) respectively. Columns 2, 3 and 4 (Table 1c) and Columns 3,4 and 5 (Table 1d) show the 95% confidence interval for the polynomial coefficients, while column 4 (Table 2a) and column 5 (Table 2b) show the a-priori variance for model B (options 1 and 2 respectively). Column 6 (Table 4.14b) shows the a-priori variance of model C (option 1). Columns 6,7 and 8 (Table 2a) and Columns 7,8 and 9 (Table 2b) shows the sum of squares of the residual gravity anomalies for options 1 and 2 using the three models. It could be inferred that the a-priori variances of all the two options using model A, B and C are unbiased estimators of the regional gravity anomaly. However, the mean variance and sum of squares of the residuals for model C (option 2) is lower than that obtained using models A and B for the two options.
2. In columns 3 and 4 (Table 3a) and columns 4 and 5 (Table 3b), the results of the multivariate hypotheses tests for the variances of all the gravity lines show that $X^2 < X^2_{\gamma/2}(n-2)$. Hence the null hypothesis (H_0) is accepted. It could be inferred that the variances are neither too large nor too small and therefore, the least squares adjustment is not distorted at 5% significance level. The null hypothesis used is one tailed at 5% significance level.
3. In columns 5, 6 and 7 (Table 3a) and columns 6, 7 and 8 (Table 3b), the results of the null hypotheses tests for the polynomial coefficients show that $H_0 > -t_{\gamma/2}(n-2)$ and $H_0 < t_{\gamma/2}(n-2)$ for all the gravity lines. In columns 2,

3 and 4 (Table 3c) and columns 3, 4 and 5 (Table 3d), the results of the null hypotheses tests for the polynomial coefficients for model B (option 2) show that $H_0 > -F_{\gamma/2}(n-3)$ and $H_0 < F_{\gamma/2}(n-3)$ for all the gravity lines. Also in columns 3 and 4 (Table 3e) columns 4 and 5 (Table 3f) the results of the null hypotheses tests for the polynomial coefficients for model C show that $H_0 > F_{\gamma/2}(n-6)$ for options 1 and 2. Hence, the polynomial functional models for computing the regional anomaly in models A, B and C for options 1 and 2 are accepted. The values of the polynomial coefficients of models A, B and C for both options all fell within the 5% significance level. The polynomial coefficients did not introduce any significant distortion in the computation of the regional gravity anomaly. The null hypothesis used is one tailed at 5% significance level.

4. In column 8 (Table 3a) shows a negative correlation coefficient for the entire basin. Also, in column 9 (Table 3b), the correlation coefficient obtained in all the gravity lines is negative except in lines 94V007 and 94V045. This is because the basin is dominated by dolomites and sandstone whose density contrast with the basement is negative and as such produces negative gravity anomaly. The average correlation coefficient for all the gravity lines for model A is -51% (option1) and -63.96% (option 2). In column 5 (Table 3c) and column 6 (Table 3d), the average multiple correlation coefficient obtained in all the gravity lines from model B is 68% (option 1) and 71.38% (option 2), while in column 5 (Table 3e) and column 6 (Table 3f), the average multiple correlation coefficient for all the gravity lines obtained from model C is 73% (option1) and 93.6% (option 2).

5. Based on the above least squares analysis of the model results, Table 4 shows the ranking of the models. From the table, model C (option 2) has the least sum of squares of residuals and variances and the highest value for correlation coefficient and as such is considered the best model for basin analysis.

4 Summary of Findings

i. The maps developed using the gravity data show that the Gongola basin consists of four zones namely: sedimentary, transition, granite pluton and basement complex zones.

ii. The anomalous mass under investigation is located along a composite profile 94V071/95D071 and 94V037 respectively. The gravity contour closures are better defined in model C option 2 than in the other models. Figures 1 and 2 show the residual gravity anomaly maps derived from option 2, models B and C respectively.

iii. The excess mass computed using option 2 is $1.613x10^{11}Kg$ for model A, $1.79727x10^{11}Kg$ for model B and $4.7x10^{10}Kg$ for model C.

iv. The regional profiles of model C (option 1) were all straight lines, while the profiles of the same model in option 2 followed the symmetric path of the Bouguer profiles thereby producing a minimized residual for basin analysis.

v. The regional field in the Gongola basin has numerous geological convolutions and as such, the second degree polynomial as applied to the individual profiles is preferred to the other polynomials for basin analysis.

5 Conclusion

The application of the multivariate statistical analyses on the residual gravity anomaly results provided a valuable tool in the determination of the most appropriate model for basin analysis. The null hypotheses provided the validity of the chosen model. The models did not introduce any distortion at 5% significant level. The map produced using model C as applied to the individual profiles should be used in the delineation of the Gongola basin for hydrocarbon exploration.

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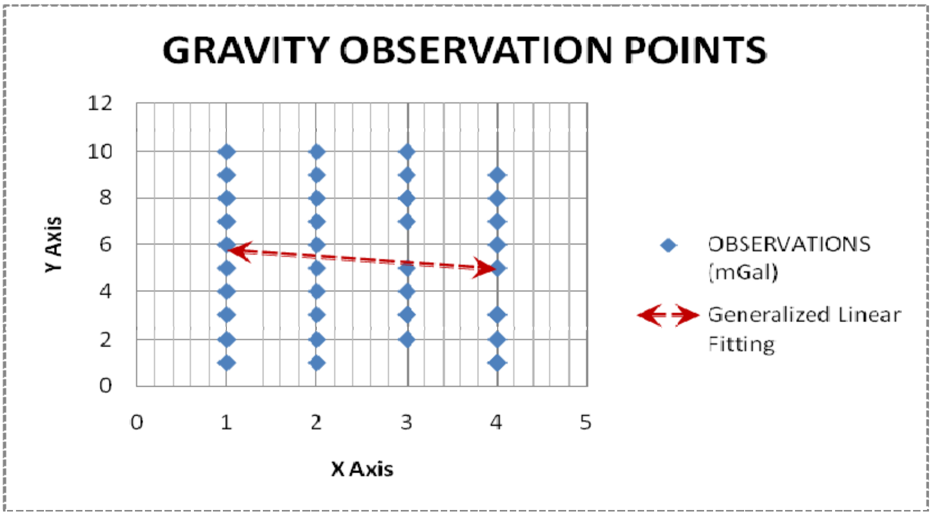

Figure 1: A Model of the Polynomial Fitting Methods

Table 1a: Results of 95% Confidence Intervals for Model A (Option 1)

S/N	Confidence Interval σ^2 $X^2_{\gamma/2}(n-2)$ $\gamma = 0.05$	Confidence Interval β $t_{\gamma/2}(n-2)$ $\gamma = 0.05$	Confidence Interval α $t_{\gamma/2}(n-2)$ $\gamma = 0.05$
1	$73.1 < \delta^2 < 96.5$	$-0.00016 < \beta < -0.00014$	$-7.83 < \alpha < -7.77$

Table 1b: Results of 95% Confidence Intervals for Model A

(Option 2)

S/N	LINE	Confidence Interval σ^2 $X^2_{\gamma/2}(n-2)$ $\gamma = 0.05$	Confidence Interval β $t_{\gamma/2}(n-2)$ $\gamma = 0.05$	Confidence Interval α $t_{\gamma/2}(n-2)$ $\gamma = 0.05$
1	94V007	$1.09 < \delta^2 < 2.65$	$-0.0038 < \beta < -0.0034$	$-0.00892 < \alpha < -0.0091$
2	94V020	$3.60 < \delta^2 < 8.51$	$-0.0006 < \beta < -0.0005$	$-0.00149 < \alpha < -0.00151$
3	94V023	$10.88 < \delta^2 < 18.08$	$-0.0006 < \beta < -0.0004$	$-0.00258 < \alpha < -0.00261$
4	94V037	$0.98 < \delta^2 < 2.38$	$-0.0017 < \beta < -0.0015$	$-23.97 < \alpha < -24.29$
5	94V039	$2.54 < \delta^2 < 4.23$	$-0.0019 < \beta < -0.0017$	$-0.99 < \alpha < -1.007$
6	94V045	$1.64 < \delta^2 < 3.61$	$-0.0024 < \beta < -0.0022$	$-5.06 < \alpha < -5.13$
7	94V055	$2.21 < \delta^2 < 3.92$	$-0.0016 < \beta < -0.0014$	$-4.02 < \alpha < -4.05$
8	94V071	$5.34 < \delta^2 < 9.48$	$-0.0011 < \beta < -0.0012$	$-9.38 < \alpha < -9.37$
9	94V080	$3.47 < \delta^2 < 6.16$	$-0.0017 < \beta < -0.0015$	$-0.00118 < \alpha < -0.00191$
10	94V120	$10.06 < \delta^2 < 17.86$	$-0.0017 < \beta < -0.0011$	$-0.00158 < \alpha < -0.00162$
11	94V146	$5.22 < \delta^2 < 11.49$	$-0.0059 < \beta < -0.0045$	$-6.431 < \alpha < -6.809$
12	94D032	$7.59 < \delta^2 < 12.61$	$-0.0007 < \beta < -0.0005$	$-0.00179 < \alpha < -0.00181$
13	94D039	$10.66 < \delta^2 < 17.72$	$-0.0013 < \beta < -0.0011$	$-0.00139 < \alpha < -0.00141$
14	94D048	$11.74 < \delta^2 < 20.85$	$-0.0016 < \beta < -0.0014$	$-0.00128 < \alpha < -0.00131$
15	94D064	$3.70 < \delta^2 < 6.58$	$-0.0018 < \beta < -0.0016$	$-0.00169 < \alpha < -0.00171$
16	94D096	$11.45 < \delta^2 < 19.04$	$-0.0015 < \beta < -0.0013$	$-0.00159 < \alpha < -0.00161$
17	95D030	$1.93 < \delta^2 < 3.21$	$0.0016 < \beta < 0.0020$	$-27.83 < \alpha < -28.34$
18	95D071	$0.23 < \delta^2 < 0.41$	$-0.00237 < \beta < -0.00231$	$-14.01 < \alpha < -14.07$
19	DA1	$0.23 < \delta^2 < 0.89$	$-0.0025 < \beta < -0.0020$	$3.98 < \alpha < 4.02$
20	DA2	$0.57 < \delta^2 < 1.27$	$-0.0016 < \beta < -0.0090$	$-1.23 < \alpha < -1.24$
21	DA3	$1.27 < \delta^2 < 2.81$	$-0.00088 < \beta < -0.0005$	$-3.08 < \alpha < -3.13$

Table 1c: Results of 95% Confidence Intervals for Model B (Option 1)

S/N	Confidence Interval α $X^2_{\gamma/2}(n-3)$ $\gamma = 0.05$	Confidence Interval β $t_{\gamma/2}(n-3)$ $\gamma = 0.05$	Confidence Interval γ $t_{\gamma/2}(n-3)$ $\gamma = 0.05$
1	$-0.00019 < \alpha < -0.00017$	$-0.0035 < \beta < -0.0033$	$13.2495 < \gamma < 13.2496$

Table 1d: Results of 95% Confidence Intervals for Model B (Option 2)

S/N	LINE	Confidence Interval α $X^2_{\gamma/2}(n-3)$ $\gamma = 0.05$	Confidence Interval β $t_{\gamma/2}(n-3)$ $\gamma = 0.05$	Confidence Interval γ $t_{\gamma/2}(n-3)$ $\gamma = 0.05$
1	94V007	$-0.0047 < \alpha < 0.0047$	$-0.0045 < \beta < 0.0049$	$-21.82 < \gamma < -21.80$
2	94V020	$0.000285 < \alpha < 0.000539$	$-0.00091 < \beta < -0.00066$	$1.5174 < \gamma < 1.5175$
3	94V023	$-0.00045 < \alpha < -2.04E-5$	$-0.0014 < \beta < -0.00099$	$15.349 < \gamma < 15.350$
4	94V037	$4.39E-6 < \alpha < 0.00040$	$0.00321 < \beta < 0.00361$	$-278.451 < \gamma < -278.450$
5	94V039	$-0.00049 < \alpha < 0.00013$	$-0.00190 < \beta < -0.00127$	$54.1493 < \gamma < 54.1499$
6	94V045	$-0.00018 < \alpha < 0.00020$	$-0.000154 < \beta < 0.000228$	$-11.762 < \gamma < -11.761$
7	94V055	$-0.00021 < \alpha < -0.00017$	$-0.000125 < \beta < -8.04E-5$	$-4.5164 < \gamma < -2.5163$
8	94V071	$-0.00047 < \alpha < 0.00044$	$-0.00085 < \beta < 6.64E-5$	$9.328 < \gamma < 9.329$
9	94V080	$-0.00337 < \alpha < -0.00327$	$-0.00105 < \beta < -0.00094$	$269.4191 < \gamma < 269.4192$
10	94V120	$0.00169 < \alpha < 0.00204$	$-0.00152 < \beta < -0.00112$	$-140.786 < \gamma < -140.785$
11	94V146	$0.000354 < \alpha < 0.00124$	$-0.00174 < \beta < -0.00085$	$-49.996 < \gamma < -49.995$
12	94D032	$-0.0030 < \alpha < -0.0037$	$-0.00368 < \beta < -0.00314$	$15.91 < \gamma < 15.92$

13	94D039	$-0.160 < \alpha < 0.159$	$-0.1614 < \beta < -0.1564$	$84.012 < \gamma < 84.332$
14	94D048	$-0.00022 < \alpha < -0.00014$	$-0.000452 < \beta < -0.000369$	$20.098 < \alpha < 20.099$
15	94D064	$-0.534 < \alpha < -0.533$	$-0.536 < \beta < -0.535$	$3.464 < \gamma < 4.534$
16	94D096	$-0.00476 < \alpha < 0.00600$	$-0.00649 < \beta < 0.00433$	$-14.158 < \gamma < -14.147$
17	95D030	$-0.00233 < \alpha < -0.00289$	$-0.00085 < \beta < -0.00073$	$49.269 < \gamma < 49.279$
18	95D071	$-0.0031 < \alpha < 0.0033$	$-0.00571 < \beta < 0.00073$	$119.059 < \gamma < 119.066$
19	DA1	$-0.0006 < \alpha < 0.00124$	$-0.0023 < \beta < -0.00047$	$45.056 < \gamma < 45.070$
20	DA2	$-0.00071 < \alpha < -0.00070$	$-0.00038 < \beta < -0.00010$	$-10.163 < \gamma < -10.160$
21	DA3	$-0.00019 < \alpha < 0.003$	$-0.00044 < \beta < 8.7E-5$	$-12.8759 < \alpha < -12.8753$

Table 2a: Results for Variance and Sum of Squares Residuals for the three Models (Option 1)

S/N	Number of stations (n)	Variance Model A σ_A^2	Variance Model B σ_B^2	Variance Model C σ_C^2	Sum of Squares of Residuals: Model A $\sum v_A^2$	Sum of Squares of Residuals: Model B $\sum v_B^2$	Sum of Squares of Residuals: Model C $\sum v_C^2$
1	1813	82.5	39.2	12.3	246447.3	255443.8	178376.6

Table 2b: Results for Variance and Sum of Squares Residuals for the three Models (Option 2)

S/N	LINE	Number of stations (n)	Variance Model A σ_A^2	Variance Model B σ_B^2	Variance Model C σ_C^2	Sum of Squares of Residuals: Model A $\sum v_A^2$	Sum of Squares of Residuals: Model B $\sum v_B^2$	Sum of Squares of Residuals: Model C $\sum v_C^2$
1	94V007	42	1.62	1.23	0.46	2114.62	886.39	329.23
2	94V020	144	5.78	5.48	5.16	13649.3	7571.37	2986.58
3	94V023	217	15.56	16.64	14.65	48651.26	55320.35	48702.96
4	94V037	46	1.32	0.18	0.01	1344.23	1494.66	77.21
5	94V039	73	3.87	1.49	0.35	2263.91	2055.64	479.92
6	94V045	62	2.02	1.49	0.07	1699.39	841.50	39.90
7	94V055	81	3.29	4.24	0.27	1495.82	1918.04	119.98
8	94V071	96	6.69	6.49	0.40	3056.20	4360.23	266.44
9	94V080	78	5.37	2.76	1.19	23198.19	13911.18	6005.78
10	94V120	81	15.38	18.74	12.07	22863.3	11227.99	7234.22
11	94V146	44	8.87	7.38	1.16	2862.15	6550.19	1026.06
12	94D032	133	8.81	14.40	8.89	11693.82	7163.52	4421.32
13	94D039	131	12.58	11.79	9.93	31157.8	20428.79	17213.05
14	94D048	113	12.45	5.49	3.13	4463.16	3799.22	2165.32
15	94D064	94	4.74	5.39	6.92	10853.9	6816.84	8746.28
16	94D096	121	14.65	16.44	15.39	43610.62	26561.8	24857.62
17	95D030	64	2.94	2.20	0.38	1705.03	2353.03	404.70
18	95D071	76	0.39	0.18	0.01	467.76	1937.30	139.97
19	DA1	20	0.41	0.18	0.02	70.22	281.34	30.27
20	DA2	52	0.82	1.2	2.38	364.64	78.07	155.10
21	DA3	46	2.07	2.29	0.18	1246.76	1141.71	90.09
Total Sum of Squares of Residuals						228832.1	176699.2	125492

Table 3a: Results for Hypotheses Test and Correlation Coefficients for Model A (Option 1)

S/N	Number of stations (n)	$H_0 :$ $\sigma^2 = \sigma_0^2$	$X^2_{\gamma/2}(n-2)$ $\gamma = 0.05$	$H_0 : \beta = \beta$	$H_0 : \alpha = \alpha_0$	$t_{\gamma/2}(n-2)$ $\gamma = 0.05$	Correlation Coefficient (%) $\hat{p}$
1	1813	92	124.3	1.53	0.78	± 1.96	-51

Table 3c: Results for Hypotheses Test and Correlation Coefficients for Model B (Option 1)

S/N	Number of stations (n)	$X^2_{\gamma/2}(n-3)$ $\gamma = 0.05$	$H_0 : \alpha = \beta = 0$	$F_{\gamma/2}(n-3)$ $\gamma = 0.05$	Multiple Correlation Coefficient (%) $\hat{R}^2$
1	1813	140.57	19.19	3.00	68

Table 3b: Results for Hypotheses Test and Correlation Coefficients for Model A (Option2)

S/N	LINE	Number of stations (n)	$H_0 :$ $\sigma^2 = \sigma_0^2$	$X^2_{\gamma/2}(n-2)$ $\gamma = 0.05$	$H_0 : \beta = \beta$	$H_0 :$ $\alpha = \alpha_0$	$t_{\gamma/2}(n-2)$ $\gamma = 0.05$	Correlation Coefficient (%) $\widehat{p}$
1	94V007	42	24	55.76	1.92	0.19	±2.01	22.4
2	94V020	144	92	124.3	0.86	0.31	±1.96	-95.8
3	94V023	217	92	124.3	0.81	0.47	±1.96	-94.0
4	94V037	46	24	61.6	1.20	0.20	±2.01	-87.4
5	94V039	73	65	96.22	1.20	0.26	±1.99	-84.5
6	94V045	62	32	79.08	1.60	0.30	±2.01	3.6
7	94V055	81	66	101,9	2.70	0.34	±1.99	-86.5
8	94V071	96	66	118.7	1.15	0.15	±1.96	-7.9
9	94V080	78	66	96.12	1.15	0.23	±1.99	-96.4
10	94V120	81	77	107.5	0.54	0.11	±1.99	-88.5
11	94V146	44	42	55.76	0.22	0.04	±2.01	-75.7
12	94D032	133	92	140.57	0.64	0.28	±1.96	-92.0
13	94D039	131	92	140.57	1.60	0.24	±1.96	-92.5
14	94D048	113	66	109.83	1.00	0.14	±1.99	-72.4
15	94D064	94	66	118.70	1.40	0.24	±1.99	-92.5
16	94D096	121	92	140.57	1.30	0.20	±1.96	-94.6
17	95D030	64	57	84.81	2.40	0.12	±1.99	-79.2
18	95D071	76	32	96.22	2.80	0.54	±1.99	99.1
19	DA1	20	8	28.87	0.64	0.17	±2.11	-92.8
20	DA2	52	32	67.50	1.60	0.71	±2.01	-86.9
21	DA3	46	32	61.63	1.40	0.22	±2.01	-48.8
							Average Correlation Coefficient	-63.97

Table 3d: Results for Hypotheses Test and Correlation Coefficients for Model B (Option 2)

S/N	LINE	$X^2_{\gamma/2}(n-3)$ $\gamma=0.05$	$H_0 : \alpha = \beta = 0$	$F_{\gamma/2}(n-3)$ $\gamma=0.05$	Multiple Correlation Coefficient (%) $\widehat{R}^2$
1	94V007	49.77	8.19	3.19	29.6
2	94V020	124.3	69.33	3.00	92.7
3	94V023	124.3	60.1	3.00	94.0
4	94V037	61.63	62.7	3.19	96.7
5	94V039	96.22	28.5	3.11	89.1
6	94V045	73.29	10.83	3.15	27.6
7	94V055	96.22	82.73	3.11	87.9
8	94V071	118.7	2.33	3.11	4.0
9	94V080	96.12	100.6	3.11	96.4
10	94V120	96.22	20.1	3.11	84.1
11	94V146	61.63	94.75	3.19	82.2
12	94D032	140.57	19.3	3.00	74.8
13	94D039	140.57	41.49	3.00	86.6
14	94D048	109.83	20.89	3.11	79.2
15	94D064	118.70	23.4	3.11	83.7
16	94D096	140.57	25.6	3.00	81.3
17	95D030	84.81	77.71	3.11	71.8
18	95D071	96.22	60.2	3.11	95.7
19	DA1	27.59	13.4	3.59	94.1
20	DA2	61.63	9.39	3.19	62.1
21	DA3	61.63	4.56	3.19	17.5
Average Multiple Correlation Coefficient					73

Table 3e: Results for Hypotheses Test and Correlation Coefficients for Model C (Option 1)

S/N	Number of stations (n)	$H_0 : \alpha = \beta = \gamma = \lambda = \xi = 0$	$F_{\gamma/2}(n-6)$ $\gamma = 0.05$	Multiple Correlation Coefficient (%) $\widehat{R}^2$
1	1813	9.33	3.00	73

Table 4: Ranking of Models Using Least Squares Criteria

Ranking	**Option**	**Model**
1	2	C
2	2	B
3	1	C
4	2	A
5	1	A
6	1	B

Table 3f: Results for Hypotheses Test and Correlation Coefficients for Model C (Option 2)

S/N	LINE	Number of stations (n)	$H_0 : \alpha = \beta = \gamma = \lambda = \xi = 0$	$F_{\gamma/2}(n-6)$ $\gamma = 0.05$	Multiple Correlation Coefficient (%) $\widehat{R}^2$
1	94V007	42	8.23	3.26	90.5
2	94V020	144	3.36	3.00	97.6
3	94V023	217	4.04	3.00	94.4
4	94V037	46	3.24	3.23	99.7
5	94V039	73	9.51	3.11	94.4
6	94V045	62	3.36	3.15	99.3
7	94V055	81	3.75	3.11	99.6
8	94V071	96	3.17	3.11	99.5
9	94V080	78	3.85	3.11	97.2
10	94V120	81	6.7	3.11	88.3
11	94V146	44	3.45	3.24	93.2
12	94D032	133	6.44	3.00	96.4
13	94D039	131	3.84	3.00	93.1
14	94D048	113	3.62	3.11	94.6
15	94D064	94	7.81	3.11	67.1
16	94D096	121	8.55	3.11	94.5
17	95D030	64	4.65	3.19	96.6
18	95D071	76	5.77	3.11	99.6
19	DA1	20	4.48	3.74	95.7
20	DA2	52	3.33	3.19	79.6
21	DA3	46	3.52	3.23	95.5
Average Multiple Correlation Coefficient					93.6

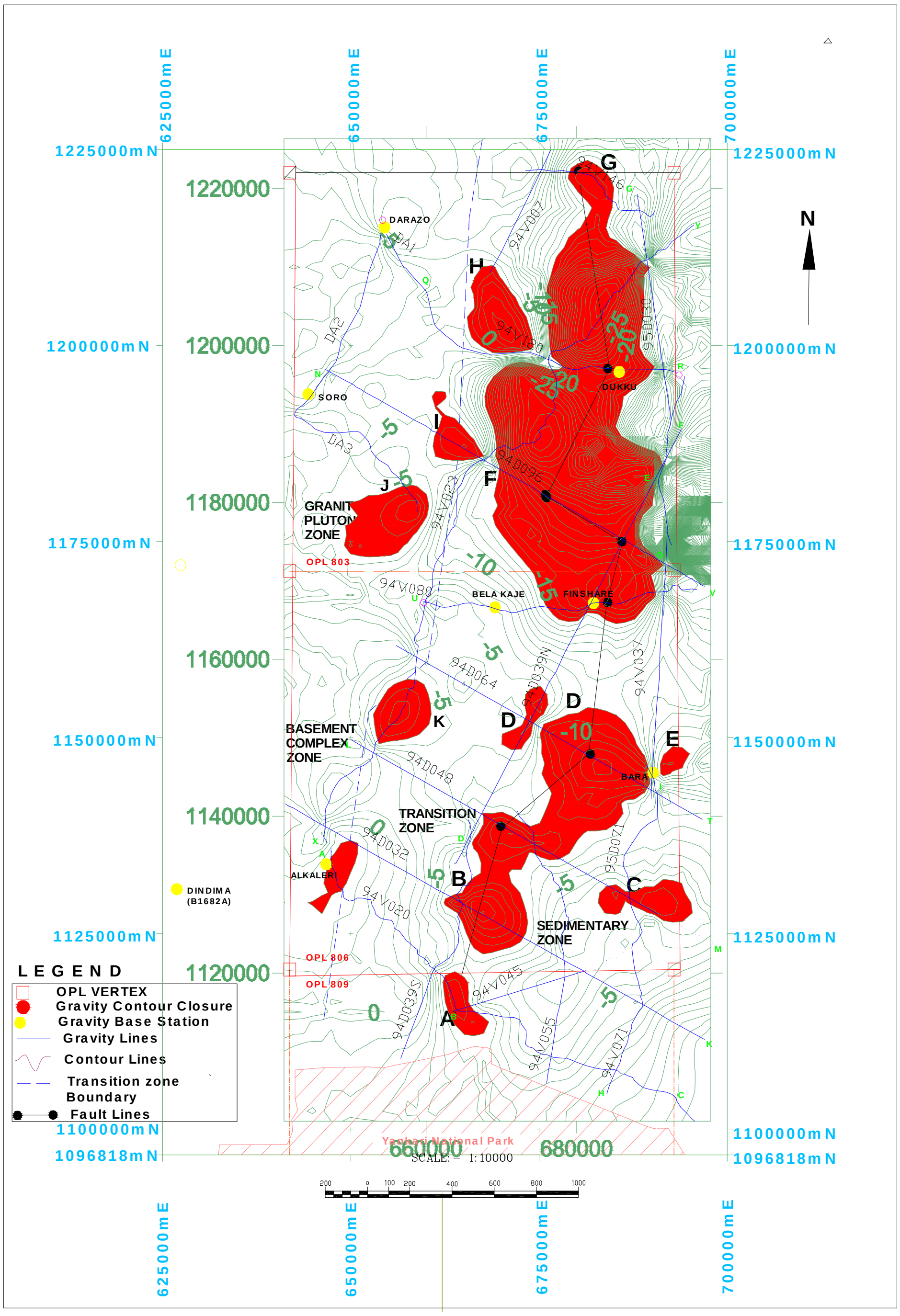

Figure 1: Residual Gravity Anomaly of Option 2 Model B. Contour interval =1mGal

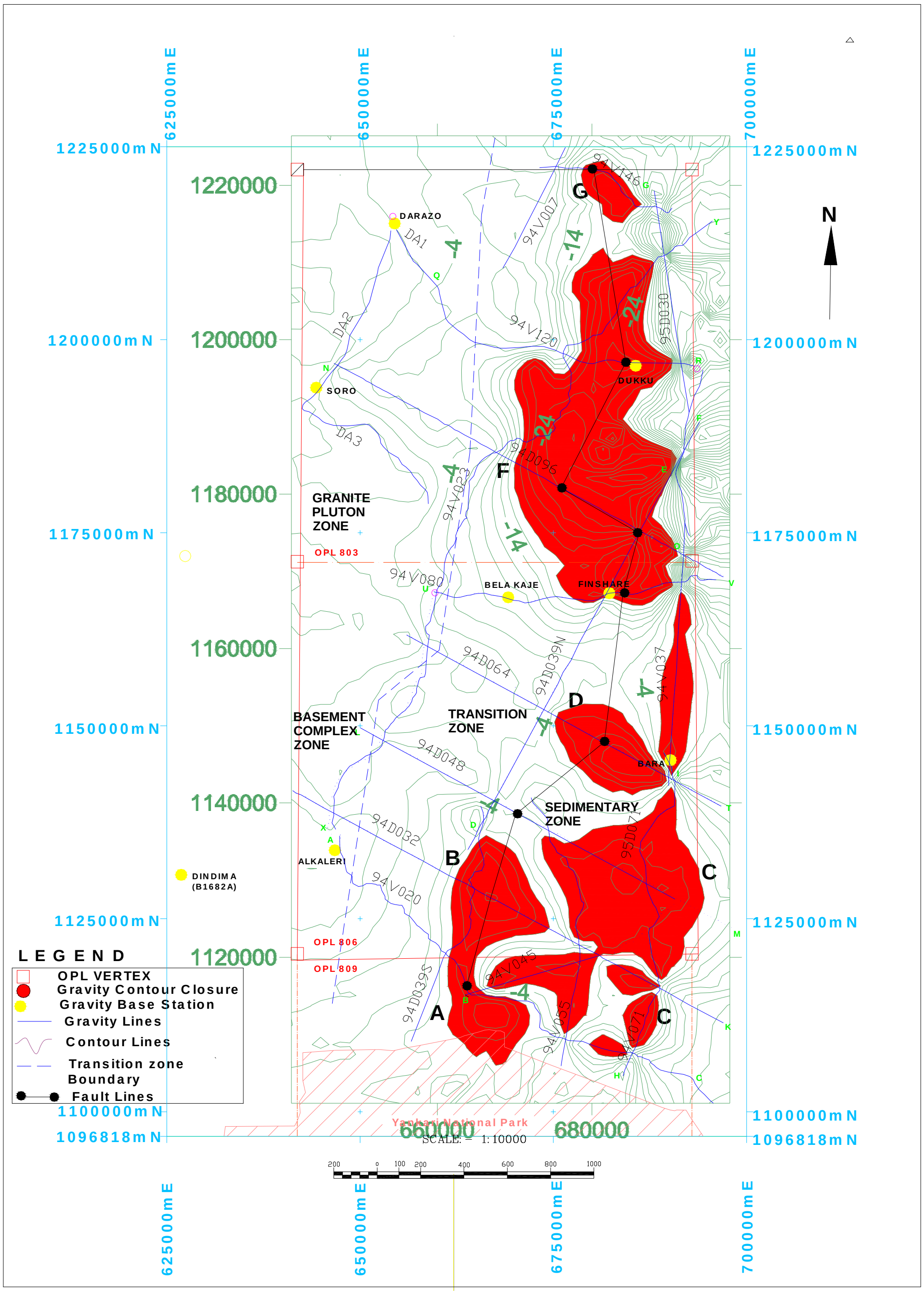

Figure 2: Residual Anomaly Map of option 2 Model C. C.I =1mGa