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L-Q-Fuzzy Quotient ζ - Group

Mourad Oqla Massa'deh[1] and Hazem"Moh'd said" Hatamleh[2]

Abstract

In this paper, we define a new algebraic structure of L-Q-fuzzy sub ζ-groups and L -Q-fuzzy quotient ζ-groups and discussed some properties. We also defined ζ-Q-homomorphism over L-Q-fuzzy quotient ζ–groups. Some related results have been derived.

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1 Introduction

Zadeh [12] introduced the notion of a fuzzy subset of a set X as a function from X into [0, 1]. Goguen in [5] replaced the lattice [0, 1] by a complete lattice L and studied L–fuzzy subsets. Rosenfeld [1] used this concept and developed some

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[1]Department of Applied Science, Ajloun College, Al – Balqa' Applied University – Jordan. E-mail: mourad.oqla@bau.edu.jo
[2]Department of Applied Science, Ajloun College, Al – Balqa' Applied University – Jordan. E-mail: hazim-hh@bau.edu.jo

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results in fuzzy group theory. Solairaju and Nagarajan [2, 3] introduced and defined a new algebraic structure of Q – fuzzy groups. Saibaba [4] introduced the concept of L – fuzzy sub ζ – groups and L – fuzzy ζ – ideal of ζ – groups. Sundrerrajan et al [11] studied the concepts of anti Q – L–fuzzy ζ – group, we invite the reader to consult the cited work [6, 7, 8, 9, 10] a non gathers. Here in this paper we introduce the notion of L – Q–fuzzy quotient ζ – group and there define ζ – Q – homomorphism over L – Q–fuzzy quotient ζ – groups.

2 Preliminary Notes

2.1 Definition:[5] A post (L, $\leq$) is called a lattice if supremum of a, b and infimum of a, b exist for all a, b $\in$ L.

2.2 Definition: A lattice ordered group (ζ – group) is a system G = (G, +, $\leq$) where

1. (G, +) is a group
2. (G, $\leq$) is a lattice
3. The inclusion is invariant under all translations $x \leq y \Rightarrow a + x + b \leq a + y + b$ for all a ,b $\in$ G.

2.3 Definition:[5] Let X be a non empty set L = (L, $\leq$) be a lattice with least element 0 and greatest element 1.

An L – fuzzy subset μ of X is a function $\mu : X \rightarrow L$.

2.4 Definition: Let X be a non empty set L = (L, $\leq$) be a lattice with least element 0 and greatest element 1 and Q be a non empty set . A L – Q – fuzzy subset μ of X is a function $\mu : X \times Q \rightarrow L$.

2.5 Definition: An L – Q – fuzzy subset μ of G is said to be an L – Q – fuzzy sub ζ – group (LQFSζG) of G if for any $x, y \in G$.

1. $\mu(xy, q) \geq \min\{\mu(x, q), \mu(y, q)\}$
2. $\mu(x^{-1}, q) = \mu(x, q)$
3. $\mu(x \vee y, q) \geq \min\{\mu(x, q), \mu(y, q)\}$
4. $\mu(x \wedge y, q) \geq \min\{\mu(x, q), \mu(y, q)\}$.

2.6 Theorem: If μ is an L – Q – fuzzy sub ζ – group of G, then $\mu(x, q) \leq \mu(e, q)$ for $x \in G$ and e is the identity element in G.

2.7 Theorem: Let μ be an L – Q – fuzzy sub ζ – group of G, then $H = \{x \in G, q \in Q; \mu(x, q) = \mu(e, q)\}$ is either empty or a sub ζ – group of G.

Proof: If no element satisfies this condition, then H is empty. If x, y satisfy this condition, then $\mu(xy^{-1}, q) \geq \min\{\mu(x, q), \mu(y^{-1}, q)\} = \min\{\mu(e, q), \mu(e, q)\} = \mu(e, q)$ and $\mu(e, q) \geq \mu(xy^{-1}, q)$, since μ is an L – Q – fuzzy sub ζ – group of G hence $\mu(e, q) = \mu(xy^{-1}, q)$ thus $xy^{-1} \in H$, let $x, y \in H$ then $\mu(x, q) = \mu(e, q)$ and $\mu(y, q) = \mu(e, q)$. $\mu(x \vee y, q) \geq \min\{\mu(x, q), \mu(y, q)\} \geq \min\{\mu(e, q), \mu(e, q)\} = \mu(e, q)$ then $\mu(x \vee y, q) = \mu(e, q)$ hence $x \vee y \in H$, also $\mu(x \wedge y, q) \geq \min\{\mu(x, q), \mu(y, q)\} \geq \min\{\mu(e, q), \mu(e, q)\} = \mu(e, q)$ then $\mu(x \vee y, q) = \mu(e, q)$ hence $x \wedge y \in H$, therefore H is sub ζ – group of an ζ – group G.

2.8 Definition: An L – Q – fuzzy sub ζ – group μ of G is called an L – Q – fuzzy normal sub ζ – group (LQFNSζG) of G if for any $x, y \in G$ $\mu(xyx^{-1}, q) \geq \mu(y, q)$.

2.9 Theorem: Let G be an ζ – group and μ be an L – Q – fuzzy sub ζ – group of G then the following conditions are equivalent.

1. μ is an L – Q – fuzzy normal sub ζ – group of G
2. $\mu(xyx^{-1}, q) = \mu(y, q)$ for all $x, y \in G$.

3. μ (xy, q) = μ (yx, q) for all x, y $\in$ G.

2.10 Corollary: Let μ be an L – Q – fuzzy normal sub ζ – group of G, then H = {x $\in$G, q $\in$ Q; μ(x, q) = μ(e, q)} is either empty or a normal sub ζ – group of G.

Proof: By Theorem 2.7 H is a sub ζ – group of G, then for any x $\in$ G and y $\in$ H

$$\mu (xyx^{-1}, q) = \mu (y, q) = \mu(e, q)$$

since μ is an L – Q – fuzzy normal sub ζ – group of G and y $\in$ H hence $xyx^{-1} \in$ H thus H is a normal sub ζ – group of G, therefore H is either empty or a normal sub ζ – group of G.

2.11 Lemma: Let μ be an L – Q – fuzzy sub ζ – group of G**.** Then xμ **=** yμ if and only if $\mu(x^{-1}y, q) = \mu(y^{-1}x, q) = \mu(e, q)$ for all x, y $\in$ G and q $\in$ Q.

Proof: Straightforward.

2.12 Definition: Let G_1, G_2 be any two ζ – groups. Then the function $\Psi: G_1 \to G_2$ is said to be ζ – Q – homomorphism if for all x, y $\in$ G_1

1. Ψ (xy, q) = Ψ(x, q) Ψ(y, q)
2. Ψ (x $\vee$ y, q) = max{ Ψ(x, q), Ψ(y, q)}
3. Ψ (x $\wedge$ y, q) = min{ Ψ(x, q), Ψ(y, q)}.

2.13 Definition: An L – Q – fuzzy subset μ of X is said to bare sup property if, for any subset A of X, if there exist $a_0 \in$ A such that $\mu(a_0, q) = \vee_{a \in A} \mu (a, q)$.

2.14 Definition: Let Φ be a function from a set X into a set Y. An L – Q – fuzzy subset μ of X is called Φ - invariant if Φ(x, q) = Φ(y,q) then μ(x,q) = μ(y,q) where x, y $\in$ X and q $\in$Q.

2.15 Definition: Let G_1, G_2 be any two ζ – groups. Then the function $\Psi: G_1 \to G_2$ is said to be ζ – Q – isomorphism if for all $x, y \in G_1$

1. $\Psi(xy, q) = \Psi(x, q)\Psi(y, q)$
2. Ψ is bijection.

3 Main Results

3.1 Theorem: Let μ be an L – Q – fuzzy sub ζ – group of G with identity e. Let H = $\{x \in G, q \in Q; \mu(x, q) = \mu(e, q)\}$. Consider the map $\mu^* : G / H \to L$ defined by $\mu^*(xh, q) = \vee \mu(xh, q)$ for all $h \in H$, $x \in G$ and $q \in Q$. Then

1. H is a normal sub ζ – group of G.
2. The map μ^* is well defined.
3. μ^* is an L – Q – fuzzy sub ζ – group of G/ H.

Proof: Since μ is an L – Q – fuzzy normal sub ζ – group of G.

1. $H = \{x \in G, q \in Q; \mu(x, q) = \mu(e, q)\}$let $y \in H$, $x \in G$ and $q \in Q$ then $\mu(y, q) = \mu(e, q)$, now$\mu(xyx^{-1}, q) = \mu(y, q) = \mu(e, q)$, since μ is an L – Q – fuzzy normal sub ζ – group of G, Hence $xyx^{-1} \in H$.

Let $x, y \in H$ then $\mu(x, q) = \mu(e, q) = \mu(y, q)$

$\mu(x \vee y, q) \geq \min\{\mu(x, q), \mu(y, q)\}= \min\mu(e, q), \mu(e, q)\} = \mu(e, q)$ hence $\mu(x \vee y, q) \geq \mu(e, q)$, then $\mu(x \vee y, q)\ \mu(e, q)$ thus $x \vee y \in H$.

and $\mu(x \wedge y, q) \geq \min\{\mu(x, q), \mu(y, q)\}= \min\mu(e, q), \mu(e, q)\} = \mu(e, q)$ hence $\mu(x \wedge y, q) \geq \mu(e, q)$, then $\mu(x \wedge y, q)\ \mu(e, q)$ thus $x \wedge y \in H$.

Therefore H is a normal sub ζ – group of G.

2. Consider the map $\mu^* : G / H \to L$ defined by $\mu^*(xh, q) = \vee \mu(xh, q)$ for all $h \in H$, $x \in G$ and $q \in Q$ then $xy^{-1} \in k$ that is, $\mu(xy^{-1}, q) = \mu(e, q)$ thus $\mu(xh, q) = \mu(yh, q)$ and hence $\mu^*(xh, q) = \mu^*(yh, q)$ therefore, the map μ^* is well – defined.

3. (i)$\mu^*(xh\ yk, q) = \mu^*(xyh, q) = \vee\ \mu(xyh, q)$ for all $h \in H$, $x, y \in G$ and $q \in Q$.

$\geq \vee \min\{\mu(xh_1, q), \mu(yh_2, q)\}; h_1, h_2 \in H$

$\geq \min\{\vee\mu(xh_1, q), \vee\mu(yh_2, q)\}; h_1, h_2 \in H$

$\geq \min\{\mu^*(xh, q), \mu^*(yh, q)\}$

(ii)$\mu^*((xh)^{-1}, q) = \mu^*(x^{-1}h, q) = \vee\ \mu(x^{-1}h, q)$ for all $h \in H$, $x \in G$ and $q \in Q$.

$= \vee\ \mu(x\ h, q) = \mu^*(x\ h, q)$.

(iii)$\mu^*(xh \vee yk, q) = \mu^*((x\vee y)h, q) = \vee\ \mu((x\vee y)h, q)$ for all $h \in H$, $x, y \in G$ and $q \in Q$.

$\geq \vee \min\{\mu(xh_1, q), \mu(yh_2, q)\}; h_1, h_2 \in H$

$\geq \min\{\vee\mu(xh_1, q), \vee\mu(yh_2, q)\}; h_1, h_2 \in H$

$\geq \min\{\mu^*(xh, q), \mu^*(yh, q)\}$

(iv)$\mu^*(xh \wedge yk, q) = \mu^*((x\wedge y)h, q) = \vee\ \mu((x\wedge y)h, q)$ for all $h \in H$, $x, y \in G$ and $q \in Q$.

$\geq \vee \min\{\mu(xh_1, q), \mu(yh_2, q)\}; h_1, h_2 \in H$

$\geq \min\{\vee\mu(xh_1, q), \vee\mu(yh_2, q)\}; h_1, h_2 \in H$

$\geq \min\{\mu^*(xh, q), \mu^*(yh, q)\}$

Hence μ^* is an L – Q – fuzzy sub ζ – group of G/H.

3.2 Definition: Let μ be an L – Q – fuzzy sub ζ – group of G with identity e. Let $H = \{x \in G, q \in Q; \mu(x, q) = \mu(e, q)\}$. Consider the map $\mu^* : G / H \rightarrow L$ defined by

$$\mu^*(xh, q) = \vee\ \mu(xh, q) \text{ for all } h \in H,\ x \in G \text{ and } q \in Q.$$

Then, the L – Q – fuzzy sub ζ – group μ^* of G is called an L – Q – fuzzy quotient ζ – group of μ by H.

3.3 Remark:

(1) μ^* is not L – Q – fuzzy normal quotient ζ – group of G/ H.

(2) Consider the map $\mu^* : G / H \to L$ defined by $\mu^*(xh, q) = \vee \mu(xh, q)$ for all h $\in$ H , x $\in$ G and q $\in$ Q. Then, μ^* is an L – Q – fuzzy normal quotient ζ – group of G/ H.

3.4 Theorem: If μ^* is an L – Q – fuzzy quotient ζ – group of G/ H, then $\mu^*(xh, q) \leq \mu^*(eh, q)$.

Proof: Let x $\in$ G, $\mu^*(eh, q) = \mu^*(xx^{-1}h, q)$

$$\geq \min\{\mu^*(xh, q), \mu^*(x^{-1}h, q)\}$$
$$= \mu^*(xh, q)$$

3.5 Theorem: μ^* is an L- Q-fuzzy quotient ζ-group of G/ H iff for all x, y $\in$ G

1. $\mu^*(xhy^{-1}h, q) \geq \min\{\mu^*(xh, q), \mu^*(yh, q)\}$
2. $\mu^*(xk \vee yk, q) \geq \min\{\mu^*(xh, q), \mu^*(yh, q)\}$
3. $\mu^*(xk \wedge yk, q) \geq \min\{\mu^*(xh, q), \mu^*(yh, q)\}$.

Proof:

($\Rightarrow$) $\mu^*(xhy^{-1}h, q) \geq \min\{\mu^*(xh, q), \mu^*(y^{-1}h, q)\}$
$\geq \min\{\mu^*(xh, q), \mu^*(y h, q)\}$

As, μ^* is an L – Q – fuzzy quotient ζ – group of G/ H, then (2), (3) are hold

($\Leftarrow$) If (1) hold then $\mu^*(x^{-1}h, q) = \mu^*(e x^{-1}h, q) \geq \min\{\mu^*(eh, q), \mu^*(x^{-1}h, q)\} = \min\{\mu^*(eh, q), \mu^*(x h, q)\} = \mu^*(x h, q)$, therefore $\mu^*(x^{-1}h, q) \geq \mu^*(xh, q)$ for all x $\in$ G. Hence $\mu^*((x^{-1})^{-1} h, q) \geq \mu^*(x^{-1}h, q)$ and $\mu^*(x^{-1}h, q) \leq \mu^*(xh, q)$ thus$\mu^*(x^{-1}h, q) = \mu^*(xh, q)$ for all x $\in$ G.

Now, by (1) replace y by y^{-1} then $\mu^*(xy h, q) = \mu^*(x(y^{-1})^{-1} h, q) \geq \min\{\mu^*(xh, q), \mu^*(y^{-1}h, q)\} = \min\{\mu^*(xh, q), \mu^*(yh, q)\}$ for all x, y $\in$ G.

Also $\mu^*(xk \vee yk, q) \geq \min\{\mu^*(xh, q), \mu^*(yh, q)\}$ and $\mu^*(xk \wedge yk, q) \geq$

min{ $\mu^*(xh, q)$, $\mu^*(yh, q)$}. Therefore μ^* is an L – Q – fuzzy quotient ζ – group of G/ H.

3.6 Theorem: If μ^*, λ^* are two L – Q – fuzzy quotient ζ – group of G/ H then their intersection is an L – Q – fuzzy quotient ζ – group of G/ H**.**

3.7 Corollary: The intersection of any collection of L – Q – fuzzy quotient ζ – group of G/ H is an L – Q – fuzzy quotient ζ – group of G/ H**.**

3.8 Theorem: Let G_1, G_2 be any two ζ – groups, $\Psi : G_1 \rightarrow G_2$ be an ζ – Q - epimorphism and $\mu^* : G_1 / H \rightarrow L$ be an L – Q – fuzzy quotient ζ – group of G_1/ H. Then $\Psi(\mu^*)$ is an L – Q – fuzzy quotient ζ – group of G_2/ H, if μ^* has a sup property and μ^* is Ψ - invariant and $\Psi(\mu^*) = (\Psi(\mu))^*$.

Proof:

1.$\Psi(\mu^*)(\Psi(x)\Psi(y)h, q) = \Psi(\mu^*)(\Psi(xy)h, q)$

$= \mu^*(xyh, q)$

$\geq \min\{\mu^*(xh, q), \mu^*(y h, q)\}$

$\geq \min\{\Psi(\mu^*)(\Psi(x)h, q), \Psi(\mu^*)(\Psi(y) h, q)\}$

2.$\Psi(\mu^*)((\Psi(x))^{-1} h, q) = \Psi(\mu^*)(\Psi(x^{-1})h, q)$

$= \mu^*(x^{-1}h, q)$

$= \mu^*(xh, q)$

$= \Psi(\mu^*)(\Psi(x) h, q)$

3. $\Psi(\mu^*)(\Psi(x) \vee \Psi(y)h, q) = \Psi(\mu^*)(\Psi(x \vee y)h, q)$

$= \mu^*((x \vee y)h, q)$

$\geq \min\{\mu^*(xh, q), \mu^*(y h, q)\}$

$\geq \min\{\Psi(\mu^*)(\Psi(x)h, q), \Psi(\mu^*)(\Psi(y) h, q)\}$

4. $\Psi(\mu^*)(\Psi(x) \wedge \Psi(y)h, q) = \Psi(\mu^*)(\Psi(x \wedge y)h, q)$

$= \mu^*((x \wedge y)h, q)$

$\geq \min\{ \mu^*(xh, q), \mu^*(y h, q)\}$

$\geq \min\{ \Psi(\mu^*) (\Psi(x)h, q), \Psi(\mu^*) (\Psi(y) h, q)\}$

Hence $\Psi(\mu^*)$ is an L – Q – fuzzy quotient ζ – group of G_2/ H.

$(\Psi(\mu))^* (yh, q) = \vee \Psi(\mu)(yh, q) \ \forall\ h \in H , y \in G_2$ and $q \in Q$.

$= \vee \Psi(\mu)(\Psi(x)h, q) \ \forall\ h \in H , x \in G_1$ and $q \in Q$.

$= \vee \mu(xh, q)$

$= \mu^*(xh, q)$

$= \Psi(\mu^*) (\Psi(x)h, q)$

$= \Psi(\mu^*) (yh, q)$

3.9 Theorem: Let G_1, G_2 be any two ζ – groups, $\Psi : G_1 \to G_2$ be an ζ – Q homomorphism and $\lambda^* : G_2 / H \to L$ be an L – Q – fuzzy quotient ζ – group of G_2/ H. Then $\Psi^{-1}(\lambda^*)$ is an L – Q – fuzzy quotient ζ – group of G_1/ H and

$\Psi^{-1}(\lambda^*) = (\Psi^{-1}(\lambda))^*$.

Proof:

1.$\Psi^{-1}(\lambda^*)(xyh, q) = \lambda^* (\Psi(xy)h, q)$

$= \lambda^*(\Psi(x) \Psi(y) h, q)$

$\geq \min\{ \lambda^*(\Psi(x)h, q), \lambda^*(\Psi(y)h, q)\}$

$\geq \min\{ \Psi^{-1}(\lambda^*) (xh, q), \Psi^{-1}(\lambda^*) (y h, q)\}$

2.$\Psi^{-1}(\lambda^*)(x^{-1}h, q) = \lambda^* (\Psi(x^{-1})h, q)$

$= \lambda^*((\Psi(x))^{-1} h, q)$

$= \lambda^*(\Psi(x) h, q)$

$= \Psi^{-1}(\lambda^*) (xh, q)$.

3.$\Psi^{-1}(\lambda^*)((x\vee y)h,q) = \lambda^*(\Psi(x\vee y)h,q)$

$= \lambda^*((\Psi(x) \vee\Psi(y)) h, q)$

$\geq \min\{ \lambda^*(\Psi(x)h, q), \lambda^*(\Psi(y)h, q)\}$

$\geq \min\{ \Psi^{-1}(\lambda^*) (xh, q), \Psi^{-1}(\lambda^*) (y h, q)\}$

4.$\Psi^{-1}(\lambda^*)((x\wedge y)h,q) = \lambda^*(\Psi(x\wedge y)h,q)$

$= \lambda^*((\Psi(x) \wedge \Psi(y))\, h, q)$

$\geq \min\{ \lambda^*(\Psi(x)h, q), \lambda^*(\Psi(y)h, q)\}$

$\geq \min\{ \Psi^{-1}(\lambda^*)\,(xh, q), \Psi^{-1}(\lambda^*)\,(y\, h, q)\}$

Hence $\Psi^{-1}(\lambda^*)$ is an L – Q – fuzzy quotient ζ – group of G_1/ H.

$(\Psi^{-1}(\lambda))^*\,(xh, q) = \vee\, \Psi^{-1}(\lambda)(xh, q)\ \forall\, h \in H$, $x \in G_1$ and $q \in Q$.

$= \vee\, \lambda\, (\Psi(x)h, q)\ \forall\, h \in H$, $x \in G_1$ and $q \in Q$.

$= \lambda^*(\Psi(x)h, q)$

$= \Psi^{-1}(\lambda^*)\,(xh, q)$

3.10 Theorem: Let G_1, G_2 be any two ζ – groups, $\Psi : G_1 \rightarrow G_2$ be an ζ – Q - homomorphism and λ be an L – Q fuzzy normal sub ζ – group of G_2 such that $\mu = \Psi^{-1}(\lambda)$, then $\Phi: G_1 / \mu \rightarrow G_2 / \lambda$ such that $\Phi(x\mu, q) = \Psi(x, q)\lambda$ for every $x \in G_1$ and $q \in Q$ is an ζ – Q- isomorphism.

Proof:

Clearly Φ is onto as Ψ is onto

Let $x\mu, y\mu \in G_1 / \mu$, $\Phi(x\mu, q) = \Phi(y\mu, q)$ then $\Psi(x, q)\lambda = \Psi(y, q)\lambda$ and

$$\lambda(\Phi^{-1}(x)\Phi(y), q) = \lambda(\Phi^{-1}(y)\Phi(x), q) = \lambda(\Phi(e), q)$$

hence

$$\lambda(\Phi\,(x^{-1}y), q) = \lambda(\Phi\,(y^{-1}x), q) = \lambda(\Phi(e), q)$$

then $x\mu = y\mu$ by 2.11 Lemma, therefore Φ is one– one.

$$\Phi((x\mu)(y\mu), q) = \Psi((xy)\mu, q) = \Psi(xy, q)\lambda = (\Psi(x, q).\ \Psi(x, q))\lambda$$
$$=(\Psi(x, q)\lambda).\ (\Psi(x, q)\,\lambda) = \Phi(x\mu, q)\,\Phi(y\mu, q).$$

Now

$$\Phi((x\mu\vee y\mu), q) = \Psi((x\vee y)\mu, q) = \Psi(x\vee y, q)\lambda = (\Psi(x, q)\vee \Psi(x, q))\lambda$$
$$= (\Psi(x, q)\lambda)\vee (\Psi(x, q)\,\lambda)= \Phi(x\mu, q) \vee\Phi(y\mu, q).$$

And

$$\Phi((x\mu\wedge y\mu), q) = \Psi((x\wedge y)\mu, q) = \Psi(x\wedge y, q)\lambda = (\Psi(x, q)\wedge \Psi(x, q))\lambda$$

$$= (\Psi(x, q)\lambda) \wedge (\Psi(x, q)\ \lambda) = \Phi(x\mu, q) \wedge \Phi(y\mu, q)$$

Clearly Φ is an ζ – Q- homomorphism and hence Φ is an ζ – Q- isomorphism.

4 Conclusions

Further work is in progress in order to develop the L-Q-intuitionistic fuzzy quotient ζ- groups and L-Q-intuitionistic anti fuzzy quotient ζ-groups.

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