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Analysis of a Production Order Quantity Model With Declining Unit Cost

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Abstract

This paper improves upon the existing literature surrounding the production order quantity inventory model in which unit cost and daily production are assumed to be constant. By including economies of scale into the model, we examine its impact on production order quantity and total cost. The results suggest that the minimal cost solution derived from the production order quantity model needs to

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balance out holding, setup and production costs. As a result, a smaller inventory level corresponding to a minimum unit production cost is found to be preferred.

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1 Introduction

A common inventory model heavily utilized in the literature is the economic order quantity model where fixed daily demand, zero lead time, constant holding cost and ordering cost are assumed. Inventory velocity, the speed at which components move through the operation chains, is sometimes used as a measure of the company's performance index [9]. For example, it is claimed by the company CEO, Michael Dell, that the Dell computer is of a higher quality if fewer inventories are on the floor of company warehouses. [10] In this line of analysis, some authors have even gone so far as to refer to existing product inventories as the "root of all evil" in the business world. [1] At the other end of the spectrum concerning inventories, a high stock of heating oil in the presence of a severe winter and rising price typically enhances profit. As a consequence, the industry type and business characteristics are important considerations when applying any empirical model to real business analysis. The variation in how inventories are viewed by management reflects the important role inventory itself plays in an increasingly competitive production environment. It is worth noting that this is an environment in which a large stock of inventory is often discouraged given higher production costs and greater degrees of business uncertainty. In this scenario, lean production and a minimum level of product inventory necessary to meet consumer demand is preferred. [4] [12] [16] An application of this approach is described in Uzsoy, Lee, and Martin-Vega [15] by illustrating the importance of production planning and scheduling models in the semiconductor industry. The problems of random yields, complex product flows and rapidly changing technologies render performance evaluation and product planning extremity difficult in this environment.

While the economic order quantity model applies to the situation in which managers of a continuous review system choose the best order quantity, it does not address the production scenario where one does not rely on purchasing from outside. Internal production will circumvent and reduce the uncertainty emanating from problems associated with suppliers and transportation companies. In this case, one may apply the production order quantity inventory model in which a floor manager can activate machines to produce on the same day the merchandise is sold. A significant number of studies on variations from the original product order quantity model utilized on a wide array of industries outline applications of this empirical model. De Castro, Tabucanan, and Nagaruv [3], for example, used a product order quantity model with a stochastic demand curve on production data from the chocolate milk industry. Chakravarty and Balakrishnan [2] showed that a rank-ordering of products can be used when real-time revisions are introduced into the model and that industry profits increased when stronger buyer-supplier linkages existed.

Two recent studies are more closely linked to the work done in the current analysis. Jeang [7] showed that the product order quantity model leads to overproduction if there is a change in the production technology owing to process deterioration. This deterioration may be due to varied or increased quality costs at different points of time along with changes in the production run length. Rather than adding a measure for process deterioration, Tao, Guiffrida, and Troutt [13] included a green cost variable to the production order quantity model. This green cost variable incorporated company actions that reflect a growing awareness in business and society of how decisions impact environmental conditions. Costs of pollution prevention programs, for example, are included as a part of this green cost measure. As predicted, their work showed that a smaller production quantity results when green costs are included in the empirical model; a result supported in introductory business and management courses.

The current analysis goes beyond the existing literature and represents a significant innovation to the field by considering how the existence of declining unit costs influence the optimal outcome for the production order quantity model. Diminishing unit costs are often observed in rapidly growing and technologically-advanced industries so that this improvement to the production quantity model has real applications to current economic conditions. The next section of this paper introduces the improved production order quantity inventory model while the third section presents a simulation to illustrate the different scenarios of declining and increasing unit cost. A summarizing conclusion is provided in final section of this paper.

2 An Improved Production Order Quantity Model

To meet customer demand without delay requires keeping on hand some amount of stock that is awaiting sale. Three types of inventory cost are usually involved in this scenario: ordering and/or setup cost, holding or carrying cost; and unit production or purchasing cost. It is commonly agreed that ordering or setup cost does not depend on the size of the order or lot size of a production run. If the product is produced internally, the labor costs of setting up and shutting down a machine, lubrication, maintenance are included. In contrast, holding or carrying cost is directly associated with the size of production, and is often expressed in dollars per unit per year. Included in this cost category are storage costs, insurance costs, taxes on inventory, spoilage and theft costs. Of course, obsolescence cost and the opportunity cost of tying up capital in inventory (evaluated at cost of capital) seem to be more substantial in this category. The third cost component, unit production cost, generally constitutes a lion's share of average total cost, [11] [14]. As a consequence, a greater emphasis must be placed on lean production or diminishing the cost of producing goods when possible. With this in mind, the impact that economies and diseconomies of scale have on production cannot be discounted if cost minimization is among the important management goals.

Consider a production order quantity inventory model when internal manufacturing takes place simultaneously with daily selling activity, [5]. It is useful to formulate this model into a minimization problem as following:

Minimize Total Cost = Setup Cost + Holding Cost + Production Cost

$$Q \in I+ = \frac{DS}{Q_i} + \frac{1}{2} Q_i \left(1 - \frac{d}{p_i}\right) H + AC_i \cdot Q_i + \frac{D}{Q_i} \quad (1)$$

Where

D = estimated annual quantity demanded

S = setup cost per production run

Q_i = number if unit per production run

I+ = set of positive integers

d = daily demand or D/number of working days

p_i = daily production rate or $Q_i = p_i t$, where t denotes number of days in a production run

H = constant holding cost per unit of inventory per year

AC_i = unit production cost which varies with Q_i or $AC_i = AC_i(Q_i)$

Note that the second term on the right side of equation (1), $\frac{1}{2} Q_i \left(1 - \frac{d}{p_i}\right)$, is the average net inventory level when machine is on for t days. The number of days when a production continues is exogenous to the model: it is in the domain of an engineering division.

The necessary condition for the minimization problem requires the first derivative to vanish or

$$-\frac{DS}{Q_i^2}+\frac{1}{2}(1-\frac{d}{p_i})H+AC_i'D=0 \quad (2)$$

which leads to

$$Q_i=\sqrt{\frac{DS}{\frac{1}{2}(1-\frac{d}{p_i})H+AC_i'D}} \quad (3)$$

where AC' is the derivative of the unit production cost evaluated at optimality. We present it in equation (3) just for comparison purpose. That is, if average cost is constant, as is the case in a standard text [5]; equation (3) is reduced to

$$Q_i^*=\sqrt{\frac{DS}{\frac{1}{2}(1-\frac{d}{p_i})H}} \quad (4)$$

for AC' = 0 and thus the AC' D term drops out of the equation.

An examination of (3) indicates the optimum production order quantity Q^* is greater than that with a constant minimum average cost if there exists economies of scale (or AC' < 0). The result of equation (3) is in agreement with the common wisdom indicating that the saving in production cost enables managers to hold a larger Q^* (optimal quantity produced), but tat this larger quantity entails a higher holding cost. Conversely, if AC' > 0 indicating a condition of diseconomies of scale, the optimum production order quantity level is to be kept smaller than that of (4) when average cost is constant. As a consequence, the optimum production order quantity value needs to be balanced between these costs.

The second- order condition of the minimization problem can be derived from differentiating equation (2) with respect to Q^* or

$$\frac{DS}{Q_i^3}+AC_i''D>0 \quad (5)$$

The first term of (5) is positive. In a typical U-shaped AC curve, the first half of the curve had less steep slopes (negative) and as such, AC" is positive. In this case, one indeed has a true minimum cost solution. For the second half of the average cost curve, positive slopes increase at increasing rates, which again guarantees a minimum total cost solution. This is to say, the addition of a U-shaped average cost curve reinforces the second-order condition and as such ensures a stable solution.

3 A Simulation of the Improved Model

In order to illustrate the production order quantity model with flexible unit costs, we choose the following parameter values: S = 10, D = 1,000, H = 1, d = 5. Given the various production level and unit cost combinations $P_1 = 10$, $AC_1 = 10$; $P_2 = 20$, $AC_2 = 9$; and $P_3 = 30$, $AC_3 = 8$. A quick substitution into (1) yields immediately $Q_1^* = 200$, $Q_2^* = 163.3$ and $Q_3^* = 154.92$ with the corresponding total cost of $TC_1 = 10{,}100$, $TC_2 = 9{,}122.5$ and $TC_3 = 8{,}129$ (see Table 1). Evidently, a presence of economies of scale leads to an optimum production order quantity that balances out with other costs. If we assume an increasing AC, i.e., AC = 9 for $P_i = 40$, a higher production order quantity is not appropriate as incremental production cost outweighs the savings on the setup cost. The optimum solutions are reported in Table 1.

In these simulations, we find readily that the minimum cost solution is when average cost is at its lowest and daily production takes on its lower value for a given average cost. That is, one prefers a lean production over mass production for a given minimum average cost or minimum efficient scale (MES). The MES is specifically defined as the output level at which long-run unit costs are lowest. Economics texts often recommend MES in pure production setting for it shares minimum unit cost with other larger output levels [6]. Within the wider framework of inventory and production, MES is actually required instead of a matter of choice in order to minimize total cost. In addition, a perusal of Table 1 reveals that the MES solution dominates others regardless of the parameter values. The production order quantity for a given unit cost (AC' = 0) indeed obeys equation (3). That is, a doubling on H from 1 to 2, and a halving of S from 20 to 10 reduces Q^* by 50%.

Table 1: Optimum Solutions to the POQ Model

(S = 10, D = 1,000, H = 1, d = 5)

	Parameter / Variables	P=10 AC=10	P=20 AC=9	P=30 AC=8	P=40 AC=8	P=50 AC=9	P=60 AC=10
S=10 H=1	Q	200	163.3	154.9*	151.2	149.1	147.7
	TC	10100	9122.5	8129.1*	8132.2	9134.164	10135.4
S=20 H=1	Q	282.8	230.9	219.1*	213.8*	210.8	208.9
	TC	10141	9173.2	8182.6*	8187.1	9189.7	10191.5
S=10 H=2	Q	141.4	115.5	109.5*	106.9	105.4	104.4
	TC	10141.4	9173.2	8182.6	8187.1	9189.7	10191.5

Q = cost-minimizing production order quantity

* = minimum cost solution of the 6 different scenarios

4 Suggestions and Conclusion

While a continuous unit cost function is mathematically convenient, it is empirically irrelevant. In a world where there exist only finite numbers of production lines in a factory, estimated daily production and corresponding unit cost are needed to calculate the optimum production order quantity. If one employs a statistically estimated unit cost function (e.g., a cubic or quadratic function of output levels), and substitutes it into (1), one needs to replace P_i with $\frac{Q_1}{t}$, where t (approximated number of days to produce a lot) is determined exogenously. A recommended approach is to identify several pairs of unit costs (AC_i) and daily production levels (P_i) from the estimated statistical cost function before substituting them into (1). The values of t do not matter in our model as the average net inventory level is always

$$\frac{1}{2}(P_i\ t - dt) = \frac{1}{2}[P_i \frac{Q_i}{p_i} - d(\frac{Q_i}{p_i})] = \frac{1}{2}[Q_i - \frac{d}{p_i} Q_i] = \frac{1}{2} Q_i [1 - \frac{d}{p_i}]$$

as long as t is predetermined (Heizer and Render, 2007). If, however, t as well as P_i are decision variables, a nonlinear constraint $P_i\ t_i = Q$ needs to be added to (1) to ensure solution consistency. In addition, mathematical programming software such as Mathematica or Lingo makes such simulations rather convenient; it can handle a large-scale simulation in empirical applications.

The production order quantity inventory model is essential to determine the production order quantity, which at a later stage of production may be used to calculate numbers of kanbans needed to meet a probabilistic demand while accommodating for the problem of product cycle. This paper expands the traditional production order quantity inventory model to take economies of scale into consideration. Needless to say, economies of scale if significant, is still a force to be reckoned with even in the era of lean production, just-in-time inventory. Out results suggest a smaller production order quantity is preferred to the production order quantity with identical minimum unit cost: the concept of MES in economics. Of course, the expanded production order quantity model can be improved upon, similar to that observed in literature surrounding the economic order quantity model, by including probabilistic demand and statistically estimated cost functions into the model. This is a project that will be addressed in future research.

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