Journal of Finance and Investment Analysis, Vol. 12, No. 2, 2023, 11-28
ISSN: 2241-0988 (print version), 2241-0996(online)
https://doi.org/10.47260/jfia/1222a
Scientific Press International Limited

Modeling the Time Variation in Factor Exposures

Johan Knif[1], James W. Kolari[2], Gregory Koutmos[3] and Seppo Pynonen[4]

Abstract

This paper offers new evidence on the dynamic behavior of multifactor models. Specifically, we investigate the significance and temporal stability of conditional factor betas in the context of multifactor asset pricing models. Using a Kalman filter approach, we find that conditional factor betas are dynamic and their statistical significance varies over time. Furthermore, the inclusion of more factors improves that statistical significance and time stability of the market factor. Overall, our empirical results support the view that multifactors may not be independent risk factors but help to better identify the market factor.

JEL classification numbers: G11; G12.
Keywords: Asset Pricing, Risk Factors.

[1] Professor Emeritus, Hanken School of Economics.
[2] JP Morgan Chase Professor of Finance, Texas A&M University.
[3] Gerald M. Levin Professor of Finance, Fairfield University.
[4] University of Vaasa.

Article Info: *Received:* April 18, 2023. *Revised*: May 18, 2023.
Published online: May 23, 2023.

1. Introduction and Literature Review

Research on multifactors accelerated after Fama and French's (1992, 1993) papers on the shortcomings of the CAPM and their now famous three-factor model. In light of earlier work by Basu (1977), Stattman (1980), Banz (1981), Rosenberg, Reid, and Lanstein (1985), and Bhandari (1988) on patterns in the cross-section of stock returns missed by market betas, Fama and French (1992, 1993) augment the market factor with size and value (book-to-market equity) risk factors. The intuition is that the additional factors compensate for financial risks that are not fully captured by the market factor. This line of reasoning has been further explored by Amihud (2002), Griffin and Lemmon (2002), Arshanapalli, Fabozzi, and Nelson (2006), Campbell, Hilscher, and Szilagyi (2008), and Simpson and Ramchander (2008).

Based on momentum findings identified by Jagadeesh and Titman (1993), Carhart (1997) further augments Fama and French's three-factor model by including a momentum factor (see also Grinblatt and Moskowitz, 2004). Subsequent studies have shown that momentum is both market dependent (see Chordia and Shivakumar, 2002 and Cooper, Gutierrez, and Hameed, 2004) and credit quality dependent (see Avramov, Chordia, Jostova, and Philipov, 2007).

It is possible that the market factor is comprised of a variety of risks. Hamada's (1972) famous corporate finance paper on capital structure shows that levered betas are a function of unlevered beta and risk associated with financial leverage. Merton's (1973) Intertemporal CAPM (ICAPM) hypothesizes that the market beta of a firm can be decomposed into two different parts, wherein one part stems from covariance with cash flows and the other part from covariance with discount rates. Campbell and Vuolteenaho (2004) empirically find that the former cash flow covariance is priced by the market. Recently, Armstrong, Knif, Kolari and Pynnönen (2012) demonstrate that market beta can be decomposed into a universal risk component with no exchange rate risk exposure and another component capturing the exchange rate risk of the asset. Perhaps multifactors are components of the market factor also. In this regard, some authors continue to support the CAPM and/or suggest that the augmented Fama-French factors might not be risk factors after all (Chan and Lakonishok, 1993; Grundy and Malkiel, 1996; Daniel and Titman, 1996; Loughran, 1997; Griffin, Ji, and Martin, 2003; Tai, 2003; Arnott, Hsu, Liu, and Markowitz, 2006; Petkova, 2006; and Levy, 2009; Da, Guo and Jagannathan 2012).

Chen, Novy-Marx, and Zhang (2010) offer an alternative three-factor model containing the market factor, an investment size factor, and a return-on-assets size factor. They argue that this new model outperforms other asset pricing models and therefore should be used to obtain expected returns in practice. Ammann, Odoni, and Oesch (2012) present supporting international empirical evidence for the Chen et al. (2010) model. Other studies by Fung and Hsieh (2004) and Ammann, Huber and Schmid (2010) recommend seven-factor and eight-factor models, respectively. Also, studies by Chan, Chen, and Hsieh (1985) and Chen, Roll, and Ross (1986) test different sets of macroeconomic variables as factors in asset pricing.

In the literature there is little consensus of how the loadings on the risk factors should be estimated. In most studies, the loadings are estimated using time series regression with the coincident problem of time variation in both loadings and premiums. Ghysels (1998) points out that misspecification of the time variation in beta may lead to a model with larger pricing errors than an unconditional beta model even if beta in fact is time varying. Ferson and Harvey (1999) uses macroeconomic variables to model the time variation in the loadings. Assuming that loadings are expected to be constant over short horizon returns, Lewellen and Nagel (2006) conclude that the conditional CAPM cannot account for size and value effects.

Alternative approaches have been designed to explicitly take into account time variation in factor loadings. Knif (1990), Berglund and Knif (1999), Ang and Chen (2007), Brennan and Wang (2007), and Trecroci (2009) model time variation in conditional loadings as stochastic AR(1) variables and employ Kalman filtering procedures for estimation. Adrian and Franzoni (2009) model conditional loadings using the Kalman filter and show that, for low-frequency variation in beta, the CAPM passes size- and value-based specification tests.

Ang and Kristensen (2009) develop a new methodology for testing conditional factor models and reject the null of long-run alphas being zero for the CAPM and Fama-French models, although they find substantial variation in the conditional factor loadings of their portfolios.

Additional evidence of time variation in market betas is provided by Koutmos and Knif (2002). They report substantial variation in market betas that follow stationary mean-reverting processes. Interestingly, accounting for time variation in betas reduces unsystematic risk by approximately 10%, which in turn implies that the static model underestimates the market risk premium.

The rest of this paper is organized as follows: Section 2 presents the empirical framework used for the estimation of time-varying factor loadings. Section 3 describes the data. Section 4 reports the empirical findings. Lastly, Section 5 gives the summary and conclusion.

2. Methodology

2.1 Empirical Asset Pricing Models

In line with Zhang (2005), Ang and Chen (2007), and Adrian and Franzoni (2009), we specify the conditional CAPM model such that the expected return on an asset is proportional to the conditional market factor loading and the corresponding conditional market factor risk premium, or

$$E_{t-1}\left[r_{i,t}^{e}\right] = E_{t-1}\left[\beta_{i,t}\right] E_{t-1}\left[r_{m,t}^{e}\right] \tag{1}$$

where $r_{i,t}^{e}$ denotes the excess return on asset *i*, *i=1,…,N*, $\beta_{i,t}$ is the market factor loading for asset *i*, and $E_{t-1}\left[r_{m,t}^{e}\right]$ is the corresponding conditional market risk premium. The conditional market factor loading (beta) is defined as

$$\beta_{i,t|t-1} = E_{t-1}[\beta_{i,t}] = cov_{t-1}(r^e_{i,t}, r^e_{m,t})/var_{t-1}(r^e_{m,t}) \quad (2)$$

An empirical version of the conditional CAPM model will then take the form

$$r^e_{i,t} = \alpha_{i,t} + \beta_{i,t} r^e_{m,t} + \varepsilon_{i,t} \quad (3)$$

where $E_{t-1}[\alpha_{i,t}] = 0$, and $\varepsilon_{i,t}$ are i.i.d. normal and uncorrelated with $r^e_{m,t}$. A corresponding empirical multifactor asset pricing model is given by

$$r^e_{i,t} = \alpha_{i,t} + \sum_{k=1}^{K} \beta_{i,k,t} f_{k,t} + \varepsilon_{i,t}\,, \quad (4)$$

where again $\beta_{i,k,t}$ is the time-varying loading on factor *k, k=1,…,K,* and $f_{k,t}$ is the corresponding value of the risk factor at time *t*.

2.2 Time-varying factor loading characteristics

Following the approach of Knif (1990), Berglund and Knif (1999), Ang and Chen (2007), Brennan and Wang (2007), and Trecoci (2009), we model time variation in conditional risk factor loadings as stochastic AR(1) variables and use Kalman filtering procedures for estimation. For the empirical multifactor model (4), we specify the dynamics of mispricing $\alpha_{i,t}$ and factor loadings $\beta_{i,k,t}$ as

$$\alpha_{i,t} = \gamma^0_i + \gamma^1_i \alpha_{i,t-1} + \vartheta_{i,t} \quad (5)$$
$$\beta_{i,k,t} = \gamma^0_{i,k} + \gamma^1_{i,k} \beta_{i,k,t-1} + \vartheta_{i,k,t}\,, \quad k = 1, \dots, K \quad (6)$$

where $\vartheta_{i,t}$ and $\vartheta_{i,k,t}$ are assumed to be zero mean, i.i.d. normally distributed, and mutually independent with variances $\sigma^2_{\vartheta,i}$ and $\sigma^2_{\vartheta,i,k}$ respectively. Furthermore, $\vartheta_{i,t}$ and $\vartheta_{i,k,t}$ are assumed to be independent of $\varepsilon_{i,t}$ from equation (4). The corresponding conditional components to $\alpha_{i,t}$ and $\beta_{i,k,t}$ can be represented as

$$\alpha_{i,t|t-1} = E_{t-1}[\alpha_{i,t}] = \gamma^0_i + \gamma^1_i \alpha_{i,t-1} \quad (7)$$
$$\beta_{i,k,t|t-1} = E_{t-1}[\beta_{i,k,t}] = \gamma^0_{i,k} + \gamma^1_{i,k} \beta_{i,k,t-1}\,, \quad k = 1, \dots, K. \quad (8)$$

Using these AR(1) representations as state equations and the multifactor model (4) as a signal equation, the Kalman filer algorithm will generate estimates of the conditional alphas, $\alpha_{i,t|t-1}$, and the conditional risk factor loadings, $\beta_{i,k,t|t-1}$, *k=1,…,K,* along with the corresponding conditional variance estimates and conditional mean squared errors, $\sigma^2_{\alpha,i,t|t-1}$ and $\sigma^2_{\beta,i,k,t|t-1}$, *k=1,…,K*, respectively. Using these variances, the statistical significance of the conditional risk factors can be monitored over time *t*, *t=1,…,T*.

The unconditional parameters of the model specification, the coefficients in the AR(1) models, or γ^0_i, $\gamma^1_i, \gamma^0_{i,k}, \gamma^1_{i,k}$, and the variances, σ^2_ε, $\sigma^2_{\vartheta,i}$ and $\sigma^2_{\vartheta,i,k}$ are estimated using the log likelihood function

$$-\frac{T}{2}\log(2\pi)-\frac{1}{2}\sum_{t=1}^{T}\log(\boldsymbol{F}_t'\boldsymbol{\Sigma}_{\alpha,\beta,i,t|t-1}\boldsymbol{F}_t+\ \sigma_\varepsilon^2)-\frac{1}{2}\sum_{t=1}^{T}\left(r_{i,t}^e-\hat{r}_{i,t|t-1}^e\right)^2/(\boldsymbol{F}_t'\boldsymbol{\Sigma}_{\alpha,\beta,i,t|t-1}\boldsymbol{F}_t+\ \sigma_\varepsilon^2), \tag{9}$$

where $\boldsymbol{F}_t$ is the *(K+1)x1* vector of observations on factors $f_{k,t}$, *k=1,…,K* including the constant as the first element, $\boldsymbol{\Sigma}_{\alpha,\beta,i,t|t-1}$ is the diagonal *(K+1)x(K+1)* covariance matrix including $\sigma_{\alpha,i,t|t-1}^2$ and $\sigma_{\beta,i,k,t|t-1}^2$, *k=1,…,K* as elements. The term $\left(r_{i,t}^e-\hat{r}_{i,t|t-1}^e\right)$ is simply the residual using the conditional parameter estimates of $\alpha_{i,t|t-1}$, and $\beta_{i,k,t|t-1}$, *k=1,…,K*.

The recursive Kalman filter algorithm for this model specification is given by

$$\begin{aligned}\boldsymbol{\beta}_{t|t-1}&=\boldsymbol{\gamma}^{\mathbf{0}}+\boldsymbol{\gamma}^{\mathbf{1}}\boldsymbol{\beta}_{t-1|t-1}\\ \boldsymbol{\Sigma}_{\alpha,\beta,i,t|t-1}&=\ \boldsymbol{\gamma}^{\mathbf{1}}\boldsymbol{\Sigma}_{\alpha,\beta,i,t-1|t-1}\boldsymbol{\gamma}^{\mathbf{1}\prime}+\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}\\ \boldsymbol{\kappa}_t&=\boldsymbol{\Sigma}_{\alpha,\beta,i,t|t-1}\boldsymbol{F}_t\left(\boldsymbol{F}_t'\boldsymbol{\Sigma}_{\alpha,\beta,i,t|t-1}\boldsymbol{F}_t+1\right)^{-1}\\ \boldsymbol{\beta}_{t|t}&=\boldsymbol{\beta}_{t|t-1}+\boldsymbol{\kappa}_t(r_{i,t}^e-\hat{r}_{i,t|t-1}^e)\\ \boldsymbol{\Sigma}_{\alpha,\beta,i,t|t}&=\ \boldsymbol{\Sigma}_{\alpha,\beta,i,t|t-1}-\boldsymbol{\kappa}_t\boldsymbol{F}_t'\boldsymbol{\Sigma}_{\alpha,\beta,i,t|t-1},\end{aligned} \tag{10}$$

Where,

$$\begin{aligned}\boldsymbol{\beta}_{t|t-1}&=\left[\alpha_{i,t|t-1},\beta_{i,1,t|t-1},\dots,\beta_{i,K,t|t-1}\right]',\\ \boldsymbol{\Sigma}_{\alpha,\beta,i,t|t-1}&=diag\left[\sigma_{\alpha,i,t|t-1}^2,\sigma_{\beta,i,1,t|t-1}^2,\dots,\sigma_{\beta,i,K,t|t-1}^2\right],\\ \boldsymbol{\gamma}^{\mathbf{0}}&=\left[\gamma_i^0,\gamma_{i,1}^0,\dots,\gamma_{i,K}^0\right]'\\ \boldsymbol{\gamma}^{\mathbf{1}}&=\left[\gamma_i^1,\gamma_{i,1}^1,\dots,\gamma_{i,K}^1\right]'.\end{aligned} \tag{11}$$

The (*K+1*) vector $\boldsymbol{\kappa}_t$ is the so-called Kalman gain that distributes the correction imposed by the estimation error, or $(r_{i,t}^e-\hat{r}_{i,t|t-1}^e)$, over the elements of $\boldsymbol{\beta}_{t|t-1}$. A thorough description of the Kalman filter is found in Hamilton (1994).

The above specification does not impose any prior restrictions regarding stationarity of the factor loading. Hence, the stationarity of the loading can be monitored and tested empirically. If the variances $\sigma_{\vartheta,i}^2$ and $\sigma_{\vartheta,i,k}^2$ are statistically significant, the time dynamic is stochastic. Insignificant γ_i^1 or $\gamma_{i,k}^1$ will indicate that the corresponding time variation follows a constant mean model. Statistically significant parameters γ_i^1 or $\gamma_{i,k}^1$ will further indicate the type of autocorrelation structure, i.e., mean reverting if negative and otherwise positive autocorrelation. In the case of constant mean or mean reversion, OLS will provide fair estimates of the average loading. However, in the case of positive autocorrelation, OLS will not be an optimal approach for the estimation of unconditional risk factor loadings.

3. Data

As recommended by Lewellen, Nagel, and Shanken (2010) and Grauer and Janmaat (2010), the present study does not use grouped portfolios based on size, value, or other potential risk factors. Instead, we utilize industry portfolios to examine different empirical approaches and models. Monthly returns for 49 US industry portfolios are sampled over the time period January 1972 to December 2019. As a natural benchmark, we use the traditional market model (MM) with the market excess return as the single risk factor. We also use the augmented five-factor Fama-French model (FF) with excess market return, size, value, profitability, investment and momentum (see Fama and French 2015 and Carhart 1997).
A summary of descriptive statistics for the six risk factors are presented in Table 1.

Table 1: Summary of descriptive statistics for factors

	Market	Size	Value	Momentum	Investment	Profitability
Mean	0.42	0.19	0.44	0.76	0.29	0.77
Median	0.78	0.05	0.41	0.89	0.29	0.81
Maximum	16.05	21.99	13.87	18.35	6.77	22.65
Minimum	-23.14	-16.85	-12.37	-34.69	-6.85	-20.30
Std.deviation	4.65	3.22	3.08	4.61	1.87	4.20
Skewness	-0.59	0.59	-0.03	-1.50	0.08	0.10
Kurtosis	5.24	9.55	5.28	13.67	3.52	9.52
Jarque-Bera	120.47	829.47	97.21	2302.68	5.63	798.17
Probability	0.00	0.00	0.00	0.00	0.06	0.00
Value of $1	176.68	84.78	192.85	349.71	127.43	342.09

Monthly returns from January 1972 to December 2019. Data are obtained from Kenneth French's website (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

The average return is positive for all risk factors. The corresponding average monthly returns are significantly different from zero for the market, value, momentum, investment, and profitability factors. The standard deviation, skewness, and kurtosis for the investment factor is low compared to the other factors. By contrast, skewness is high and negative for market and momentum but positive and high for size. Also, kurtosis is high for size, momentum, and profitability. This last characteristic is apparent in Figure 1 that graphs the time series of the six risk factors over the sample period. As shown there, a large increase in variance occurred around the dot.com bubble in the late 1990s. The return on momentum and profitability are remarkably high, with a large part of their returns earned during the dot.com bubble.

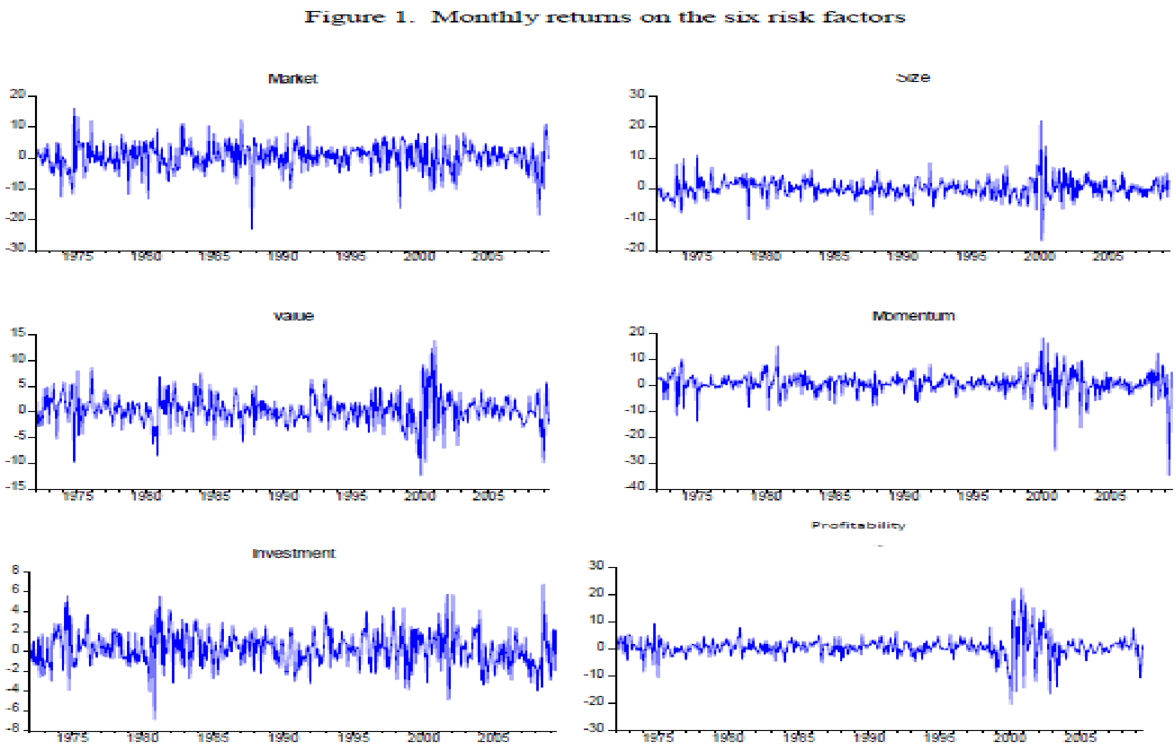

Figure 1: Monthly returns on the six risk factors

The contemporaneous unconditional correlations between the six risk factors are reported in Table 2. Panel A shows that all multifactors are significantly correlated with the market factor and that all are negative except for size. The value factor has the strongest correlation of -0.36 with the market. Also, the value factor is significantly correlated with all other factors, and its correlation with investment of 0.45 is especially high. The return on profitability, is significantly correlated with all FF factors (e.g., its correlation with size is -0.43). However, the profitability factor is uncorrelated with the factor investment. These results suggest that none of the six risk factors carry 100% unique information. The information in the risk factors is to a large extent overlapping. This issue is confirmed by the multiple correlations reported in Panel B of Table 2. Multiple correlations are obtained using the positive square root of OLS *R*-squares from a regression of the factor on a set of other factors with the dependent variable excluded from the right-hand-side of the regression. The profitability and FF multifactors share more information with each other than with the market factor, i.e, about 20%-30%.The market factor shares about 14% of information with the FF multifactors and about 20% with the profitability factor. The value factor contains the least unique information with a multiple correlation of 0.61 with all other factors indicating about 63% unique information which is still high.

Panel C of Table 2 presents the first order autocorrelation and cross-autocorrelation structure among the risk factors. The market factor as well as value, investment, and profitability factors exhibit significant first order autocorrelation. A bi-directional lead-lag relation exists between the market and profitability factors. The size factor leads the market and momentum factors and has a bi-directional lead-lag relation with the value factor. Additionally, the profitability factor seems to lead the value factor. In unreported results, higher order auto- or cross-autocorrelations are not statistically significant.

4. Empirical Results

Evidently, the loadings on the multifactors are generally not robust over the different parts of the return distribution. Hence, it might be expected that assessing the outcome of the return distribution over time would indicate time-varying (deterministic or stochastic) factor loadings. To evaluate the time-varying characteristics of risk factor loadings, the Kalman filter is applied to the factor models (signal equations) with an AR(1) model for the factor loadings (the states). Table 3 reports on the average level of stochasticity in the factor loadings. The first column of Table 3 presents the average values of the ML estimates of the standard deviations, or $\sigma_{\vartheta,i}$ and $\sigma_{\vartheta,i,k}$, of the error component in the AR(1) specification. A lower value of $\sigma_{\vartheta,i}$ or $\sigma_{\vartheta,i,k}$ indicates less stochastic behavior.

Table 2: Unconditional correlation between risk factors

Panel A. Unconditional contemporaneous correlations
Bold indicates significance at the 5% level or less.

Factor	Market	Size	Value	Momentum	Investment	Profitability
Market	1.00					
Size	**0.26**	1.00				
Value	**-0.36**	**-0.26**	1.00			
Momentum	**-0.14**	-0.01	**-0.15**	1.00		
Investment	**-0.26**	0.00	**0.45**	0.04	1.00	
Profitability	**-0.26**	**-0.43**	**0.20**	**0.31**	-0.03	1.00

Panel B. Multiple correlations among factors
Estimates are obtained using the positive square root of OLS R-squares from a regression of the factor on a set of other factors with the dependent variable excluded from the right-hand side of the regression.

Factor	Market	Market, size, value, and momentum	Market, investment, and profitability	All other
Market		0.37	0.45	0.47
Size	0.26	0.32	0.46	0.50
Value	0.36	0.45	0.54	0.61
Momentum	0.14	0.26	0.31	0.44
Investment	0.26	0.49	0.27	0.51
Profitability	0.26	0.55	0.27	0.57

Panel C. Unconditional cross autocorrelations
Autocorrelations are shown in the diagonal and cross autocorrelations in off diagonal positions.

Lead one month

Factor	Market	Size	Value	Momentum	Investment	Profitability
Lag one month						
Market	**0.09**	**0.20**	0.04	**-0.11**	0.06	**-0.10**
Size	0.08	0.01	**0.11**	0.06	-0.04	0.01
Value	-0.06	**0.21**	**0.14**	-0.03	0.01	**0.11**
Momentum	-0.04	**-0.10**	-0.07	0.07	0.02	0.00
Investment	-0.06	-0.04	**0.09**	0.05	**0.10**	-0.02
Profitability	**-0.10**	-0.05	-0.04	-0.01	-0.02	**0.17**

Monthly returns from January 1972 to December 2019. Data are obtained from Kenneth French's website.

Table 3: Average ML estimates of standard errors in AR(1) models for alpha and factor loadings

Panel A	**Kalman filter average ML estimates of AR(1) error std. deviation**	**Kalman filter average ML estimates of AR(1) error std. deviation in percentage of mean**
Market model (MM)		
Alpha	2.26	4146.61
Market	0.40	42.48
FF model		
Alpha	2.04	2313.44
Market	0.18	17.41
Size	0.32	373.14
Value	0.28	567.18
Momentum	0.23	1781.52
ALL factors		
Alpha	1.90	2443.82
Market	0.21	20.68
Size	0.30	157.86
Value	0.18	168.18
Momentum	0.16	472.71
Investment	0.36	537.45
Profitability	0.28	262.13

Results are based on the Fama-French 49 industry portfolios using monthly returns from January 1972 to December 2019. Data are obtained from Kenneth French's website. The asset pricing model is: $r_{i,t}^{e} = \alpha_{i,t} + \sum_{k=1}^{K} \beta_{i,k,t} f_{k,t} + \varepsilon_{i,t}$, where the AR(1) models are $\alpha_{i,t} = \gamma_i^0 + \gamma_i^1 \alpha_{i,t-1} + \vartheta_{i,t}$ for alpha, and $\beta_{i,k,t} = \gamma_{i,k}^0 + \gamma_{i,k}^1 \beta_{i,k,t-1} + \vartheta_{i,k,t}$, $k = 1, \dots, K$ for the risk factor loadings.

Again, using the market model (MM) as a benchmark and studying the effect of including multifactors, we see that the standard deviation in the AR(1) model for the market factor significantly decreases. For the FF model, the reduction is 55% (i.e., from 0.40 down to 0.18). The corresponding reduction the ALL factor model is 48% (i.e., down to 0.21). Consequently, the multifactors seem to significantly reduce the stochastic behavior of the market factor loading. This effect can be clearly seen in Figure 2. In first graph shown there, the conditional loading on the market factor in the MM model for the food industry is obviously time varying. The corresponding conditional loading on the market factor in the FF model appears to be more robust, almost constant, as shown in the second graph of Figure 2.

Figure 2. Kalman filter conditional factor loading estimates for the market model (MM) as well as FF model with 95% confidence limits for the food industry.

Figure 2: Kalman filter conditional factor loading estimates for the market model (MM) as well as FF model with 95% confidence for the food industry

From Table 3 we also see a small reduction of the average standard error of the AR(1) model for alpha. The reduction is 10% for the FF model and 16% for ALL models, respectively. Hence, a part of the stochastic behavior in alpha is reduced by the multifactors. In this respect we would normally expect the alpha of a good asset pricing model to exhibit unpredictable stochastic behavior around zero, as the error component in the AR(1) of alpha is dominating the mispricing behavior.

The second column of Table 3 gives the corresponding coefficients of variation, or the average standard deviation in percentage of the estimated long-run mean of the factor loading. This long-run mean for factor *k* is calculated as $\gamma_{i,k}^{0}/(1-\gamma_{i,k}^{1})$. For the mispricing parameter alpha, the long-run average is statistically significant for only the drugs industry in both the MM and FF models. For the ALL model, it is significant for the electricity equipment industry. In all other cases there is no significant long-run mispricing.

From the results in the second column of Table 3, it is evident that the multifactors on average exhibit time variation crossing zero and alternatively produce both positive and negative risk factor loadings. Different from the risk factor loadings, it would be preferable to have a mispricing alpha on average close to zero with a large enough standard deviation to guarantee the insignificance of the mispricing.

The significance of the individual conditional risk factor loadings over the sample period is summarized in Table 4. Here the distribution of the 49 industry portfolios over the number of statistically significant months is presented for each of the six individual factor loadings and for the mispricing alpha. Generally, the inclusion of multifactors in the asset pricing model makes the conditional loading on the market factor more statistically significant. For the MM model, only 11-out-of-49 industry portfolios have significant market factor loadings for the entire period. When multifactors are included, this number rises to 45, for the FF and ALL models, respectively. Moreover, the number of portfolios with no significant mispricing alphas throughout the sample period changes from 45 for the MM model to 46 the FF, and 47 for the ALL models, respectively.

Based on our findings in Table 4, it is also evident that for several of the portfolios the conditional loadings on the multifactors are not significant over the sample period. In the FF model, the conditional market factor loading is very strong and never insignificant throughout the sample period. On the other hand, the conditional loading on size is not significant in any of the 564 months for 61% (30-out- of-49) of the portfolios. The corresponding proportions are 33% (16-out-of-49) for value and 49% (24-out-of-49) for momentum.

For the FF model, the conditional size and value factor loadings are significant throughout the 564 months for only one of the 49 respective portfolios (viz., the fabricated products industry for size and the real estate industry for value).

It is also evident that, when the conditional multifactor loadings have a significant impact, it usually only occurs during a small fraction of the 564 months of the sample period. In the FF model, the conditional loading on size is significant for 19 portfolios. For 63% (12-out-of-19) of these, the loading is significant during less than 5% (i.e., 24 months out of the total sample) of the conditional time periods. The corresponding proportions are 58% (19-out-of- 33) for the value and as high as 96% (24-out-of-25) for the conditional momentum factor loading.

A similarly clear interpretation for the ALL model is difficult due to the fact that in many cases at least one of the factor loadings needed to be estimated unconditionally to guarantee a nonsingular conditional variance-covariance matrix.

This problem arose in 33-out-of-49 portfolios. However, in most of the cases, when the loadings on the multifactors are significant during all the 564 months, the corresponding loading had to be restricted to a constant to guarantee a nonsingular conditional variance-covariance matrix.

Table 4: Statistically significant conditional risk factor loadings by the number of significant months out of 564 months

	Number of portfolios with significant states based on number of months out of 450 months					
	All	More than 24 months	7-to-24 months	2-to-6 months	Only 1 month	None
Market model (MM)						
Alpha	0	0	1	3	0	45
Market	11	23	3	1	2	9
FF model						
Alpha	0	0	3	0	0	46
Market	45(2)[1]	3	0	1	0	0
Size	1	6	1	3	8	30
Value	1	13	8	9	2	16
Momentum	0	1	7	13	4	24
ALL factors						
Alpha	0	0	1	1	0	47
Market	45(6)[1]	2	0	0	0	2
Size	12(9)[1]	3	0	3	3	28
Value	9(8)[1]	8	5	2	3	22
Momentum	1(1)[1]	3	13	8	3	21
Investment	3(3)[1]	1	4	4	2	35
Profitability	9(7)[1]	3	4	2	3	28

[1] The number in parenthesis indicates for how many of the portfolios this factor loading was estimated unconditionally as a constant in order to guarantee a nonsingular variance-covariance matrix. For the FF model and the market factor, 45(2) is interpreted to mean that for 45-out-of-49 portfolios the market factor loading was statistically significant for all months and for 2-out-of-45 portfolios the market loading had to be restricted to be constant (i.e., estimated using the maximum likelihood function along with the other unconditional parameters of the model).

Results are based on the Fama-French 49 industry portfolios using monthly returns from January 1972 to December 2019, obtained from Kenneth French's website. The asset pricing model is: $r_{i,t}^{e} = \alpha_{i,t} + \sum_{k=1}^{K} \beta_{i,k,t} f_{k,t} + \varepsilon_{i,t}$, where the AR(1) models are $\alpha_{i,t} = \gamma_i^0 + \gamma_i^1 \alpha_{i,t-1} + \vartheta_{i,t}$ for alpha, and $\beta_{i,k,t} = \gamma_{i,k}^0 + \gamma_{i,k}^1 \beta_{i,k,t-1} + \vartheta_{i,k,t}$, $k = 1, \dots, K$ for the risk factor loadings.

In order to assess the effects of multifactors on their time-varying characteristics, the significance of the parameters of the AR(1) model for the factor loadings is reported in Table 5.

If none of the parameters γ^1 and σ_ϑ are statistically significant, then the constancy of the factor loading or alpha cannot be rejected. If σ_ϑ is significant but γ^1 is not, then the constant mean model cannot be rejected. If both γ^1 and σ_ϑ are statistically significant, then the AR(1) model cannot be rejected and the factor loading exhibits significant first order autocorrelation over time. This first order autocorrelation can be positive or negative. In the case when γ^1 is significantly negative, then the time variation is mean reverting. When γ^1 is positive and significant, then the time variation is not necessarily always close to a long run mean, especially if σ_ϑ is small and γ^1 is large. In the first three cases -- namely, constancy, constant mean model, and mean reversion -- the application of OLS may be satisfactory to estimate the expected loading. In the latter case, however, OLS might not be an optimal approach for estimating expected factor loadings.

In all 49 cases, all estimates of γ^1 are in absolute value less than unity. However, in many cases, due to the high standard deviation, the null hypothesis that $\gamma^1 = 0$ cannot be rejected.

Overall, the general message of Table 5 is that augmenting the market model with multifactors makes the loading on the market factor more robust over time. The number of portfolios with significant positive autocorrelation reduces by 62% (viz., from 26 to 10) for the FF model. Augmenting with multifactors has a similar effect on the mispricing alpha. For the FF model the reduction is 63% (viz., from16 to 6). Conversely, the proportion of constant or constant mean models for the market factor loading increases by 58% (viz., from19 to 30)

In general, the results of the conditional asset pricing model estimation using the Kalman filter approach show that the multifactors improve on the statistical significance of the market factor and help stabilize the time variation in the traditional market model parameters, including both the alpha and loading on the market risk factor. Furthermore, the conditional multifactor loadings are rarely significant over long periods; instead, it appears that the significance of these factors occurs during distinct periods in time.

5. Summary and conclusion

This study revisits the role of multifactors in traditional asset pricing models by using the Kalman filter approach with time-varying risk factor loadings. The evidence indicates that inclusion of multifactors in the conditional asset pricing model strengthens the statistical significance of the market risk factor loading and improves on the time stability of the market risk factor. Also, reduction of mispricing variance using multifactors is minimal.

Overall, the empirical results of the paper support the view that multifactors might not be separate risk factors but instead help to jointly in combination with a market index identify the market factor. Consistent with the notion that the market portfolio is an unobservable equilibrium benchmark that requires other factors to locate (e.g., see Shanken ,1987 and Shanken and Weinstein, 2006), the multifactors appear to empirically compensate (to some extent) for the effect of a poor single-factor proxy for the market portfolio. Their impact on the characteristics of the conditional market risk factor loading is evident. Hence, the role of multifactors in asset pricing models stems primarily from overlapping information with the market risk rather than from unique information. Their unique information reduces mispricing but should not have an impact on the market risk loading. In times of market turmoil or distress, the impact of multifactors appears to be more prominent.

Table 5: Factor loading stability over time: Time-varying factor loading characteristics

Factor model	Constancy not rejected	Constant mean model	Significant negative autocorrelation	Significant positive autocorrelation
Market model (MM)				
Alpha	32	1	0	16
Market	3	16	4	26
FF model				
Alpha	40	1	2	6
Market	7	23	9	10
Size	8	20	4	17
Value	4	10	3	32
Momentum	1	10	3	35
ALL factors				
Alpha	37	1	5	6
Market	13	24	7	5
Size	19	14	3	13
Value	18	9	3	19
Momentum	5	8	2	34
Investment	28	4	1	16
Profirability	13	20	1	15

Results are based on the Fama-French 49 industry portfolios using monthly returns from January 1972 to December 2019. The asset pricing model is: $r_{i,t}^{e} = \alpha_{i,t} + \sum_{k=1}^{K} \beta_{i,k,t} f_{k,t} + \varepsilon_{i,t}$, where the AR(1) models are $\alpha_{i,t} = \gamma_i^0 + \gamma_i^1 \alpha_{i,t-1} + \vartheta_{i,t}$ for alpha, and $\beta_{i,k,t} = \gamma_{i,k}^0 + \gamma_{i,k}^1 \beta_{i,k,t-1} + \vartheta_{i,k,t}$, $k = 1, \ldots, K$ for the risk factor loadings.

References

[1] Adrian, T, and Franzoni, F. (2009). Learning about beta: Time-varying factor loadings, expected returns, and the conditional CAPM, *Journal of Empirical Finance,* 16, 537–556.

[2] Amihud, Y. (2002). Illiquidity and stock returns: Cross-section and time-series effects, *Journal of Financial Markets,* 5, 31-56.

[3] Ammann, M, Odoni, S. and Oesch, D. (2012). An alternative three-factor model for international markets: Evidence from the European Monetary Union, *Journal of Banking & Finance,* (forthcoming)

[4] Ammann, M., Huber, O and Schmid, M. (2010). Benchmarking hedge funds: The choice of factor model, *SSRN Working paper*, Electronic copy available at: http://ssrn.com/abstract=1672543

[5] Ang, A, and Chen, J. (2007). CAPM over the long run: 1926-2001, *Journal of Empirical Finance*,14, 1-40.

[6] Ang, A., and Kristensen, D. (2009). Testing conditional factor models, *SSRN Working* paper, Electronic copy available at: http://ssrn.com/abstract=1353358

[7] Armstrong, W., Knif, J, Kolari, J.W. and S Pynnönen, S. (2012). Exchange risk and universal returns: A test of international arbitrage pricing theory, *Pacific-Basin Finance Journal*, 20, 1, 24-40.

[8] Arnott, R., Hsu, J., Liu, J. and H. Markowitz, (2006). Does noise create the size and value factors? Working Paper.

[9] Arshanapalli, B., Fabozzi, F.J. and Nelson, W. (2006). The value, size, and momentum spread during distressed economic periods, *Finance Research Letters*, 3, 244-252.

[10] Avramov, D., Chordia, T., Jostova, G. and Philipov, A. (2007). Momentum and credit rating, *Journal of Finance*, 62, 2503-2520.

[11] Banz, R.W. (1981). The relationship between return and market value of common stocks, *Journal of Financial Economics*, 9, 3-18.

[12] Basu, S. (1977). The investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis, *Journal of Finance*, 32, 663-682.

[13] Berglund, T. and Knif, J. (1999). Accounting for the accuracy of beta estimates in CAPM-tests on assets with time-varying risks, *European Financial Management*, 5, 25-42.

[14] Bhandari, L. C. (1988). Debt/equity ratio and expected common stock returns: Empirical evidence, *Journal of Finance*, 43, 507-528.

[15] Brennan, M., and Wang, A. (2007). The mispricing return premium, *SSRN Working paper*. Electronic copy available at: http://ssrn.com/abstract=1232484

[16] Campbell, J., Hilscher, J. and Szilagyi, J. (2008). In search of distress risk, *Journal of Finance*, 63, 2899-2939.

[17] Campbell, J., and Vuolteenaho, T. (2004). Bad beta, good beta, *American Economic Review*, 94, 1249-1275.

[18] Carhart, M. (1997). On persistence in mutual fund performance, *Journal of Finance*, 52, 57–82.
[19] Chan, L., and Lakonishok, J. (1993). Are the reports of beta's death premature?, *Journal of Portfolio Management*, 20, 51-62.
[20] Chan, K.C., Chen, N. and Hsieh, D. (1985). An exploratory investigation of the firm size effect, *Journal of Financial Economics*, 14, 451–-471.
[21] Chen, N., Roll, R., Ross, S. (1986). Economic forces and the stock market, *Journal of Business*, 59, 383-403.
[22] Chen, L., Novy-Marx, R. and Zhang, L. (2010) (Revised version 2011). An alternative three-factor model, *SSRN Working paper*, Electronic copy available at: http://ssrn.com/abstract=1418117
[23] Chordia, T., and Shivakumar, L. (2002). Momentum, business cycle, and time-varying expected returns, *Journal of Finance*, 57, 985-1019.
[24] Cooper, M., Gutierrez, R. and Hameed, A. (2004). Market states and momentum, *Journal of Finance*, 59, 1345-1365.
[25] Da, Z., Guo R.-J., and Jagannathan, R. (2012). CAPM for estimating the cost of equity capital: Interpreting the empirical evidence, *Journal of Financial Economics*, 103, 204-220.
[26] Daniel, K., and Titman, S. (1996). Evidence on the characteristics of cross sectional variation in stock returns, *Journal of Finance*, 52, 1-33.
[27] Fama, E. F., and French, K.R. (1992). The cross-section of expected stock returns, *Journal of Finance*, 47, 427-465.
[28] Fama, E.F. and French, K.R. (2015). A five-factor asset pricing model. *Journal of Finance*, 116, 1, 1-22.
[29] Fama, E. F., and French, K.R. (1993). Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, 33, 3-56.
[30] Ferson, W., and Harvey, C. (1999). Conditioning variables and cross-section of stock returns, *Journal of Finance*, 54, 1325-1360.
[31] Fung, W. and Hsieh, D. A. (2004), Hedge fund benchmarks: A risk based approach. *Financial Analysts Journal*, 60, 65-80.
[32] Ghysels, E. (1998). On stable factor structures in the pricing of risk: Do time-varying betas help or hurt*?* *Journal of Finance*, 53, 549-573.
[33] Grauer, R. R., and Janmaat, J. A. (2010). Cross-sectional tests of the CAPM and Fama–French three factor model, *Journal of Banking &Finance*, 34, 457-470.
[34] Grinblatt, M., and Moskowitz, T. (2004). Predicting stock price movements from past returns: The role of consistency and tax-loss selling, *Journal of Financial Economics*, 71, 541-579.
[35] Griffin, J., and Lemmon, M. (2002). Book-to-market equity, distress risk, and stock returns, *Journal of Finance*, 57, 2317-2336.
[36] Griffin, J., Ji, S. and Martin, S. (2003). Momentum investing and business cycle risk: Evidence from pole to pole, *Journal of Finance*, 58, 2515-2547
[37] Grundy, K., and Malkiel, M. (1996). Reports of beta's death have been greatly exaggerated, *Journal of Portfolio Management*, 2, 36-44.

[38] Hamada, R.S. (1972). The effect of the firm's capital structure on the systematic risk of common stocks, *Journal of Finance*, 27, 435-452.
[39] Hamilton, J. (1994). *Time Series Analysis*, Princeton University Press, Princeton, New Jersey.
[40] Jagadeesh, N., and Titman, S. (1993). Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance,* 48, 65-92.
[41] Knif, J. (1990). Parameter variability in the single factor market model: An empirical comparison of tests and estimation procedures using data from the Helsinki Stock Exchange, *Commentationes Scientiarum Socialium* ,40, The Finnish Society of Science and Letters.
[42] Koutmos, G., and Knif, J. (2002). Estimating systematic risk using time varying distributions. *European Financial Management*, 8, No.1, 59-73.
[43] Levy, H. (2009). The CAPM is alive and well: A review and synthesis, *European Financial Management*, 16, 43-71.
[44] Lewellen, J., and Nagel, S. (2006). The conditional CAPM does not explain asset-pricing anomalies, *Journal of Financial Economics,* 82, 289-314.
[45] Lewellen, J., Nagel, S., and Shanken, J. (2010). A skeptical appraisal of asset-pricing tests, *Journal of Financial Economics*, 96, 175-194.
[46] Loughran, T. (1997). Book-to-market across firm size, exchange, and seasonality: Is there an effect? *Journal of Financial and Quantitative Analysis*, 32, 249-268.
[47] Merton, R. C. (1973). An intertemporal capital asset pricing model, *Econometrica*, 41, 867-887.
[48] Petkova, R. (2006). Do the Fama–French factors proxy for innovations in predictive variables? *Journal of Finance*, 61, 581-612.
[49] Rosenberg, B., Reid, K. and Lanstein, R. (1985). Persuasive evidence of market inefficiency, *Journal of Portfolio Management*, 11, 9-17.
[50] Shanken, J. (1987). Multivariate proxies and asset pricing relations: Living with the Roll critique, *Journal of Financial Economics*, 18, 91-110.
[51] Shanken, J., and Weinstein, M.I. (2006). Economic forces and the stock market revisited, *Journal of Empirical* Finance, 13, 129-144.
[52] Simpson, M.W., and Ramchander, S. (2008). An inquiry into the economic fundamentals of the Fama and French equity factors, *Journal of Empirical Finance*, 15, 801-815.
[53] Stattman, D. (1980). Book values and stock returns, *The Chicago MBA: A Journal of Selected Papers*, 4, 25-45.
[54] Tai, C.-S. (2003). Are Fama-French and momentum factors really priced? *Journal of Multinational Financial Management*, 13, 359-384.
[55] Trecroci, C. (2009). How do alphas and betas move? Uncertainty, learning and time variation in systematic risk, SSRN Working paper, Electronic copy available at: http://ssrn.com/abstract=1498752
[56] Zhang, L., (2005). The value premium, *Journal of Finance,* 60, 67-103.