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# EULER-MARUYAMA APPROXIMATION OF STOCHASTIC DEPENDENT POISSON-JUMP IN BLACK-SCHOLES ASSET PRICE MODEL

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**Abstract**

In this study, the Gaussian white noise and the differential Poisson of the Stochastic Differential Equation(SDE) with distributed jump are examined. Using Ito integral as a tool, a one step Euler-Maruyama (E-M) method is considered for the approximation of Stochastic Dependent Poisson Analysis (SDPA) in finanace. The Deterministic Quadrature Rule (DQR) was used in the establishment of the method for easy examination of the Black-Scholes asset price model for stock investors; MATLAB package was used for simulation of the method. However the Mean Absolute Error (MAE) as well as Strong Order of Convergence (SOC) method was considered to ascertain its usability. The result clearly shows entry points and exit points of stock market. Consequently, the findings of this research is strongly recommended.

**Keywords**: Euler-Maruyama method, Stochastic differential equation, Ito integral, Poisson distributed jump, Random variables, Deterministic model.

# 1 INTRODUCTION

## 1.1 Preamble

Differential Equation (DE) is a type of equation which relates rate of change of a dependent variable with respect to one or more independent variable.
mathematically,

$$y' = f(x, y) \tag{1.1}$$

## 1.2 Types of Differential Equation

Some of the types of DE that was studied in this research are listed below

- Ordinary Differential Equation ODE
- Partial Differential Equation PDE
- Stochastic Differential Equation SDE

Some Authors (example: Kayode S. J. and Abejide K. S. 2019) [12] had developed several numerical methods to solve some kind of differential equations, the methods established were found to be accurate using the test criterion widely studied from Lambert, J. D. 1991 [17]
In this study SDEs as it related to fiancial mathematics were studied using knowledge from Operation Research [15]

## 1.3 Stochastic Differential Equation

Stochastic differential equation (SDE) is a branch of mathematics which includes random variables in the deterministic model. Stochastic models arise from different fields of studies such as biology, telecommunications, engineering, space science, financial market, vibrations from bomb detonations, etc.
Below is an example of SDE with jump.

$$dQ = z(t, Q(t))dt + y(t, Q(t))dW(t) + f(t, Q(t))dP(t, Q(t)), \;\; Q(t_0) = Q_0 \tag{1.2}$$

The equation above are the mathematical representation of random phenomenon relating to the financial market. In most cases, the analytical solution might not be readily available, this calls for other solution techniques which brings about

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the E-M method as an approximate solution. Some related research on SDEs are reviewed as follows

Kayode, Ganiyu and Ajiboye (2016) [14]developed a onestep method of Euler-Maruyama. This method was used to solve the general form of stochastic differential equations without jump. In their method, effect of varying stepsizes was inspected as against the general simulation approach which uses a random step size. This method was found to be accurate and recommended for further researches in the field of SDE.

On the other hand, Kayode and Ganiyu (2015)[13] developed a one step Milstien method on the approximation of the general SDEs. In the work, graphical representation was used for result comparison and the effect of varying stepsizes was also monitored. It was found that the order of convergence is 1 which implies accuracy of the method; and the method is further recommended for applications and research purpose.

Some authors research were carefully studied to establish links between the Wiener process, Poisson Jumps and the differential equation that form the SDEs; few of the authors are Higham and Kloeden P. E. (2000)[11], Lamba, Mattingly and Stuart (2006)[16], Bayram, Partal and Byukoz(2018)[4] and Briand, Labart and Ghannoum (2018)[5].
The work of these authors was studied for the possibility of developing new methods in next chapters.

Akinbo, Faniran and Ayoola(2015)[1] produced a paper which provided an introduction to the main concepts and techniques necessary for someone who wishes to carryout numerical experiments involving Stochastic Differential Equation (SDEs).
SDEs are generally randomized in formulation and the solutions are continuous stochastic process that represent diffusive dynamic especially in finance, the authors took into account random effects and influences in real life systems which are essential in the accurate description of such situations. case studies around financial mathematics was experimented using Taylor approach and their results were found to be accurate.

Araposthasis, Biswas and Caffarelli (2014)[2] studied stochastic differential equations with jumps with no diffusion part. In their work, they provided some basic stochastic characterizations of solutions of the corresponding non-local partial differential equations and prove the Harnack inequality for a class of these operators. They also established key connections between the recurrence properties of these jump processes and the non-local partial differential operator, this made working on SDEs with no diffussion part very easy to use.
The Poisson distrubution properties was also studied by Araposthasis, Hmedi and Pang (2016)[3]. Further studies on probabilty distributon [8], [9] was examined as required by this research.

Burrage and Burrage (1996)[7] worked on the classical Runge-Kutta method as an approximate solution of SDEs. in their work, it was stated that the pioneering work of Runge and Kutta about hundred years ago has ultimately led to suites of sophisticated numerical methods suitable for solving complex systems of deterministic ordinary differential equations. However, it was gathered that in many modeling situations, the appropriate representation is a stochastic differential equation and here numerical methods are much less sophisticated. This was the motivation for this work where a very general class of stochastic Runge-Kutta methods is presented and much more efficient classes of explicit methods than previous extant methods are constructed. In particular, a method of strong order 2 with a deterministic component based on the classical Runge-Kutta method is constructed and some numerical results are presented to demonstrate the efficacy of this approach.
Some other authors who worked of SDEs are (Burrage, Burrage, Mitsui (2001) [6] ), ( Rio, Setiyo, Putri, Fajar, Andi and Mudrik (2021)[19]), (Setiyo, Lukman, Freddy, Ketty, Fajar, Iwan and Septi (2020)[20]), (Floyed (2007)[10]) and (Moshe (2020)[18]).

# 2 DERIVATION OF METHOD

In Kayode and Ganiyu 2015[13], Euler Maruyama methods with varying step-sizes was developed for stochastic differential equation of the form

$$dQ(t) = z(t, Q(t)) + y(t, Q(t)) \tag{2.1}$$

This SDE is independent of any form of perturbations or any other factor affecting the usual direction of the equation and the wiener process.
In this research, there is focus on the internal disturbances which affects the usual movement of the resulting chart of the equation relating to the asset price in stock market. This disturbances are called the Poisson distributed jumps.

## 2.1 Ito Integral

This part is the basis on which the SDE is solved, it is the essential procedure to be established before the research can proceed.

$$\int_0^t Q(s)ds = \int_0^t z(s,Q(s))ds + \int_0^t y(s,Q(s))dW(s) + \int_0^t f(s,Q(s))dP(s,Q(s))ds, \;\; Q(t_0) = Q_0 \tag{2.2}$$

$$[Q(s)]_0^t = \int_0^t z(s,Q(s))ds + \int_0^t y(s,Q(s))dW(s) + \int_0^t f(s,Q(s))dP(s,Q(s))ds, \;\; Q(t_0) = Q_0 \tag{2.3}$$

$$Q(t) - Q(0) = \int_0^t z(s,Q(s))ds + \int_0^t y(s,Q(s))dW(s) + \int_0^t f(s,Q(s))dP(s,Q(s))ds, \;\; Q(t_0) = Q_0 \tag{2.4}$$

note that $Q(0) = Q(t_0) = Q_0$, so that 3.6 becomes

$$Q(t) = Q_0 + \int_0^t z(s,Q(s))ds + \int_0^t y(s,Q(s))dW(s) + \int_0^t f(s,Q(s))dP(s,Q(s))ds, \;\; Q(t_0) = Q_0 \tag{2.5}$$

## 2.2 Euler-Maruyama Approximation For Method One

Next is to obtain Euler-Maruyama approximation for equation 3.7
Represent $t$ by $\tau j$ in 3.8 to get

$$Q(\tau j) = Q_0 + \int_0^{\tau j} z(s,Q(s))ds + \int_0^{\tau j} y(s,Q(s))dW(s) + \int_0^{\tau j} f(s,Q(s))dP(s,Q(s))ds, \;\; Q(t_0) = Q_0 \tag{2.6}$$

in same manner, Represent $t$ by $\tau j + 1$ in 3.8 to get

$$Q(\tau j+1) = Q_0 + \int_0^{\tau j+1} z(s,Q(s))ds + \int_0^{\tau j+1} y(s,Q(s))dW(s) + \int_0^{\tau j+1} f(s,Q(s))dP(s,Q(s))ds \tag{2.7}$$

subtract equation 3.9 from equation 3.10 to obtain the following

$$\begin{aligned} Q(\tau j+1) - Q(\tau j) = \int_0^{\tau j+1} z(s,Q(s))ds - \int_0^{\tau j} z(s,Q(s))ds + \int_0^{\tau j+1} y(s,Q(s))dW(s) - \int_0^{\tau j} y(s,Q(s))dW(s) \\ + \int_0^{\tau j+1} f(s,Q(s))dP(s,Q(s))ds - \int_0^{\tau j} f(s,Q(s))dP(s,Q(s))ds \end{aligned} \tag{2.8}$$

## Theorem (Additivity of Riemann integral )

Let $Q_t$:[a,b] $\rightarrow$ R be bounded on say [a,b] and let c $\subset$ (a,b). If $Q_t$ is Riemann integrable on both [a,c] and [c,b], then $Q_t$ is Riemann integrable on [a,b] and $\int_a^b Q_t = \int_a^c Q_t + \int_c^b Q_t$
by the theorem, the following can be established.

$$Q(\tau j+1) - Q(\tau j) = \int_{\tau j}^{\tau j+1} z(s,Q(s))ds + \int_{\tau j}^{\tau j+1} y(s,Q(s))dW(s) + \int_{\tau j}^{\tau j+1} f(s,Q(s))dP(s,Q(s))ds \tag{2.9}$$

which will eventually become

$$Q(\tau j+1) = Q(\tau j) + \int_{\tau j}^{\tau j+1} z(s,Q(s))ds + \int_{\tau j}^{\tau j+1} y(s,Q(s))dW(s) + \int_{\tau j}^{\tau j+1} f(s,Q(s))dP(s,Q(s))ds \tag{2.10}$$

In differential calculus, there are some integrals that may not have an exact integration due to its complexity. such integrals are approximated using the DETERMINISTIC QUADRATURE RULE; The rule states mathematically as follows

$$\int_a^b Q(s)ds \approx w_1 Q(x_1) + w_2 Q(x_2) + ... + w_n Q(x_n) = \sum_{i=1}^{n} w_i Q(x_i) \tag{2.11}$$

if $n = 1$, $x_1 = a$ and $w_1 = b - a$; we can then assume the following by approximation

$$\int_a^b Q(s)ds \approx \sum_{i=1}^{n} w_i Q(x_i) = (b-a)Q(a) \tag{2.12}$$

next, using the deterministic quadrature rule on each term of equation 2.10 as follows

$$\int_{\tau j}^{\tau j+1} z(s, Q(s))ds \approx z(j, Q(\tau j)) \int_{\tau j}^{\tau j+1} ds \approx z(j, Q(\tau j))((\tau j+1) - (\tau j)) \tag{2.13}$$

$$\int_{\tau j}^{\tau j+1} y(s, Q(s))dw(s) \approx y(\tau j, Q(\tau j)) \int_{\tau j}^{\tau j+1} dw(s) \approx y(\tau j, Q(\tau j))(w_{\tau j+1} - w_{\tau j}) \tag{2.14}$$

$$\int_{\tau j}^{\tau j+1} f(s, Q(s))d(p(s), Q(s)) \approx f(\tau j, Q(\tau j)) \int_{\tau j}^{\tau j+1} d(p(s), Q(s))$$

$$\approx f(\tau j, Q(\tau j))((p_{\tau j+1}, Q_{\tau j+1}) - (p_{\tau j}, Q_{\tau j})) \tag{2.15}$$

substituting equations 2.13, 2.14, 2.15 into equation 2.10 as follows

$$Q(\tau j+1) = Q(\tau j) + z(j, Q(\tau j))((\tau j+1) - (\tau j)) + y(\tau j, Q(\tau j))(w_{\tau j+1} - w_{\tau j})$$

$$+f(\tau j, Q(\tau j))((p_{\tau j+1}, Q_{\tau j+1}) - (p_{\tau j}, Q_{\tau j})) \tag{2.16}$$

from the equation above, we can make some simple definitions

1. $Q(\tau j + r) \equiv Q_{j+r} \quad r = 0, 1, ..$
2. $dt = \tau j + 1$ - $\tau j$
3. $dw_j = w_{\tau j+1}$ - $w_{\tau j}$
4. $d\rho Q_j = \rho Q_{\tau j+1}$ - $\rho Q_{\tau j} = (p_{\tau j+1}, Q_{\tau j+1}) - (p_{\tau j}, Q_{\tau j})$

as enumerated above, inserting each term into equation 2.10 to have

$$Q_{j+1} = Q_j + z(\tau_j, Q_j)dt + y(\tau_j, Q_j)dw_j + f(\tau_j, Q_j)d\rho Q_{\tau j} \tag{2.17}$$

equation 2.17 is the derived Euler-Maruyama method for Black-Scholes Stochastic Dependent Poisson Analysis. The procedures to evaluate this method is stated as follows
for j = 0

$$Q_1 = Q_0 + z(\tau_0, Q_0)dt + y(\tau_0, Q_0)dw_0 + f(\tau_0, Q_0)d\rho Q_{\tau_0} \tag{2.18}$$

for j=1

$$Q_2 = Q_1 + z(\tau_1, Q_1)dt + y(\tau_1, Q_1)dw_1 + f(\tau_1, Q_1)d\rho Q_{\tau_1} \tag{2.19}$$

for j=2

$$Q_3 = Q_2 + z(\tau_2, Q_2)dt + y(\tau_2, Q_2)dw_2 + f(\tau_2, Q_2)d\rho Q_{\tau_2} \tag{2.20}$$

for j=3

$$Q_4 = Q_3 + z(\tau_3, Q_3)dt + y(\tau_3, Q_3)dw_3 + f(\tau_3, Q_3)d\rho Q_{\tau_3} \tag{2.21}$$

and so on.
This procedure is not easy to evaluate manually because of the Wiener process and the Levy process which needed to be simulated; consequently, MATLAB is the application tool for simulation in this work.

# 3 ANALYSIS OF METHOD

In this section, stock market model for assest analysis was investigated for stochastic dependent poisson analysis (SDPA). Let us consider the famous Black-Scholes asset price model without a Levy process $\rho$

$$dQ(t) = \mu Q(t)dt + \sigma Q(t)dW(t) \tag{3.1}$$

where $\mu$ and $\sigma$ are arbitrary non negative values.
Suppose there is a little perturbation in this asset model as a result of some embedded Technical analysis in the chart; to produce a levy process called the jump in stock prices and other financial asset studies. Then we can have the following equation.

$$dQ(t) = \mu Q(t)dt + \sigma Q(t)dW(t) + \alpha Q(t)d\rho(t) \tag{3.2}$$

where $\mu$, $\sigma$ and $\alpha$ are arbitrary non negative values.
Equation 3.2 is a standard Black-Scholes asset price model(Technical Analysis) (SDPA) and the following assumptions were made

- $Q(t)d\rho(t)$ is the Jump process
- $Q(t)$ is present in the system because the root cause of the jump is embedded in the market chart
- $d\rho(t)$ is the changes in the levy process that produces the jump

Exact solution of equation 3.2 is given as

$$Q_0(1 + v_0^{p_t}) \exp^{(\mu_0 - \sigma_0^2/2)t + \sigma_0 \omega(t)} \tag{3.3}$$

Next is to solve equation 3.2 using the method generated in equation 2.17. The levy process and the geometrically distributed wiener process are randomly generated using MATLAB simulation with the following steps.

$$Q_{j+1} = Q_j + \mu(Q_j)dt + \sigma(Q_j)dw_j + \alpha(Q_j)d\rho Q_{\tau j} \tag{3.4}$$

the starting points required for the MATLAB simulation are as follows
for j=0

$$Q_1 = Q_0 + \mu(Q_0)dt + \sigma(Q_0)dw_0 + \alpha(Q_0)d\rho Q_{\tau_0} \tag{3.5}$$

for j=1

$$Q_2 = Q_1 + \mu(Q_1)dt + \sigma(Q_1)\delta w_1 + \alpha(Q_1)d\rho Q_{\tau_1} \tag{3.6}$$

for j=2

$$Q_3 = Q_2 + \mu(Q_2)dt + \sigma(Q_2)dw_2 + \alpha(Q_2)d\rho Q_{\tau_1} \tag{3.7}$$

for j=3

$$Q_4 = Q_3 + \mu(Q_3)dt + \sigma(Q_3)dw_3 + \alpha(Q_3)d\rho Q_{\tau_3} \tag{3.8}$$

take $\mu = 0.5$, $\sigma = 0.8$ and $\alpha = 1$
imposing the available data

$$Q_{j+1} = Q_j + 0.5(Q_j)dt + 0.8(Q_j)dw_j + (Q_j)d\rho Q_{\tau j} \tag{3.9}$$

the starting points required for the MATLAB simulation are as follows
for j=0

$$Q_1 = Q_0 + 0.5(Q_0)dt + 0.8(Q_0)dw_0 + (Q_0)d\rho Q_{\tau_0} \tag{3.10}$$

for j=1

$$Q_2 = Q_1 + 0.5(Q_1)dt + 0.8(Q_1)\delta w_1 + (Q_1)d\rho Q_{\tau_1} \tag{3.11}$$

for j=2

$$Q_3 = Q_2 + 0.5(Q_2)dt + 0.8(Q_2)dw_2 + (Q_2)d\rho Q_{\tau_1} \tag{3.12}$$

for j=3

$$Q_4 = Q_3 + 0.5(Q_3)dt + 0.8(Q_3)dw_3 + (Q_3)d\rho Q_{\tau_3} \tag{3.13}$$

equations 3.9, 3.10, 3.11, 3.12 and 3.13 are the SDPA for Asset price model to be simulated with MATLAB. taking $t = 2^{-8}$ The following graph was generated for the Asset price problem, this solution clearly shows the weiner process as well as the jump in the system and also shows entry points for potential investors in asset.

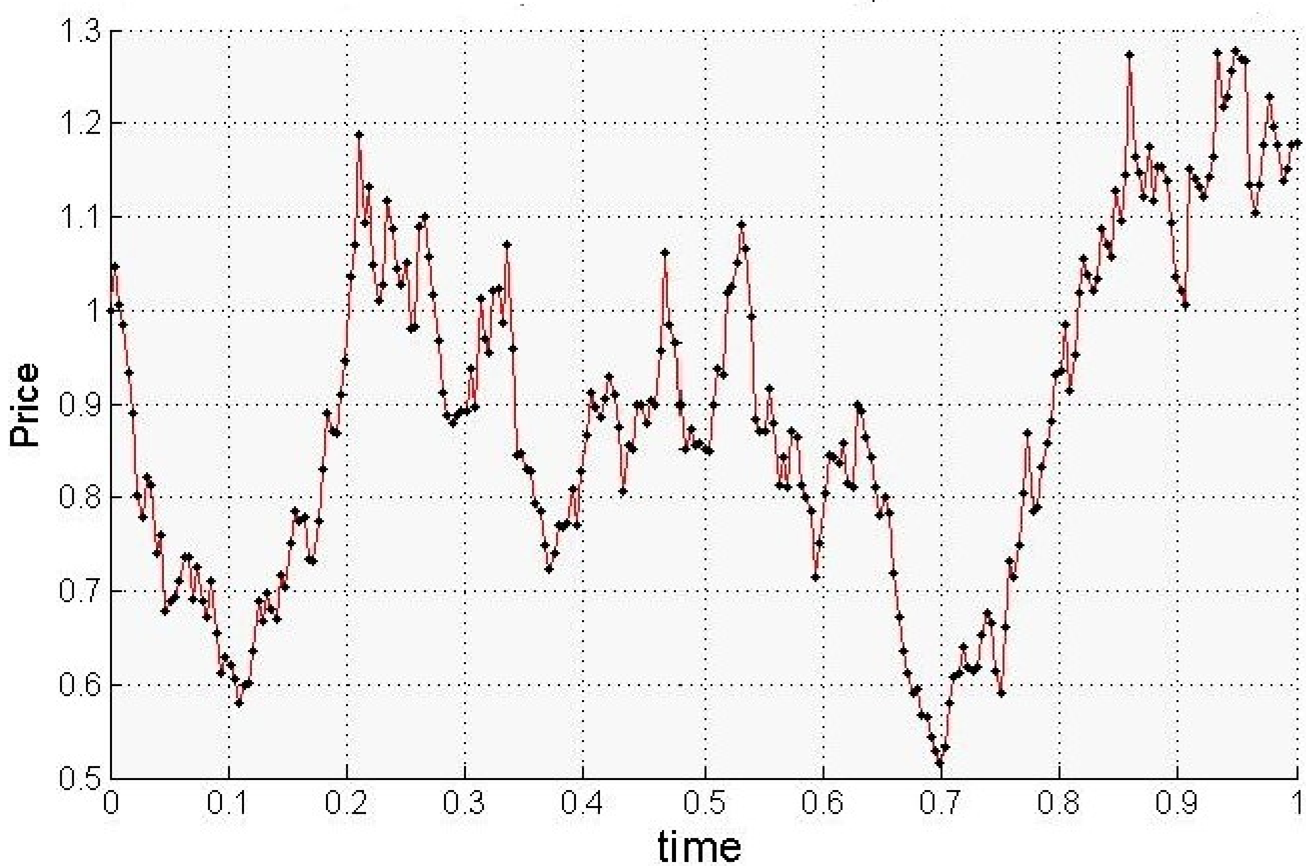


Figure 1: Graph of jump process for Black-Scholes SDPA Model.

The following table was generated for the Asset price $Q(t)$ and time interval of $t = 2^{-8}$ Error table for Black-Scholes model (Technical Analysis).

Table 1: Error between the analytical solution and developed method

| $t = 2^{-8}$ | Q Euler-Maruyama | Q Exact Solution | Error |
|---|---|---|---|
| $\frac{1}{256}$ | 1.000000000000000 | 1.000000000000000 | 0.0000000000000000 |
| $\frac{10}{256}$ | 0.812908199215707 | 0.814537619624799 | 0.0016294204090920 |
| $\frac{20}{256}$ | 0.726216432102308 | 0.732005284815325 | 0.0057888527130170 |
| $\frac{30}{256}$ | 0.598892303324720 | 0.604742397803245 | 0.0058500944785250 |
| $\frac{40}{256}$ | 0.751691073205030 | 0.757542027157013 | 0.0058509539519830 |
| $\frac{50}{256}$ | 0.869067180529991 | 0.870944529623458 | 0.0018773490934670 |
| $\frac{60}{256}$ | 1.027459259177800 | 1.040964676541320 | 0.0135054173635201 |
| $\frac{70}{256}$ | 1.057119444725450 | 1.076053640100730 | 0.0189341953752800 |
| $\frac{80}{256}$ | 0.897443354897564 | 0.903887980900941 | 0.0064446260033770 |
| $\frac{90}{256}$ | 0.846875047308351 | 0.871249902130323 | 0.0243748548219721 |
| $\frac{100}{256}$ | 0.772607019597246 | 0.794590692551104 | 0.0219836729538581 |
| $\frac{110}{256}$ | 0.909496579532232 | 0.931853869807439 | 0.0223572902752071 |
| $\frac{120}{256}$ | 0.956278014483808 | 0.972059100165068 | 0.0157810856812600 |
| $\frac{130}{256}$ | 0.849192986683570 | 0.862721604467754 | 0.0135286177841840 |
| $\frac{140}{256}$ | 0.884361156179665 | 0.902170922704207 | 0.0178097665245419 |
| $\frac{150}{256}$ | 0.812381490317926 | 0.829832126603794 | 0.0174506362858681 |
| $\frac{160}{256}$ | 0.815058405700508 | 0.832237599687052 | 0.0171791939865440 |
| $\frac{1700}{256}$ | 0.718274886575970 | 0.732615732523400 | 0.0143408459474299 |
| $\frac{180}{256}$ | 0.516418120788345 | 0.522728109258762 | 0.0063099884704170 |
| $\frac{190}{256}$ | 0.676897914315295 | 0.681231896569507 | 0.0043339822542120 |
| $\frac{200}{256}$ | 0.784808464308656 | 0.808282753571219 | 0.0234742892625630 |
| $\frac{210}{256}$ | 1.019730119601270 | 1.054328499269870 | 0.0345983796686000 |
| $\frac{220}{256}$ | 1.144575760664370 | 1.179610644713260 | 0.0350348840488899 |
| $\frac{230}{256}$ | 1.093090150530610 | 1.131793924211570 | 0.0387037736809599 |
| $\frac{240}{256}$ | 1.274364659616550 | 1.324676569666680 | 0.0503119100501299 |
| $\frac{250}{256}$ | 1.176438831160300 | 1.216056628977870 | 0.0396177978175700 |

The table above shows the comparison between the numerical solution and the exact solution.

# 4 CONVERGENCE ANALYSIS

This section covers the process of which the method was tested for validity. In numerical methods of solving Stochastic Differential Equation with distributed jumps, the test for convergence entails Mean Absolute Error (MAE) analysis.

## 4.1 Theorem: Strong Order of Convergence of Euler-Maruyama Scheme for Jump Processes

Let $Q_t^E$ and $Q_t^{N1}$ be the exact solution and Euler-Maruyama Method for the stochastic jump process respectively; let $\Delta t$ be the time step in the methods. Then there exist a constant $\Pi$ such that

$$\frac{E|Q_t^{N1} - Q_t^E}{j}| \leq \Pi.\Delta t \tag{4.1}$$

$$E|Q_t^{N1} - Q_t^E| \leq \Pi.j\Delta t \tag{4.2}$$

for $\frac{1}{j}$ is halving the time step $\forall j \geq 1$
Proof:
it is sufficient to clearly state the Exact Solution (ES) of method and the Euler-Maruyama(EM) method.

$$Q_t^E = Q_0(1 + V_0)^{P_t} \exp^{(\mu_0 - \sigma_0^2/2)t} + \sigma_0\omega(t) \ldots\ldots\ldots ES \tag{4.3}$$

obtain the expected value of the ES follows

$$E(Q_t^E) = E(Q_0(1 + V_0)^{P_t} \exp^{(\mu_0 - \sigma_0^2/2)t} + \sigma_0\omega(t)) \tag{4.4}$$

For Poisson Distribution(discrete distribution), take the Probability Density Function(PDF) as follows

$$P(t) = \exp^{-\lambda_0 t} \sum_{k=0}^{\infty} \frac{(\lambda_0 t)^k}{k!} (1 + v_0)^k \tag{4.5}$$

let the Wiener process $\omega(t)$ be normally distributed with PDF as follows

$$\frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} \exp^{-\omega^2/2t} \exp^{\sigma_0 \omega} d\omega \tag{4.6}$$

suppose equation 4.5 converges to the following at k=0; as the starting point

$$\exp^{-\lambda_0 t} \exp^{\lambda_0 t(1+v_0)} \tag{4.7}$$

and in same manner equation 4.6 converges to

$$\exp^{(\sigma_0^2 t/2)} \tag{4.8}$$

so that we can have the following

$$Q_0 \exp^{(\mu_0 - \sigma_0^2/2)t} \exp^{-\lambda_0 t} \exp^{\lambda_0 t(1+v_0)} \exp^{(\sigma_0^2 t/2)} \tag{4.9}$$

and finally to obtain the following for $1 + v_0 = \lambda_0 v_0$

$$Q_0 \exp^{(\mu_0 + \lambda_0 v_0)\Delta t} \tag{4.10}$$

the equation above represent the convergent form of the exact solution as required.
Next is to obtain the convergent form of the Euler-Maruyama method as follows.
Let the approximate value of equation 1.2 be given as follows

$$Q_k = Q_{k-1}(1 + \mu_0 \Delta t + \sigma_0 \Delta\omega_{k-1} + v_0 \rho_{k-1}) \tag{4.11}$$

$\forall$ k=1;$N_t$
NB: Expectation of the Poisson coefficient $v_0 = 0$
Expected value of the E-M method is of equation 4.11 is given below

$$E(Q_k) = Q_{k-1}(1 + \mu_0 \Delta t) \tag{4.12}$$

such that for k=1

$$E(Q_1) = Q_0(1 + \mu_0 \Delta t) \tag{4.13}$$

for k=2

$$E(Q_2) = Q_1(1 + \mu_0 \Delta t) \tag{4.14}$$

for k=3

$$E(Q_3) = Q_2(1 + \mu_0 \Delta t) \tag{4.15}$$

using backward substitution technique, equation 4.15 can be expressed as follows

$$E(Q_3) = Q_1(1 + \mu_0 \Delta t)(1 + \mu_0 \Delta t) \tag{4.16}$$

$\succ$

$$E(Q_3) = Q_0(1 + \mu_0 \Delta t)(1 + \mu_0 \Delta t)(1 + \mu_0 \Delta t) \tag{4.17}$$

implies

$$E(Q_3) = Q_0(1 + \mu_0 \Delta t)^3 \tag{4.18}$$

and for $N_t$ term

$$E(Q_{N_t}) = Q_0(1 + \mu_0 \Delta t)^{Nt} \tag{4.19}$$

insert equation 4.19 and equation 4.10 in LHS of equation 4.2

$$E|Q_0 \exp^{(\mu_0 + \lambda_0 v_0)\Delta t} - Q_0(1 + \mu_0 \Delta t)^{Nt}| \tag{4.20}$$

from the assumption that expectation of the Poisson Coefficient equals zero. i.e $v_0 = 0$

$$E|Q_0 \exp^{\mu_0 \Delta t} - Q_0(1 + \mu_0 \Delta t)^{Nt}| \tag{4.21}$$

## Definiton 1

suppose A and B are $\subset \Re$ then, the subtraction of the absolute value of A and B are commutative. i.e.
$|A - B| = |B - A|$
Consequently, equation 4.21 can take the following form

$$E|Q_0(1 + \mu_0\Delta t)^{Nt} - Q_0 \exp^{\mu_0\Delta t}| \tag{4.22}$$

will give the following

$$E|Q_0[(1 + \mu_0\Delta t)^{Nt} - Q_0 \exp^{\mu_0\Delta t}]| \tag{4.23}$$

for expectations of Q at $Q = Q_0$ the values in equation 4.23 can be given as

$$|Q_0||(1 + \mu_0\Delta t)^{Nt} - \exp^{\mu_0\Delta t}| \tag{4.24}$$

## Definiton 2

suppose a, b and c takes any value within the domain of $\Re$. i.e. $a, b, c \subset \Re$
then
$|(a + b)^c| = |\exp^{c\ln(a+b)}|$

So that equation 4.24 can become

$$|Q_0||\exp^{Nt\ln(1+\mu_0\Delta t)} - \exp^{\mu_0\Delta t}| \tag{4.25}$$

from the transformation of stochastic equation the term
$\exp^{Nt\ln(1+\mu_0\Delta t)}$ converges to $\exp^{\mu_0 tf} \exp^{-0.5\mu_0^2 tf\Delta t}$
so that equation 4.25 can become

$$|Q_0||\exp^{\mu_0 tf} \exp^{-0.5\mu_0^2 tf\Delta t} - \exp^{\mu_0 tf}| \tag{4.26}$$

$$|Q_0|\exp^{\mu_0 tf} |\exp^{-0.5\mu_0^2 tf\Delta t} - 1| \tag{4.27}$$

it is worthy to note that $\exp^{-0.5\mu_0^2 tf\Delta t}$ converges to 1 as the power approaches zero from the negative side.
Then it can be assumed that $(\exp^{-0.5\mu_0^2 tf\Delta t} - 1)$ converges to $-0.5\mu_0^2 tf\Delta t$
then the folowing can be established

$$|Q_0|\exp^{\mu_0 tf} |-0.5\mu_0^2 tf\Delta t| = |Q_0|\exp^{\mu_0 tf} 0.5\mu_0^2 tf\Delta t \tag{4.28}$$

compare 4.28 to 4.2

$$\Pi = |Q_0|\exp^{\mu_0 tf} 0.5\mu_0^2 \tag{4.29}$$

it can be explicitly expressed as

$$\frac{1}{j}E|Q_t^{N1} - Q_t^E| \le |Q_0|\exp^{\mu_0 tf} 0.5\mu_0^2 tf\Delta t \tag{4.30}$$

but

$$\frac{1}{j}E|Q_t^{N1} - Q_t^E| = \Pi\Delta jt \tag{4.31}$$

which implies

$$E|Q_t^{N1} - Q_t^E| \le |Q_0|\exp^{\mu_0 tf} 0.5\mu_0^2 tfj\Delta t \tag{4.32}$$

Equation 4.32 is the Strong Order of Convergence(SOC).
if MAE $\le$ SOC $\forall$ j; then, it converges at those points.
In this research, $j = 1, 2, 3, 4, 5...$, $Q_0 = 1$, $\Delta t = 0.0625$ (to reduce large data to smaller bits for easier analysis) and $tf =$ max (t) =1
was used for the convergent analysis

Table examining the convergence bounds and conditions for the problem
The convergence analysis of this problem-method relationship determines when the chart is said to be stable enough for entry or exit of the financial market.

Table 2: Convergence analysis using MAE and SOC

| S/N | MAE FOR PROBLEM SDPA | J | SOC |
|---|---|---|---|
| 1 | 0.0186291999 | 1 | 0.01288063493 |
| 2 | 0.0186291999 | 2 | 0.02576126985 |
| 3 | 0.0186291999 | 3 | 0.03864190479 |
| 4 | 0.0186291999 | 4 | 0.05152253972 |
| 5 | 0.0186291999 | 5 | 0.06440317465 |

(4.33)

from the table above it is seen that the convergence analysis failed at the point one because MAE$\not\leq$ SOC
Therefore, the region of stability is $j \geq 2$

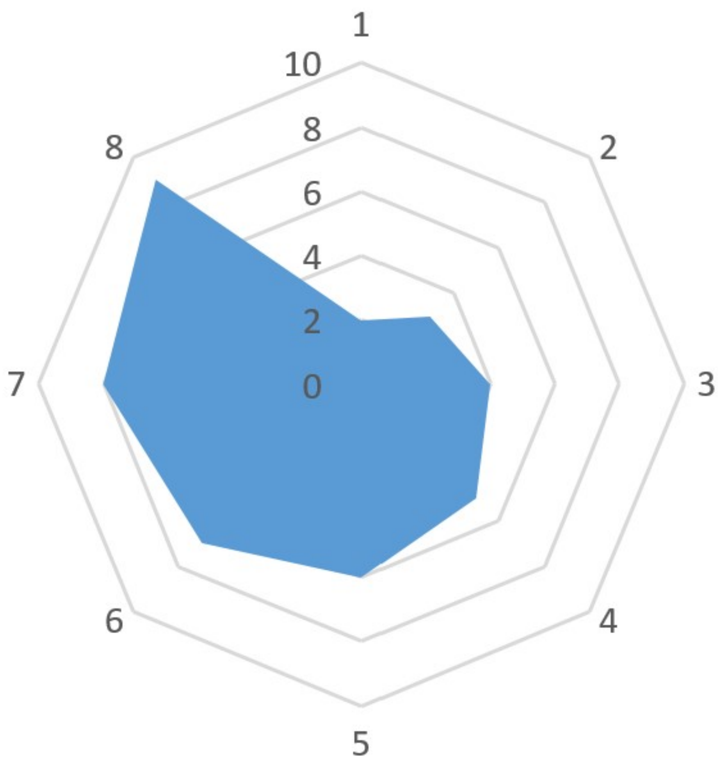


Figure 2: Graph of convergence analysis for Black-Scholes model S/N plotted against J values

# 5 RESULTS

The method developed in this work was tested with numerical problem in stock market. The approximate solution from the experiment shows the efficiency and less human effort MATLAB programs; it is observed from the table that the results obtained from the method are close to the exact solution thereby establishing a reasonable link between the numerical method and the analytical method. The region of stability also shows clearly that the market is stable enough for entry or exit at $j \geq 2$. The unstable region is marked blue.

# 6 CONTRIBUTION TO KNOWLEDGE

The graph of this method revealed the jumps in the system as well as the Wiener process, recommendations can be made based on this revelation.

The strong order of convergence and the region of convergence and the region of stability for j is a critical breakthrough in the method generated.

# 7 CONCLUSION

In this research a standard stochastic differential equation with distributed jumps was studied, where E-M methods for stochastic dependent Poisson analysis was developed for Black-Scholes Asset price model; the behaviour of the jumps were properly studied to give an acceptable result.

The property of the convergence analysis was taken into consideration and region of stability was established for the method developed.

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