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Pythagorean Relation In Triangles and Fermat's Last Theorem

Md. Razib Talukder[1]

Abstract

This paper derives *n*-th Pythagorean relation from the edges of right triangle and the result be applied to other triangles as well as with the properties of binomial equations to discover the truly marvelous proof of Fermat's Last Theorem which the famous quotation French mathematician Pierre de Fermat quoted on the margin of his favorite book Diophantus' Arithmatica but the proof he never expressed. When the value of power *n* is equal to 2 FLT turns to Pythagorean Theorem, so the proof should be there [1]. If we can make a *n*-th power relation among the edges of right triangle, then by applying this to any triangle we will find our desire first step. For, non-triangle integers [Appendix 7.1] general form of binomial equation is sufficient.

Mathematics Subject Classification: 11D41, 11L03, 11B65.
Keywords: Fermat's Last Theorem, Trigonometry, Binomial Equations.

Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos & generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.
– Pierre de Fermat ∼ 1637

[1] Mathematics Discipline, SET School, Khulna University, Khulna 9208, Bangladesh.

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1. Introduction

The mathematical presentation of Fermat's Last Theorem is that the equation $x^n + y^n = z^n$ has no integer solutions for $n > 2$, where *x, y, z* are three non-zero positive integers. Since, the variables *x, y, z* are non-zero positive integers, then they must satisfy one of the relations - summation of two non-zero positive integers is greater than, equal or less than the other positive integer. If we can proof FLT for these three relations [Corollary 5.1] of non-zero positive integers then we will find our desire proof.

2. Fermat's Actions

For $x + y > z$

If the summation of two non-zero positive integers is greater than the other positive integer, they forms a triangle. There are two types of triangles - right angled and non-right angled (acute, obtuse) triangle. First, if we assume *x, y, z* are the edges of any right triangle, where *z* is hypotenuse. From trigonometry [2],

$$\frac{x}{z} = \sin A; \frac{y}{z} = \cos A \tag{1}$$

Here, *A* is an acute angle of the right triangle. Now applying *n*-th power and addition we find $x^n + y^n = z^n(\sin^n A + \cos^n A)$. For *n* = 2, this presents,

$$x^2 + y^2 = z^2 \tag{2}$$

because when the value of *n* is equal to 2 then the value of $\sin^n A + \cos^n A$ is equal to **1** but when *n* is greater than 2 the value is not equal to **1**[Lemma]. For this, when *n* is greater than 2, z^n be greater than or less than $x^n + y^n$ but from [Lemma] we find the following,

$$z^n = x^n + y^n + k_r \tag{3}$$

Now, if *x, y, z* are edges of non-right-angled triangle then the triangle must be divided into two right angled triangle by drawing a perpendicular. Now applying the above equation on these two right angled triangles separately and addition, then by calculation we will find for *n* > 2,

$$x^n + y^n \neq z^n$$

For $x + y = z$

Applying n-th power on the both sides this relation turns $(x+y)^n = z^n$, then from the binomial equation [3] we find,

$$z^n = x^n + \sum_{i=1}^{n-1} {}^nC_i x^{n-i} y^i + y^n \tag{4}$$

Since, x, y are non-zero positive integers, then z^n must be greater than $x^n + y^n$ that means [4] for $n > 2$,

$$x^n + y^n \neq z^n \tag{5}$$

For $x + y < z$

For this relation we can write $x + y + k_s = z$, where k_s is a constant. Since, it is obvious from the calculation of equation (4) we find z^n is greater than $x^n + y^n$, that means for $n > 2$,

$$x^n + y^n \neq z^n \tag{6}$$

3. Elaboration

The above section presents the probable Fermat's thinking which also may be the verbal form of the proof and this section is the elaboration. Assume *ABC* is a non-right-angled triangle, and *x, y, z* represents [5] the lengths of the edges of the triangle *ABC* then by drawing perpendicular on suitable edge [Appendix 6.2] from opposite top point will divide the non-right-angled triangle into two right triangles.

Then that edge, assume *z* will be divided into two parts *b* and (*z* - *b)*. Let, *a* is the length of the perpendicular. Therefore, the length of the edges of one right triangle will be *x, a, b* and another will be *y, a,* (*z* - *b)* where *x, y* will be the hypotenuse of the right triangles respectively.

3.1 First case

For the right triangle having edges *x, a, b* where *A* as acute angle,

$$\frac{a}{x} = \sin A; \frac{b}{x} = \cos A \tag{7}$$

Applying *n*-th power on above both equations of (7) and addition, then after our childhood calculation we find the following equation,

$$a^n + b^n = x^n(\sin^n A + \cos^n A) \tag{8}$$

For $n = 2$, this equation [6] presents,

$$a^2 + b^2 = x^2 \tag{9}$$

because when the value of n is equal to 2 then the value of $\sin^n A + \cos^n A$ is equal to **1** but when n is greater than 2 the value is not equal to **1** (in real calculation the value is always be less than **1**) [Lemma]. For this, when n is greater than 2, x^n will be greater than $a^n + b^n$. Hence, we can write,

$$x^n = a^n + b^n + k_1 \tag{10}$$

Here, k_1 is a constant and also for another triangle we will get,

$$y^n = a^n + (z - b)^n + k_2 \tag{11}$$

Now applying binomial theorem [7] we find,

$$(z-b)^n = z^n + \sum_{i=1}^{n-1} {}^nC_i z^{n-i} (-b)^i + (-b)^n \tag{12}$$

Now, after setting this value from (12) into (11) and adding to (10), we will find the following calculated equation,

$$x^n + y^n = z^n + 2a^n + b^n + k_1 + \sum_{i=1}^{n-1} {}^nC_i z^{n-i} (-b)^i + (-b)^n + k_2 \tag{13}$$

Since, a, b, k_1, k_2 are non-zero positive integers then for $n > 2$,

$$x^n + y^n \neq z^n$$

3.2 Second case

Applying n-th power on the both sides of $z = x + y$, it turns into $z^n = (x + y)^n$ after applying the binomial equation we find,

$$z^n = x^n + \sum_{i=1}^{n-1} {}^nC_i x^i y^{n-i} + y^n \tag{14}$$

This equation presents for non-zero positive integers x, y. So, it is obvious z^n is greater than $x^n + y^n$ that means for $n > 2$,

$$x^n + y^n \neq z^n \tag{15}$$

3.3 Third case

For the relation, when $x + y$ is less than z we can write $x + y + k_s = z$, where k_s is a constant. Subsection 3.2 proves z^n is greater than $x^n + y^n$, then it is obvious for $x + y + k_s = z$ from the calculation of equations (14), (15), we will get for $n > 2$,

$$x^n + y^n \neq z^n$$

4. Conclusion

In introduction it is described that when the value of n is 2 then Fermat's Last Theorem turns to Pythagorean Theorem and in the subsection 3.1 it is proved that for only right triangle when the value of n is 2, only

$$x^2 + y^2 = z^2 \tag{16}$$

but for the value of n greater than 2, for any other triangle even right triangle and also any other relations [Corollary 5.2] among x, y, z for $n > 2$,

$$x^n + y^n \neq z^n \tag{17}$$

5. Lemma

For the value of n is equal to 2 then $\sin^n A + \cos^n A = 1$ but when n is greater than 2, then there be $\sin^n A + \cos^n A < 1$.

Proof. If u, v, w be three edges of a right triangle where w is the hypotenuse, A is the acute angle of the right triangle then we always find from the basic [8] principles of trigonometry,

$$\frac{u}{w} = \sin A; \frac{v}{w} = \cos A \tag{18}$$

Applying n-th power and addition,

$$\sin^n A + \cos^n A = \frac{u^n + v^n}{w^n} \tag{19}$$

Since, w is the hypotenuse and u, v be the other edges of a right triangle, we find from Pythagorean Theorem $w^2 = u^2 + v^2$. Hence, applying n-th power we find [9] $w^n = u^n (1 + \frac{v^2}{u^2})^{\frac{n}{2}}$ and we know for the fraction power of binomial equation [10] the value of w^n will be following,

$$u^n + v^n + \frac{\frac{n}{2}(\frac{n}{2}-1)}{2!}\frac{u^4}{v^4} + ... + \infty \tag{20}$$

Since *u*, *v* are non-zero positive integers,

$$w^n > u^n + v^n \tag{21}$$

For the value of *n* is equal to 2 then, $\sin^n A + \cos^n A = 1$ but when *n* is greater than 2, then there be,

$$\sin^n A + \cos^n A < 1 \tag{22}$$

6. Corollary

6.1 The possible relations

Now, we find that if *x, y, z* are three non-zero positive integers there may be other relations among them except described in introduction,

$$xy = z; \frac{x}{y^2} = z; x^3 y = z \tag{23}$$

and so on, but these relations must satisfy one of the three relations described in introduction. For example, 2, 3, 6 be three integers and multiplication of 2 and 3 is equal to 6 but they satisfy following,

$$2 + 3 < 6$$

If $x^n + y^n = z^n$ is true

For the non-right triangles [of Section 3] presented in [5], when *n* is equal to 2 then from the principles of Pythagorean Theorem,

$$x^2 + y^2 = [a^2 + b^2] + [a^2 + (z-b)^2] \tag{24}$$

From the above equation (24) we find, for *x, y, z* positive integers for no other relations FLT is true except right triangle,

$$x^2 + y^2 = z^2$$

7. Appendix

7.1 Non-Triangle Integers

The integers which are not equal to the length of edges of a triangle because to be edges of any triangle they must have to satisfy the condition - The summation of two edges of any triangle is greater than the other. For example, 3, 4, 5 are triangle as well as hypotenuse integers but 2, 3, 15 are non-triangle integers.

7.2 Suitable Edge

For obtuse angled triangle we cannot draw perpendicular from any top to the just opposite edge, we can only draw the perpendicular from only the obtuse angle top point [5]

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