Journal of Applied Mathematics & Bioinformatics, vol.7, no.1, 2017, 71-142
ISSN: 1792-6602 (print), 1792-6939 (online)
Scienpress Ltd, 2017

Mathematical Description of Cyber-Attacks and Proactive Defences

Nicholas J. Daras[1] and Argyrios Alexopoulos[2]

Abstract

The main purpose of this paper is to document a holistic modeling background and set up a corresponding mathematical theory in order to provide a rigorous description of cyber-attacks and cyber-security. The starting point is to determine the concepts of valuations and vulnerabilities of parts of a node constituent. Based on these two concepts, one may be led to consider the fundamental concept of node supervision and subsequently to give the definition of cyber-effects and from this the definition of cyber-interaction. As we shall see a germ of cyber-attack can be viewed as a family of cyber-interactions with coherence properties and depending strongly on subjective purposes, information and/or estimates on the valuations and the vulnerabilities of parts of the involved nodes. In general the germs of cyber-attacks can be distinguished in three types: the germs of correlated cyber-attacks, the germs of absolute cyber-attacks and the germs of partial

[1] Department of Mathematics and Engineering Sciences, Hellenic Military Academy, 166 73, Vari Attikis, Greece. E-mails: ndaras@sse.gr, njdaras@gmail.com
[2] Hellenic Army General Staff, 41500 Chaparral Dr. Temecula, CA92592, USA (permanent address) and Rue Stone 8D/6, 7010, Mons, Belgium (current address). E-mail: a.d.alexopoulos@army.gr

Article Info: *Received* : January 7, 2017. *Revised* : February 22, 2017.
Published online : April 30, 2017.

cyber-attacks. This approach provides immediate possibility of rigorous determination of the concepts of proactive cyber defense and proactive cyber protection.

Mathematics Subject Classifications: 00A71; 68M11, 28A05, 28A12, 32C15
Keywords: Mathematical modeling (models of systems); internet topics; measure theory; complex spaces; valuation of a part of node constituent; vulnerability of a part of node constituent; node supervision, cyber-effect; cyber-interaction; germ of cyber attack; cyber defense; proactive cyber protection

1 Introduction

In many modern scientific studies, quantifying assumptions, data and variables can contribute to the accurate description of the phenomena through appropriate mathematical models. So, in many disciplines, the analysts resort to a mathematical foundation of the concepts, in order to create a solid base for the theoretical formulation and solving all relevant problems. As classic examples of such an integrated mathematization, we can mention Mechanics, Physics, Biology, Earth Science, Meteorology, Medicine, Statistics and Operations Research. In recent years, it has begun an effort to mathematical modeling of the social sciences, such as Economics ([3-5 14, 15, 22 and 24], Psychology (see, for instance, [*6*, 18 and 19]), Sociology (see, indicatively, [7]), Political Science (see, for instance, [17 and 32]) and Geopolitics ([12-13]).

In this direction, there have been numerous significant contributions on the mathematical modeling of several branches of Theoretical Engineering disciplines, such as Theoretical Computer Science, Network Security, Electronics, and Artificial Intelligence etc. Especially, in the case of cyber-security, we may mention several descriptive papers ([21]) or papers containing several partial research results. All these scientific approaches emphasize mainly on some of stochastic modeling applications, leaving open the question of introducing a full mathematical theory of cyber-security. See, for instance, the papers [23, 27, 29-31]. One can also consult the books [1 and 20] and the references therein. These two books provide in-depth coverage of the mathematical prerequisites and assemble a complete presentation of how computer networks function. The interested reader may also consult the chapter [28] and the references therein and/or the report of President's Information Technology Advisory Committee ([25]) which explicitly states that "*we urgently need to expand our focus on short-term patching to also include longer-term development of new methods for designing and engineering secure systems. Addressing cyber security for the longer term requires a vigorous ongoing program of fundamental research to explore the science and develop the technologies necessary to design security into computing and networking systems and software from the ground up. Fundamental research is characterized by its potential for broad, rather than specific, application and includes farsighted, high-payoff research that provides the basis for technological progress*". Indeed, starting from this consideration, Daniel M. Dunlavy, Bruce Hendrickson, and Tamara G. Kolda gave three challenge areas that are, in their opinion, the major mathematical challenges in cyber security ([16]).

Indicative of the great interest shown for the mathematization of cyber-security is the regular organization of international conferences of major interest. Examples include the two Workshops "*Mathematics of Data Analysis in Cyber-Security*" (https://icerm.brown.edu/topical_workshops/tw14-8-mdac/) and "*Mathematics of Lattices and Cyber Security*" (https://icerm.brown.edu /topical_workshops/tw15-7-mlc/; also in https://sinews.siam.org/DetailsPage /tabid/607/ArticleID/397/ICERM-Workshop-Mathematics-of-Lattices-and-Cybers ecurity.aspx) held in Brown University, at *October 22-24, 2014 and April 21-24, 2015, respectively*. The purpose of first workshop was to bring together mathematical scientists and cyber- security practitioners with expertise in several main areas, including especially high dimensional data analysis and cryptography, to establish a road map for bringing more mathematicians into the field of cyber-security. The goal of the second workshop was on the one hand to stimulate activity between different groups interested in lattice problems, such as mathematicians, computer scientists, and experts in cyber-security, and, on the other hand, to give recent results on densest lattice packings, the geometry of lattice moduli space and its connections with automorphic forms and algebraic number theory, cryptographic applications of lattices, and the state of the art of lattice reduction in high dimensions.

However, many authors do not fail to highlight the importance of creating a ***whole*** mathematical theory of cyber-security. For instance, one can mention the abstract [26] in a workshop sponsored by the Department of Energy (DOE) Office of Advanced Scientific Computing, Applied Mathematics Research Program, where Dwayne Ramsey of Lawrence Berkeley National Laboratory found that "*significant fundamental mathematical research is needed to characterize the network in new meaningful ways and subsequently assess risk for the DOE cyber infrastructure in order to make informed decisions with regard to cyber security policy*". In the same spirit, Wendelberger, Griffin, Wilder, Yu Jiao and Kolda made a remarkable comment on the Current Landscape and Need for Fundamental Research. In this comment, it was pointed out that "*cyber-security, as currently practiced, is a mixed bag of electronic patches and reactionary physical and administrative controls aimed at fixing the crisis of the day. …. As the cyber threat continues to grow, it becomes increasingly clear that the Department of Energy (DOE) must embark on a scientific process of inquiry, investigation, and sound decision-making. Rather than waiting to discover a cyber attack (perhaps days, weeks, or months after it has happened), we need to implement a science-based approach to cyber-security with a rigorous technical foundation. Here, we propose a mathematical research that will pave the way for the interdisciplinary advances needed to thwart the growing cyber threat and transform the DOE approach for protecting electronic resources*" ([33]). Finally, Juan Meza, Scott

Campbell and David Bailey noted that "*the role of mathematics in a complex system such as the Internet has yet to be deeply explored. In this paper, we summarize some of the important and pressing problems in cyber security from the viewpoint of open science environments. We start by posing the question \What fundamental problems exist within cyber security research that can be helped by advanced mathematics and statistics?" Our first and most important assumption is that access to real-world data is necessary to understand large and complex systems like the Internet. Our second assumption is that many proposed cyber security solutions could critically damage both the openness and the productivity of scientific research. After examining a range of cyber security problems, we come to the conclusion that the field of cyber security poses a rich set of new and exciting research opportunities for the mathematical and statistical sciences*" ([23]).

Although these presentations are innovative and promising, it seems that they lack a holistic view of the cyber environment. Moreover, there is no predictability of cyber attacks, nor any opportunity to have given a strict definition of defensive protection so that we can look for an optimal design and organization of cyber defense. As a consequence thereof, one can not build a solid foundation for a complete theory containing assumptions, definitions, theorems and conclusions. But, this prevents the researcher to understand deeper behaviours, and requires limiting ourselves solely to practical techniques.

The aim of the present paper is to document a holistic modeling background and set up a corresponding mathematical theory in order to provide a rigorous description of cyber-attacks and cyber-security. The text that follows comes as a follow-up of the forthcoming article [9] in which it has been given a mathematical definition of cyberspace. In Section 2, we will first introduce general assumptions and basic notation that we will use later. Bearing this in mind, the starting point will be to determine, in Sections 3 and 4, the concepts of *valuations* and *vulnerabilities* of the parts of a node constituent. Based on these two concepts, we will give, in Section 5, the fundamental concept of a *node* *supervision* and subsequently, in Sections 6 and 7, the definition of a *cyber-effect* and, from this, the definition of a *cyber-interaction*. As we shall see, in Section 9, a *germ of cyber-attack* can be viewed as a family of cyber-interactions having coherence properties (described in Section 8) and depending strongly on subjective aims, information and/or estimates on the valuations and the vulnerabilities of parts of the involved nodes. The subjectivity in evaluation and vulnerabilities of a cyber-node is studied in deep length in [11]. In general the germs of cyber-attacks can be distinguished in three types: the *germs of correlated cyber-attacks*, the *germs of absolute cyber-attacks* and the *germs of partial cyber-attacks*. The above described approach provides the immediate possibility of a rigorous determination of the concepts of *proactive cyber defense* and *proactive cyber protection* in Section 10. A systematic effort to introduce and give a practical definition, description and technical organization of the concept of preventive cyber-defense has become by [8] and the references therein. Here, we discuss the theoretical foundation of this concept. A mathematical study of the proactive defense against different special types of germs of cyber attacks is given in [2].

2 General Assumptions and Basic Notations

Having already mentioned in [9] an adequate supportive theoretical background for cyberspace modeling, we can proceed to the consideration of the concepts of cyber-attack and cyber-defense. In order to rigorously define these two concepts, we will adopt the following approach. At any moment t, a node $V = V_{(x_1,x_2,x_3,t,)}$ in location (x_1, x_2, x_3) of the cyber-domain $\left(|ob(W_e)|, d_{W_e}\right)$ is composed of cyber constituents (or cyber characteristics) consisting in devices $dev_j^{(V)}$ (:sensors, regulators of information flow, etc) and resource elements $res_k^{(V)}$ (:services, data, messages etc), the number of which depend potentially from the three geographical coordinates x_1, x_2, x_3 and the time t. Here, the order of any used quote of devices $dev_1^{(V)}$, $dev_2^{(V)}$,… and the order of any used quote of resource elements $res_1^{(V)}$, $res_2^{(V)}$,… are assumed to be given, pre-assigned and well defined. For instance, one can order the devices $dev_1^{(V)}$, $dev_2^{(V)}$,… as well as the resource elements $res_1^{(V)}$, $res_2^{(V)}$,… alphabetically.

Assumption 2.1. We will assume uninterruptedly that:

- the potential number of all possible devices of V is equal to $\mathcal{M}_V \gg 0$, while
- the number of V's available devices is only $m_V = m_V(t)$, with $m_V < \mathcal{M}_V$.

Similarly, we will assume that

- the potential quantity (or number) of all possible resource elements of V is equal to $\mathcal{L}_V \gg 0$, while
- the quantity (or number) of V's available resource elements is only $\ell_V = \ell_V(t)$, in the sense that $\ell_V < \mathcal{L}_V$.

3 Valuations of Parts of a Node Constituent

Let us now turn to the definition of valuation measures, as well as the definition of the vulnerability measures, of an available constituent $\mathcal{A}^{(V)}$ in a cyber node V:

$$\mathcal{A} = \begin{cases} dev, if\ the\ constituent\ is\ a\ device\ , \\ res, if\ the\ constituent\ is\ a\ resource\ element. \end{cases}$$

Obviously, $\mathcal{A}^{(V)}$ may be viewed as a nonempty collection of a number of elements.

Lemma 3.1. *One can make as much finite σ −algebras as partitions on $\mathcal{A}^{(V)}$.*

Recall that a partition of a set Σ is defined as a set of nonempty, pairwise disjoint subsets of Σ whose union is Σ.

Proof. Let $\mathcal{G}$ be the collection of all the algebras over $\mathcal{A}^{(V)}$. Let also Π be the set of all the partitions of $\mathcal{A}^{(V)}$. There is a bijective correspondence between $\mathcal{G}$ and Π. Indeed, for a partition $\mathcal{P} \in \Pi$, consider the algebra $\mathfrak{U}_{\mathcal{P}}$ generated by $\{A_1, \dots, A_k\}$, the elements of $\mathcal{P}$. Then $\mathfrak{U}_{\mathcal{P}}$ consists of the set $\bigcup_{j \in J} A_j$, where $J \subset \{1, \dots, k\}$. To see that this correspondence is bijective, given an algebra $\mathfrak{U}$, one can define, for all $x \in \mathcal{A}^{(V)}$, the set $A_x := \bigcap_{A \in \mathfrak{U}, x \in A} A$ (it is a finite intersection), and that will give a unique partition. Indeed, define the equivalence relation $x \sim y$ if and only if $A_x = A_y$. It gives a partition, and it is the unique one. If $\mathcal{P} = \{S_1, \dots, S_m\}$ works, then $A_x = S_{i(x)}$ for some $i(x)$, and you can check that this partition consists of the equivalence classes of $\sim$. So the problem is to enumerate the number of partitions of the set $\mathcal{A}^{(V)}$.

Definition 3.2 *Let $W,\ V \in ob\big(cy(t)\big)$ be two cyber nodes and let $\mathcal{A}^{(V)}$ be an available constituent in V. For every partition $\mathcal{P}$ of $\mathcal{A}^{(V)}$, let us consider the corresponding $\sigma-$algebra $\mathfrak{U}_{\mathcal{P}}$ of subsets of $\mathcal{A}^{(V)}$ as well as a monotonic measure μ defined on $\mathfrak{U}_{\mathcal{P}}$. Let also $Cr_1, Cr_2, \dots, Cr_{\mathfrak{N}}$ be $\mathfrak{N} = \mathfrak{N}\big(\mathcal{A}^{(V)}, \mathcal{P}\big)$ objective quantifiable Criteria for the assessment of the points of $\mathcal{A}^{(V)}$. Denoting by $Cr_j(p) = Cr_j[x_1, x_2, x_3, t\,](p) \in \mathbb{R}$ the value of Cr_j on $p \in \mathcal{A}^{(V)}$ at a point $(x_1, x_2, x_3, t\,) \in \mathbb{R}^3 \times [0,1]$, representing location of V at time t, suppose*

1) *the functions $Cr_j(p)$ are measurable and*
2) *an importance of valuation weight $w_j(p)$ is attributed by the (user(s) of) node W to the Criterion Cr_j on $p \in \mathcal{A}^{(V)}$ at $(x_1, x_2, x_3, t\,) \in \mathbb{R}^4$ (; of course, if the users of W are indifferent or not at all informed on the situation of part p in V relative to the Criterion Cr_j, then the relevant valuation weight $w_j(p)$ will be 0).*

If $E \in \mathfrak{U}_{\mathcal{P}}$ is a part of $\mathcal{A}^{(V)}$and $\mathfrak{n} \leq \mathfrak{N}$, then a relative valuation of E from the *viewpoint of the (user(s) of) node* W *at the spatiotemporal point* $(x_1, x_2, x_3, t) \in \mathbb{R}^4$ *is any vector*

$$S_W(E) = S_W[x_1, x_2, x_3, t](E) := \left(s_{W,1}(E), s_{W,2}(E), \dots, s_{W,\mathfrak{n}}(E)\right) \in \mathbb{R}^{\mathfrak{n}}$$

where

$$s_{W,j}(E) = s_{W,j}^{(\mathcal{A}^{(V)},\mathcal{P})}[x_1, x_2, x_3, t](E) := \int_E Cr_j(p) w_j(p)\, d\mu(p).$$

Each one indefinite integral

$$s_{W,j} = s_{W,j}^{(\mathcal{A}^{(V)},\mathcal{P})}[x_1, x_2, x_3, t] = \int Cr_j(p) w_j(p)\, d\mu(p)$$

is called a producing valuation component of part E *from the viewpoint of the (user(s) of) node* W *into the constituent* $\mathcal{A}^{(V)}$ *at* (x_1, x_2, x_3, t) *with respect to the quantifiable Criterion that represents, while the component values* $s_{W,j}(E)$ *are called component valuations of* E *from the viewpoint of the (user(s) of) node* W *into the constituent* $\mathcal{A}^{(V)}$ *at the spatiotemporal point* (x_1, x_2, x_3, t). *The number* $\mathfrak{n}$ *is the dimension of the valuation.*

For simplicity and without loss of generality, in what follows, we will always assume that the dimension of the valuation is fixed over the set of all cyber nodes and equal to $\mathfrak{n} = \mathfrak{N}$.

Remark 3.3 It is possible that all of the components $s_{W,k}(E)$ belong to a fixed discrete or finite set in $\mathbb{R}$. In such a case, the valuation is said to be discrete or finite, respectively. It is also possible to consider the extending of component valuations $s_{W,k}(E)$ onto the Alexandroff one-point compactification $\mathbb{RP}^1$ of $\mathbb{R}$, so that

$s_{W,k}(E) > 0$ means "positive valuation in activated part_E"
$s_{W,k}(E) = 0$ means "valuation in disabled /non-existent/non-available part E"
$s_{W,k}(E) < 0$ means "negative valuation in activated (ενεργοποιημένο) part E"
$s_{W,k}(E) = \infty$ means "part E takes its extreme (maximal or minimal) valuation".

If no reference is made to node W and there is no risk of confusion, we can omit the notation of the node W into the indices used.

Let us give an example of the particular case where the component valuations belong to a finite set.

Example 3.4 Given an available constituent $\mathcal{A}^{(V)}$ (: device $dev^{(V)}$ and/or resource element $res^{(V)}$) in a node V, let us consider a partition $\mathcal{P}$ of $\mathcal{A}^{(V)}$. Let us consider the corresponding $\sigma-$algebra $\mathfrak{U}_{\mathcal{P}}$ of subsets of $\mathcal{A}^{(V)}$. A valuation of a part $E \in \mathfrak{U}_{\mathcal{P}}$ can be parameterized and measured using segmentation in subparts and issues concerning stochastic as well as administrative processes. Specifically, a valuation of E can be broken down to $\mathfrak{n} = \mathfrak{N} = 22$ component (continuous or discrete) valuations on $\mathfrak{U}_{\mathcal{P}}$: $s_j = s_j^{(\mathcal{A}^{(V)})}$ ($j = 1,2,\dots,22$ and $\mathcal{A} = dev, res$). In fact, taking equal valuation weights $w_j = 1$ and a normalized measure $\mu(E) = 1$, we may consider the following component valuations, many of which can be the parameters for calculating the reliability of the constituent $\mathcal{A}^{(V)}$.

1) Cr_1: "Aging of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation of part E into the constituent $\mathcal{A}^{(V)}$ is $s_1(E)$, so, if, for instance, $s_1(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for recent, $(1/\kappa)$ stands for not recent and 1 for old.
2) Cr_2: "Level of patching of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_2(E)$, so, if, for instance, $s_2(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for unpatched, $(1/\kappa)$ for not adequately patched and ν for fully patched.
3) "Amount of compromises of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_3(E)$, so,

if, for instance, $s_3(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for low amount, $(1/\kappa)$ for moderate amount and ν for large amount.

4) "Criticality of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_4(E)$, so, if, for instance, $s_4(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for trivial, $(1/\kappa)$ for not so critical and ν for very critical.

5) Cr_5: "Indication of over-load of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_5(E)$, so, if, for instance, $s_5(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for a limited low, $(1/2)$ for a moderate load and ν for a big load.

6) Cr_6: "Is part E in the constituent $\mathcal{A}^{(V)}$ of node V of k nown manufacturer/Brand that can support it uninterruptedly?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_6(E)$, so, if, for instance, $s_6(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for a little known manufacturer/Brand, $(1/2)$ for a known manufacturer/Brand and ν for a big manufacturer/Brand.

7) Cr_7: "Has part E in the constituent $\mathcal{A}^{(V)}$ of node V been adequately tested?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_7(E)$, so, if, for instance, $s_7(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$ and $\nu, \kappa > 1$, then ε stands for a bit tested, $(1/\kappa)$ for quite tested and ν for too well tested.

8) Cr_8: "Is part E in the constituent $\mathcal{A}^{(V)}$ of node V in the first line of defense? Or is it protected by another defense component?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_8(E)$, so, if, for instance, $s_7(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε

stands for a little protected, $(1/\kappa)$ stands for moderately protected and ν for very well protected.

9) Cr_9: "Degree of complexity of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_8(E)$, so, if, for instance, $s_9(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for non-complex, $(1/2)$ for neutral and ν for complex.

10) Cr_{10}: "Is the part E in the constituent $\mathcal{A}^{(V)}$ of node V adequately monitored?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{10}(E)$, so, if, for instance, $s_{10}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for a little monitorated, $(1/\kappa)$ for moderately monitorated and ν for very well monitorated.

11) Cr_{11}: "What is the price of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{11}(E)$, so, if, for instance, $s_{11}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for low cost, $(1/\kappa)$ for moderate cost and ν for high cost.

12) Cr_{12}: "Failure rate of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{12}(E)$, so, if, for instance, $s_{12}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for low failure rate, $(1/\kappa)$ for moderate failure rate and ν for high failure rate.

13) Cr_{13}: "Proximity of part E in the constituent $\mathcal{A}^{(V)}$ of node V to its health tolerance". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{13}(E)$, so, if, for instance, $s_{13}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for too close, $(1/\kappa)$ for not so close and ν for far from health tolerance.

14) Cr_{14}: "MTBF (Mean Time Between Failure) of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{14}(E)$, so, if, for instance, $s_{14}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for low MTBF, $(1/\kappa)$ for moderate MTBF and ν for high MTBF.
15) Cr_{15}: "Is the average user of part E in the constituent $\mathcal{A}^{(V)}$ of node V trained?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{15}(E)$, so, if, for instance, $s_{15}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for untrained, $(1/\kappa)$ for not so trained and ν for fully trained.
16) Cr_{16}: "Is any Information Awareness training in place into the part E of constituent $\mathcal{A}^{(V)}$ in node V?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{16}(E)$, so, if, for instance, $s_{16}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for low Information Awareness training, $(1/\kappa)$ for moderate Information Awareness training and ν for high Information Awareness training.
17) Cr_{17}: "Are all security functions automated or there is human-in-the-loop process?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{17}(E)$, so, if, for instance, $s_{17}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for few automated safety functions, $(1/\kappa)$ for several automated safety functions and ν for many automated safety functions.
18) Cr_{18}: "Is average user of part E in the constituent $\mathcal{A}^{(V)}$ of node V experienced?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{18}(E)$, so, if, for instance, $s_{18}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for little experience of the average user, $(1/\kappa)$ for moderate experience of the average user and ν for great experience of the average user.

19) Cr_{19}: "Strictness of security Law and regulations in the wide area of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{19}(E)$, so, if, for instance, $s_{19}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for looseness of regulations and security law in the wide area of node, $(1/\kappa)$ for typical regulations and security law in the wide area of node and ν for strictness of regulations and security law in the wide area of node.

20) Cr_{20}: "Is a detailed security policy in place?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{20}(E)$, so, if, for instance, $s_{20}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for a little detailed security police, $(1/\kappa)$ stands for a sufficiently detailed security police and ν for a very detailed security police.

21) Cr_{21}: "Are there any back up processes?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{21}(E)$, so, if, for instance, $s_{21}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for the existence of not so successful back up procedures, $(1/\kappa)$ stands for the existence of quite successful back up procedures and ν for the existence of successful back up procedures.

22) Cr_{22}: "How much risk can the organization accept?" The corresponding component valuation for the part E of $\mathcal{A}^{(V)}$ is $s_{22}(E)$, so, if, for instance, $s_{22}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then ε stands for no risk, $(1/\kappa)$ stands some risk and ν for full risk acceptance.

Both effectiveness states

$$S_W[x_1, x_2, x_3, t\,]\left(fr\left(dev_1^{(V)}\right)\right), \dots, S_W[x_1, x_2, x_3, t\,]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)$$

and applicability situations

$$S_W[x_1, x_2, x_3, t\,]\left(fr\left(res_1^{(V)}\right)\right), \dots, S_W[x_1, x_2, x_3, t\,]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)$$

are called cyber node valuations of V from the viewpoint of the (user(s) of) node

W at the spatiotemporal point (x_1, x_2, x_3, t). They are also denoted separately by $fr\left(\beta_\kappa^{(W \rightsquigarrow V)}\right) = fr\left(\beta_\kappa^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t]$, $\kappa = 1,2, \dots, \mathcal{M}_V + \mathcal{L}_V$, or by the vector valuation representation

$$fr\left(\beta^{(W \rightsquigarrow V)}\right) = fr\left(\beta^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t] := \left(fr\left(\beta_1^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t], \dots, fr\left(\beta_{\mathcal{M}_V + \mathcal{L}_V}^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t]\right)^T.$$

If there is no risk of confusion, we will prefer write simply $\beta_\kappa^{(W \rightsquigarrow V)} = \beta_\kappa^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t]$, $\kappa = 1,2, \dots, \mathcal{M}_V + \mathcal{L}_V$, or use by the joint vector valuation representation

$$\beta^{(W \rightsquigarrow V)} = \beta^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t] := \left(\beta_1^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t], \dots, \beta_{\mathcal{M}_V + \mathcal{L}_V}^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t]\right)^T.$$

In the total case, the effectiveness states $S_W[x_1, x_2, x_3, t]\left(dev_1^{(V)}\right), \dots, S_W[x_1, x_2, x_3, t]\left(dev_{\mathcal{M}_V}^{(V)}\right)$ and applicability situations $S_W[x_1, x_2, x_3, t]\left(res_1^{(V)}\right), \dots, S_W[x_1, x_2, x_3, t]\left(res_{\mathcal{L}_V}^{(V)}\right)$ are called cyber node valuations of V from the viewpoint of the (user(s) of) node W at the spatiotemporal point (x_1, x_2, x_3, t). As above, they are again denoted separately by

$$\beta_\kappa^{(W \rightsquigarrow V)} = \beta_\kappa^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t], \ \kappa = 1,2, \dots, \mathcal{M}_V + \mathcal{L}_V,$$

or jointly by the vector valuation representation

$$\beta^{(W \rightsquigarrow V)} = \beta^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t] :=. \left(\beta_1^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t], \dots, \beta_{\mathcal{M}_V + \mathcal{L}_V}^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t]\right)^T.$$

By analogy, both available effectiveness states

$$S_W[x_1, x_2, x_3, t]\left(fr\left(dev_1^{(V)}\right)\right), \dots, S_W[x_1, x_2, x_3, t]\left(fr\left(dev_{m_V}^{(V)}\right)\right)$$

and available applicability situations

$$S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_1^{(V)}\right)\right)\ ,\ldots,\ S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_{\ell_V}^{(V)}\right)\right)$$

are called available cyber node fractional valuations of V from the viewpoint of the (user(s) of) node W at the spatiotemporal point $(x_1,x_2,x_3,t\,)$. They are denoted separately by

$$fr\left(b_\kappa^{(W\rightsquigarrow V)}\right)=fr\left(b_\kappa^{(W\rightsquigarrow V)}\right)[x_1,x_2,x_3,t\,],\ \ \kappa=1,2,\ldots,m_V+\ell_V,$$

or jointly by the available vector valuation representation

$$fr\left(b^{(W\rightsquigarrow V)}\right)=fr\left(b^{(W\rightsquigarrow V)}\right)[x_1,x_2,x_3,t\,]:=.$$
$$\left(fr\left(b_1^{(W\rightsquigarrow V)}\right)[x_1,x_2,x_3,t\,],\ldots,fr\left(b_{m_V+\ell_V}^{(W\rightsquigarrow V)}\right)[x_1,x_2,x_3,t\,]\right)^T.$$

As before, if there is no risk of confusion, we may adopt the simpler notation

$$b_\kappa^{(W\rightsquigarrow V)}=b_\kappa^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,],\ \ \kappa=1,2,\ldots,m_V+\ell_V,$$

or use the joint vector valuation representation

$$b^{(W\rightsquigarrow V)}=b^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,]:=$$
$$\left(b_1^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,],\ldots,b_{\mathcal{M}_V+\mathcal{L}_V}^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,]\right)^T.$$

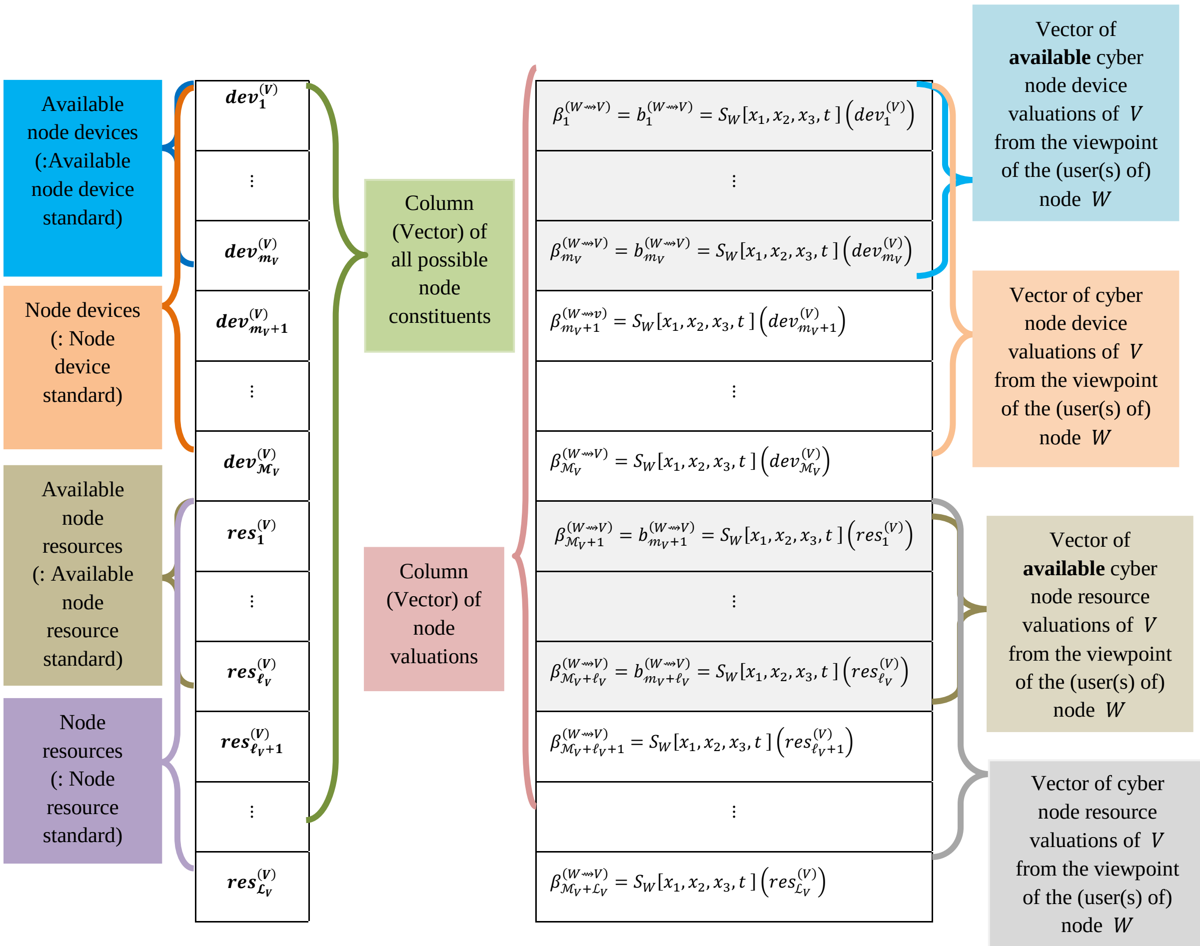
In particular, in total case, the effectiveness states $S_W[x_1,x_2,x_3,t\,]\left(dev_1^{(V)}\right),\ldots,S_W[x_1,x_2,x_3,t\,]\left(dev_{m_V}^{(V)}\right)$ and applicability situations $S_W[x_1,x_2,x_3,t\,]\left(res_1^{(V)}\right),\ldots,S_W[x_1,x_2,x_3,t\,]\left(res_{\ell_V}^{(V)}\right)$ are called available cyber node valuations of V from the viewpoint of the (user(s) of) node W at the spatiotemporal point $(x_1,x_2,x_3,t\,)$. They are also denoted separately by

$$b_\kappa^{(W\rightsquigarrow V)}=b_\kappa^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,],\ \ \kappa=1,2,\ldots,m_V+\ell_V,$$

or jointly by the available vector valuation representation

$$b^{(W\rightsquigarrow V)}=b^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,]:$$
$$\left(b_1^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,],\ldots,b_{m_V+\ell_V}^{(W\rightsquigarrow V)}[x_1,x_2,x_3,t\,]\right)^T.$$

In order to be more understandable, let us give a schematic example only for the indicative case of some of the above definitions in the **total** case.



4 Vulnerabilities of Parts of a Node Constituent

There is a special category of valuations of particular interest. This category refers to those valuations that are determined in regards to the low degree of "security" of the constituents of the node. The low degree of security is described completely by the concept of vulnerability. Vulnerability, as used in cyber context, is the property of a constituent (device or resource element) in a given state that may be exploited in the relative future. This exploitation at time t may actually lead to a constituent (device or resource element) of any node to be compromised and the valuation of this component to be degraded proportionally.

Definition 4.1. *Let* $W, V \in ob(cy(t))$ *be two cyber nodes and let* $\mathcal{A}^{(V)}$ *be an available constituent in* V*. For every partition* $\mathcal{P}$ *of* $\mathcal{A}^{(V)}$*, let us consider the corresponding* $\sigma-$*algebra* $\mathfrak{U}_{\mathcal{P}}$ *of subsets of* $\mathcal{A}^{(V)}$*as well as a monotonic measure* λ *defined on* $\mathfrak{U}_{\mathcal{P}}$ *. Let also* $SeCr_1, SeCr_2, \dots, SeCr_{\mathfrak{M}}$ *be* $\mathfrak{M} = \mathfrak{M}(\mathcal{A}^{(V)}, \mathcal{P})$ *objective quantifiable Security Criteria for the security assessment of the points of* $\mathcal{A}^{(V)}$*. Denoting by* $SeCr_j(p) = SeCr_j[x_1, x_2, x_3, t\,](p) \in \mathbb{R}$ *the value of* $SeCr_j$ *on* $p \in \mathcal{A}^{(V)}$ *at a spatiotemporal point* $(x_1, x_2, x_3, t\,) \in \mathbb{R}^3 \times [0,1]$*, representing location of node* V *at time* t*, suppose*

1) *the functions* $SeCr_j(p)$ *are measurable and*
2) *an importance of vulnerability weight* $w_j(p)$ *is attributed by the (user(s) of) node* W *to the Security Criterion* $SeCr_j$ *on* $p \in \mathcal{A}^{(V)}$ *at* $(x_1, x_2, x_3, t\,) \in \mathbb{R}^4$ *(; of course, if the users of* W *are indifferent or not at all informed on the situation of part* p *in* V *relative to the Criterion* $SeCr_j$*, then* $w_j(p) = 0$*).*

If $E \in \mathfrak{U}_{\mathcal{P}}$ *is a part of* $\mathcal{A}^{(V)}$*and* $\mathfrak{m} \leq \mathfrak{M}$*, then a relative vulnerability of* E *from the viewpoint of the (user(s) of) node* W *at* $(x_1, x_2, x_3, t\,) \in \mathbb{R}^4$ *is any vector*

$$U_W(E) = U_W[x_1, x_2, x_3, t\,](E) := \left(u_{W,1}(E), u_{W,2}(E), \dots, u_{W,\mathfrak{m}}(E)\right) \in \mathbb{R}^m$$

where

$$u_{W,j}(E) = u_{W,j}^{(\mathcal{A}^{(V)},\mathcal{P})}[x_1, x_2, x_3, t\,](E) := \int_E SeCr_j(p) w_j(p)\, d\lambda(p).$$

Each one indefinite integral

$$u_{W,j} = u_{W,j}^{(\mathcal{A}^{(V)},\mathcal{P})}[x_1, x_2, x_3, t\,] = \int SeCr_j(p) w_j(p)\, d\lambda(p)$$

is called a producing vulnerability component of part E *from the viewpoint of the (user(s) of) node* W *into the constituent* $\mathcal{A}^{(V)}$ *at* $(x_1, x_2, x_3, t\,)$ *with respect to the quantifiable Security Criterion that represents, while the component values*

$u_{W,j}(E)$ *are called component vulnerabilities of* E *from the viewpoint of the (user(s) of) node* W *into the constituent* $\mathcal{A}^{(V)}$ *at* (x_1, x_2, x_3, t). *The number* $\mathfrak{m}$ *is the dimension of the vulnerability.*

For simplicity and without loss of generality, in what follows, we will always assume that the dimension of the vulnerability is fixed over the set of all cyber nodes and equal to $\mathfrak{m} = \mathfrak{M}$.

Remark 4.2. It is possible that the components $u_{W,j}(E)$ belong to a fixed discrete or finite set in $\mathbb{R}$. In such a case, the vulnerability is said to be discrete or finite, respectively. It is also possible to consider the extending of component vulnerabilities $u_{W,j}(E)$ onto the Alexandroff one-point compactification $\mathbb{RP}^1$ of $\mathbb{R}$, so that

$u_{W,j}(E) > 0$ means "vulnerability in activated part E"
$u_{W,j}(E) = 0$ means "invulnerability in disabled/non-existent/non-available part E"
$u_{W,j}(E) < 0$ means "invulnerability in activated part E"
$u_{W,j}(E) = \infty$ means "extreme vulnerability situation: completely immune part E".

If no reference is made to node W and there is no risk of confusion, we can omit the notation of the node W into the indices used. Let us give an example.

Example 4.3. Following the notation in the Example 3.4, and taking equal vulnerability weights $w_j = 1$ and normalized measure $\lambda(E) = 1$, vulnerability can be broken down to the following 5 parameters.

1) $SeCr_1$: "Level of patching of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component vulnerability of part E into the constituent $\mathcal{A}^{(V)}$ is $u_1(E)$ that is the inverse of the valuation $s_2(E)$ in Example 3.4. In the discrete case, if $s_2(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then $u_1(E) = 1/\varepsilon$ stands great vulnerability for unpatched

part E, $u_1(E) = \kappa$ moderate vulnerability for not adequately patched part E and $u_1(E) = 1/\nu$ small vulnerability for fully patched part E.

2) "Amount of compromises of part E in the constituent $\mathcal{A}^{(V)}$ of node V". The corresponding component vulnerability of part E into the constituent $\mathcal{A}^{(V)}$ is $u_2(E)$ that is the inverse of the valuation $s_3(E)$ in Example 3.4. Note that in the discrete case, if $s_2(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then $u_2(E) = 1/\varepsilon$ stands great vulnerability for low amount of compromises of part E, $u_2(E) = \kappa$ moderate vulnerability for moderate amount of compromises of part E and $u_2(E) = 1/\nu$ small vulnerability for large amount of compromises of part E.

3) $SeCr_3$: "Is part E in the constituent $\mathcal{A}^{(V)}$ of node V in the first line of defense? Or is it protected by another defense component? " The corresponding component vulnerability of part E into the constituent $\mathcal{A}^{(V)}$ is $u_3(E)$ that is the inverse of the valuation $s_8(E)$ in Example I.1. In the discrete case, if $s_8(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then $u_3(E) = 1/\varepsilon$ stands great vulnerability for a little protected part E, $u_3(E) = \kappa$ moderate vulnerability for a moderately protected part E, while $u_3(E) = 1/\nu$ small vulnerability for a very well protected part E.

4) $SeCr_4$: "Are all security functions automated or there is human-in-the-loop process?" The corresponding component vulnerability of part E into the constituent $\mathcal{A}^{(V)}$ is $u_4(E)$ that is the inverse of the valuation $s_{17}(E)$ in Example 3.4. In the discrete case, if $s_{17}(E) \in \{\varepsilon, (1/\kappa), \nu\}$, with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then $u_4(E) = 1/\varepsilon$ stands great vulnerability for few automated safety functions, $u_4(E) = \kappa$ moderate vulnerability for several automated safety functions and $u_4(E) = 1/\nu$ small vulnerability for many automated safety functions.

5) $SeCr_5$:Is any security police (cryptographic process) in place? " The corresponding component vulnerability of part E into the constituent $\mathcal{A}^{(V)}$ is $u_5(E)$ that is the inverse of the valuation $s_{20}(E)$ in Example 3.4. In the discrete case, if $s_{20}(E) \in \{\varepsilon, (1/\kappa), \nu\}$ with $0 < \varepsilon \ll \infty$, $1 < \kappa \ll \infty$ and $\nu \gg 1$, then $u_5(E) = 1/\varepsilon$ stands great vulnerability for a little detailed security police, $u_5(E) = \kappa$ moderate vulnerability for a sufficiently detailed security police and $u_5(E) = 1/\nu$ small vulnerability for a very detailed security police.

Remark 4.4. A basic and reasonable question arises immediately and may be constitute the central subject of discussion in subsequent additional scientific studies. The question relates to the objectivity and/or subjectivity in the choice of the numerical characteristics (:objective quantifiable Criteria) of a device and a resource element (see Definitions 3.2 and 4.1): given that it is very doubtful whether the considered set of numerical characteristics could be considered as exhaustive, one wonders if the above approach is ultimately reliable. Equivalently, *if a scientific entity considers a set of numerical characteristics and if another scientific entity considers a different set of numerical characteristics, then how much the two approaches will differ or diverge?* Certainly, the issue of rational choice of specifications, characteristics and criteria is more general. An initial attempt to set up an appropriate theory has begun in [13] for the choice of characteristics and associated numerical values in a systemic geopolitical modeling. However, the question is much general and as such will be considered at a forthcoming article. At present, for the purposes of the present work, we will make the following technical and often realistic assumption.

> **Assumption 4.5.** We will uninterruptedly assume that the numerical characteristics in Definitions 3.2 and 4.1 are always chosen rationally and

> objectively, using an exhaustive algorithmic process which is commonly accepted, documented and tested.

Both effectiveness states

$$U_W[x_1, x_2, x_3, t\,]\left(fr\left(dev_1^{(V)}\right)\right), \dots, U_W[x_1, x_2, x_3, t\,]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)$$

and applicability situations

$$U_W[x_1, x_2, x_3, t\,]\left(fr\left(res_1^{(V)}\right)\right), \dots, U_W[x_1, x_2, x_3, t\,]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)$$

are called cyber node fractional vulnerabilities of V from the viewpoint of the (user(s) of) node W, at the spatiotemporal point $(x_1, x_2, x_3, t\,)$. They are also denoted separately by $fr\left(\phi_\kappa^{(W \rightsquigarrow V)}\right) = fr\left(\phi_\kappa^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,]$, $\kappa = 1,2, \dots, \mathcal{M}_V + \mathcal{L}_V$, or by a vector vulnerability representation

$$fr\left(\phi^{(W \rightsquigarrow V)}\right) = fr\left(\phi^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,]$$
$$:= \left(fr\left(\phi_1^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,],\ \dots, fr\left(\phi_{\mathcal{M}_V + \mathcal{L}_V}^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,]\right)^T.$$

If there is no risk of confusion, we will prefer write simply $\phi_\kappa^{(W \rightsquigarrow V)} = \phi_\kappa^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]$, $\kappa = 1,2, \dots, \mathcal{M}_V + \mathcal{L}_V$, or use the vector vulnerability representation

$$\phi_\kappa^{(W \rightsquigarrow V)} = \phi_\kappa^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,] :=$$
$$\left(\phi_1^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,], \dots, \phi_{\mathcal{M}_V + \mathcal{L}_V}^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]\right)^T.$$

In the total case, effectiveness states $U_W[x_1, x_2, x_3, t\,]\left(dev_1^{(V)}\right), \dots, U_W[x_1, x_2, x_3, t\,]\left(dev_{\mathcal{M}_V}^{(V)}\right)$ and applicability situations $U_W[x_1, x_2, x_3, t\,]\left(res_1^{(V)}\right), \dots, U_W[x_1, x_2, x_3, t\,]\left(res_{\mathcal{L}_V}^{(V)}\right)$ are called cyber node vulnerabilities of V from the viewpoint of the (user(s) of) node W at the spatiotemporal point $(x_1, x_2, x_3, t\,)$ and they are again denoted separately by $\phi_\kappa^{(W \rightsquigarrow V)} = \phi_\kappa^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]$, $\kappa = 1,2, \dots, \mathcal{M}_V + \mathcal{L}_V$, or by the joint vector vulnerability representation

$$\phi^{(W \rightsquigarrow V)} = \phi^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,] := \left(\phi_{V,1}^{(W)}[x_1, x_2, x_3, t\,], \dots, \phi_{V,\mathcal{M}_V+\mathcal{L}_V}^{(W)}[x_1, x_2, x_3, t\,]\right)^T.$$

By analogy, both available effectiveness states

$$U_W[x_1, x_2, x_3, t\,]\left(fr\left(dev_1^{(V)}\right)\right), \dots,\ U_W[x_1, x_2, x_3, t\,]\left(fr\left(dev_{m_V}^{(V)}\right)\right)$$

and available applicability situations

$$U_W[x_1, x_2, x_3, t\,]\left(fr\left(res_1^{(V)}\right)\right), \dots,\ U_W[x_1, x_2, x_3, t\,]\left(fr\left(res_{\ell_V}^{(V)}\right)\right)$$

are called available cyber node fractional vulnerabilities from the viewpoint of the (user(s) of) node W, at the spatiotemporal point $(x_1, x_2, x_3, t\,)$. They are denoted separately by $fr\left(c_\kappa^{(W \rightsquigarrow V)}\right) = fr\left(c_\kappa^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,]$, $\kappa = 1,2, \dots, m_V + \ell_V$, or jointly by a corresponding available node vector vulnerability representation

$$fr\left(c^{(W \rightsquigarrow V)}\right) = fr\left(c^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,] \coloneqq \left(fr\left(c_1^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,], \dots, fr\left(c_{m_V+\ell_V}^{(W \rightsquigarrow V)}\right)[x_1, x_2, x_3, t\,]\right)^T.$$

If there is no risk of confusion, we will prefer write simply $c_\kappa^{(W \rightsquigarrow V)} = c_\kappa^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]$, $\kappa = 1,2, \dots, m_V + \ell_V$, or adopt the vector vulnerability representation

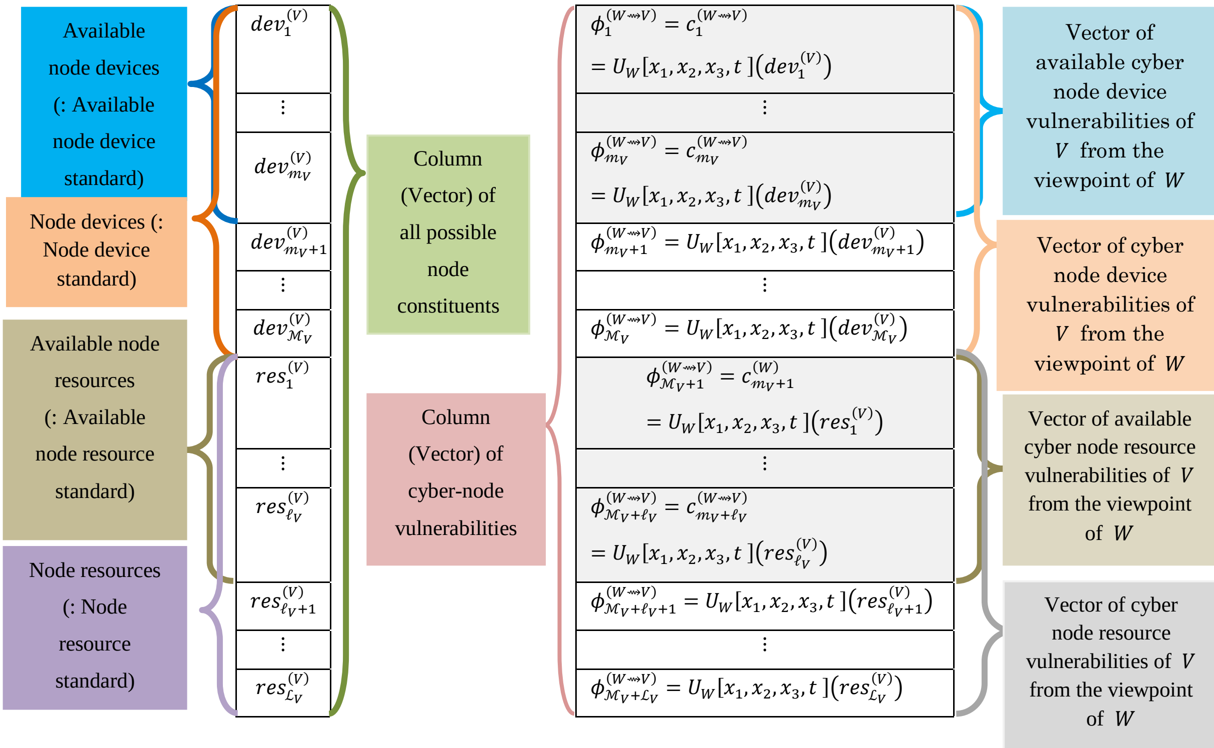
$$c_\kappa^{(W \rightsquigarrow V)} = c_\kappa^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,] \coloneqq \left(c_1^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,], \dots, c_{m_V+\ell_V}^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]\right)^T.$$

In total case, effectiveness states $U_W[x_1, x_2, x_3, t\,]\left(dev_1^{(V)}\right), \dots, U_W[x_1, x_2, x_3, t\,]\left(dev_{m_V}^{(V)}\right)$ and applicability situations $U_W[x_1, x_2, x_3, t\,]\left(res_1^{(V)}\right), \dots,$ $U_W[x_1, x_2, x_3, t\,]\left(res_{\ell_V}^{(V)}\right)$ are called available cyber node vulnerabilities from the viewpoint of the (user(s) of) node W at the spatiotemporal point $(x_1, x_2, x_3, t\,)$ and they are also denoted separately by $c_\kappa^{(W \rightsquigarrow V)} = c_\kappa^{(W \rightsquigarrow V)}[x_1, x_2, x_3, t\,]$, $\kappa = 1,2, \dots, m_V + \ell_V$, or jointly by the available cyber node vector vulnerability representation

$$c_V^{(W)} = c_V^{(W)}[x_1, x_2, x_3, t\,] \coloneqq$$

$$\left(c_{V,1}^{(W)}[x_1,x_2,x_3,t\,],\ldots,c_{V,m_V+\ell_V}^{(W)}[x_1,x_2,x_3,t\,]\right)^T.$$

In order to be more understandable, let us give a schematic example only for the indicative case of some of the above definitions in the **total** case.



5 Node Supervisions

We are now in position to proceed towards a qualitative/quantitative description of homorphisms between cyber nodes. Let W and V be two cyber nodes. We will presume the following notations for the sets of relative valuations of parts (fractions) of possible constituents:

1) $\mathfrak{C}^{(fraction)}(V)=\left\{\left(fr\left(dev_1^{(V)}\right),\ldots,fr\left(dev_{\mathcal{M}_V}^{(V)}\right),fr\left(res_1^{(V)}\right),\ldots,fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)^T:\right.$

$$fr\left(dev_k^{(V)}\right)\,is\ part\ of\ possible\ device\ dev_k^{(V)}\ of\ V,$$

$$k=1,2,\ldots,\mathcal{M}_V\ ,with\ \ \mathcal{M}_V\in\mathbb{N}$$

$$and\ fr\left(res_{\xi}^{(V)}\right) is\ part\ of\ possible\ resource\ res_{\xi}^{(V)}\ of\ V,$$

$\xi = 1,2,\dots,\ \mathcal{L}_V, with\ \mathcal{L}_V \in \mathbb{N}\}$: the set of all ordered columns of possible parts (fractions) of constituents $\left(fr\left(dev_1^{(V)}\right),\dots,fr\left(dev_{\mathcal{M}_V}^{(V)}\right),fr\left(res_1^{(V)}\right),\dots,fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)^T$ of V;

2) $\mathcal{S}_W\mathfrak{C}^{(fraction)}(V) =$

$$\Big\{\Big(S_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_1^{(V)}\right)\right),\dots,S_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right),$$

$$S_W[x_1,x_2,x_3,t,\ id_t\,]\left(fr\left(res_1^{(V)}\right)\right),\dots,S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)\Big)^T :$$

$$S_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_k^{(V)}\right)\right)\ is\ valuation\ of\ part$$

$$of\ possible\ device\ in\ V\ subject\ to\ W,\ \ k \le \mathcal{M}_V\ \ with\ \mathcal{M}_V \in \mathbb{N}$$

$$S_W[x_1,x_2,x_3,t\,]\left(fr\left(res_{\xi}^{(V)}\right)\right)\ is\ valuation$$

$$of\ possible\ resource\ \ in\ V\ subject\ to\ W, \xi \le \mathcal{L}_V\ with\ \ \mathcal{L}_V \in \mathbb{N}\ ,$$

$$at\ the\ spatiotemporal\ point\ (x_1,x_2,x_3,t)\ \in \mathbb{R}^3 \times [0,1]\Big\}\ :$$

the set of all ordered columns of relative valuations of parts (fractions) of possible constituents of V, from the viewpoint of the (user(s) of) node W, over the space time $\mathbb{R}^3 \times [0,1]$;

3) $\mathcal{U}_W\mathfrak{C}^{(fraction)}(V) =$

$$\Big\{\Big(U_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_1^{(V)}\right)\right),\dots,U_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right),$$

$$U_W[x_1,x_2,x_3,t,\ id_t\,]\left(fr\left(res_1^{(V)}\right)\right),\dots,U_W[x_1,x_2,x_3,t\,]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)\Big)^T :$$

$$U_W[x_1,x_2,x_3,t\,]\left(fr\left(dev_k^{(V)}\right)\right)\ is\ vulnerability\ of\ part$$

$$of\ possible\ device\ in\ V\ subject\ to\ W,\ \ k \le \mathcal{M}_V\ \ with\ \mathcal{M}_V \in \mathbb{N}$$

$$U_W[x_1,x_2,x_3,t\,]\left(fr\left(res_{\xi}^{(V)}\right)\right)\ is\ vulnerability$$

$$of\ possible\ resource\ \ in\, V\ subject\ to\, W, \xi \leq \mathcal{L}_V\ with\ \ \mathcal{L}_V \in \mathbb{N},$$
$$at\ the\ spatiotemporal\, point\ (x_1, x_2, x_3, t)\ \in \mathbb{R}^3 \times [0,1]\}:$$

the set of all ordered columns of relative vulnerabilities of parts (fractions) of possible constituents in V, from the viewpoint of the (user(s) of) node W, over $\mathbb{R}^3 \times [0,1]$.

Definition 5.1. *Let W and V be two cyber nodes. The combinatorial triplet*

$$\mathcal{P} = \mathcal{P}(V) = \left(\mathfrak{C}^{(fraction)}(V), \mathcal{S}_W \mathfrak{C}^{(fraction)}(V), \mathcal{U}_W \mathfrak{C}^{(fraction)}(V)\ \right)$$

will be called the cyber-field of V from the viewpoint of the users of W.Its elements are threefold cyber situations which will be represented by $\wp$. Especially, if $W = V$, the cyber-field $\mathcal{P} = \mathcal{P}(V)$ will be called the cyber-purview of V and will be denoted $\mathcal{P}^{(self)} = \mathcal{P}^{(self)}(V)$. Its elements are special threefold cyber situations called self-perceived sites and they are represented by the general form $\hat{\wp}$.

Let now W be a given cyber node and $fr\left(C^{(V)}\right)$ be a given cyber-vector in a *fixed constituent*

$$C^{(V)} = \left(dev_1^{(V)}, \dots, dev_{m_V}^{(V)}, \dots, dev_{\mathcal{M}_V}^{(V)}, res_1^{(V)}, \dots, res_{\ell_V}^{(V)}, \dots, res_{\mathcal{L}_V}^{(V)}\right)^T$$

of V. Its cyber states are

$$\left(dev_1^{(V)}\right), \dots, fr\left(dev_{m_V}^{(V)}\right), \dots fr\left(dev_{\mathcal{M}_V}^{(V)}\right), fr\left(res_1^{(V)}\right), \dots, fr\left(res_{\ell_V}^{(V)}\right), \dots, fr\left(res_{\mathcal{L}_V}^{(V)}\right).$$

Then any two threefold cyber situations $\wp$ and $\hat{\wp}$ on the node $V \in ob\big(cy(t)\big)$ from the viewpoint of the users of node W, situated in the cyber fields

$$\mathcal{P} \equiv (\mathfrak{U}_{\mathcal{P}})^{\mathcal{M}_V + \mathcal{L}_V} \times \mathbb{R}^{(\mathcal{M}_V + \mathcal{L}_V) \times \mathfrak{n}} \times \mathbb{R}^{(\mathcal{M}_V + \mathcal{L}_V) \times \mathfrak{m}} \text{ and}$$
$$\mathcal{P}^{(self)} \equiv (\mathfrak{U}_{\mathcal{P}})^{\mathcal{M}_V + \mathcal{L}_V} \times \mathbb{R}^{(\mathcal{M}_V + \mathcal{L}_V) \times \mathfrak{n}} \times \mathbb{R}^{(\mathcal{M}_V + \mathcal{L}_V) \times \mathfrak{m}}$$

respectively, can simply be viewed as two ordered pairs

$$\wp = (\,\mathbb{S}_{W \to V}, \mathbb{U}_{W \to V}) = \left(\left(s_{i,j}\right), \left(u_{i,j}\right)\right) \in \mathbb{R}^{(\mathcal{M}_V + \mathcal{L}_V) \times \mathfrak{n}} \times \mathbb{R}^{(\mathcal{M}_V + \mathcal{L}_V) \times \mathfrak{m}}$$

and

$$\hat{\wp} = \left(\widehat{\mathbb{S}}_{V\to V}, \widehat{\mathbb{U}}_{V\to V}\right) = \left(\left(\hat{s}_{i,j}\right), \left(\hat{u}_{i,j}\right)\right) \in \mathbb{R}^{(\mathcal{M}_V+\mathcal{L}_V)\times \mathfrak{n}} \times \mathbb{R}^{(\mathcal{M}_V+\mathcal{L}_V)\times \mathfrak{m}}$$

respectively, with

$$\mathbb{S}_{W\to V} = \mathbb{S}_{W\to V}\left(fr\left(C^{(V)}\right)\right) =$$

$$\begin{pmatrix}
S_W\left(fr\left(dev_1^{(V)}\right)\right) = S_W[x_1,x_2,x_3,t]\left(fr\left(dev_1^{(V)}\right)\right) = \left(\underbrace{s_{W,1}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\beta_{1,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{W,2}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\beta_{1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{W,\mathfrak{n}}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\beta_{1,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_W\left(fr\left(dev_{m_V}^{(V)}\right)\right) = S_W[x_1,x_2,x_3,t]\left(fr\left(dev_{m_V}^{(V)}\right)\right) = \left(\underbrace{s_{W,1}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\beta_{m_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{W,2}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\beta_{m_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{W,\mathfrak{n}}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\beta_{m_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_W\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = S_W[x_1,x_2,x_3,t]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = \left(\underbrace{s_{W,1}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{W,2}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{W,\mathfrak{n}}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
S_W\left(fr\left(res_1^{(V)}\right)\right) = S_W[x_1,x_2,x_3,t]\left(fr\left(res_1^{(V)}\right)\right) = \left(\underbrace{s_{W,1}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{W,2}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{W,\mathfrak{n}}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+1,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_W\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = S_W[x_1,x_2,x_3,t]\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = \left(\underbrace{s_{W,1}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{W,2}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+\ell_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{W,\mathfrak{n}}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_W\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = S_W[x_1,x_2,x_3,t]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = \left(\underbrace{s_{W,1}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+\mathcal{L}_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{W,2}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+\mathcal{L}_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{W,\mathfrak{n}}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\beta_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right)
\end{pmatrix}$$

$$\mathbb{U}_{W\to V} = \mathbb{U}_{W\to V}\left(fr\left(C^{(V)}\right)\right) =$$

$$\begin{pmatrix}
U_W\left(fr\left(dev_1^{(V)}\right)\right) = U_W[x_1,x_2,x_3,t]\left(fr\left(dev_1^{(V)}\right)\right) = \left(\underbrace{u_{W,1}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\phi_{1,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{W,2}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\phi_{1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{W,\mathfrak{m}}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\phi_{1,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
U_W\left(fr\left(dev_{m_V}^{(V)}\right)\right) = U_W[x_1,x_2,x_3,t]\left(fr\left(dev_{m_V}^{(V)}\right)\right) = \left(\underbrace{u_{W,1}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\phi_{m_V,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{W,2}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\phi_{m_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{W,\mathfrak{m}}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\phi_{m_V,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
U_W\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = U_W[x_1,x_2,x_3,t]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = \left(\underbrace{u_{W,1}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{W,2}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{W,\mathfrak{m}}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
U_W\left(fr\left(res_1^{(V)}\right)\right) = U_W[x_1,x_2,x_3,t]\left(fr\left(res_1^{(V)}\right)\right) = \left(\underbrace{u_{W,1}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{W,2}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{W,\mathfrak{m}}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+1,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
U_W\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = U_W[x_1,x_2,x_3,t]\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = \left(\underbrace{u_{W,1}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{W,2}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+\ell_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{W,\mathfrak{m}}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
U_W\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = U_W[x_1,x_2,x_3,t]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = \left(\underbrace{u_{W,1}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+\mathcal{L}_V,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{W,2}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+\mathcal{L}_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{W,\mathfrak{m}}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\phi_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right)
\end{pmatrix},$$

$$\widehat{\mathbb{S}}_{V\to V} = \widehat{\mathbb{S}}_{V\to V}\left(fr\left(C^{(V)}\right)\right) =$$

$$\begin{pmatrix}
S_V\left(fr\left(dev_1^{(V)}\right)\right) = S_V[x_1,x_2,x_3,t]\left(fr\left(dev_1^{(V)}\right)\right) = \left(\underbrace{s_{V,1}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\hat{\beta}_{1,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{V,2}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\hat{\beta}_{1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{V,\mathfrak{n}}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\hat{\beta}_{1,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_V\left(fr\left(dev_{m_V}^{(V)}\right)\right) = S_V[x_1,x_2,x_3,t]\left(fr\left(dev_{m_V}^{(V)}\right)\right) = \left(\underbrace{s_{V,1}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\hat{\beta}_{m_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{V,2}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\hat{\beta}_{m_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{V,\mathfrak{n}}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\hat{\beta}_{m_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_V\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = S_V[x_1,x_2,x_3,t]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = \left(\underbrace{s_{V,1}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\hat{\beta}_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{V,2}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\hat{\beta}_{\mathcal{M}_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{V,\mathfrak{n}}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\hat{\beta}_{\mathcal{M}_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
S_V\left(fr\left(res_1^{(V)}\right)\right) = S_V[x_1,x_2,x_3,t]\left(fr\left(res_1^{(V)}\right)\right) = \left(\underbrace{s_{V,1}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\hat{\beta}_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{V,2}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\hat{\beta}_{\mathcal{M}_V+1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{V,\mathfrak{n}}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\hat{\beta}_{\mathcal{M}_V+1,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_V\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = S_V[x_1,x_2,x_3,t]\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = \left(\underbrace{s_{V,1}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\hat{\beta}_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{V,2}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\hat{\beta}_{\mathcal{M}_V+\ell_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{V,\mathfrak{n}}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\hat{\beta}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
S_V\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = S_V[x_1,x_2,x_3,t]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = \left(\underbrace{s_{V,1}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\hat{\beta}_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)}}, \underbrace{s_{V,2}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\hat{\beta}_{\mathcal{M}_V+\ell_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{s_{V,\mathfrak{n}}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\hat{\beta}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(W\rightsquigarrow V)}}\right)
\end{pmatrix},$$

$$\widehat{\mathbb{U}}_{V\to V} = \widehat{\mathbb{U}}_{V\to V}\left(fr\left(C^{(V)}\right)\right) =$$

$$\begin{pmatrix}
U_V\left(fr\left(dev_1^{(V)}\right)\right) = U_V[x_1,x_2,x_3,t]\left(fr\left(dev_1^{(V)}\right)\right) = \left(\underbrace{u_{V,1}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\hat{\phi}_{1,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{V,2}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\hat{\phi}_{1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{V,\mathfrak{m}}\left(fr\left(dev_1^{(V)}\right)\right)}_{=:\hat{\phi}_{1,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
U_V\left(fr\left(dev_{m_V}^{(V)}\right)\right) = U_V[x_1,x_2,x_3,t]\left(fr\left(dev_{m_V}^{(V)}\right)\right) = \left(\underbrace{u_{V,1}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\hat{\phi}_{m_V,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{V,2}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\hat{\phi}_{m_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{V,\mathfrak{m}}\left(fr\left(dev_{m_V}^{(V)}\right)\right)}_{=:\hat{\phi}_{m_V,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
U_V\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = U_V[x_1,x_2,x_3,t]\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right) = \left(\underbrace{u_{V,1}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\hat{\phi}_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{V,2}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\hat{\phi}_{\mathcal{M}_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{V,\mathfrak{m}}\left(fr\left(dev_{\mathcal{M}_V}^{(V)}\right)\right)}_{=:\hat{\phi}_{\mathcal{M}_V,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
U_V\left(fr\left(res_1^{(V)}\right)\right) = U_V[x_1,x_2,x_3,t]\left(fr\left(res_1^{(V)}\right)\right) = \left(\underbrace{u_{V,1}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\hat{\phi}_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{V,2}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\hat{\phi}_{\mathcal{M}_V+1,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{V,\mathfrak{m}}\left(fr\left(res_1^{(V)}\right)\right)}_{=:\hat{\phi}_{\mathcal{M}_V+1,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
U_V\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = U_V[x_1,x_2,x_3,t]\left(fr\left(res_{\ell_V}^{(V)}\right)\right) = \left(\underbrace{u_{V,1}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\hat{\phi}_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{V,2}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\hat{\phi}_{\mathcal{M}_V+\ell_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{V,\mathfrak{m}}\left(fr\left(res_{\ell_V}^{(V)}\right)\right)}_{=:\hat{\phi}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right) \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
U_V\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = U_V[x_1,x_2,x_3,t]\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right) = \left(\underbrace{u_{V,1}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\hat{\phi}_{\mathcal{M}_V+\mathcal{L}_V,1}^{(W\rightsquigarrow V)}}, \underbrace{u_{V,2}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\hat{\phi}_{\mathcal{M}_V+\mathcal{L}_V,2}^{(W\rightsquigarrow V)}}, \dots, \underbrace{u_{V,\mathfrak{m}}\left(fr\left(res_{\mathcal{L}_V}^{(V)}\right)\right)}_{=:\hat{\phi}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}}^{(W\rightsquigarrow V)}}\right)
\end{pmatrix}.$$

Without any loss of generality, we may suppose the numbers $\mathcal{M}_V + \mathcal{L}_V$ and $\mathcal{M}_W + \mathcal{L}_W$ are enough large, so that $\mathcal{M}_V + \mathcal{L}_V = \mathcal{M}_W + \mathcal{L}_W$, for any two cyber nodes W and V. To simplify the notation, we set

$$\mathcal{N} \coloneqq \mathcal{M}_V + \mathcal{L}_V = \mathcal{M}_W + \mathcal{L}_W.$$

Definition 5.2. *Let W and V be two cyber nodes. The supervision of V in the system of the two nodes V and W at a given time moment $t \in [0,1]$ is defined to be the pair*

$$(z_1, \zeta_1) = (z_1, \zeta_1)(t) \in \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}}$$

with

$$z_1 = \mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V},\ \zeta_1 = \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V},$$

and such that

- $i := \sqrt{-1} = (0,1) \in \mathbb{C}$,
- $(\mathbb{S}_{W\to V}, \mathbb{U}_{W\to V}) = \left((s_{i,j}), (u_{i,j})\right) \in \mathbb{R}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{R}^{\mathcal{N}\times\mathfrak{m}}$ *and*
- $\left(\widehat{\mathbb{S}}_{V\to V}, \widehat{\mathbb{U}}_{V\to V}\right) = \left((\hat{s}_{i,j}), (\hat{u}_{i,j})\right) \in \mathbb{R}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{R}^{\mathcal{N}\times\mathfrak{m}}$. *]*

The complex matrices z_1 and ζ_1 are called supervisory perceptions of V in the system of nodes V and W at the moment t. The piecewise continuous mapping $\delta_V \equiv \delta_{[(V,W)\rightsquigarrow V]}$ defined by

$$\delta_V\colon [0,1] \to \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}}\colon t \mapsto \delta_V(t) = (z_1, \zeta_1)(t)$$
$$\equiv \left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right)(t)$$

is the supervisory perception curve of V in the node system (V, W). Its image $\delta_V^* = \delta_V\ ([0,1])$ is called universal supervision of V in the node system (V, W), while any subset $\delta_V(I) = \{\delta_V(t)\colon t \in I \subset [0,1]\}$ of $\delta_V([0,1])$ is said to be a partial supervisory perception of V in the system of the two nodes V and W.

If, according to Remarks 3.3 and 4.2, the component valuations $s_{W,k}\left(fr\left(C^{(V)}\right)\right)$ or vulnerabilities $u_{W,j}\left(fr\left(C^{(V)}\right)\right)$ of a given part $fr\left(C^{(V)}\right)$ in the cyber-node V extent onto the real projective line $\mathbb{RP}^1$ of $\mathbb{R}$, then any two threefold cyber situations $\mathscr{p}$ and $\hat{\mathscr{p}}$ in the corresponding cyber fields $\mathcal{P} \equiv (\mathfrak{U}_{\mathcal{P}})^{\mathcal{N}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{m}}$ and $\mathcal{P}^{(self)} \equiv (\mathfrak{U}_{\mathcal{P}})^{\mathcal{N}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{m}}$ can be viewed as two ordered pairs

$$\mathscr{p} = (\mathbb{S}_{W\to V}, \mathbb{U}_{W\to V}) = \left(\left(s_{i,j}\right), \left(u_{i,j}\right)\right) \in (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{m}} \text{ and}$$

$$\hat{\mathscr{p}} = \left(\widehat{\mathbb{S}}_{V\to V}, \widehat{\mathbb{U}}_{V\to V}\right) = \left(\left(\hat{s}_{i,j}\right), \left(\hat{u}_{i,j}\right)\right) \in (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{m}}$$

respectively. In such a case, the set δ_V^* of extended universal supervisions of V in the system of the two nodes V and W consists of all ordered pairs $\big(\mathbb{S}_{W\to V} + i\mathbb{S}V\to V, \mathbb{U}W\to V + i\mathbb{U}V\to V \in \mathbb{CP}1\mathcal{N}\times\mathfrak{n}\times \mathbb{CP}1\mathcal{N}\times\mathfrak{m}$, which are defined in such a way that a column in the matrices $(\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{n}}$ and $(\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{m}}$ is considered to be infinite if and only if the real or the imaginary part of an element of the column becomes infinite. Here $\mathbb{CP}^1$ denotes, as usually, the complex projective line (: the Riemann sphere S^3). We need the following.

Theorem 5.2. *The* $\mathcal{N}$ *−fold symmetric product of* $\mathbb{CP}^1$ *is homeomorphic to* $\mathbb{CP}^{\mathcal{N}}$.

Sketch of Proof. One can be trying to understand the space obtained by taking the Cartesian product $\mathbb{CP}^1 \times \mathbb{CP}^1$ and identifying some of its points by the rule $(x, y) \sim (y, x)$. Viewing $\mathbb{CP}^1$ as a CW complex with one 0-cell and one 2-cell, we can compute the homology of $\mathbb{CP}^1 \times \mathbb{CP}^1/\sim$ which matches that of $\mathbb{CP}^2$ but we can't seem to visualize an "obvious" homeomorphism between the two spaces. The question is the following:

- is $\mathbb{CP}^1 \times \mathbb{CP}^1/\sim$ homeomorphic to $\mathbb{CP}^2$ and,
- if so, how?

We believe we are on the right track, and a homeomorphism from $\mathbb{CP}^1 \times \mathbb{CP}^1/\sim$ to $\mathbb{CP}^2$ is given by

$$[((z_1{:}\,z_2), (w_1{:}\,w_2))] \mapsto (z_1w_1{:}\,z_2w_2{:}\,z_1w_2 + z_2w_1).$$

Note that elements of the form $[(1{:}\,z), (1{:}\,w)]$ map to $(1{:}\,zw{:}\,z + w)$, i.e., the coordinates are given by the elementary symmetric functions of z and w, so the map is a homeomorphism restricted to this subspace onto the subspace of $\mathbb{CP}^2$ given by points with non-zero first coordinate. We have not worked out all the details, but we are pretty sure that this argument can be promoted to show that the map is actually a homeomorphism between your spaces. To see this in the 2−fold case: consider homogeneous polynomials of degree two $\mathbb{C}[x,y]^{(2)}$ whose elements are of the form $ax^2 + bxy + cy^2$ and notice that for $\lambda \in \mathbb{C}^*$, it holds

$$\lambda[ax_0^2 + bx_0y_0 + cy_0^2] = 0 \Leftrightarrow ax_0^2 + bx_0y_0 + cy_0^2 = 0.$$

This allows us to identify points of $\mathbb{CP}^2$ with elements of $\mathbb{C}[x,y]^{(2)}/\sim$, where $\sim$ identifies polynomials having the same roots. The map from $\mathbb{CP}^2$ to the symmetric product of two copies of $\mathbb{CP}^1$ is then given by

$$(a{:}\,b{:}\,c) \mapsto ax^2 + bxy + cy^2 = (\alpha x + \beta y)(\alpha' x + \beta' y) \mapsto [(\alpha{:}\,\beta), (\alpha'{:}\,\beta')]$$

where the equality comes from the fundamental theorem of algebra.

In view of this result, we are led to the following definition.

Definition 5.3. *Let W and V be two cyber nodes. The extended supervision of V in the system of the two nodes V and W at a given time moment $t \in [0,1]$ is defined to be the pair*

$$(z_1, \zeta_1) = (z_1, \zeta_1)(t) \in (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{n}} \times (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{m}} \equiv (\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{m}}$$

with

$$z_1 = \mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V},\ \zeta_1 = \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V},$$

and such that

- $i := \sqrt{-1} = (0,1) \in \mathbb{C}$,
- $(\mathbb{S}_{W\to V}, \mathbb{U}_{W\to V}) = \left(\left(s_{i,j}\right), \left(u_{i,j}\right)\right) \in (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{m}}$ *and*
- $\left(\widehat{\mathbb{S}}_{V\to V}, \widehat{\mathbb{U}}_{V\to V}\right) = \left(\left(\hat{s}_{i,j}\right), \left(\hat{u}_{i,j}\right)\right) \in (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{RP}^1)^{\mathcal{N}\times\mathfrak{m}}$.

The complex projective points z_1 and ζ_1 are called extended supervisory perceptions of V in the system of nodes V and W at the moment t. The piecewise continuous mapping

$$\delta\mathbb{P}_V \equiv \delta\mathbb{P}_{[(V,W)\rightsquigarrow V]}$$

defined by

$$\delta\mathbb{P}_V: [0,1] \to (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{n}} \times (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{m}}: t \mapsto \delta\mathbb{P}_V(t) = (z_1, \zeta_1)(t)$$
$$\equiv \left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right)(t)$$

is the extended supervisory perception curve of V *in the node system* (V,W)*. Its image* $\delta\mathbb{P}_V([0,1])$ *is called extended universal supervision of* V *in the node system* (V,W)*, while any subset* $\delta\mathbb{P}_V(I) = \{\delta\mathbb{P}_V(t): t \in I \subset [0,1]\}$ *of* $\delta\mathbb{P}_V([0,1])$ *is said to be a partial extended supervisory perception of* V *in the system of nodes* V *and* W*.*

Provided there is no risk of confusion, we will denote indiscriminately with $\mathbb{CM}$ either $\mathbb{C}$ or $\mathbb{CP}$. Further, in what will follow, we will adopt the common notation

$$\gamma_V \equiv \gamma_{[(V,W)\rightsquigarrow V]}[0,1] \to \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}: t \mapsto \gamma_V(t) = (z_1, \zeta_1)(t)$$
$$\equiv \left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right)(t)$$

for the two supervisory perception curves δ_V and $\delta\mathbb{P}_V$. Similarly, we will adopt the common notation $\gamma_V(I) = \{\gamma_V(t): t \in I \subset [0,1]\}$ for the two supervisory perception sets $\delta_V(I)$ and $\delta\mathbb{P}_V(I)$. In particular, we will write γ_V^* for the two universal supervisions $\delta_V([0,1])$ and $\delta\mathbb{P}_V([0,1])$. With this notation, we are now in position to proceed further, as in the following Session.

6 Cyber-Effects

A momentary homomorphism $g: W \to V$ between the two cyber nodes $V, W \in ob\big(cy(t)\big)$ is defined as a collection of mappings from a cyber field of W at time $t \in\,]\alpha,\beta[\, \subset\subset [0,1]$ into a cyber field of V at other times $t' \in [\alpha,\beta]$.

Definition 6.1. *Let us consider the two supervisory perception sets*

$$\Omega_V = \Omega_{[(V,W)\rightsquigarrow V]}([0,1]) \subset \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}} \text{ and}$$

$$\Omega_W = \Omega_{[(V,W)\rightsquigarrow W]}([0,1]) \subset \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}.$$

The momentary homomorphism $g\colon W \to V$ *can be rather understandable as an "adaptive" movement* $\mathcal{g}$ *between time-shifted partial (extended or not) supervisory perceptions of* W *and* V*:*

$$\mathcal{g}\colon [\alpha,\beta] \mapsto \Omega_W \times \Omega_V\colon t \mapsto \mathcal{g}(t) := \big(\gamma_W(t), \gamma_V(t+\Delta t)\big).$$

The shifted curve $\mathcal{g}$ *is called cyber-effect of* W *on* V*.*

It is more appropriate to represent a cyber-effect as a collection of point-wise correspondences

$$\Big(\mathcal{g}_t\colon \gamma_W(t) \longmapsto \gamma_V^{'}(t^{'})\Big)_{t\in]\alpha,\beta[} \quad (t^{'} := t + \Delta t),$$

where we denote by $\gamma_W(t)$ and $\gamma_V^{'}(t^{'})$ the curves $\gamma_{[(V,W)\rightsquigarrow W]}(t)$ and $\gamma_{[(V,W)\rightsquigarrow V]}(t+\Delta t)$, respectively. With this notation, at time t, a supervisory perception of W in the system of nodes V, W:

$$\gamma_W(t) = \big(\mathbb{S}_{V\to W} + i\widehat{\mathbb{S}}_{W\to W}, \mathbb{U}_{V\to W} + i\widehat{\mathbb{U}}_{W\to W}\big) =$$

$$\left(\begin{pmatrix} \beta_{1,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{1,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{1,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{1,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{m_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{m_W,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{m_W,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{m_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{M}_W,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \beta_{\mathcal{M}_W+1,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+1,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{M}_W+1,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+1,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_W+\ell_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+\ell_W,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{M}_W+\ell_W,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+\ell_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{N},1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{N},1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{N},\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{N},\mathfrak{n}}^{(W\rightsquigarrow W)} \end{pmatrix}, \begin{pmatrix} \phi_{1,1}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{1,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{1,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{1,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \phi_{m_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{m_W,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{m_W,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{m_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{M}_W,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{\mathcal{M}_W,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{M}_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \phi_{\mathcal{M}_W+1,1}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{M}_W+1,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{\mathcal{M}_W+1,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{M}_W+1,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_W+\ell_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{M}_W+\ell_W,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{\mathcal{M}_W+\ell_W,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{M}_W+\ell_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{N},1}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{N},1}^{(W\rightsquigarrow W)} & \cdots & \phi_{\mathcal{N},\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\phi}_{\mathcal{N},\mathfrak{m}}^{(W\rightsquigarrow W)} \end{pmatrix}\right) \in \Omega_W$$

is depicted, by means of the cyber-effect $\mathcal{g} = \mathcal{g}_t$, at the supervisory perception of V in the system of nodes V and W at a next time $t^{'} := t + \Delta t$:

$$\gamma_V^{'}(t^{'}) = \big(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\big) =$$

$$\left(\begin{pmatrix} \beta_{1,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{1,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{m_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{m_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{m_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{M}_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \beta_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{M}_V+1,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{N},1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{N},1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{N},\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{N},\mathfrak{n}}^{(V\rightsquigarrow V)} \end{pmatrix}, \begin{pmatrix} \phi_{1,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{1,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \phi_{m_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{m_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{V,1}^{(W\rightsquigarrow V)} & \cdots & \phi_{\mathcal{M}_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \phi_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{\mathcal{M}_V+1,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{N},1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{N},1}^{(V\rightsquigarrow V)} & \cdots & \phi_{\mathcal{N},\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{N},\mathfrak{m}}^{(V\rightsquigarrow V)} \end{pmatrix}\right)$$

$$\in \Omega_V.$$

Remark 6.2 . The case $\Delta t = 0$ is not excluded.

Let us give two indicative examples showing the alteration diversity and combinatorial suppleness of this flexible concept.

Example 6.3.i. In practice, often, we prefer to reduce only to available constituents and available valuations. Then, the momentary homomorphism g transforms only available quantities of W at a time t into available quantities of V at a next time $t' = t + \Delta t$ and we write $g = g_t: \mathcal{Q}_7^{(V)}(W)(t) \to \mathcal{P}_7^{(W)}(V)(t')$, where the combinatorial triplet

$\mathcal{Q}_7^{(V)}(W) = \mathcal{Q}_7^{(V)}(W)(t) = (\mathfrak{C}_{available}(W), \mathcal{S}_V\mathfrak{C}_{available}(W), \mathcal{U}_V\mathfrak{C}_{available}(W)\ )$

represents the set of available components of node W at time t, as evaluated in terms of their valuations and their vulnerabilities by the users of node V:

$$\mathfrak{C}_{available}(W) = \Big\{\Big(dev_1^{(W)}, \dots, dev_{m_W}^{(W)}, res_1^{(W)}, \dots, res_{\ell_W}^{(W)}\Big)^T:$$
$$dev_k^{(W)}\ is\ available\ device\ of\ W, with\ m_V \in \mathbb{N}\ and$$
$$res_k^{(W)}\ is\ available\ resource\ \ of\ W, with\ \ell_V \in \mathbb{N}\Big\}:$$

the set of all ordered columns of available constituents $\Big(dev_1^{(W)}, \dots, dev_{m_W}^{(W)}, res_1^{(W)}, \dots, res_{\ell_V}^{(W)}\Big)^T$ of W,

$$\mathcal{S}_V\mathfrak{C}_{available}(W) = \Big\{\Big(S_V[x_1, x_2, x_3, t\,]\Big(dev_1^{(W)}\Big), \dots, S_V[x_1, x_2, x_3, t\,]\Big(dev_{m_W}^{(W)}\Big),$$
$$S_V[x_1, x_2, x_3, t\,]\Big(res_1^{(W)}\Big), \dots, S_V[x_1, x_2, x_3, t\,]\Big(res_{\ell_W}^{(W)}\Big)\Big)^T:$$

$$S_V[x_1, x_2, x_3, t\,]\left(dev_k^{(W)}\right) \; is\; valuation\; of\; available\; device$$
$$in\; W\; subject\; to\; V,$$
$$k = 1,2, \dots, m_W \quad , \quad with\; m_W \in$$
$$\mathbb{N}\; S_V[x_1, x_2, x_3, t\,]\left(res_\xi^{(W)}\right) \; is\; valuation\; of\; available\; resource$$
$$in\; W\; subject\; to\; V, \xi = 1,2, \dots, \ell_W\; with\; \ell_W$$
$$\in \mathbb{N}, at\; the\; spatiotemporal\; point\; (x_1, x_2, x_3, t)$$
$$\in \mathbb{R}^3 \times [0,1]\}:$$

the set of all ordered columns of relative valuations of available constituents in W, from the viewpoint of the (user(s) of) node V, over the space time $\mathbb{R}^3 \times [0,1]$,

$$\mathcal{U}_V \mathfrak{C}_{available}(W) = \left\{\left(U_V[x_1, x_2, x_3, t\,]\left(dev_1^{(W)}\right), \dots, U_V[x_1, x_2, x_3, t\,]\left(dev_{m_W}^{W}\right),\right.\right.$$
$$\left.U_V[x_1, x_2, x_3, t\,]\left(res_1^{(W)}\right), \dots, U_V[x_1, x_2, x_3, t\,]\left(res_{\ell_W}^{(W)}\right)\right)^T :$$
$$U_V[x_1, x_2, x_3, t\,]\left(dev_k^{(W)}\right) \; is\; vulnerability\; of\; available\; device$$
$$in\; W\; subject\; to\; V, k = 1,2, \dots, m_W \quad , \quad with\; m_W \in$$
$$\mathbb{N}\; S_V[x_1, x_2, x_3, t\,]\left(res_\xi^{(W)}\right) \; is\; vulnerability\; of\; available\; resource$$
$$in\; W\; subject\; to\; V,$$
$$\xi = 1,2, \dots, \ell_W\; with\; \ell_W \in$$
$$\mathbb{N}, \quad at\; the\; spatiotemporal\; point\; (x_1, x_2, x_3, t) \in$$
$$\left.\mathbb{R}^3 \times [0,1]\right\}:$$

the set of all ordered columns of relative vulnerabilities of available constituents in W, from the viewpoint of the (user(s) of) node V, over $\mathbb{R}^3 \times [0,1]$.

Similarly, the combinatorial triplet $\mathcal{P}_7^{(W)}(V) = \mathcal{P}_7^{(W)}(V)(t') = (\mathfrak{C}_{available}(V), \mathcal{S}_W \mathfrak{C}_{available}(V), \mathcal{U}_W \mathfrak{C}_{available}(V)\,)$ represents the set of available components of node V at time t', as evaluated in terms of their valuations and their vulnerabilities by the users of node W. In view of the above Definition 6.1, the correspondence $\mathcal{g} = \mathcal{g}_t$ can be seen as a mapping between (extended or not) supervisory perceptions $\mathcal{g} = \mathcal{g}_t: \gamma_W(t) \longmapsto \gamma_V'(t')$, in such a way that each (extended or not) supervisory perception of W in the system of nodes V and W at a time moment t, of the form

$$\gamma_W(t) = \left(\mathbb{S}_{V\to W} + i\widehat{\mathbb{S}}_{W\to W}, \mathbb{U}_{V\to W} + i\widehat{\mathbb{U}}_{W\to W}\right) =$$

$$\left(\begin{pmatrix} \beta_{1,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{1,1}^{(W\rightsquigarrow W)} & \dots & \beta_{1,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{1,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{m_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{m_W,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{m_W,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{m_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ 0 & & 0 \\ \cdots & \cdots & \cdots \\ 0 & & 0 \\ \beta_{\mathcal{M}_W+1,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+1,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{M}_W+1,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+1,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_W+\ell_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+\ell_W,1}^{(W\rightsquigarrow W)} & \dots & \beta_{\mathcal{M}_W+\ell_W,\mathfrak{n}}^{(V\rightsquigarrow W)} + i\ \hat{\beta}_{\mathcal{M}_W+\ell_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ 0 & \cdots & 0 \\ \cdots & & \cdots \\ 0 & & 0 \end{pmatrix}, \begin{pmatrix} \gamma_{1,1}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{1,1}^{(W\rightsquigarrow W)} & \dots & \gamma_{1,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{1,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \gamma_{m_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{m_W,1}^{(W\rightsquigarrow W)} & \cdots & \gamma_{m_W,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{m_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ 0 & & 0 \\ \cdots & \cdots & \cdots \\ 0 & & 0 \\ \gamma_{\mathcal{M}_W+1,1}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{\mathcal{M}_W+1,1}^{(W\rightsquigarrow W)} & \cdots & \gamma_{\mathcal{M}_W+1,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{\mathcal{M}_W+1,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \gamma_{\mathcal{M}_W+\ell_W,1}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{\mathcal{M}_W+\ell_W,1}^{(W\rightsquigarrow W)} & \dots & \gamma_{\mathcal{M}_W+\ell_W,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \hat{\gamma}_{\mathcal{M}_W+\ell_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ 0 & \cdots & 0 \\ \cdots & & \cdots \\ 0 & & 0 \end{pmatrix}\right)$$

$$\in \Omega_W$$

is depicted, via the correspondence g, at an (extended or not) supervisory perception of V in the system of nodes V and W at the next time moment $t' := t + \Delta t$, of the form:

$$\gamma_V'(t') = \left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right) =$$

$$\left(\begin{pmatrix} \beta_{1,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{1,1}^{(V\rightsquigarrow V)} & \dots & \beta_{1,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{m_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{m_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{m_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ 0 & & 0 \\ \cdots & \cdots & \cdots \\ 0 & & 0 \\ \beta_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{M}_V+1,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \dots & \beta_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ 0 & \cdots & 0 \\ \cdots & & \cdots \\ 0 & & 0 \end{pmatrix}, \begin{pmatrix} \gamma_{1,1}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{1,1}^{(V\rightsquigarrow V)} & \dots & \gamma_{1,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \gamma_{m_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \gamma_{m_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ 0 & & 0 \\ \cdots & \cdots & \cdots \\ 0 & & 0 \\ \gamma_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \gamma_{\mathcal{M}_V+1,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{\mathcal{M}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \gamma_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \dots & \gamma_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\gamma}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ 0 & \cdots & 0 \\ \cdots & & \cdots \\ 0 & & 0 \end{pmatrix}\right)$$

$$\in \Omega_V.$$

ii. Similarly, if the momentary homomorphism $g: W \to V$ acts only on all the resources of W by transforming and transferring fractions of the available resources of W at a time t into the node resource standard $\left(r_1^{(V)}, \dots, r_{\mathcal{L}_V}^{(V)}\right)$ of V at a next time $t' = t + \Delta t$, then the cyber-effect g is a mapping of the form $g = g_t: \mathcal{Q}_9^{(V)}(W)(t) \to \mathcal{P}_3^{(W)}(V)(t')$. Here, as usually, the combinatorial triplet $\mathcal{Q}_9^{(V)}(W) = \mathcal{Q}_9^{(V)}(W)(t') = (\mathfrak{R}_{available}(W), \mathcal{S}_V\mathfrak{R}_{available}(W), \mathcal{U}_V\mathfrak{R}_{available}(W)\ )$

represents a set of available resources of node W, at the time moment t, as evaluated in terms of their valuations and their vulnerabilities by the users of node V:

$$\mathfrak{R}_{available}(W) = \left\{\left(res_1^{(W)}, \dots, res_{\ell_W}^{(W)}\right)^T : res_k^{(W)}\ possible\ resource\ of\ W, k = 1,2,..,\ell_W, \ell_W \in \mathbb{N}\right\}:$$

the set of all ordered columns of available resources of W,

$$\mathcal{S}_V\mathfrak{R}_{available}(W) = \left\{\left(S_V[x_1, x_2, x_3, t]\left(res_1^{(W)}\right), \dots, S_V[x_1, x_2, x_3, t]\left(res_{\ell_W}^{(W)}\right)\right)^T : S_V[x_1, x_2, x_3, t]\left(res_\xi^{(W)}\right)\ is\ valuation\ of\ possible\ resource\ in\ W\ subject\ to\ V, \xi = 1,2,\dots,\ell_W\ with\ \ell_W \in \mathbb{N}, at\ the\ spatiotemporal\ point\ (x_1, x_2, x_3, t) \in \mathbb{R}^3 \times [0,1]\right\}:$$

the set of all ordered columns of relative valuations of available constituents in W, from the viewpoint of the (user(s) of) node V, over $\mathbb{R}^3 \times [0,1]$,

$$\mathcal{U}_V\mathfrak{R}_{available}(W) = \left\{\left(U_V[x_1, x_2, x_3, t]\left(res_1^{(W)}\right), \dots, U_V[x_1, x_2, x_3, t]\left(res_{\ell_W}^{(W)}\right)\right)^T : U_V[x_1, x_2, x_3, t]\left(res_\xi^{(W)}\right)\ is\ vulnerability\ of\ possible\ resource\ in\ W\ subject\ to\ V, \xi = 1,2,\dots,\ell_W\ with\ \ell_W \in \mathbb{N}, at\ the\ spatiotemporal\ point\ (x_1, x_2, x_3, t) \in \mathbb{R}^3 \times [0,1]\right\}:$$

the set of all ordered columns of relative vulnerabilities of available constituents in W, from the viewpoint of the (user(s) of) node V, over $\mathbb{R}^3 \times [0,1]$.

Similarly, the combinatorial triplet

$$\mathcal{P}_3^{(W)}(V) = \mathcal{P}_3^{(W)}(V)(t') = \left(\mathfrak{R}(V), \mathcal{S}_W\mathfrak{R}(V), \mathcal{U}_W\mathfrak{R}(V)\right)$$

represents a set of resources of node V, at the next time moment t', as evaluated in terms of their valuations and their vulnerabilities by the users of node W. In view of Definition 6.1, the correspondence $\mathscr{g} = \mathscr{g}_t$ can be seen as a mapping between (extended or not) supervisory perceptions $\mathscr{g} = \mathscr{g}_t : \gamma_W(t) \longmapsto \gamma_V^{'}(t^{'})$, in such a way that each (extended or not) supervisory perception of W in the system of nodes V and W at time moment t

$$\gamma_W(t) = \left(\mathbb{S}_{V\to W} + i\hat{\mathbb{S}}_{W\to W}, \mathbb{U}_{V\to W} + i\hat{\mathbb{U}}_{W\to W}\right) =$$

$$\left(\left(\begin{array}{ccc} 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \\ \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,1} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & \cdots & \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} \\ \vdots & \vdots & \vdots \\ \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} \\ 0 & \vdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{array}\right), \left(\begin{array}{ccc} 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \\ \gamma^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,1} + i\ \hat{\gamma}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & \cdots & \gamma^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}} + i\ \hat{\gamma}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}} \\ \vdots & \vdots & \vdots \\ \gamma^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} + i\ \hat{\gamma}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \gamma^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} + i\ \hat{\gamma}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} \\ 0 & \vdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{array}\right)\right) \in \Omega_W$$

is depicted, via the correspondence g, at an (extended or not) supervisory perception of V in the system of nodes V and W at the moment $t^{'} := t + \Delta t$ of the form

$$\gamma_V^{'}(t^{'}) = \left(\mathbb{S}_{W\to V} + i\hat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\hat{\mathbb{U}}_{V\to V}\right) =$$

$$\left(\left(\begin{array}{ccc} 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \\ \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\ \vdots & \vdots & \vdots \\ \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \vdots & \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\ \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,1} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,1} = \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,1} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & & \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,\mathfrak{n}} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,\mathfrak{n}} = \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} \\ \vdots & \vdots & \vdots \\ \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,1} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,1} = \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \beta^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,\mathfrak{n}} + i\ \hat{\beta}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,\mathfrak{n}} = \beta^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} + i\ \hat{\beta}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} \\ 0 & \vdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{array}\right),\right.$$

$$\left.\left(\begin{array}{ccc} 0 & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \\ \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\ \vdots & \vdots & \vdots \\ \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \vdots & \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\ \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,1} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,1} = \phi^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,1} + i\ \hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & & \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,\mathfrak{m}} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+1,\mathfrak{m}} = \phi^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}} + i\ \hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}} \\ \vdots & \vdots & \vdots \\ \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,1} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,1} = \phi^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} + i\ \hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \phi^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,\mathfrak{m}} + i\ \hat{\phi}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V+\ell_W,\mathfrak{m}} = \phi^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} + i\ \hat{\phi}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} \\ 0 & \vdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & 0 \end{array}\right)\right) \in \Omega_V.$$

Although the concept of cyber-effect at a time moment t seems to be rather sufficient, sometimes we care to describe the interaction that has one cyber-node on each other, as well as the mutual effects resulting at a later time $t^{'} = t + \Delta t$. In this case, the putative mutuality directly is influenced by the subjectivity of the users of the two cyber nodes. So, frequently, instead of the concept of a momentary cyber-effect, we are forced to consider mappings describing mutual influences between cyber-nodes.

7 Cyber-Interactions

As in Definition 6.1, let us consider the sets

$\Omega_V = \Omega_{[(V,W)\rightsquigarrow V]}([0,1]) \subset \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}$ and

$$\Omega_W = \Omega_{[(V,W)\rightsquigarrow W]}([0,1]) \subset \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}$$

of supervisory perception curves of V and W in the node system (V, W).

Definition 7.1. *If* $]\alpha,\beta[\subset\subset [0,1]$, *an interplay of the ordered cyber pair* (V, W) *over the time* $t \in]\alpha,\beta[$ *or, simply, a cyber-interplay, is an open*[3] *shift curve*

$$\mathcal{g}:]\alpha,\beta[\to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V:$$

$$t \mapsto \mathcal{g}(t) := \big(\gamma_W(t), \gamma_V(t),\ \gamma_W(t+\Delta t),\ \gamma_V(t+\Delta t)\big).$$

If the cyber-interplay $\mathcal{g}$ *is composition of several separate interplays, we say that the cyber-interplay* $\mathcal{g}$ *is sequential; otherwise is called elementary.*

It is more appropriate to represent a cyber-interplay as a collection of point-wise correspondences

$$\left(\mathcal{g}_t: \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V: (\gamma_W(t), \gamma_V(t)\ ) \longmapsto \left(\gamma_W^{'}(t^{'}), \gamma_V^{'}(t^{'})\right)\right)_{t\in]\alpha,\beta[}$$

---

[3] Open intervals are used for so called open curves (line, parabola, hyperbola...). Closed intervals are used for closed curves (circles, ellipse...). The reason for use of open intervals for open curves and closed intervals for closed curves is that parameterization is a homeomorphism between to "shapes". Circle is not homeomorphic to the line, for example. But it is to any closed loop (http://math.stackexchange.com/questions/209309/open-interval-in-definition-of-curve).

$$(t^{'} := t + \Delta t),$$

where, as usually, we denote by $\gamma_X(t)$ and $\gamma_X^{'}(t^{'})$ the curves $\gamma_{[(V,W)\rightsquigarrow X]}(t)$ and $\gamma_{[(V,W)\rightsquigarrow X]}(t+\Delta t)$, respectively (with $X = V, W$) and we say that the interplay is a cyber- activity of W on V over the time $t \in]\alpha, \beta[$. If the cyber-interplay is sequential, we say that the cyber-activity of W on V is sequential; otherwise the cyber-activity is called elementary.

Definition 7.2. *A cyber-interaction or simply interaction between W and V at a given time moment $t_0 \in]\alpha, \beta[$ is a tetrad*

$$Z = Z_{(W,V)}(t_0) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}})^4$$

for which there is an associated cyber-activity of W on V:

$$\left(\mathcal{g}_t = \mathcal{g}_t^{(Z)}: \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V : (\gamma_W(t), \gamma_V(t)\,) \mapsto \left(\gamma_W'(t'), \gamma_V'(t')\right)\right)_{t\in]\alpha,\beta[}$$

$$(t' := t + \Delta t),$$

such that

$$(z_1,\zeta_1) = \gamma_W(t_0) = \left(\mathbb{S}_{V\to W} + i\widehat{\mathbb{S}}_{W\to W}, \mathbb{U}_{V\to W} + i\widehat{\mathbb{U}}_{W\to W}\right) \in \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}},$$

$$(z_2,\zeta_2) = \gamma_V(t_0) = \left(\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}\right) \in \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}},$$

$$(z_3,\zeta_3) = \gamma_W^{'}(t_0^{'}) = \left(\mathbb{S}_{V\to W}^{'} + i\widehat{\mathbb{S}}_{W\to W}^{'}, \mathbb{U}_{V\to W}^{'} + i\widehat{\mathbb{U}}_{W\to W}^{'}\right) \in \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}},$$

$$(z_4,\zeta_4) = \gamma_V^{'}(t_0^{'}) = \left(\mathbb{S}_{W\to V}^{'} + i\widehat{\mathbb{S}}_{V\to V}^{'}, \mathbb{U}_{W\to V}^{'} + i\widehat{\mathbb{U}}_{V\to V}^{'}\right) \in \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}.$$

If the corresponding interplay

$$\mathcal{g} = \mathcal{g}^{(Z)}:]\alpha, \beta[\to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V:$$

$$t \mapsto \mathcal{g}(t) := \left(\gamma_W(t), \gamma_V(t), \gamma_W^{'}(t^{'}), \gamma_V^{'}(t^{'})\right)$$

is sequential, we say that the cyber-interaction is sequential; otherwise the cyber-interaction is called elementary.

Obviously, in Definition 7.1, keeping a fixed supervisory perception $\gamma_V(t_0)$ in the archetype component Ω_V and a fixed supervisory perception $\gamma_W(t+\Delta t)$ in the component image Ω_W, the corresponding cyber-interaction becomes a cyber-effect in the sense of Definition 6.1. And, as we shall see, proper management of cyber-effects is enough to study cyber navigations ([2). However, in most cases, as in the case of cyber attacks (see again [2]), it is necessary to consider cyber-interactions. So, because cyber-effects are a partial case of cyber-interactions, we will give a slight priority in the most general context of cyber-interactions.

It is easily verified that the most detailed general form of a cyber-interaction is as follows.

$$Z = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big)(t_0)$$

$$= \Bigg(\underbrace{\mathbb{S}_{V\to W} + i\widehat{\mathbb{S}}_{W\to W}, \mathbb{U}_{V\to W} + i\widehat{\mathbb{U}}_{W\to W}}_{\gamma_W(t_0)}, \underbrace{\mathbb{S}_{W\to V} + i\widehat{\mathbb{S}}_{V\to V}, \mathbb{U}_{W\to V} + i\widehat{\mathbb{U}}_{V\to V}}_{\gamma_V(t_0)},$$

$$\underbrace{\mathbb{S}'_{V\to W} + i\widehat{\mathbb{S}}'_{W\to W}, \mathbb{U}'_{V\to W} + i\widehat{\mathbb{U}}'_{W\to W}}_{\gamma'_W(t'_0)}, \underbrace{\mathbb{S}'_{W\to V} + i\widehat{\mathbb{S}}'_{V\to V}, \mathbb{U}'_{W\to V} + i\widehat{\mathbb{U}}'_{V\to V}}_{\gamma'_V(t'_0)}\Bigg)$$

$$=$$

$$\left(\left(\underbrace{\begin{pmatrix} \beta_{1,1}^{(V\to W)} + i\ \hat{\beta}_{1,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{1,\mathfrak{n}}^{(V\to W)} + i\ \hat{\beta}_{1,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{m_W,1}^{(V\to W)} + i\ \hat{\beta}_{m_W,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{m_W,\mathfrak{n}}^{(V\to W)} + i\ \hat{\beta}_{m_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_W,1}^{(V\to W)} + i\ \hat{\beta}_{\mathcal{M}_W,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{M}_W,\mathfrak{n}}^{(V\to W)} + i\ \hat{\beta}_{\mathcal{M}_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \beta_{\mathcal{M}_W+1,1}^{(V\to W)} + i\ \hat{\beta}_{\mathcal{M}_W+1,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{M}_W+1,\mathfrak{n}}^{(V\to W)} + i\ \hat{\beta}_{\mathcal{M}_W+1,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_W+\ell_W,1}^{(V\to W)} + i\ \hat{\beta}_{\mathcal{M}_W+\ell_W,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{M}_W+\ell_W,\mathfrak{n}}^{(V\to W)} + i\ \hat{\beta}_{\mathcal{M}_W+\ell_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_W+\mathcal{L}_W,1}^{(V\to W)} + i\ \hat{\beta}_{\mathcal{M}_W+\mathcal{L}_W,1}^{(W\rightsquigarrow W)} & \cdots & \beta_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{n}}^{(V\to W)} + i\ \hat{\beta}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \end{pmatrix}}_{z_1=\mathbb{S}_{V\to W}+i\widehat{\mathbb{S}}_{W\to W}}, \underbrace{\begin{pmatrix} \phi_{1,1}^{(V\to W)} + i\ \hat{\phi}_{1,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{1,\mathfrak{m}}^{(V\to W)} + i\ \hat{\phi}_{1,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \phi_{m_W,1}^{(V\to W)} + i\ \hat{\phi}_{m_W,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{m_W,\mathfrak{m}}^{(V\to W)} + i\ \hat{\phi}_{m_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_W,1}^{(V\to W)} + i\ \hat{\phi}_{\mathcal{M}_W,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{\mathcal{M}_W,\mathfrak{m}}^{(V\to W)} + i\ \hat{\phi}_{\mathcal{M}_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \phi_{\mathcal{M}_W+1,1}^{(V\to W)} + i\ \hat{\phi}_{\mathcal{M}_W+1,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{\mathcal{M}_W+1,\mathfrak{m}}^{(V\to W)} + i\ \hat{\phi}_{\mathcal{M}_W+1,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_W+\ell_W,1}^{(V\to W)} + i\ \hat{\phi}_{\mathcal{M}_W+\ell_W,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{\mathcal{M}_W+\ell_W,\mathfrak{m}}^{(V\to W)} + i\ \hat{\phi}_{\mathcal{M}_W+\ell_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_W+\mathcal{L}_W,1}^{(V\to W)} + i\ \hat{\phi}_{\mathcal{M}_W+\mathcal{L}_W,1}^{(W\rightsquigarrow W)} & \cdots & \phi_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{m}}^{(V\to W)} + i\ \hat{\phi}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \end{pmatrix}}_{\zeta_1=\mathbb{U}_{V\to W}+i\widehat{\mathbb{U}}_{W\to W}}\right),\right.$$

$$\left(\underbrace{\begin{pmatrix} \beta_{1,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{1,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{m_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{m_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{m_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{M}_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \beta_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{M}_V+1,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \beta_{\mathcal{M}_V+\mathcal{L}_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+\mathcal{L}_V,1}^{(V\rightsquigarrow V)} & \cdots & \beta_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{\beta}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \end{pmatrix}}_{z_2=\mathbb{S}_{W\to V}+i\widehat{\mathbb{S}}_{V\to V}}, \underbrace{\begin{pmatrix} \phi_{1,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{1,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \phi_{m_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{m_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{\mathcal{M}_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \phi_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{\mathcal{M}_V+1,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_V+\ell_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \phi_{\mathcal{M}_V+\mathcal{L}_V,1}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+\mathcal{L}_V,1}^{(V\rightsquigarrow V)} & \cdots & \phi_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{\phi}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \end{pmatrix}}_{\zeta_2=\mathbb{U}_{W\to V}+i\widehat{\mathbb{U}}_{V\to V}}\right),$$

$$\left(\underbrace{\begin{pmatrix} \beta'^{(V\to W)}_{1,1}+i\,\widehat{\beta}'^{(W\rightsquigarrow W)}_{1,1} & \cdots & \beta'^{(V\to W)}_{1,\mathfrak{n}}+i\,\widehat{\beta}'^{(W\rightsquigarrow W)}_{1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \beta'^{(V\to W)}_{m_W,1}+i\,\widehat{\beta}'^{(W\rightsquigarrow W)}_{m_W,1} & \cdots & \beta'^{(V\to W)}_{m_W,\mathfrak{n}}+i\,\widehat{\beta}'^{(W\rightsquigarrow W)}_{m_W,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \beta'^{(V\to W)}_{\mathcal{M}_W,1}+i\,\widehat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W,1} & \cdots & \beta'^{(V\to W)}_{\mathcal{M}_W,\mathfrak{n}}+i\,\widehat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W,\mathfrak{n}} \\ \beta'^{(V\to W)}_{\mathcal{M}_W+1,1}+i\,\widehat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & \cdots & \beta'^{(V\to W)}_{\mathcal{M}_W+1,\mathfrak{n}}+i\,\widehat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \beta'^{(V\to W)}_{\mathcal{M}_W+\ell_W,1}+i\,\widehat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \beta'^{(V\to W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}}+i\,\widehat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \beta'^{(V\to W)}_{\mathcal{M}_W+\mathcal{L}_W,1}+i\,\widehat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,1} & \cdots & \beta'^{(V\to W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{n}}+i\,\widehat{\beta}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{n}} \end{pmatrix}}_{z_3=\mathbb{S}'_{V\to W}+i\widehat{\mathbb{S}}'_{W\to W}}, \underbrace{\begin{pmatrix} \phi'^{(V\to W)}_{1,1}+i\,\widehat{\phi}'^{(W\rightsquigarrow W)}_{1,1} & \cdots & \phi'^{(V\to W)}_{1,\mathfrak{m}}+i\,\widehat{\phi}'^{(W\rightsquigarrow W)}_{1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \phi'^{(V\to W)}_{m_W,1}+i\,\widehat{\phi}'^{(W\rightsquigarrow W)}_{m_W,1} & \cdots & \phi'^{(V\to W)}_{m_W,\mathfrak{m}}+i\,\widehat{\phi}'^{(W\rightsquigarrow W)}_{m_W,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \phi'^{(V\to W)}_{\mathcal{M}_W,1}+i\,\widehat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W,1} & \cdots & \phi'^{(V\to W)}_{\mathcal{M}_W,\mathfrak{m}}+i\,\widehat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W,\mathfrak{m}} \\ \phi'^{(V\to W)}_{\mathcal{M}_W+1,1}+i\,\widehat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & \cdots & \phi'^{(V\to W)}_{\mathcal{M}_W+1,\mathfrak{m}}+i\,\widehat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \phi'^{(V\to W)}_{\mathcal{M}_W+\ell_W,1}+i\,\widehat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,1} & \cdots & \phi'^{(V\to W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}}+i\,\widehat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\ell_W,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \phi'^{(V\to W)}_{\mathcal{M}_W+\mathcal{L}_W,1}+i\,\widehat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,1} & \cdots & \phi'^{(V\to W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{m}}+i\,\widehat{\phi}'^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{m}} \end{pmatrix}}_{\zeta_3=\mathbb{U}'_{V\to W}+i\widehat{\mathbb{U}}'_{W\to W}}\right),$$

$$\left(\underbrace{\begin{pmatrix} \beta'^{(W\rightsquigarrow V)}_{1,1}+i\,\widehat{\beta}'^{(V\rightsquigarrow V)}_{1,1} & \cdots & \beta'^{(W\rightsquigarrow V)}_{1,\mathfrak{n}}+i\,\widehat{\beta}'^{(V\rightsquigarrow V)}_{1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \beta'^{(W\rightsquigarrow V)}_{m_V,1}+i\,\widehat{\beta}'^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & \beta'^{(W\rightsquigarrow V)}_{m_V,\mathfrak{n}}+i\,\widehat{\beta}'^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V,1}+i\,\widehat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V,1} & \cdots & \beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{n}}+i\,\widehat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{n}} \\ \beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+i\,\widehat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}}+i\,\widehat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1}+i\,\widehat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & \beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}+i\,\widehat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1}+i\,\widehat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1} & \cdots & \beta'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}}+i\,\widehat{\beta}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}} \end{pmatrix}}_{z_4=\mathbb{S}'_{W\to V}+i\widehat{\mathbb{S}}'_{V\to V}}, \underbrace{\begin{pmatrix} \phi'^{(W\rightsquigarrow V)}_{1,1}+i\,\widehat{\phi}'^{(V\rightsquigarrow V)}_{1,1} & \cdots & \phi'^{(W\rightsquigarrow V)}_{1,\mathfrak{m}}+i\,\widehat{\phi}'^{(V\rightsquigarrow V)}_{1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \phi'^{(W\rightsquigarrow V)}_{m_V,1}+i\,\widehat{\phi}'^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & \phi'^{(W\rightsquigarrow V)}_{m_V,\mathfrak{m}}+i\,\widehat{\phi}'^{(V\rightsquigarrow V)}_{m_V,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V,1}+i\,\widehat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V,1} & \cdots & \phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{m}}+i\,\widehat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{m}} \\ \phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+i\,\widehat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}}+i\,\widehat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1}+i\,\widehat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & \phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}+i\,\widehat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ \phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1}+i\,\widehat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1} & \cdots & \phi'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}}+i\,\widehat{\phi}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}} \end{pmatrix}}_{\zeta_4=\mathbb{U}'_{W\to V}+i\widehat{\mathbb{U}}'_{V\to V}}\right)\right).$$

Remark 7.3. The key sets

$$\Omega_V = \Omega_{[(V,W)\rightsquigarrow V]}([0,1]) \text{ and } \Omega_W = \Omega_{[(V,W)\rightsquigarrow W]}([0,1])$$

of (extended or not) supervisory perceptions of two cyber nodes V and W into the system of themselves, that are used in critical definitions given up to now, are subsets of the product spaces

$$\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}} \text{ and } (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{n}} \times (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{m}} = (\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{n}} \times (\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{m}}.$$

The spaces $\mathbb{C}^{\mathcal{N}\times\mathfrak{n}}$ and $\mathbb{C}^{\mathcal{N}\times\mathfrak{m}}$ will be called complex multi-coordinate spaces. Each element of a complex multi-coordinate space $\mathbb{C}^{\mathcal{N}\times\nu}$ is of the form

$$\left(z^{(1)}, \dots, z^{(\nu)}\right)$$

with $z^{(r)} = \left(z_1^{(r)}, \dots, z_{\mathcal{N}}^{(r)}\right)^T \in \mathbb{C}^{\mathcal{N}}$. Similarly, the spaces $(\mathbb{CP}^{\mathcal{N}})^{\mathfrak{n}} = (\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{n}}$ and $(\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{m}} = (\mathbb{CP}^{\mathcal{N}})^{\mathfrak{m}}$ are called complex multi-projective spaces. Each element of a complex multi-projective space $(\mathbb{CP}^1)^{\mathcal{N}\times\nu} = (\mathbb{CP}^{\mathcal{N}})^{\nu}$ has the form

$$\left(\zeta^{(1)}, \dots, \zeta^{(\nu)}\right)$$

with $\zeta^{(r)} = \left(\zeta_1^{(r)}, \dots, \zeta_{\mathcal{N}}^{(r)}\right)^T \in \mathbb{CP}^{\mathcal{N}}$.

Below, for terminology consolidation purposes, we will prefer not make any distinction between the spaces $\mathbb{C}^{\mathcal{N}\times\mathfrak{n}}$ and $(\mathbb{CP}^1)^{\mathcal{N}\times\mathfrak{n}}$, and we will call them using the common name complex multi-spaces. As usually, if there is no risk of confusion, the complex multi-spaces may also be represented using the common notation

$$\mathbb{CM}^{\mathcal{N}\times\mathfrak{n}}.$$

On the other hand, by Definition 1.8, we are also interested for the twofold Cartesian products of complex multi spaces. In fact, each momentary cyber interaction $\mathcal{g}$ can be considered as a correspondence derived from a map transforming a subset $\mathcal{D}$ of the twofold Cartesian product $\mathbb{CM}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}$ of complex multi-spaces within its own self:

$$\mathcal{g}\colon \mathcal{D}(\subset \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}) \to \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}:$$

$$\left(\begin{pmatrix} z_1^{(1)} & \cdots & z_1^{(\mathfrak{n})} \\ \vdots & \cdots & \vdots \\ z_{\mathcal{N}}^{(1)} & \cdots & z_{\mathcal{N}}^{(\mathfrak{n})} \end{pmatrix}, \begin{pmatrix} \zeta_1^{(1)} & \cdots & \zeta_1^{(\mathfrak{m})} \\ \vdots & \cdots & \vdots \\ \zeta_{\mathcal{N}}^{(1)} & \cdots & \zeta_{\mathcal{N}}^{(\mathfrak{m})} \end{pmatrix}\right) \longmapsto$$

$$\mathcal{g}\left(\begin{pmatrix} z_1^{(1)} & \cdots & z_1^{(\mathfrak{n})} \\ \vdots & \cdots & \vdots \\ z_{\mathcal{N}}^{(1)} & \cdots & z_{\mathcal{N}}^{(\mathfrak{n})} \end{pmatrix}, \begin{pmatrix} \zeta_1^{(1)} & \cdots & \zeta_1^{(\mathfrak{m})} \\ \vdots & \cdots & \vdots \\ \zeta_{\mathcal{N}}^{(1)} & \cdots & \zeta_{\mathcal{N}}^{(\mathfrak{m})} \end{pmatrix}\right) =$$

$$\left(\begin{pmatrix} z'^{(1)}_1 & \cdots & z'^{(\mathfrak{n})}_1 \\ \vdots & \cdots & \vdots \\ z'^{(1)}_{\mathcal{N}} & \cdots & z'^{(\mathfrak{n})}_{\mathcal{N}} \end{pmatrix}, \begin{pmatrix} w'^{(1)}_1 & \cdots & w'^{(\mathfrak{m})}_1 \\ \vdots & \cdots & \vdots \\ w'^{(1)}_{\mathcal{N}} & \cdots & w'^{\mu}_{\mathcal{N}} \end{pmatrix}\right).$$

Such a mapping will be called (complex) twofold multi-mapping. In particular, a cyber-navigation is a chain of twofold multi-mappings ([10]).

8 Coherent Interactive Families

We now intend to look at the areas in which occurs an increase or decrease in cyber-valuations and/or cyber-vulnerabilities during a interplay of the cyber pair (V, W) over the time $t \in]\alpha, \beta[\subset\subset [0,1]$. Under this approach, we will see when an interaction is evolving into an attack.

For simplification purposes, we will limit ourselves only to the case where $\mathbb{CM} = \mathbb{C}$. A study of the general case will remain open.

In the finite case, we will distinguish two cases. The first case deals with interactions occurring in parts of interacting nodes, while the second case refers to interactions that are assumed throughout entire nodes To this end, suppose $X, Y \in \{V, W\}$ and $\mathscr{r} > 0$. Let $fr\left(dev_{\mu_1}^{(X)}\right)$,…, $fr\left(dev_{\mu_\nu}^{(X)}\right)$ be given $(\mu_1, \dots, \mu_\nu) -$ device parts in X. Let also $fr\left(res_{\kappa_1}^{(X)}\right)$,,…, $fr\left(res_{\kappa_\lambda}^{(X)}\right)$ be given $(\kappa_1, \dots, \kappa_\lambda) -$ resource parts in X. Let finally $\mathbb{I}$ be a given set into the time subinterval $]\alpha, \beta[\subset\subset [0,1]$. We need to introduce a certain terminology. A family of interactions $\mathcal{F} = \{Z = Z_{(Y,X)}(t) = \left((z_1, \zeta_1), (z_2, \zeta_2), (z_3, \zeta_3), (z_4, \zeta_4)\right) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4, t \in \mathbb{I}\}$, with associated family of cyber-interplays of the ordered cyber pair (Y, X) over the time $t \in]\alpha, \beta[$

$$\mathcal{D}_{\mathcal{F}} = \Big\{\mathscr{g} = \mathscr{g}^{(Z)}: \mathbb{I} \to \Omega_Y \times \Omega_X \times \Omega_Y \times \Omega_X:$$
$$t \mapsto \mathscr{g}^{(Z)}(t) := \left(\gamma_Y^{(Z)}(t), \gamma_X^{(Z)}(t), \gamma_Y^{(Z)}(t + \Delta t), \gamma_X^{(Z)}(t + \Delta t)\right) : Z \in \mathcal{F}\Big\},$$

is called coherent interactive family in $\mathbb{I}$, if there is a homotopy

$$H: \mathbb{I} \times [0,1] \to \Omega_Y \times \Omega_X \times \Omega_Y \times \Omega_X$$

such that, for each cyber-interplay $\mathscr{g} = \mathscr{g}^{(Z)} \in \mathcal{D}_{\mathcal{F}}$ there is a $p \in [0,1]$ satisfying $H(t, p) = \mathscr{g}(t)$ at any moment time $t \in \mathbb{I}$ on which the cyber-interplay $\mathscr{g} = \mathscr{g}^{(Z)}$ implements the interaction Z. Recall that, in topology, two continuous functions from one topological space to another are called homotopic (Greek ὁμός (homós) = same, similar, and τόπος (tópos) = place) if one can be "continuously deformed" into the other, such a deformation being called a homotopy between the two functions. Formally, a homotopy between two continuous functions f and g from a topological space U to a topological space V is defined to be a continuous function $H : U \times [0,1] \to V$ from the product of the space U with the unit interval $[0,1]$ to V such that, if $x \in U$ then

$$H(x,0) = f(x) \text{ and } H(x,1) = g(x).$$

9 Subjectivity in Interactive Variations Germs of Cyber Attacks

9.1 Germs of Correlated Cyber-Attacks

Often, outside the objectivity of evaluating cyber attacks, there is also a subjective approach which sometimes can give very strong arguments in assessing the reality. In this direction, in this section, we will propose several definitions and cases for an alternate consideration based on the *subjectivity* of the users of the involved nodes. We point out that, in the following definitions, the foundation adopted was based exclusively on the Euclidean norms. However, this is not restrictive, and we can consider any other norm in place in $\mathbb{R}^{\mathfrak{n}}$ and $\mathbb{R}^{\mathfrak{m}}$.

Let us begin with the case of valuation variations relative to the norm valuation and the subjectivity of user(s) of another or same node.

Definition 9.1. *Let again* $\mathbb{I}$ *be any given set in the time subinterval* $]\alpha,\beta[\subset\subset [0,1]$. *Let also* $X,Y \in \{V,W\}$.

i. *The area* $[\mathcal{A}_Y^-(X)](\mathbb{I})$ *of correlated reduction of total valuation for node* X *as evaluated subjectively from the user(s) of* Y *over the time set* $\mathbb{I}$, *is the family of coherent interactions* $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\|Rez_4\| = \|\beta'^{(Y \rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left|\beta'^{(Y \rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the next moment* t' *is less than the (Euclidean) norm* $\|Rez_2\| = \|\beta^{(Y \rightsquigarrow X)}\| :=$

$\left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\beta_{\lambda,j}^{(Y\rightsquigarrow X)}\right|^2\right)^{1/2}$ *of the initial overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the preceding moment* t*:*

$$\|Rez_4\| = \left\| \beta'^{(Y\rightsquigarrow X)} \right\| < \left\|\beta^{(Y\rightsquigarrow X)}\right\| = \|Rez_2\|.$$

If the difference $\|Rez_2\| - \|Rez_4\|$ *exceeds a given valuation danger threshold for node* X *as evaluated by the user(s) of* Y, *we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{A}_Y^-(X)](\mathbb{I})$ *are evaluated as subjectively damaging for* X *from the viewpoint of* Y.

ii. *The area* $[\mathcal{A}_X^-(X)](\mathbb{I})$ *of correlated reduction of total valuation for node* X *as assessed subjectively by themselves the user(s) of node* X *over the time set* $\mathbb{I}$, *is the family of coherent interactions* $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\|Imz_4\| = \left\| \hat{\beta}'^{(X\rightsquigarrow X)} \right\| :=$ $\left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\hat{\beta}'^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall valuation in the node* X *as assessed by themselves the user(s) of* X *at the next moment* t' *is less than the (Euclidean) norm* $\|Imz_2\| = \left\|\hat{\beta}^{(X\rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\hat{\beta}_{\lambda,j}^{(X\rightsquigarrow X)}\right|^2\right)^{1/2}$ *of the initial overall valuation in the node* V *as assessed by themselves the user(s) of* V *at the preceding moment* t*:*

$$\|Imz_4\| = \left\| \hat{\beta}'^{(X\rightsquigarrow X)} \right\| < \left\|\hat{\beta}^{(X\rightsquigarrow X)}\right\| = \|Imz_2\|.$$

If the difference $\|Imz_2\| - \|Imz_4\|$ *exceeds a given valuation danger threshold for node* X *as assessed by themselves the user(s) of* X, *we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{A}_X^-(X)](\mathbb{I})$ *are evaluated as reflexively damaging from the viewpoint of* X.

iii. *The area* $[\mathcal{A}_Y^+(X)](\mathbb{I})$ *of correlated growth of total valuation for node* X *as evaluated subjectively from the user(s) of* Y *over the time set* $\mathbb{I}$, *is the family of coherent interactions* $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in$

$(\mathbb{C}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\|Rez_4\| = \|\beta'^{(Y\rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\beta'^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the next moment* t' *is greater than the (Euclidean) norm* $\|Rez_2\| = \|\beta^{(Y\rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\beta^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the initial overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the preceding moment* t:

$$\|Rez_4\| = \|\beta'^{(Y\rightsquigarrow X)}\| > \|\beta^{(Y\rightsquigarrow X)}\| = \|Rez_2\|.$$

If the difference $\|Rez_4\| - \|Rez_2\|$ *exceeds a given valuation benefit limit for node* X *as evaluated by the user(s) of* Y, *we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{A}_Y^+(X)](\mathbb{I})$ *are evaluated as subjectively advantageous for* X *from the viewpoint of* Y.

iv. *The area* $[\mathcal{A}_X^+(X)](\mathbb{I})$ *of correlated growth of total valuation for node* X *as evaluated subjectively from the user(s) of* X *over the time set* $\mathbb{I}$, *is the family of coherent interactions* $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\|Imz_4\| = \left\|\hat{\beta}'^{(X\rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\hat{\beta}'^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* X *at the next moment* t' *is greater than the (Euclidean) norm* $\|Imz_2\| = \|\hat{\beta}^{(X\rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\hat{\beta}^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the initial overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the preceding moment* t:

$$\|Imz_4\| \left\|\hat{\beta}'^{(X\rightsquigarrow X)}\right\| > \|\hat{\beta}^{(X\rightsquigarrow X)}\| = \|Imz_2\|.$$

If the difference $\|Imz_4\| - \|Imz_2\|$ *exceeds a given valuation danger threshold for node* V *as assessed by themselves the user(s) of* V, *we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{A}_X^+(X)](\mathbb{I})$ *are evaluated as reflexively* *advantageous from the viewpoint of* X.

Similar considerations apply to the vulnerability variations relative only to the user(s) of another or the same node.

Definition 9.2 *Let again* $\mathbb{I}$ *be any given subset of the time interval* $]\alpha,\beta[\subset\subset[0,1]$. *Let also* $X,Y\in\{V,W\}$.

i *The area* $[\mathcal{B}_Y^-(X)](\mathbb{I})$ *of correlated reduction of total vulnerability for node* X *as evaluated subjectively from the viewpoint of the user(s) of* Y *over the time set* $\mathbb{I}$ *is the family of coherent interactions* $Z=Z_{(Y,X)}(t)=\big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big)\in(\mathbb{C}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\|Re\zeta_4\|=\|\phi'^{(Y\rightsquigarrow X)}\|:=\left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\phi'^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the next moment* t' *is less than the (Euclidean) norm* $\|Re\zeta_2\|=\|\phi^{(Y\rightsquigarrow X)}\|:=\left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\phi^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the initial overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the preceding moment* t:

$$\|Re\zeta_4\|=\|\phi'^{(Y\rightsquigarrow X)}\|<\|\phi^{(Y\rightsquigarrow X)}\|=\|Re\zeta_2\|.$$

If the difference $\|Re\zeta_2\|-\|Re\zeta_4\|$ *exceeds a given vulnerability danger threshold for node* X *as evaluated by the user(s) of* Y, *we say that the interactions* $Z=Z_{(Y,X)}(t)$ *of* $[\mathcal{B}_Y^-(X)](\mathbb{I})$ *are evaluated as subjectively painless for* X *from the viewpoint of* Y.

ii *The area* $[\mathcal{B}_Y^+(X)](\mathbb{I})$ *of correlated growth of total vulnerability for node* X *as evaluated subjectively from the viewpoint of the user(s) of* Y *over the time set* $\mathbb{I}$ *is the family of coherent interactions* $Z=Z_{(Y,X)}(t)=\big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big)\in(\mathbb{C}^{\mathcal{N}\times\mathfrak{n}}\times\mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\|Re\zeta_4\|=\|\phi'^{(Y\rightsquigarrow X)}\|:=$

$\left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\phi'^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the next moment* t' *is greater than the (Euclidean) norm* $\|Re\zeta_2\| = \|\phi^{(Y\rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\phi^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the initial overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the preceding moment* t*:*

$$\|Re\zeta_4\| = \|\phi'^{(Y\rightsquigarrow X)}\| > \|\phi^{(Y\rightsquigarrow X)}\| = \|Re\zeta_2\|.$$

If the difference $\|Re\zeta_4\| - \|Re\zeta_2\|$ *exceeds a given vulnerability benefit limit for node* X *as evaluated by the user(s) of* Y, *we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{B}_Y^+(X)](\mathbb{I})$ *are evaluated as subjectively painful for* X *from the viewpoint of* Y.

iii *The area* $[\mathcal{B}_X^-(X)](\mathbb{I})$ *of correlated reduction of total vulnerability for node* X *as assessed subjectively by themselves the user(s) of node* X *over the time set* $\mathbb{I}$ *, is the family of coherent interactions* $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\|Im\zeta_4\| = \left\|\hat{\phi}'^{(X\rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\hat{\phi}'^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall vulnerability in the node* X *as assessed by themselves the user(s) of* X *at the next moment* t' *is less than the (Euclidean) norm* $\|Im\zeta_2\| = \|\hat{\phi}^{(X\rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{m}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\hat{\phi}^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the initial overall vulnerability in the node* X *as assessed by themselves the user(s) of* X *at the preceding moment* t*:*

$$\|Im\zeta_4\| = \left\|\hat{\phi}'^{(X\rightsquigarrow X)}\right\| < \|\hat{\phi}^{(X\rightsquigarrow X)}\| = \|Im\zeta_2\|.$$

If the difference $\|Im\zeta_2\| - \|Im\zeta_4\|$ *exceeds a given vulnerability danger threshold for node* X *as assessed by themselves the user(s) of* X, *we say that the* *interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{B}_X^-(X)](\mathbb{I})$ *are evaluated as subjectively painless for* X *from the viewpoint of* X *itself.*

iv *The area* $[\mathcal{B}_X^+(X)](\mathbb{I})$ *of correlated growth of total vulnerability for node* X *as evaluated subjectively from the viewpoint of the user(s) of* X *over the time set* $\mathbb{I}$ *, is the family of coherent interactions* $Z = Z_{(Y,X)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for which the (Euclidean) norm* $\|Im\zeta_4\| = \left\| \hat{\phi}'^{(X\rightsquigarrow X)} \right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left|\hat{\phi}'^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* X *at the next moment* t' *is greater than the (Euclidean) norm* $\|Im\zeta_2\| = \left\|\hat{\phi}^{(X\rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\hat{\phi}^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the initial overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the preceding moment* t*:*

$$\|Im\zeta_4\| \left\| \hat{\phi}'^{(X\rightsquigarrow X)} \right\| > \left\|\hat{\phi}^{(X\rightsquigarrow X)}\right\| = \|Im\zeta_2\|.$$

If the difference $\|Imz_4\| - \|Imz_2\|$ *exceeds a given vulnerability danger threshold for node* X *as assessed by themselves the user(s) of* X*, we say that the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{B}_X^+(X)](\mathbb{I})$ *are evaluated as subjectively painful for* X *from the viewpoint of* X *itself.*

Definition 9.3. *A germ of correlated cyber attack from* W *against* V*, during a given time set* $\mathbb{I}$ *in* $]\alpha,\beta[\subset\subset [0,1]$*, is a family of* <u>*coherent*</u> *interactions* $Z = Z_{(W,V)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *,* $t \in \mathbb{I}$ *, lying in the so called correlated danger sector* $\mathfrak{X} = \mathfrak{X}_{W\to V}(\mathbb{I})$ *to the node* V *from the node* W *during the entire time set* $\mathbb{I}$*, defined by intersection*

$$\mathfrak{X} = \big\{\big([\mathcal{A}_W^-(V)](\mathbb{I})\cap[\mathcal{A}_V^-(V)](\mathbb{I})\big)\cap\big([\mathcal{A}_V^+(W)](\mathbb{I})\cap[\mathcal{A}_W^+(W)](\mathbb{I})\big)\cap \big([\mathcal{B}_V^-(W)](\mathbb{I})\cap[\mathcal{B}_W^-(W)](\mathbb{I})\big)\cap\big([\mathcal{B}_V^+(V)](\mathbb{I})\cap[\mathcal{B}_W^+(V)](\mathbb{I})\big)\big\},$$

provided, of course, that $\mathfrak{X} \neq \emptyset$. *If each one of the coherent interactions* $Z_{(W,V)}(t)$ *is elementary, we say that the germ is elementary; otherwise, it is called sequential or complex. If* $\mathbb{I} = \{t_0\}$ *for some* $t_0 \in]0,1[$, *the germ is called momentary.*

Definition 9.4. *The node* V *is said to be affine secure from attacks of* W *during the time set* $\mathbb{I}$ *if* $\mathfrak{X} = \emptyset$.

Definition 9.5. *More generally, an affine secure area of* V *from the correlated cyber attacks of* W *during the time set* $\mathbb{I}$ *is any set in the complementary* $\mathfrak{X}^{C}$ *in* $(\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *of* $\mathfrak{X}$.

9.2 Germs of Absolute Cyber-Attacks

Next, we consider the case of valuation variations relative only to the user(s) of another or the same node and independently of the valuation variations of this node.

Definition 9.6 *Let* $\mathbb{I}$ *be any given set in the time subinterval* $]\alpha,\beta[\subset\subset [0,1]$. *Let also* $X, Y \in \{V, W\}$.

i *The area* $\left[\tilde{A}_Y^{-}(X)\right](\mathbb{I})$ *of absolute reduction of total valuation for node* X *as evaluated subjectively from the user(s) of* Y *over the time set* $\mathbb{I}$, *is the family of coherent interactions* $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\| \beta'^{(Y \rightsquigarrow X)} \| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left| \beta'^{(Y \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ *of the resulting overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at a next moment* t' *is less than a given threshold* $\mathcal{C}$:

$$\| \beta'^{(Y \rightsquigarrow X)} \| < \mathcal{C}.$$

The number $\mathcal{C}$ *is called extensibility radius of total valuation reduction in* X *from the viewpoint of* Y. *If the extensibility radius* $\mathcal{C}$ *is less than a given* *valuation damage threshold* $\mathcal{V}a\ell_Y(X)$, *we say that* $[\tilde{A}_Y^-(X)](\mathbb{I})$ *is an area of absolute danger for* X *as evaluated subjectively by the user(s) of* Y.

ii *The area* $[\tilde{A}_X^-(X)](\mathbb{I})$ *of absolute reduction of total valuation for node* X *as assessed subjectively by themselves the user(s) of node* X *themselves over the time set* $\mathbb{I}$, *s the family of coherent interactions* $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\|\beta'^{(X\rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\hat{\beta}'^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* X *at the next moment* t' *is less than a threshold* $\mathcal{C}$:

$$\left\| \hat{\beta}'^{(X\rightsquigarrow X)} \right\| < \mathcal{C}.$$

The number $\mathcal{C}$ *is called extensibility radius of the total valuation reduction in* X *from the viewpoint of* X *itself. If this extensibility radius* $\mathcal{C}$ *is less than a given valuation damage threshold* $\mathcal{V}a\ell_X(X)$, *we say that* $[\tilde{A}_X^-(X)](\mathbb{I})$ *is an area of absolute danger of node* X *as evaluated subjectively by the user(s) of* X.

iii *The area* $[\tilde{A}_Y^+(X)](\mathbb{I})$ *of absolute growth of total valuation for node* X *as evaluated subjectively from the user(s) of* Y *over the time set* $\mathbb{I}$, *is the family of coherent interactions* $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for which the (Euclidean) norm* $\| \beta'^{(Y\rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{n}}\sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X}\left|\beta'^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall valuation in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the next moment* t' *is greater that a threshold* $\mathcal{C}$:

$$\| \beta'^{(Y\rightsquigarrow X)}\| > \mathcal{C}.$$

The number $\mathcal{C}$ *is called extensibility radius of the total valuation growth in* X *from the viewpoint of* Y. *If this extensibility radius* $\mathcal{C}$ *is greater than a given valuation benefit limit* $\mathcal{B}e\mathcal{n}\mathcal{L}i\mathcal{m}_Y(X)$, *we say that* $[\tilde{A}_Y^+(X)](\mathbb{I})$ *is an area of absolute security of node* X *as evaluated subjectively by the user(s) of* Y.

iv *The area* $[\tilde{A}_X^+(X)](\mathbb{I})$ *of absolute growth of total valuation for node* X *as* *evaluated subjectively from the user(s) of* X *themselves over the time set* $\mathbb{I}$*, is the family of coherent interactions* $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$ *, for which the (Euclidean) norm* $\left\| \hat{\beta}'^{(X \rightsquigarrow X)} \right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left| \hat{\beta}'^{(X \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ *of the resulting overall valuation in node* X *as evaluated from the viewpoint of the user(s) of* X *at the next moment* t' *is greater that a threshold* $\mathcal{C}$*:*

$$\left\| \hat{\beta}'^{(X \rightsquigarrow X)} \right\| > \mathcal{C}.$$

The number $\mathcal{C}$ *is called extensibility radius of the total valuation growth in* X *from the viewpoint of* X *itself. If this extensibility radius* $\mathcal{C}$ *is greater than a given valuation benefit limit* $\mathcal{BenLim}_X(X)$*, we say that* $\left[\tilde{A}_X^+(X)\right](\mathbb{I})$ *is an area of absolute security of node* X *as evaluated subjectively from the viewpoint of* X *itself.*

Next, we consider the case of valuation variations relative only to the user(s) of another or the same node and independently of the valuation variations of this node.

Definition 9.7 *Let again* $\mathbb{I}$ *be a given set in the time subinterval* $]\alpha, \beta[\subset\subset [0,1]$*. Let also* $X, Y \in \{V, W\}$*.*

i *The area* $[\tilde{B}_Y^-(X)](\mathbb{I})$ *of absolute reduction of total vulnerability for node* X *as evaluated subjectively from the viewpoint of the user(s) of* Y *over the time set* $\mathbb{I}$*, is the family of coherent interactions* $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for which the (Euclidean) norm* $\left\| \phi'^{(Y \rightsquigarrow X)} \right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_Y+\mathcal{L}_X} \left| \phi'^{(Y \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ *of the resulting overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the next moment* t' *is less than a given threshold* $\mathcal{C}$*:*

$$\left\| \phi'^{(Y \rightsquigarrow X)} \right\| < \mathcal{C}.$$

The number $\mathcal{C}$ *is called extensibility radius of the total vulnerability reduction in* X *from the viewpoint of* Y*. If this extensibility radius* $\mathcal{C}$ *is less than a given vulnerability benefit limit* $\widetilde{\mathcal{BenLim}}_Y(X)$*, we say that* $[\tilde{B}_Y^-(X)](\mathbb{I})$ *is a secure area for node* X *as evaluated subjectively from the user(s) of* Y*.*

ii *The area* $[\tilde{B}_Y^+(X)](\mathbb{I})$ *of absolute growth of total vulnerability for node* X *as evaluated subjectively from the viewpoint of the user(s) of* Y *over the time set* $\mathbb{I}$*, is the family of coherent interactions* $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for which the (Euclidean) norm* $\|\phi'^{(Y\rightsquigarrow X)}\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left|\phi'^{(Y\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* Y *at the next moment* t' *is greater that a threshold* $\mathcal{C}$*:*

$$\|\phi'^{(Y\rightsquigarrow X)}\| > \mathcal{C}.$$

The number $\mathcal{C}$ *is called extensibility radius of the total vulnerability growth in* X*from the viewpoint of* Y*. If this extensibility radius* $\mathcal{C}$ *is greater than a given vulnerability damaging threshold* $\mathcal{Vul}_Y(X)$ *for node* X *as evaluated by the user(s) of* Y*, we say that* $[\tilde{B}_Y^+(X)](\mathbb{I})$ *is a damaging area for* X *from the viewpoint of* Y*.*

iii *The area* $[\tilde{B}_X^-(X)](\mathbb{I})$ *of absolute reduction of total vulnerability for node* X *as assessed subjectively by the user(s) of node* X *themselves over the time set* $\mathbb{I}$*, is the family of coherent interactions* $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for which the (Euclidean) norm* $\left\|\hat{\phi}'^{(X\rightsquigarrow X)}\right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_X+\mathcal{L}_X} \left|\hat{\phi}'^{(X\rightsquigarrow X)}_{\lambda,j}\right|^2\right)^{1/2}$ *of the resulting overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* X *at the next moment* t' *is less than a threshold* $\mathcal{C}$*:*

$$\left\|\hat{\phi}'^{(X\rightsquigarrow X)}\right\| < \mathcal{C}.$$

The number $\mathcal{C}$ *is called extensibility radius of total vulnerability reduction in* X

from the viewpoint of X *itself. If this extensibility radius* $\mathcal{C}$ *is less than a given vulnerability benefit* $\widetilde{\mathcal{BenLim}}_X(X)$*, we say that* $[\tilde{B}_Y^-(X)](\mathbb{I})$ *is a subjectively secure area for* X*.*

iv *The area* $[\tilde{B}_X^+(X)](\mathbb{I})$ *of absolute growth of total vulnerability for node* X *as evaluated subjectively from the viewpoint of the user(s) of* V *themselves over the time set* $\mathbb{I}$*, is the family of coherent interactions* $Z = Z_{(Y,X)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$*, for which the (Euclidean) norm* $\left\| \hat{\phi}'^{(X \rightsquigarrow X)} \right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\mathcal{M}_X + \mathcal{L}_X} \left| \hat{\phi}'^{(X \rightsquigarrow X)}_{\lambda,j} \right|^2 \right)^{1/2}$ *of the resulting overall vulnerability in the node* X *as evaluated from the viewpoint of the user(s) of* X *at the next moment* t' *is greater that a threshold* $\mathcal{C}$*:*

$$\left\| \hat{\phi}'^{(X \rightsquigarrow X)} \right\| > \mathcal{C}.$$

The number $\mathcal{C}$ *is called extensibility radius of the total vulnerability growth in* V *from the viewpoint of* X *itself. If this extensibility radius* $\mathcal{C}$ *is greater than a given vulnerability damaging threshold* $\mathcal{Vul}_X(X)$ *for node* X *as evaluated by the user(s) of* X *themselves, we say that* $[\tilde{B}_X^+(X)](\mathbb{I})$ *is a subjectively damaging area for* X*.*

Definition 9.8. *A germ of absolute cyber attack from* W *against* V*, during a given time set* $\mathbb{I}$ *in the subinterval* $]\alpha, \beta[\subset\subset [0,1]$*, is a family of* <u>*coherent*</u> *interactions* $Z = Z_{(W,V)}(t) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$*,* $t \in \mathbb{I}$*, lying in the so called absolute danger sector* $\widetilde{\mathfrak{X}} = \widetilde{\mathfrak{X}}_{W \to V}$ *to the node* V *from the node* W *during the entire time set* $\mathbb{I}$*, defined by intersection*

$$\widetilde{\mathfrak{X}} = \Big\{ \Big(\big[\tilde{A}_W^-(V)\big](\mathbb{I}) \cap \big[\tilde{A}_V^-(V)\big](\mathbb{I}) \Big) \cap \Big(\big[\tilde{A}_V^+(W)\big](\mathbb{I}) \cap \big[\tilde{A}_W^+(W)\big](\mathbb{I}) \Big) \cap \Big(\{[\tilde{B}_V^-(W)](\mathbb{I}) \cap [\tilde{B}_W^-(W)](\mathbb{I})\} \Big) \cap \Big([\tilde{B}_V^+(V)](\mathbb{I}) \cap [\tilde{B}_W^+(V)](\mathbb{I}) \Big) \Big\}.$$

provided, of course, that $\widetilde{\mathfrak{X}} \neq \emptyset$*. If each one of the coherent interactions* $Z_{(W,V)}(t)$ *is elementary, we say that the germ is said to be elementary; otherwise,* *it is called sequential or complex. If* $\mathbb{I} = \{t_0\}$ *for some* $t_0 \in]\alpha, \beta[$, *the germ is called momentary*.

Definition 9.9. *The node* V *is absolutely secure from cyber attacks of* W *during the time set* $\mathbb{I}$ *if* $\tilde{\mathfrak{X}} = \emptyset$.

Definition 9.10. *And, more generally, an absolutely secure area for node* V *from cyber attacks of* W *during the time set* $\mathbb{I}$ *is any set in the complementary* $\tilde{\mathfrak{X}}^C$ *in* $(\mathbb{C}^{\mathcal{N}\times\mathrm{n}} \times \mathbb{C}^{\mathcal{N}\times\mathrm{m}})^4$ *of* $\tilde{\mathfrak{X}}$.

9.3 Germs of partial cyber-attacks

It is known that cyber attacks carried out in a targeted or oriented manner against specific parts of particular devices or against specific parts of particular resources. So, in this section, we will consider the case of partial interactions, i.e., of cyber interactions between parts of some devices or resources cyber two nodes. To do this, let's again

$$X, Y \in \{V, W\} \text{ and } \mathscr{r} > 0.$$

Let also $(\mu_1, \dots, \mu_\nu) -$ device parts, say

$$fr\left(dev_{\mu_1}^{(X)}\right),\ fr\left(dev_{\mu_2}^{(X)}\right), \dots, fr\left(dev_{\mu_\nu}^{(X)}\right)$$

of X, and $(\kappa_1, \dots, \kappa_\lambda) -$ resource parts, say

$$fr\left(res_{\kappa_1}^{(X)}\right),\ fr\left(res_{\kappa_2}^{(X)}\right), \dots, fr\left(res_{\kappa_\lambda}^{(X)}\right)$$

of X. Let finally $\mathbb{I}$ be a given set in the time subinterval $]\alpha, \beta[\subset\subset [0,1]$.

Definition 9.10. *Let* $\mathbb{I}$ *be a given set in the time subinterval* $]\alpha, \beta[\subset\subset [0,1]$. *Let also* $X, Y \in \{V, W\}$.

i. *The region* $[\mathcal{R}_{\mathbb{S}-}(X)](\mathbb{I})$ *(or simply denoted by* $[\mathcal{R}_1(X)](\mathbb{I})$*) of partial valuation reduction of node* V *as evaluated subjectively from the viewpoint of the user(s) of* W *over the time set* $\mathbb{I}$, *with extensiveness radius* $\mathscr{r} > 0$, *in the*

$(\mu_1, \dots, \mu_\nu) -$ *device parts* $fr\left(dev_{\mu_1}^{(X)}\right)$, $fr\left(dev_{\mu_2}^{(X)}\right)$,…, $fr\left(dev_{\mu_\nu}^{(X)}\right)$ *of* X *and the* $(\kappa_1, \dots, \kappa_\lambda) -$ *resource parts* $fr\left(res_{\kappa_1}^{(X)}\right)$, $fr\left(res_{\kappa_2}^{(X)}\right)$,…, $fr\left(res_{\kappa_\lambda}^{(X)}\right)$ *of* X *is the set of all coherent interactions* $Z = Z_{(W,V)}(t) = \left((z_1, \zeta_1), (z_2, \zeta_2), (z_3, \zeta_3), (z_4, \zeta_4)\right) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for each of which the corresponding index- set:*

$$\Big\{j \in \{\mu_1, \dots, \mu_\nu, \kappa_1, \dots, \kappa_\lambda\}: \sum_{k=1}^{\mathfrak{n}} \left|\beta_{j,k}^{(Y \rightsquigarrow X)} + i\ \hat{\beta}_{j,k}^{(Y \rightsquigarrow X)}\right|^2 > \sum_{k=1}^{\mathfrak{n}} \left|\beta'^{(Y \rightsquigarrow X)}_{j,k} + i\ \widehat{\beta}'^{(Y \rightsquigarrow X)}_{j,k}\right|^2 \text{ with at least one}$$

$$\text{index } k \in \{1,2, \dots, \mathfrak{n}\} \text{ being such that } \left|\beta_{j,k}^{(Y \rightsquigarrow X)} + i\ \hat{\beta}_{j,k}^{(Y \rightsquigarrow X)}\right| - \left|\beta'^{(Y \rightsquigarrow X)}_{j,k} + i\ \widehat{\beta}'^{(Y \rightsquigarrow X)}_{j,k}\right| > r\Big\},$$

whenever $Y = V, W$. *If the extensiveness radius* r *of* $[\mathcal{R}_{\mathbb{S}-}(X)](\mathbb{I})$ *is greater that a given valuation damage threshold, the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{R}_{\mathbb{S}-}(X)](\mathbb{I})$ *are said to be damaging in* X.

ii *The region* $[\mathcal{R}_{\mathbb{S}+}(X)](\mathbb{I})$ *(or simply denoted by* $[\mathcal{R}_2(X)](\mathbb{I})$*) of partial valuation growth of node* V *as evaluated subjectively from the viewpoint of the user(s) of* W *over the time set* $\mathbb{I}$, *with extensiveness radius* r, *in the* $(\mu_1, \dots, \mu_\nu) -$ *device parts* $fr\left(dev_{\mu_1}^{(X)}\right)$, $fr\left(dev_{\mu_2}^{(X)}\right)$,…, $fr\left(dev_{\mu_\nu}^{(X)}\right)$ *of* X *and the* $(\kappa_1, \dots, \kappa_\lambda) -$ *resource parts* $fr\left(res_{\kappa_1}^{(X)}\right)$, $fr\left(res_{\kappa_2}^{(X)}\right)$,…, $fr\left(res_{\kappa_\lambda}^{(X)}\right)$ *of* X *is the set of all coherent interactions* $Z = Z_{(W,V)}(t) = \left((z_1, \zeta_1), (z_2, \zeta_2), (z_3, \zeta_3), (z_4, \zeta_4)\right) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for each of which the corresponding index- set:*

$$\Big\{j \in \{\mu_1, \dots, \mu_\nu, \kappa_1, \dots, \kappa_\lambda\}: \sum_{k=1}^{\mathfrak{n}} \left|\beta'^{(Y \rightsquigarrow X)}_{j,k} + i\ \widehat{\beta}'^{(Y \rightsquigarrow X)}_{j,k}\right|^2 > \sum_{k=1}^{\mathfrak{n}} \left|\beta_{j,k}^{(Y \rightsquigarrow X)} + i\ \hat{\beta}_{j,k}^{(Y \rightsquigarrow X)}\right|^2 \text{ with at least one index } k \in \{1,2, \dots, \mathfrak{n}\} \text{ being such that}$$

$$\left|\beta'^{(Y \rightsquigarrow X)}_{j,k} + i\ \widehat{\beta}'^{(Y \rightsquigarrow X)}_{j,k}\right| - \left|\beta_{j,k}^{(Y \rightsquigarrow X)} + i\ \hat{\beta}_{j,k}^{(Y \rightsquigarrow X)}\right| > r\Big\},$$

whenever $Y = V, W$. *If the extensiveness radius* r *of* $[\mathcal{R}_{\mathbb{S}+}(X)](\mathbb{I})$ *is greater that a given valuation benefit limit, the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{R}_{\mathbb{S}+}(X)](\mathbb{I})$ *are said to be advantageous in* X.

iii. *The region* $[\mathcal{R}_{\mathbb{U}-}(X)](\mathbb{I})$ *(or simply denoted by* $[\mathcal{R}_3(X)](\mathbb{I})$*) of partial vulnerability reduction of node* V *as evaluated subjectively from the viewpoint of the user(s) of* W *over the time set* $\mathbb{I}$, *with extensiveness radius* r, *in the* $(\mu_1, \dots, \mu_\nu)-$ *device parts* $fr\left(dev_{\mu_1}^{(V)}\right)$, $fr\left(dev_{\mu_2}^{(V)}\right)$,…, $fr\left(dev_{\mu_\nu}^{(V)}\right)$ *of* V *and the* $(\kappa_1, \dots, \kappa_\lambda)-$ *resource parts* $fr\left(res_{\kappa_1}^{(V)}\right)$, $fr\left(res_{\kappa_2}^{(V)}\right)$,…, $fr\left(res_{\kappa_\lambda}^{(V)}\right)$ *of* V *is the set of all coherent interactions* $Z = Z_{(W,V)}(t) = \left((z_1, \zeta_1), (z_2, \zeta_2), (z_3, \zeta_3), (z_4, \zeta_4)\right) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for each of which the corresponding index- set:*

$$\left\{j \in \{\mu_1, \dots, \mu_\nu, \kappa_1, \dots, \kappa_\lambda\}: \sum_{k=1}^{\mathfrak{m}} \left|\phi_{j,k}^{(Y \rightsquigarrow X)} + i\ \hat{\phi}_{j,k}^{(Y \rightsquigarrow X)}\right|^2 > \sum_{k=1}^{\mathfrak{m}} \left|\phi'^{(Y \rightsquigarrow X)}_{j,k} + \right.\right.$$

i *ϕ'j,kY⇝X2 with at least one index k∈1,2,…,m being such that*

$$\left.\left|\phi_{j,k}^{(Y \rightsquigarrow X)} + i\ \hat{\phi}_{j,k}^{(Y \rightsquigarrow X)}\right| - \left|\phi'^{(Y \rightsquigarrow X)}_{j,k} + i\ \hat{\phi}'^{(Y \rightsquigarrow X)}_{j,k}\right| > r\right\},$$

whenever $Y = V, W$. *If the extensiveness radius* r *of* $[\mathcal{R}_{\mathbb{U}-}(X)](\mathbb{I})$ *is less than a given vulnerability damage threshold, the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{R}_{\mathbb{U}-}(X)](\mathbb{I})$ *are said to be advantageous in* X.

iv. *The region* $[\mathcal{R}_{\mathbb{U}+}(X)](\mathbb{I})$ *(or simply denoted by* $[\mathcal{R}_4(X)](\mathbb{I})$*) of partial vulnerability growth of node* V *as evaluated subjectively from the viewpoint of the user(s) of* W *over the time set* $\mathbb{I}$, *with extensiveness radius* r, *in the* $(\mu_1, \dots, \mu_\nu)-$ *device parts* $fr\left(dev_{\mu_1}^{(X)}\right)$, $fr\left(dev_{\mu_2}^{(X)}\right)$,…, $fr\left(dev_{\mu_\nu}^{(X)}\right)$ *of* X *and the* $(\kappa_1, \dots, \kappa_\lambda)-$ *resource parts* $fr\left(res_{\kappa_1}^{(X)}\right)$, $fr\left(res_{\kappa_2}^{(X)}\right)$,…, $fr\left(res_{\kappa_\lambda}^{(X)}\right)$ *of* X *is the set of all coherent interactions* $Z = Z_{(W,V)}(t) = \left((z_1, \zeta_1), (z_2, \zeta_2), (z_3, \zeta_3), (z_4, \zeta_4)\right) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *between* Y *and* X *in* $\mathbb{I}$, *for each of which the corresponding index- set:*

$$\Big\{j \in \{\mu_1, \dots, \mu_\nu, \kappa_1, \dots, \kappa_\lambda\}: \sum_{k=1}^{\mathfrak{m}} \left|\phi'^{(Y \rightsquigarrow X)}_{j,k} + i\ \hat{\phi}'^{(Y \rightsquigarrow X)}_{j,k}\right|^2 > \sum_{k=1}^{\mathfrak{m}} \left|\phi^{(Y \rightsquigarrow X)}_{j,k} + i\ \hat{\phi}^{(Y \rightsquigarrow X)}_{j,k}\right|^2 \text{ with at least one index } k \in \{1,2,\dots,\mathfrak{m}\} \text{ being such that } \left|\phi'^{(Y \rightsquigarrow X)}_{j,k} + i\ \hat{\phi}'^{(Y \rightsquigarrow X)}_{j,k}\right| - \left|\phi^{(Y \rightsquigarrow X)}_{j,k} + i\ \hat{\phi}^{(Y \rightsquigarrow X)}_{j,k}\right| > r\Big\},$$

whenever $Y = V, W$. *If the extensiveness radius* r *of* $[\mathcal{R}_{\mathbb{U}+}(X)](\mathbb{I})$ *is greater than a given vulnerability damage threshold, the interactions* $Z = Z_{(Y,X)}(t)$ *of* $[\mathcal{R}_{\mathbb{U}+}(X)](\mathbb{I})$ *are said to be damaging in* X.

Based on this preliminary material, we are now able to give the following general definition.

Definition 9.11. *A germ of partial cyber attack from* W *against the* $(\mu_1, \dots, \mu_\nu)-$ *device parts* $fr\left(dev^{(V)}_{\mu_1}\right)$, $fr\left(dev^{(V)}_{\mu_2}\right)$,…, $fr\left(dev^{(V)}_{\mu_\nu}\right)$ *of* V *and the* $(\kappa_1, \dots, \kappa_\lambda)-$ *resource parts* $fr\left(res^{(V)}_{\kappa_1}\right)$, $fr\left(res^{(V)}_{\kappa_2}\right)$,…, $fr\left(res^{(V)}_{\kappa_\lambda}\right)$ *of* V, *during a given time subset* $\mathbb{I}$ *of a subinterval* $[\alpha, \beta] \subset\subset [0,1]$, *is a family of coherent interactions* $Z = Z_{(W,V)}(t) = \big((z_1, \zeta_1), (z_2, \zeta_2), (z_3, \zeta_3), (z_4, \zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$, $t \in \mathbb{I}$, *lying in the so called partial danger sector* $\mathcal{E} = \mathcal{E}_{W\to V}$ *to the node* V *from the node* W *during the entire time set* $\mathbb{I}$, *defined by intersection*

$$\mathcal{E} := [\mathcal{R}_{\mathbb{S}-}(V)](\mathbb{I}) \cap [\mathcal{R}_{\mathbb{S}+}(W)](\mathbb{I}) \cap [\mathcal{R}_{\mathbb{U}-}(W)](\mathbb{I}) \cap [\mathcal{R}_{\mathbb{U}+}(V)](\mathbb{I}).$$

If a coherent interaction is elementary, we say that the continuous cyber attack is elementary; otherwise, it is called sequential or complex. If $\mathbb{I} = \{t_0\}$ *for some* $t_0 \in \,]0,1[$, *the germ is called momentary.*

Definition 9.12. *The node* V *is partially secure from cyber attacks of* W *against the* $(\mu_1, \dots, \mu_\nu)-$ *device parts* $fr\left(dev^{(V)}_{\mu_1}\right)$, $fr\left(dev^{(V)}_{\mu_2}\right)$,…, $fr\left(dev^{(V)}_{\mu_\nu}\right)$ *of* V *and the* $(\kappa_1, \dots, \kappa_\lambda)-$ *resource parts* $fr\left(res^{(V)}_{\kappa_1}\right)$, $fr\left(res^{(V)}_{\kappa_2}\right)$,…, $fr\left(res^{(V)}_{\kappa_\lambda}\right)$ *of* V, *during a given closed time subinterval*

$\mathbb{I} \subset\subset]0,1[$, *if* $\mathcal{E} = \emptyset$.

Definition 9.13. *And, more generally, a partially secure area for node* V *from cyber attacks of* W *against the* $(\mu_1, \dots, \mu_\nu)-$ *device parts* $fr\left(dev_{\mu_1}^{(V)}\right)$, $fr\left(dev_{\mu_2}^{(V)}\right)$,…, $fr\left(dev_{\mu_\nu}^{(V)}\right)$ *of* V *and the* $(\kappa_1, \dots, \kappa_\lambda)-$ *resource parts* $fr\left(res_{\kappa_1}^{(V)}\right)$, $fr\left(res_{\kappa_2}^{(V)}\right)$,…, $fr\left(res_{\kappa_\lambda}^{(V)}\right)$ *of* V, *during a given closed time subinterval* $\mathbb{I} \subset\subset]0,1[$, *is any set in the complementary* $\mathcal{E}^C$ *in* $(\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4$ *of* $\mathcal{E}$.

10 Proactive Cyber Defense Against Cyber Attacks

10.1 Proactive Correlated Cyber Defense against Germs of Correlated Cyber-Attacks

Let $\mathcal{F} = \mathcal{F}_{W\to V}^{(correlated)}[\mathbb{I}]$ be a germ of correlated cyber attack from W against V, during a given time set $\mathbb{I}$ in a subinterval $]\alpha,\beta[\subset\subset [0,1]$. Recall that $\mathcal{F}$ is a family $\mathcal{F} = \{Z = Z_{(W,V)}(t) = \left((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\right) \in$ $\mathbb{C}^{\mathcal{N}\times n}\times\mathbb{C}^{\mathcal{N}\times m}4,\ t\in\mathbb{I}$ of <u>coherent</u> interactions lying in the correlated danger sector $\mathfrak{X} = \mathfrak{X}_{W\to V}$ to the node V from the node W during the entire time set $\mathbb{I}$, as defined in Definition 9.6, provided, of course, that $\mathfrak{X} \neq \emptyset$. Denote by

$$\mathcal{D}_{\mathcal{F}} = \{\mathscr{g} = \mathscr{g}^{(Z)}: \mathbb{I} \to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V:$$
$$t \mapsto \mathscr{g}^{(Z)}(t) := \left(\gamma_W^{(Z)}(t), \gamma_V^{(Z)}(t), \gamma_W^{(Z)}(t+\Delta t), \gamma_V^{(Z)}(t+\Delta t)\right) : Z \in \mathcal{F}\Big\},$$

the associated coherent interactive family, a proactive correlated cyber-defense $\mathscr{f}$ against the cyber attack $\mathcal{F}$ during $\mathbb{I}$ is a map defined on the space of all cyber-interplays of the ordered cyber pair (V, W) over the entire time set $\mathbb{I}$,such that the image of $\mathfrak{X}$ via any member of the coherent interactive family $\mathcal{D}_{\mathcal{F}}$ in $\mathbb{I}$ is sent, through $\mathscr{f}$ in the complement $\mathfrak{X}^c = (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4 \setminus \mathfrak{X}$ of $\mathfrak{X}$.

Specifically,

Definition 10.1. *Let X be the space of cyber activities $\mathscr{g}: \mathbb{I} \to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V$ from the node W to the node V during the entire time set $\mathbb{I}$. A mapping $\mathscr{f}: X \to X$ is called proactive correlated cyber defense against the germ of attack $\mathcal{F}$ during $\mathbb{I}$, if $\mathscr{f}\big(\mathscr{g}(\mathfrak{X})\big) \subset \mathfrak{X}^c$, whenever $\mathscr{g} \in \mathcal{D}_{\mathcal{F}}$. The method of constructing and organizing a proactive correlated cyber defense, together with the way of processing and integrating the method in the node system, is called proactive correlated protection against the germ of attack $\mathcal{F}$. We will deal later with the question of such a protection.*

10.2. Proactive Absolute Cyber Defense against Germs of Absolute Cyber-Attacks

Let $\mathcal{F} = \mathcal{F}_{W \to V}^{(absolute)}[\mathbb{I}]$ be a germ of absolute cyber attack from W against V, during a given time set $\mathbb{I}$ in a subinterval $]\alpha, \beta[\subset\subset [0,1]$. Recall that $\mathcal{F}$ is a family $\mathcal{F} = \big\{Z = Z_{(W,V)}(t) = \big((z_1, \zeta_1), (z_2, \zeta_2), (z_3, \zeta_3), (z_4, \zeta_4)\big) \in (\mathbb{C}^{\mathcal{N} \times \mathfrak{n}} \times \mathbb{C}^{\mathcal{N} \times \mathfrak{m}})^4, t \in \mathbb{I}\big\}$ of <u>coherent</u> interactions lying in the absolute danger sector $\tilde{\mathfrak{X}} = \tilde{\mathfrak{X}}_{W \to V}$ to the node V from the node W during the entire time set $\mathbb{I}$,as defined in Definition 9.6, provided, of course, that $\tilde{\mathfrak{X}} \neq \emptyset$. Denote by

$$\mathcal{D}_{\mathcal{F}} = \Big\{\mathscr{g} = \mathscr{g}^{(Z)}: \mathbb{I} \to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V: \\ t \mapsto \mathscr{g}^{(Z)}(t) := \Big(\gamma_W^{(Z)}(t), \gamma_V^{(Z)}(t), \gamma_W^{(Z)}(t + \Delta t), \gamma_V^{(Z)}(t + \Delta t)\Big) : Z \in \mathcal{F}\Big\},$$

the associated coherent interactive family, a proactive absolute cyber-defense $\mathscr{f}$ against the cyber attack $\mathcal{F}$ during $\mathbb{I}$ is a map defined on the space of all cyber-interplays of the ordered cyber pair (V, W) over the entire time set $\mathbb{I}$,such that the image of $\tilde{\mathfrak{X}}$ via any member of the coherent interactive family $\mathcal{D}_{\mathcal{F}}$ in $\mathbb{I}$

is sent, through $\not{f}$ in the complement $\widetilde{\mathfrak{X}}^c = (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4 \setminus \widetilde{\mathfrak{X}}$ of $\widetilde{\mathfrak{X}}$. Specifically,

Definition 10.2. *Let again X be the space of cyber activities $\mathscr{g}: \mathbb{I} \to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V$ from the node W to the node V during the entire time set $\mathbb{I}$. A mapping $\not{f}: X \to X$ is called proactive absolute cyber defense against the germ of attack $\mathcal{F}$ during $\mathbb{I}$, if $\not{f}\big(\mathscr{g}(\mathfrak{X})\big) \subset \mathfrak{X}^c$, whenever $\mathscr{g} \in \mathcal{D}_{\mathcal{F}}$. The method of constructing and organizing a proactive absolute cyber defense, together with the way of processing and integrating the method in the node system, is called proactive absolute protection against the germ of attack $\mathcal{F}$. We will deal later with the question of such a protection.*

10.3. Proactive partial cyber defense against germs of partial cyber-attacks

Let $\mathcal{F} = \mathcal{F}_{W\to V}^{(partial)}[\mathbb{I}]$ be a germ of partial cyber attack from W against V, during a given time set $\mathbb{I}$ in a subinterval $]\alpha, \beta[\subset\subset [0,1]$. Recall that $\mathcal{F}$ is a family $\mathcal{F} = \{Z = Z_{(W,V)}(t) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) \in (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times$ $\mathbb{C}\mathcal{N}\times m4,\ t\in\mathbb{I}$ of <u>coherent</u> interactions lying in the partial danger sector $\mathcal{E} = \mathcal{E}_{W\to V}$ to the node V from the node W during the entire time set $\mathbb{I}$,as defined in Definition 9.6, provided, of course, that $\mathcal{E} \neq \emptyset$. Denote by

$$\mathcal{D}_{\mathcal{F}} = \{\mathscr{g} = \mathscr{g}^{(Z)}: \mathbb{I} \to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V:$$

$$t \mapsto \mathscr{g}^{(Z)}(t) := \Big(\gamma_W^{(Z)}(t), \gamma_V^{(Z)}(t), \gamma_W^{(Z)}(t+\Delta t), \gamma_V^{(Z)}(t+\Delta t)\Big) : Z \in \mathcal{F}\Big\},$$

the associated coherent interactive family, a proactive partial cyber-defense $\not{f}$ against the cyber attack $\mathcal{F}$ during $\mathbb{I}$ is a map defined on the space of all cyber-interplays of the ordered cyber pair (V, W) over the entire time set $\mathbb{I}$,such that the image of $\mathcal{E}$ via any member of the coherent interactive family $\mathcal{D}_{\mathcal{F}}$ in $\mathbb{I}$

is sent, through $\mathscr{f}$ in the complement $\mathcal{E}^c = (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^4 \setminus \mathcal{E}$ of $\mathfrak{X}$. Specifically,

Definition 10.3 *Let X be the space of cyber activities $\mathscr{g}\colon \mathbb{I} \to \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V$ from the node W to the node V during the entire time set $\mathbb{I}$. A mapping $\mathscr{f}\colon X \to X$ is called proactive partial cyber defense against the germ of attack $\mathcal{F}$ during $\mathbb{I}$, if $\mathscr{f}\big(\mathscr{g}(\mathfrak{X})\big) \subset \mathfrak{X}^c$, whenever $\mathscr{g} \in \mathcal{D}_{\mathcal{F}}$. The method of constructing and organizing a proactive partial cyber defense, together with the way of processing and integrating the method in the node system, is called proactive partial protection against the germ of attack $\mathcal{F}$. We will deal later with the question of such a protection.*

10.4. Proactive Protection against Germs of Partial Cyber-Attacks

Let us finally see how to illustrate such a proactive cyber defense.

Definition 10.5. *Suppose*

$$Z = Z_{(W,V)}(t_0) = \big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),(z_4,\zeta_4)\big) =$$
$$\big((z_1,\zeta_1),(z_2,\zeta_2),(z_3,\zeta_3),\ (z_4,\zeta_4)\big)(t_0) \in \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}}$$

is a cyber interaction between W and V at a fixed time moment $t_0 \in\,]\alpha,\beta[\, \subset\subset [0,1]$ ($W, V \in ob\big(cy(t)\big)$), with corresponding cyber- interplay

$$\mathscr{g}\colon]\alpha,\beta[\, \mapsto \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V\colon t \mapsto \mathscr{g}(t) := \Big(\gamma_W(t), \gamma_V(t), \gamma'_W(t'), \gamma'_V(t')\Big)$$

and cyber-activity

$$\Big(\mathscr{g}_t\colon \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V\colon (\gamma_W(t), \gamma_V(t)\,) \longmapsto \big(\gamma'_W(t'), \gamma'_V(t')\big)\Big)_{t\in]\alpha,\beta[}$$
$$(t' := t + \Delta t).$$

A forced cyber-reflection of Z is another cyber-interaction

$$Z' = Z'_{(W,V)}(t_0) = \big((z'_1,\zeta'_1),(z'_2,\zeta'_2),(z'_3,\zeta'_3),(z'_4,\zeta'_4)\big)$$

$$= \left(\left(z_1', \zeta_1'\right), \left(z_2', \zeta_2'\right), \left(z_3', \zeta_3'\right), \left(z_4', \zeta_4'\right)\right)\left(t_0'\right) \in \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}}$$

between W *and* V *at a next time moment* $t_0' = t_0 + \Delta t_0 \in]\alpha, \beta[$ *with corresponding forced cyber- interplay*

$$\mathscr{g}':]\alpha, \beta[\mapsto \Omega_W \times \Omega_V \times \Omega_W \times \Omega_V : t \mapsto \mathscr{g}'(t) := \left(\gamma_W'(t'), \gamma_V'(t'), \gamma_W''(t''), \gamma_V''(t'')\right)$$

and associated forced cyber-activity:

$$\left(\mathscr{g}_{t'}': \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V : \left(\gamma_W'(t'), \gamma_V'(t')\right) \longmapsto \left(\gamma_W''(t''), \gamma_V''(t'')\right)\right)_{t' \in]0,1[}$$

$$(t'' := t' + \Delta t')$$

that satisfies the following property: into an open neighborhood $]t_0 - \varepsilon, t_0 + \varepsilon[$ *of* t_0, *forces activity* $\mathscr{g}$ *to push forward its composition with activity* $\mathscr{g}'$, *in such a way that the occurrence of* $\mathscr{g}$ *guarantees the appearance of the composition* $\mathscr{g}' \circ \mathscr{g}$.

Obviously, the matrices of the tetrad

$$Z' = Z'_{(W,V)}\left(t_0'\right) = \left(\left(z_1', \zeta_1'\right), \left(z_2', \zeta_2'\right), \left(z_3', \zeta_3'\right), \left(z_4', \zeta_4'\right)\right)$$

$$= \left(\left(z_1', \zeta_1'\right), \left(z_2', \zeta_2'\right), \left(z_3', \zeta_3'\right), \left(z_4', \zeta_4'\right)\right)\left(t_0'\right) \in \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}}$$

are of the form

$$\left(z_1', \zeta_1'\right) = \gamma_W'\left(t_0'\right) = \left(\mathbb{S}_{V\to W}' + i\widehat{\mathbb{S}}_{W\to W}', \mathbb{U}_{V\to W}' + i\widehat{\mathbb{U}}_{W\to W}'\right) \in \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}},$$

$$\left(z_2', \zeta_2'\right) = \gamma_V'\left(t_0'\right) = \left(\mathbb{S}_{W\to V}' + i\widehat{\mathbb{S}}_{V\to V}', \mathbb{U}_{W\to V}' + i\widehat{\mathbb{U}}_{V\to V}'\right) \in \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}},$$

$$\left(z_3', \zeta_3'\right) = \gamma_W''\left(t_0''\right) = \left(\mathbb{S}_{V\to W}'' + i\widehat{\mathbb{S}}_{W\to W}'', \mathbb{U}_{V\to W}'' + i\widehat{\mathbb{U}}_{W\to W}''\right) \in \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}},$$

$$\left(z_4', \zeta_4'\right) = \gamma_V''\left(t_0''\right) = \left(\mathbb{S}_{W\to V}'' + i\widehat{\mathbb{S}}_{V\to V}'', \mathbb{U}_{W\to V}'' + i\widehat{\mathbb{U}}_{V\to V}''\right) \in \mathbb{CM}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{CM}^{\mathcal{N}\times\mathfrak{m}}.$$

Definition 10.6. *The cyber-activity*

$$\mathscr{g} \equiv \mathscr{g}_t: \ \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V : \left(\gamma_W(t), \gamma_V(t)\right) \longmapsto \left(\gamma_W'(t'), \gamma_V'(t')\right)$$

together with its forced cyber-activity

$$\mathscr{g}' = \mathscr{g}_{t'}': \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V : \left(\gamma_W'(t'), \gamma_V'(t')\right) \longmapsto \left(\gamma_W''(t''), \gamma_V''(t'')\right)$$

is called a reflexive cyber-activity between W *and* V *during the time interval* $]\alpha, \beta[$. *Their composition*

$$\mathcal{g}' \circ \mathcal{g}: \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V : (\gamma_W(t), \gamma_V(t)) \mapsto \left(\gamma''_W(t' + \Delta t), \gamma''_V(t' + \Delta t)\right)$$

is said to be a self-inflicted cyber-activity between W *and* V *during the time interval* $]\alpha, \beta[$. *In particular, the interaction* $Z' = Z'_{(W,V)}(t'_0)$ *is called forced cyber-reflection of* $Z = Z_{(W,V)}(t_0)$ *at time moment* t_0. *A mapping*

$$\Phi: (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^2 \to (\mathbb{C}^{\mathcal{N}\times\mathfrak{n}} \times \mathbb{C}^{\mathcal{N}\times\mathfrak{m}})^2$$

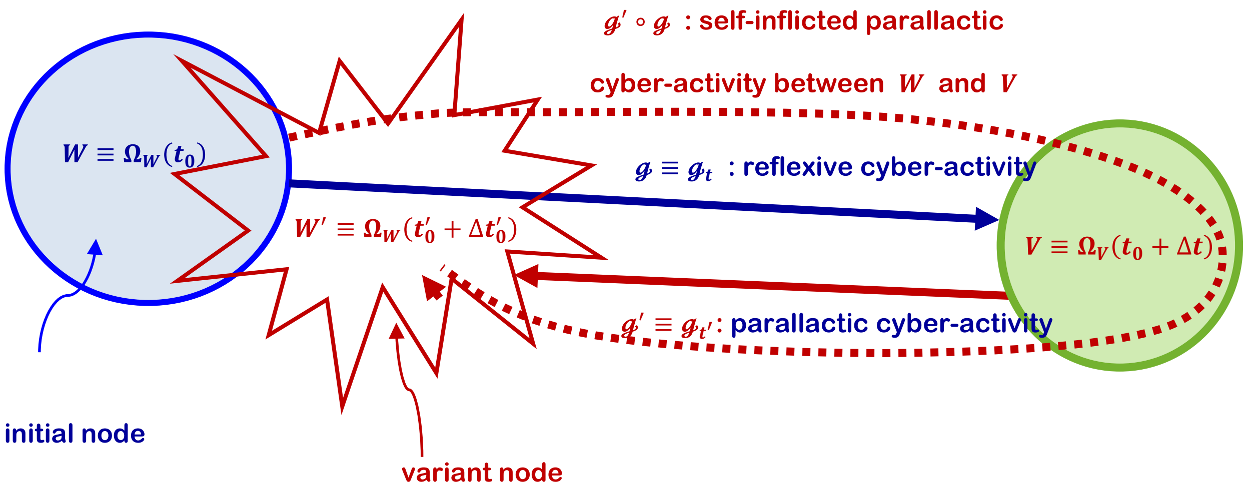
which maps the cyber-interaction $Z = Z_{(W,V)}(t_0)$ *to its forced cyber-reflection* $Z' = Z'_{(W,V)}(t'_0)$ *is called reflexive cyber-interaction mapping at time moment* t_0.

Remark 10.7 It is frequent that, under a self-inflicted cyber-activity

$$\mathcal{g}' \circ \mathcal{g}: \ \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V : (\gamma_W(t), \gamma_V(t)) \mapsto \left(\gamma''_W(t' + \Delta t), \gamma''_V(t' + \Delta t)\right)$$

between W and V during the time interval $]\alpha, \beta[$, some valuations and vulnerabilities of the initial node W change at a moment $t_0 \in]\alpha, \beta[$, in such a way to get new constituent valuations and new constituent vulnerabilities for the node W. *For emphasis*, this "new" node is called variant node of W and is denoted by W', or sometimes, without any risk of confusion, again by W. In such a case, the forced cyber-reflection $Z' = Z'_{(W,V)}(t'_0)$ is called cyber parallax of the cyber-interaction $Z = Z_{(W,V)}(t_0)$ at t_0 and the forced cyber-activity $\mathcal{g}' = \mathcal{g}'_{t'}: \Omega_W \times \Omega_V \to \Omega_W \times \Omega_V : \left(\gamma'_W(t'), \gamma'_V(t')\right) \mapsto \left(\gamma''_W(t''), \gamma''_V(t'')\right)$ is called parallactic cyber-activity. Finally, we say that *the self-inflicted parallactic cyber-activity* $\mathcal{g}' \circ \mathcal{g}: \ \Omega_W(t) \times \Omega_V(t) \to \Omega_W(t' + \Delta t) \times \Omega_V(t' + \Delta t)$ *between* W *and* V *at* t_0 *gives rise to a parallactic cyber-interaction at* t_0.

Let us give a schematic representation.



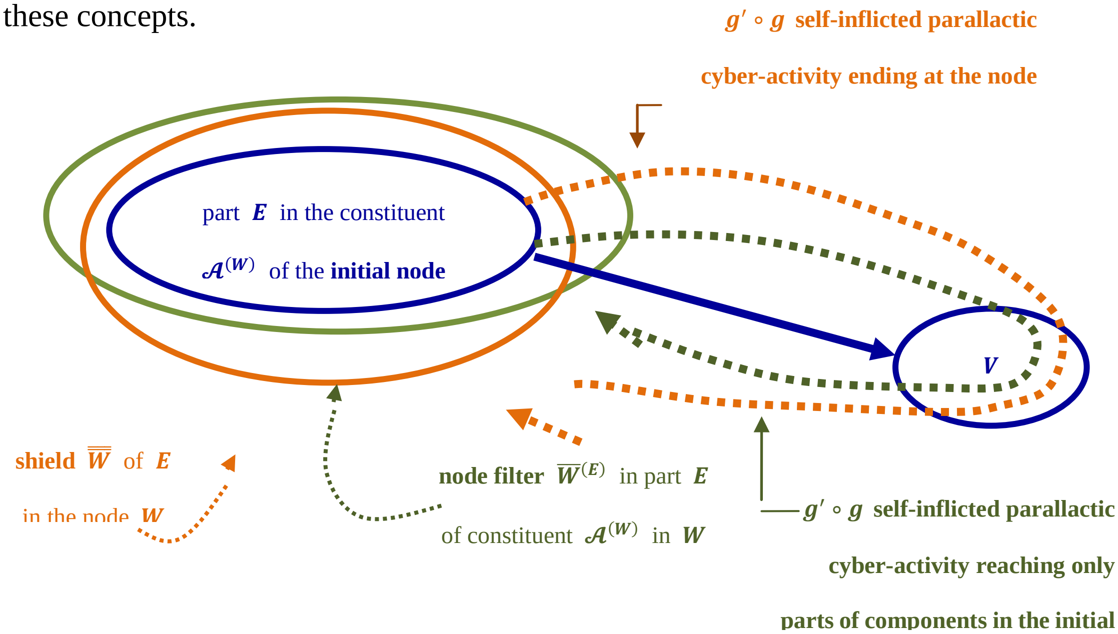
Definition 10.7 *Let* $E = fr\left(\mathcal{A}^{(W)}\right)$ *be a set in the* $\sigma-algebra$ $\mathfrak{U}_{\mathcal{P}}$ *of subsets of available or not constituents of node* W*:*

$$\mathcal{A} = \begin{cases} dev, if\ the\ constituent\ is\ a\ device\ , \\ res, if\ the\ constituent\ is\ a\ resource\ element \end{cases}$$

i *A shield of* E *in the node* W *(or a node shield containing* E*) at time* t *is an intermediate fixed node* $\overline{\overline{W}} = \overline{\overline{W}}_t$ *which, at this time, is interposed in each cyber parallax* g' *that aims at* E *in the node* W*, so that the self-inflicted parallactic cyber-activity* $g' \circ g$ *between* W *and* V *at moment time* t *ends up in the intermediate node* $\overline{\overline{W}}$*, and never can reach part* E *of the initial target* W*. The detailed process by which the node shield* $\overline{\overline{W}}$ *of a node* W *blocks the self-inflicted parallactic cyber-activity* $g' \circ g$ *and never ends up in the initial target* W*, is being analyzed in a forthcoming paper.*

ii. *Given a node* W*, a node filter in part* E *of the constituent* $\mathcal{A}^{(W)}$ *in* W *at a time moment* t *is an intermediate fixed node* $\overline{W}^{(E)}$ *which, at this time moment, is interposed in each parallactic cyber-activity* g' *that aims at part* E *of node* W*, so that the filter* $\overline{W}^{(E)}$ *allows the self-inflicted parallactic cyber-activity* $g' \circ g$ *at* t *to reach only constituent parts of the initial target* W *that are different from part* E *of the constituent* $\mathcal{A}^{(W)}$ *of* W*.*

For the convenience of the reader, let us give a schematic representation of these concepts.



References

[1] Niall M. Adams and Nicholas A. Heard (Eds.), *Data Analysis for Network Cyber-Security*. World Scientific, 200 pages, 2014.

[2] Argyrios Alexopoulos, and Nicholas J. Daras, *A mathematical study of the proactive defense against different special types of germs of cyber attacks*, In preparation.

[3] Kenneth J. Arrow, and Frank Hahn, *General Competitive Analysis*. San Francisco: Holden-Day, and Edinburgh: Oliver & Boyd. Business & Economics, 452 pages, 1971.

[4] Kenneth J. Arrow, and Michael D. Intriligator (Eds.), *Handbook of Mathematical Economics. I*. North-Holland, New York, 378 pages, 1981.

[5] Kenneth J. Arrow, and Michael D. Intriligator (Eds.), *Handbook of Mathematical Economics. II*. North-Holland, New York. 688 pages, 1982.

[6] William H. Batchelder: *Mathematical Psychology*. In *Encyclopedia of Psychology*, Alan E. Kazdin (Ed.), Washington/NY: APA/Oxford University Press, 4128 pages, 120-123, 2000.

[7] Philip Bonacich, and Phillip Lu, *Introduction to mathematical sociology*. Princeton University Press. 240 pages, 2012.

[8] Richard Colbaugh, and Kristin Glass, *Proactive defense for evolving cyber threats*. SANDIA REPORT, SAND2012-10177, Unlimited Release.2012 http://fas.org/irp/eprint/proactive.pdf

[9] Nicholas J. Daras, On the mathematical definition of cyberspace, *The Journal of Nonlinear Science and Applications* (JNSA), (submitted 2016).

[10] Nicholas J. Daras and Argyrios Alexopoulos, *Modeling cyber-security*, Modern Discrete Mathematics and Analysis with Applications in Cryptography, Springer Optimization and its Applications, to appear

[11] Nicholas J. Daras and Argyrios Alexopoulos, Preferences and Rational Choices when Evaluating the Nodes in Cyberspace to be Attacked, submitted.

[12] Nicholas J. Daras and John Th. Mazis, *Geopolitical systemic modeling, Part 1 Prediction of geopolitical events,* GeoJournal, DOI 10.1007/s10708-014-9569-3, Springer Science Verlag, Dordrecht, published online: 12 August 2014.

[13] Nicholas J. Daras, and John Th. Mazis, *Systemic geopolitical modeling. Part 2: Subjectivity in prediction of geopolitical events.* GeoJournal, **108**(81), (2017), 81-108, DOI 10.1007/s10708-015-9670-2, Springer Science Verlag, Dordrecht, published online: 04 September 2015.

[14] Gérard Debreu, *Theory of value. An axiomatic analysis of economic equilibrium* (4th Printing), Cowles Foundation for Research in Economics at Yale University. John Wiley & Sons. 126 pages, 2016 http://cowles.yale.edu/sites/default/files/files/pub/mon/m17-all.pdf

[15] Gérard Debreu, *Mathematical Economics*, *Twenty papers of Gerard Debreu*. Cambridge Cambridgeshire New York: Cambridge University Press. 264 pages, 1986.

[16] Daniel M. Dunlavy, Bruce Hendrickson, and Tamara G. Kolda, *Mathematical challenges in cyber-security*. Sandia National Laboratories. Printed February 2009 www.sandia.gov/~dmdunla/publications/SAND2009-0805.pdf

[17] Andrew Gelman, *Mathematics, Statistics, and Political Science*. 31 July 2011. http://www.stat.columbia.edu/~gelman/presentations/mathstatpshandout.pdf

[18] Joseph W. Houpt, Leslie M. Blaha (Eds.), *Mathematical Models of Perception and Cognition. I-II. A Festschrift for James T. Townsend,* Scientific Psychology Series. Psychology Press, Routledge. Taylor & Francis Group, New York and London, 554 pages, 2016

[19] Robert Duncan Luce, Robert R. Bush and Eugene Galanter (Eds.), *Handbook of mathematical psychology, Volumes I-III*. John Wiley & Sons. xiii + 491 pages, 1963.

[20] Fabrizio Luccio, Linda Pagli and Graham Stee, *Mathematical and algorithmic foundations of the Internet*. CRC Press. July 6 2011, 221 pages.

[21] Joe Malkevitch, *Mathematics and Internet security*. December 2015 http://www.ams.org/samplings/feature-column/fcarc-internet

[22] Andreu Mas-Colell, *The theory of general economic equilibrium: A differentiable approach*. Econometric Society Monographs 9, Cambridge University Press, Cambridge, U.K. 392 pages, 1985.

[23] Juan Meza, Scott Campbell and David Bailey, *Mathematical and statistical opportunities in cyber security*, 2009 http://arxiv.org/pdf/0904.1616v1.pdf . Also http://www.davidhbailey.com/dhbpapers/CyberMath.pdf

[24] Abraham Neyman, *Mathematical Economics, Lecture Notes*. Fall 2006, Jerusalem,

https://scholars.huji.ac.il/sites/default/files/abrahamn/files/matheco0607.pdf

[25] President's Information Technology Advisory Committee (PITAC), *Report to the President, cyber security: A crisis of prioritization*, February 2005, https://www.nitrd.gov/pitac/reports/20050301_cybersecurity/cybersecurity.pdf

[26] Dwayne Ramsey, *Mathematical problems in cyber security*. In *DOE Workshop on Mathematical Research Challenges in Optimization of Complex Systems*, December 7-8, 2006. http://www.courant.nyu.edu/ComplexSystems/

[27] Neil C. Rowe, E. John Custy and Binh T. Duong, *Defending cyberspace with fake honeypots*, Journal of Computers, **2**(2), (April 2007), 25-36 http://www.academypublisher.com/jcp/vol02/no02/jcp02022536.pdf

[28] Peter Y. A. Ryan, *Mathematical models of computer security.* In *Foundations of Security Analysis and Design: Tutorial Lectures (FOSAD 2000),* Riccardo Focardi and Roberto Gorrieri (Eds.). LNCS 2171. Springer-Verlag Berlin Heidelberg 2001, 1-62. http://www.ece.cmu.edu/~ece732/readings/mathematical-models-of-computer-security.pdf

[29] Dinesh K. Saini, *Cyber Defense: Mathematical modeling and simulation,* International Journal of Applied Physics and Mathematics, **2**(5), (2012), 312-315 http://www.ijapm.org/papers/121-P00009.pdf

[30] Dan Shen, Genshe Chen, Jose B. Cruz, Jr., Erik Blasch and Khanh Pham, *An adaptive Markov game model for cyber threat intent inference*. In *Theory and novel applications of machine learning*, Meng Joo Er and Yi Zhou Eds.), I-Tech, Vienna, Austria. 376 pages, February 2009 http://cdn.intechweb.org/pdfs/6200.pdf

[31] Abraham Subil and Nair Suku, *Cyber security analytics: A stochastic model for security quantification using absorbing Markov chains*, Journal of Communications **9**(12), (2014), 899-907 http://www.jocm.us/uploadfile/2014/1231/20141231022619157.pdf

[32] Andranik Tangian, *Mathematical theory of democracy,* Studies in social choice and welfare, Springer, Berlin-Heidelberg. 615 pages, 2014.

[33] Joanne Wendelberger, Christopher Griffin, Louis Wilder, Yu Jiao and Tamara Kolda, *A mathematical basis for science-based cyber-security* (with alternate title *Mathematical Underpinnings for Science-Based Cyber-Security*), February 2008, Sandia National Laboratories, Chad Scherrer, Pacific Northwest National Laboratory, https://wiki.cac.washington.edu/download/attachments/7479040/doecybermath25feb08.doc