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Some fixed point results for random operators in Hilbert spaces

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Abstract

The present paper deals with establishment of some common fixed point results in Hilbert spaces for random operators. One of them contains rational relation.

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1 Introduction

The study of fixed points of random operators forms a central topic in Probabilistic functional analysis. The Prague school of probabilistic initiated its

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study in the 1950. However, the research in this area flourished after the publication of the survey article of Bharucha-Reid [4]. Since then many interesting random fixed point results and several applications have appeared in the literature; for example the work of Beg and Shahazad [2, 3], Lin [10], O'Regan [11], Papageorgiou [12] Xu [15].

In recent years, the study of random fixed points have attracted much attention some of the recent literatures in random fixed points may be noted in [1, 2, 3, 5, 12, 15]. In particular ,random iteration schemes leading to random fixed point of random operators have been discussed in [5, 6, 7].

The present paper deals with some fixed point theorems for two random operators in Hilbert spaces. We find unique common random fixed point of two random operators in closed subset of a separable Hilbert space by considering a sequence of measurable functions satisfying implicit conditions.

2 Preliminaries Notes

Throughout this chapter, (Ω, Σ) denotes a measurable space, *H* stands for a separable Hilbert space, and *C* is non empty subset of *H*.

2.1. *Measurable function:* A function $f:\Omega \to C$ is said to be measurable if $f^{-1}(B \cap C) \in \Sigma$ foe every Borel subset *B* of *H*.

2.2. *Random operator:* A function $F:\Omega \times C \to C$ is said to be random operator, if $F(.,X):\Omega \to C$ is measurable for every X $\subset$ *C*.

2.3. *Continuous Random operator:* A random operator $F:\Omega \times C \to c$ is said to be continuous if for fixed $t \in \Omega$, $F(t,.):C \to C$ is continuous.

2.4. *Random fixed point:* A measurable function $g:\Omega \to C$ is said to be random fixed point of the random operator $F:\Omega \times C \to C$, if $F\left(t, g(t)\right) = g(t)$, $\forall t \in \Omega$.

Condition (2.4): Let $E, F : \Omega \times C \to C$ be two random operators, when C is non empty subset of a Hilbert space H, is said to satisfy the condition (2.4) if

$$|| \ E(\xi, \ x(\xi)) - \ F(\xi, y(\xi)) \, ||^2 \le \phi \begin{bmatrix} || \, x(\xi) - y(\xi) \, ||^2, \ \{|| \, x(\xi) - E(\xi, x(\xi)) \, ||^2 + || \, y(\xi) - F(\xi, y(\xi)) \, ||^2\}, \\ \frac{1}{2}\{|| \, x(\xi) - F(\xi, y(\xi)) \, ||^2 + || \, y(\xi, -E(\xi, x(\xi)) \, ||^2\} \end{bmatrix}$$

Condition (2.5): Let $E, F : \ \Omega \times C \to C$ be two random operator, when C is non empty subset of a Hilbert space H, is said to satisfy the condition 2.5, if

$$|| \ E(\xi, \ x(\xi)) - F(\xi, y(\xi)) \, ||^2 \le \phi \begin{bmatrix} \|x(\xi) - y(\xi)\|^2, \left[\ \|x(\xi) - E(\xi, x(\xi))\|^2 + \|y(\xi) - F(\xi, y(\xi))\|^2 \ \right], \\ \frac{1}{2}[\|x(\xi) - F(\xi, y(\xi))\|^2 + \|y(\xi) - E(\xi, x(\xi))\|^2], \\ \dfrac{\frac{1}{2}\left[\|x(\xi) - y(\xi)\|^2 + \|x(\xi) - E(\xi, x(\xi))\|^2 + \|y(\xi) - E(\xi, x(\xi))\|^2 \right]}{1 + \|x(\xi) - y(\xi)\|^2 \, \|x(\xi) - F(\xi, y(\xi))\|^2 \, \|x(\xi) - E(\xi, x(\xi))\|^2}, \\ \dfrac{\|y(\xi) - E(\xi, x(\xi))\|^2 + \|y(\xi) - F(\xi, y(\xi))\|^2}{1 + \|y(\xi) - E(\xi, x(\xi))\|^2 \, \|y(\xi) - F(\xi, y(\xi))\|^2 \, \|x - F(\xi, y(\xi,))\|^2} \end{bmatrix}$$

2.6 Implicit Relation :

Let Φ be the class of all real-valued continuous functions $\varphi : (R^+)^3 \to R^+$ non -decreasing in the first argument and satisfying the following conditions:

$x \le \varphi(y, \ x + y, x)$ or $x \le \varphi(y, \ x, \ ½(x + y))$ or $x \le \varphi(y, x + \ y, x + y)$

there exists a real number $0 < k < 1$ such that $x \le ky$, for all $x, y \ge 0$.

Similarly for $(R^+)^5$, let Φ be the class of all real-valued continuous functions $\varphi : (R^+)^5 \to R^+$ non -decreasing in the first argument and satisfying the following conditions for all $x, y \ge 0$, $x \le \varphi(y, \ x + y, \ x + y, \ x + y, \ y)$ or $x \le \varphi(y, x + y, \ x + y, x + y, x + y)$ or $x \le \varphi(y, 1/2(x + \ y), \ x + y, \ x + y, x + y)$

there exists a real number $0 < k < 1$ such that $x \le ky$.

3 Main Results

Theorem 3.1 *Let C be a non empty subset of Hilbert Space H. Let E and F be continuous random operators defined on C such that for* $\xi \in \Omega$*,* $E(\xi, \cdot)$ *and* $F(\xi, \cdot) : C \to C$ *satisfying condition* (2.4).

Then the sequence $\{g_n\}$ *converges to the unique common random fixed point of E and F.*

Proof. $\{g_n\}$ is sequence of function defined

$g_{2n+1}(\xi) = E(\xi, g_{2n}(\xi))$, $g_{2n+2}(\xi) = F(\xi, g_{2n+1})$, for $\xi \in \Omega$ and $n = 0,1,2,\dots$

$$\| g_{2n+1}(\xi) - g_{2n}(\xi) \|^2 = \| E(\xi, g_{2n}(\xi)) - F(\xi, g_{2n-1}(\xi)) \|^2 \le$$

$$\le \phi \begin{bmatrix} \| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2, \{\| g_{2n}(\xi) - E(\xi, g_{2n}(\xi)) \|^2 + \| g_{2n-1}(\xi) - F(\xi, g_{2n-1}(\xi)) \|^2\}, \\ \tfrac{1}{2} \{\| g_{2n}(\xi) - F(\xi, g_{2n-1}(\xi)) \|^2 + \| g_{2n-1}(\xi) - E(\xi, g_{2n}(\xi)) \|^2\}] \end{bmatrix}$$

$$= \phi \begin{bmatrix} [\| \{g_{2n}(\xi) - g_{2n-1}(\xi) \|^2, \{\| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 + \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2\}, \\ \tfrac{1}{2}\{\| \{g_{2n}(\xi) - g_{2n}(\xi) \|^2 + \| g_{2n-1}(\xi) - g_{2n+1}(\xi) \|^2\}] \end{bmatrix}$$

$$= \phi \begin{bmatrix} [\| \{g_{2n}(\xi) - g_{2n-1}(\xi) \|^2, \{\| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 + \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2\}, \\ \tfrac{1}{2}\{\| \{g_{2n}(\xi) - g_{2n}(\xi) \|^2 + \| g_{2n-1}(\xi) - g_{2n+1}(\xi) \|^2\}] \end{bmatrix}$$

$$= \phi \begin{bmatrix} \| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2, \{\| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 + \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2\}, \\ \tfrac{1}{2}\{\| g_{2n-1}(\xi) - g_{2n+1}(\xi) \|^2\}] \end{bmatrix}$$

$$= \phi \begin{bmatrix} \| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2, \{\| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 + \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2, \\ \tfrac{1}{2} \{\| (g_{2n-1}(\xi) - g_{2n}(\xi) + g_{2n}(\xi) - g_{2n+1}(\xi) \|^2\} \end{bmatrix}$$

$$= \phi \begin{bmatrix} \| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2, \{\| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 + \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2\}, \\ \tfrac{1}{2} \{2 \| (g_{2n-1}(\xi) - g_{2n}(\xi) \|^2 + 2 \| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 \\ - \| g_{2n-1}(\xi) - g_{2n}(\xi) - (g_{2n}(\xi) - g_{2n+1}(\xi)) \|^2\}] \end{bmatrix}$$

$$\le \phi \begin{bmatrix} \| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2, \{\| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2 + \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2\}, \\ \tfrac{1}{2} \{2 \| (g_{2n-1}(\xi) - g_{2n}(\xi) \|^2 + 2 \| g_{2n}(\xi) - g_{2n+1}(\xi) \|^2\} \end{bmatrix}$$

$$= k \| g_{2n-1}(\xi) - g_{2n}(\xi) \|^2$$

Also,

$$\| g_{2n}(\xi) - g_{2n-1}(\xi) \|^2 \le k \| g_{2n-1}(\xi) - g_{2n-2}(\xi) \|^2.$$

Hence in general

$$|| g_n(\xi) - g_{n+1}(\xi) ||^2 \leq k || g_{n-1}(\xi) - g_n(\xi) ||^2$$

Since $0 < k < 1$, therefore $\{g_n(\xi)\}$ be a Cauchy sequence and hence convergent in H. Therefore,. $g_n(\xi) \to g(\xi)$ as $n \to \infty$. Since C is closed, $g: C \to C$ be a function.

For all $\xi \in \Omega$

$$|| g(\xi) - F(\xi, g(\xi)) ||^2 = || g(\xi) - g_{2n+1}(\xi) + g_{2n+1}(\xi) - F(\xi, g(\xi)) ||^2$$

$$\leq 2|| g(\xi) - g_{2n+1}(\xi) ||^2 + 2 || g_{2n+1}(\xi) - F(\xi, g(\xi)) ||^2 \quad \text{by parallelogram law}$$

$$= 2|| g(\xi) - g_{2n+1}(\xi) ||^2 + 2 || E(\xi, g_{2n}(\xi)) - F(\xi, g(\xi)) ||^2$$

$$= 2 || g(\xi) - g_{2n+1}(\xi) ||^2 + 2\, \phi\, [|| g_{2n}(\xi) - g(\xi)) ||^2, \{|| g_{2n}(\xi) - E(\xi, g_{2n}(\xi)) ||^2 + || g(\xi) - F(\xi, g(\xi)) ||^2\}, ½\{|| g_{2n}(\xi) - F(\xi, g(\xi)) ||^2 + || g(\xi) - E(\xi, g_{2n}(\xi)) |^2\}]$$

$$= 2 || g(\xi) - g_{2n+1}(\xi) ||^2 + 2\, \phi\, [|| g_{2n}(\xi) - g(\xi)) ||^2, \{|| g_{2n}(\xi) - g_{2n+1}(\xi)) ||^2 + || g(\xi) - F(\xi, g(\xi)) ||^2\}, ½\{|| g_{2n}(\xi) - F(\xi, g(\xi)) ||^2 + || g(\xi) - g_{2n+1}(\xi)) ||^2\}]$$

$$= 2 || g(\xi) - g_{2n+1}(\xi) ||^2 + 2\, \phi\, [|| g_{2n}(\xi) - g(\xi)) ||^2, \{|| g_{2n}(\xi) - g_{2n+1}(\xi)) ||^2 + || g(\xi) - F(\xi, g(\xi)) ||^2\}, ½\{|| g_{2n}(\xi) - F(\xi, g(\xi)) ||^2 + || g(\xi) - g_{2n+1}(\xi)) ||^2\}]$$

As $n \to \infty$

$$|| g(\xi) - F(\xi, g(\xi)) ||^2 = 2k || g(\xi) - F(\xi, g(\xi)) ||^2$$

Hence for all $F(\xi, g(\xi)) = g(\xi)$.

Similarly $E(\xi, g(\xi)) = g(\xi)$.

Now, if $G: \Omega \times C \to C$ is a continuous random operation on a non empty subset of a separated Hilbert space H, then for any measurable formula $f: \Omega \to C$ the function $g(\xi) = G(\xi, f(\xi))$ is also measurable. Therefore the sequence of measurable functions $\{g_n\}$ converge to measurable function of this fact along with

$$E(\xi, g(\xi)) = g(\xi) = F(\xi, g(\xi))$$

shows that $g: \Omega \to C$ is common random fixed point of E and F.

Uniqueness: Let $h: \Omega \to C$ be another common random fixed point of E and F.

$$\| g(\xi)-h(\xi)\|^2 = \| E(\xi, g(\xi)) - F(\xi, h(\xi))\|^2$$
$$\leq \phi[\| g(\xi)-h(\xi)\|^2, \| g(\xi)-E(\xi,g(\xi))\|^2 + \| h(\xi) \ -F(\xi,g(\xi))\|^2$$
$$½\{\| g(\xi)-F(\xi,\ h(\xi))\|^2 + \| h(\xi)-F(\xi,\ g(\xi))\|^2\}]$$
$$= k \| g(\xi)-h(\xi)\|^2$$
$$\Rightarrow g(\xi)=h(\xi) \qquad \forall \xi \in \Omega \quad (\text{as } \ 0<k<1)\ .$$
□

Theorem 3.2 *Let C be a non empty subset of separable Hilbert Space H. Let E and F be two continuous random operations defined on C such that for* $\xi \in \Omega$, $E(\xi,\ \cdot)$ *and* $F(\xi,\ \cdot): C \to C$ *satisfying condition* (2.5). *Then the sequence* $\{g_n\}$ *converges to the unique common random fixed point of E and F.*

Proof. $\{g_n\}$ is sequence of function defined for $\xi \in \Omega$ and $n = 0,1,2,\ldots$

$$g_{2n+1}(\xi) = E(\xi, g_{2n}(\xi)),\quad g_{2n+2}(\xi) = F(\xi, g_{2n+1})$$

$$\left\|g_{2n+1}(\xi)-g_{2n}(\xi)\right\|^2 = \left\|E(\xi,g_{2n}(\xi))-F(\xi,g_{2n-1}(\xi))\right\|^2$$

$$\leq \phi\begin{bmatrix} \left\|g_{2n}(\xi)-g_{2n-1}(\xi)\right\|^2, \left[\ \left\|g_{2n}(\xi)-Eg_{2n}(\xi)\right\|^2 + \left\|g_{2n-1}(\xi)-Fg_{2n-1}(\xi)\right\|^2\ \right], \\ \frac{1}{2}[\left\|g_{2n}(\xi)-Fg_{2n-1}(\xi)\right\|^2 + \left\|g_{2n-1}(\xi)-Eg_{2n}(\xi)\right\|^2], \\ \dfrac{\frac{1}{2}\left[\left\|g_{2n}(\xi)-g_{2n-1}(\xi)\right\|^2 + \left\|g_{2n-1}(\xi)-Eg_{2n}(\xi)\right\|^2 + \left\|g_{2n}(\xi)-Eg_{2n}(\xi)\right\|^2\right]}{1+\left\|g_{2n}(\xi)-g_{2n-1}(\xi)\right\|^2 \left\|g_{2n}(\xi)-Fg_{2n-1}(\xi)\right\|^2 \left\|g_{2n}(\xi)-Eg_{2n}(\xi)\right\|^2}, \\ \dfrac{\left\|g_{2n-1}(\xi)-Fg_{2n-1}(\xi)\right\|^2 + \left\|g_{2n}(\xi)-Fg_{2n-1}(\xi)\right\|^2}{1+\left\|g_{2n-1}(\xi)-Eg_{2n}(\xi)\right\|^2 \left\|g_{2n-1}(\xi)-Fg_{2n-1}(\xi)\right\|^2 \left\|g_{2n}(\xi)-Fg_{2n-1}(\xi)\right\|^2} \end{bmatrix}$$

$$= \phi\begin{bmatrix} \left\|g_{2n}(\xi)-g_{2n-1}(\xi)\right\|^2, \left[\ \left\|g_{2n}(\xi)-g_{2n+1}(\xi)\right\|^2 + \left\|g_{2n-1}(\xi)-g_{2n}(\xi)\right\|^2\ \right], \\ \frac{1}{2}\left\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\right\|^2, \\ \dfrac{\frac{1}{2}\left[\left\|g_{2n}(\xi)-g_{2n-1}(\xi)\right\|^2 + \left\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\right\|^2 + \left\|g_{2n}(\xi)-g_{2n+1}(\xi)\right\|^2\right]}{1+\left\|g_{2n}(\xi)-g_{2n-1}(\xi)\right\|^2 \left\|g_{2n}(\xi)-g_{2n}(\xi)\right\|^2 \left\|g_{2n}(\xi)-g_{2n+1}(\xi)\right\|^2}, \\ \dfrac{\left\|g_{2n-1}(\xi)-g_{2n}(\xi)\right\|^2 + \left\|g_{2n}(\xi)-g_{2n}(\xi)\right\|^2}{1+\left\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\right\|^2 \left\|g_{2n-1}(\xi)-g_{2n}(\xi)\right\|^2 \left\|g_{2n}(\xi)-g_{2n}(\xi)\right\|^2} \end{bmatrix}$$

$$= \phi \begin{bmatrix} \|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2, \left[\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2 + \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2 \right], \\ \frac{1}{2}\|g_{2n-1}(\xi)-Eg_{2n+1}(\xi)\|^2, \\ \frac{1}{2}\left[\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2 + \|g_{2n-1}(\xi)-g_{2n+1}(\xi)\|^2 + \|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2 \right], \\ \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2 \end{bmatrix} \tag{1}$$

Now

$$\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\|^2 = \| [g_{2n-1}(\xi)-g_{2n}(\xi)] + [g_{2n}(\xi)-g_{2n+1}(\xi)] \|^2$$

By using parallelogram law we can write

$$\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2 \Rightarrow \|x+y\|^2 = 2\|x\|^2 + 2\|y\|^2 - \|x-y\|^2, \forall\, x, y \in C$$

$$\begin{aligned} &\|[g_{2n-1}(\xi)-g_{2n}(\xi)]+[g_{2n}(\xi)-g_{2n+1}(\xi)]\|^2 \\ &\quad = 2\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2 + 2\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2 - \| [g_{2n-1}(\xi)-g_{2n}(\xi)]-[g_{2n}(\xi)-g_{2n+1}(\xi)]\|^2 \\ &\quad \le 2\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2 + 2\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2 \end{aligned}$$

On putting this in (1), we get

$$\|g_{2n+1}(\xi)-g_{2n}(\xi)\|^2 \le \phi \begin{bmatrix} \|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2, \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2 + \|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2, \\ \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2 + \|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2, \\ \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2 + \|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2, \\ \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2 \end{bmatrix}$$

$$\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2 \le k\ \|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2 \tag{2}$$

Similarly we can find for

$$\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2 \le k\ \|g_{2n-1}(\xi)-g_{2n-2}(\xi)\|^2 \qquad \forall\, \xi \in \Omega \tag{3}$$

Equations (2) and (3) jointly implies that

$$\|g_n(\xi)-g_{n+1}(\xi)\|^2 \le k\ \|g_{n-1}(\xi)-g_n(\xi)\|^2 \qquad \forall\, \xi \in \Omega \tag{4}$$

It is clear that $g_n(\xi)$ is a Cauchy sequence and hence it is convergent in the Hilbert spaces *H*. So

$$g_n(\xi) \to g(\xi) \text{ as } n \to \infty \tag{5}$$

Since C is closed and $g : C \to C$, so for $\xi \in \Omega$

$$\|g(\xi) - E(\xi, g(\xi))\|^2 = \|(g(\xi) - g_{2n}(\xi)) + (g_{2n}(\xi) - E(\xi, g(\xi))\|^2$$

$$\leq 2\|g(\xi) - g_{2n}(\xi)\|^2 + 2\|g_{2n}(\xi) - E(\xi, g(\xi))\|^2 \qquad \text{by parallelogram law}$$

$$= 2\|g(\xi) - g_{2n}(\xi)\|^2 + 2\|F(\xi, g_{2n-1}(\xi)) - E(\xi, g(\xi))\|^2$$

$$= 2\|g(\xi) - g_{2n}(\xi)\|^2 + 2\|E(\xi, g(\xi)) - F(\xi, g_{2n-1}(\xi))\|^2$$

$$= 2\|g(\xi) - g_{2n}(\xi)\|^2 + 2\begin{bmatrix} \|g(\xi) - g_{2n-1}(\xi)\|^2, \left[\|g(\xi) - Eg(\xi)\|^2 + \|g_{2n-1}(\xi) - Fg_{2n-1}(\xi)\|^2\right], \\ \frac{1}{2}[\|g(\xi) - Fg_{2n-1}(\xi)\|^2 + \|g_{2n-1}(\xi) - Eg(\xi)\|^2] \\ \dfrac{\frac{1}{2}\left[\|g(\xi) - g_{2n-1}(\xi)\|^2 + \|g(\xi) - Eg(\xi)\|^2 + \|g_{2n-1}(\xi) - E(g(\xi))\|^2\right]}{1 + \|g(\xi) - g_{2n-1}(\xi)\|^2 \|g(\xi) - Fg_{2n-1}(\xi)\|^2 \|g(\xi) - Eg(\xi)\|^2} \\ + \dfrac{\|g_{2n-1}(\xi) - Eg(\xi)\|^2 + \|g_{2n-1}(\xi) - Fg_{2n-1}(\xi)\|^2}{1 + \|g_{2n-1}(\xi) - Eg(\xi)\|^2 \|g_{2n-1}(\xi) - Fg_{2n-1}(\xi)\|^2 \|g(\xi) - Fg_{2n-1}(\xi)\|^2} \end{bmatrix} \tag{6}$$

Making $n \to \infty$ and by the help of (6)

$$\|g(\xi) - E(\xi, g(\xi)\|^2 \leq \|g(\xi) - Eg(\xi)\|$$

So $\forall \xi \varepsilon \Omega$

$$E(\xi, g(\xi)) = g(\xi) \tag{7}$$

Similarly we can prove that

$$F(\xi, g(\xi)) = g(\xi) \tag{8}$$

Again, if $A : \xi \times C \to C$ is a random operator on a nonempty subset C of a separable spaces H, then for any measurable function $f : \Omega \to C$, the function $h(\xi) = A(\xi, f(\xi))$ is also measurable. It follows from the construction of $\{g_n\}$ and the above considerations that $\{g_n\}$ is sequence of measurable functions it follows that g is also a measurable function.

From (7) and (8) shows that $g : \Omega \to C$ is a common fixed point of E and F.

Uniqueness: Let $h:\Omega\to C$ be another common random fixed point of E and F that is for $\xi\,\varepsilon\,\Omega$,

$$E(\xi,h(\xi))=h(\xi),F(\xi,h(\xi))=h(\xi) \tag{9}$$

$$\|g(\xi)-h(\xi)\|^2=\|E(\xi,g(\xi))-F(\xi,h(\xi))\|^2$$

$$\le\phi\begin{bmatrix}\|g(\xi)-h(\xi)\|^2,\left[\|g(\xi)-Eg(\xi)\|^2+\|h(\xi)-Fh(\xi)\|^2\right],\\ \frac{1}{2}[\|g(\xi)-Fh(\xi)\|^2+\|h(\xi)-Eg(\xi)\|^2],\\ \dfrac{\frac{1}{2}\left[\|g(\xi)-h(\xi)\|^2+\|g(\xi)-Eg(\xi)\|^2+\|h(\xi)-Eg(\xi)\|^2\right]}{1+\|g(\xi)-h(\xi)\|^2\|g(\xi)-Fh(\xi)\|^2\|g(\xi)-Eg(\xi)\|^2},\\ \dfrac{\|h(\xi)-Eg(\xi)\|^2+\|h(\xi)-Fh(\xi)\|^2}{1+\|h(\xi)-Eg(\xi)\|^2\|h(\xi)-Fh(\xi)\|\|g(\xi)-Fh(\xi)\|^2}\end{bmatrix}$$

$$=\|h(\xi)-g(\xi)\|^2$$

which is a contradiction therefore $g(\xi)=h(\xi)$. □

4 Example

Let $H=R$, $\Omega=[0,1]$ and Σ be the sigma algebra of Lebesgue's measurable subset of [0,1].

Let $C=[0,\infty)$ and define a mapping $d:(\Omega\times X)\times(\Omega\times X)\to E$ by

$$d(x,y)=|x(\omega)-y(\omega)|\ .$$

Define random operator $E,F:\ \Omega\times X\to X$ as

$$E(\xi,x)=(1-\xi^2)x\ \text{ and }\ F(\xi,x)=(1-\xi^2)\frac{x}{2}$$

Also sequence of mapping $g_n:\Omega\to X$ is defined by $g_n(\xi)=(1-\xi^2)^{1+1/n}$, for every $\xi\in\Omega$ and $n\in N$. Define measurable mapping $g:\Omega\to X$ as $g(\xi)=1-\xi^2$, for every $\xi\in\Omega$, which is fixed point of E and F. □

5 Conclusion

Firstly, we ensure the unique fixed point without continuity of random mappings with rational relation analogue of a plane contractive. Secondly, we provide measurable sequence of function which converse to measurable function to ensure the existence of a common fixed point.

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