Communications in Mathematical Finance, vol. 6, no. 1, 2017, 61-85
ISSN: 2241-195X (print), 2241- 1968 (online)
Scienpress Ltd, 2017

A Production Inventory Model Consisting Time Dependent Linear Demand and Constant Production Rate

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Abstract

In this proposed model the products are considered to have finite life with a small amount of decay. The market demand is assumed to be linear and time dependent. It is also assumed that the production starts with zero inventories without any backlogs and the production rate is constant, stopping after inventories reach a desired highest level of inventories. Inventory depletes to zero level from where the production cycle starts. A numerical illustration for the proof of the proposed model has been shown. The objective of this model is to obtain the total optimum inventory cost.

Mathematics Subject Classification: 90B05
Keywords: Production inventory, Time dependent linear form, Decay and Constant production rate.

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1 Introduction

To be profitable it is necessary to reduce inventory cost and the business enterprises concentrate on obtaining the economic order quantity (EOQ). The innovative inventory model is, therefore, in high demand for reduction of inventory cost. All products have finite life and market demand s. As a result, the inventory continuously decreases and some items in the inventory deteriorate. This deterioration raises inventory cost. To minimize the inventory cost, an inventory model with time dependent linear demand, small amount of decay, and constant production rate has been proposed and justified with the convex property in this paper.

The researchers have been devising suitable inventory models to face the challenges in inventory cost management, which would meet the demand in real life. Many researchers have structured various types of inventory models based on the situation or the market demand. There may be different types of demands in the market such as linear, quadratic, exponential, time dependent, level or stock dependent, price dependent etc. Generally, there are two types of models in the inventory management problems. One is deterministic model which deals with the constant demand and lead time; the other one is probabilistic model which deals randomly with the variable demand and lead time. In this paper, the proposed inventory model has been developed with a deterministic demand. The objective of formulating the model is to minimize the inventory cost by finding the EOQ.

This is Harris [1] who for the first time studied inventory model. The new horizon is opened in the field of the inventory control and management since his presentation of the famous EOQ formula. Whitin [2] made an inventory model suitable for fashionable goods that decay little in inventory. Skouri and Papachristos [3] discussed a continuous review inventory model considering five costs such as deterioration, holding, shortage, opportunity cost of lost sales, and replenishment cost due to the linear dependency on the lot size. Chund and Wee [4] developed an integrated two stages production inventory deteriorating model for the buyer and the supplier on the basis of stock dependent selling rate considering imperfect items and just in time (JIT) multiple deliveries. Applying the inventory replenishment policy, Mingbao and Wang [5] expressed an inventory model for deteriorating items with trapezoidal type demand rate, where the demand rate is a piecewise linear functions. Sarker et al. [6] explained an inventory model where demand was a composite function consisting of constant component and variable component proportional to inventory level in a period in which decay was exponential and inventory was positive. Tripathy and Mishra [7] discussed an inventory model with ordering policy for weibull deteriorating items, quadratic demand, and permissible delay in payments. Islam [8, 9] considered various production rates under constant demand. Islam et. al [10, 11] developed a production inventory model for different classes of demands with constant production rate. Mishra et al. [12] explained an inventory model for deteriorating items in time dependent demand and time varying holding cost under partial backlogging. Sivazlian and Stenfel [13] determined the optimum value of time cycle by using the graphical solution of the equation to obtain the economic order quantity model. Billington [14] discussed classic economic production quantity (EPQ) model without backorders or backlogs. Pakkala and Achary [15] established a deterministic inventory model for deteriorating items with two warehouses, while the replenishment rate was finite, demand was uniform and shortage was allowed. Abad [16] discussed regarding optimal pricing and lot sizing under conditions of perishable and partial backordering.

Amutha and Chandrasekaran [17] formulated an inventory model with deterioration items, quadratic demand and time dependent holding cost. However, in our proposed model, we have emphasized on the production rate, linear demand, and constant holding cost. Ouyang and Cheng [18] explained the inventory model for deteriorating items with exponential declining demand and partial backlogging. Ukil et al. [19] considered a Production Inventory Model of Power Demand and Constant Production Rate where the Products have finite Shelf-life. Ukiland Uddin

[20]constructed a production inventory model with constant production rate and level dependent linear trend demand. Min and Zhou [21] also discussed an inventory model with stock dependent demand. Urban [22] considered a model where demand is partially dependent on instantaneous stock level. Montgomery et al. [23] and Rosenberg [24] assumed that unsatisfied demand is backlogged at a fixed fraction of the constant demand rate. Krishnamoorthy and Ekramol [25] discussed inventory model considering postponed demands. Mingbao and Wang [26] studied an inventory model for deteriorating items with trapezoidal type demand rate and then proposed a replenishment policy for this type of model. Mandal and Pal [27] extended an inventory model with power demand pattern for the perishable items. Jain et al. [28] developed an inventory model with level-dependent demand rate. They assumed the demand for items in a decreasing form and allowed partial backlogs. Dash et al. [29] discussed an inventory model using exponential decreasing demand and time varying holding cost. Ouyang et al. [30] developed an inventory model for deteriorating items with exponential declining demand and partial backlogs. Raj et al. [31] also used exponential demand to make an inventory model. Kishan and Mishra [32] developed an inventory model with exponential demand and constant deterioration. Teng et al [33] developed an inventory model with deteriorating items and shortages assuming that the demand function was positive and fluctuating with respect to time.

Section 2 describes Mathematical model of the inventory problem. In section 3 we give numerical illustration. In Section 4 sensitivity analysis is presented. Section 5 presents a conclusion.

2 Mathematical Model of the Inventory Problem

2.1 The assumptions and notations used to formulate the model are discussed below:

Assumptions

(i) Production rate λ is always constant and greater than demand rate at all time.

(ii) For a unit of inventory, the amount of decay rate, μ is very small and constant.

(iii) Production starts with zero inventories.

(iv) Inventory level is highest at a specific level and after this point, the inventory depletes quickly due to demand and deterioration.

(iv) Shortages are not allowed.

Notations

$I(\theta)$ Inventory level at instant θ

I_1 Un-decayed inventory at $T = 0$ to t_1

I_2 Un-decayed inventory at $T = t_1$ to T_1

D_1 Deteriorating inventory at $T = 0$ to t_1

D_2 Deteriorating inventory at $T = t_1$ to T_1

0 Inventory level at time $T = 0$ and T_1

Q_1 Inventory level at time $T = t_1$

$d\theta$ A very small portion of instant θ

K_0 Set up cost

h Average holding cost

TC $TC(T_1)$ = Total inventory cost in terms of T_1

t_1 Time when inventory gets maximum level

T_1 Total time cycle

Q_1^* Optimum order quantity

t_1^* Optimum time at maximum inventory

T_1^* Optimum total time

TC^* Total optimum inventory cost.

2.2 Development of the inventory model

The objective of business firm is to maximize profit and minimize cost of production. As a result, all various decisions have to be taken using suitable models. The demand pattern and production plan dictate the decision of how and which model to be used. This proposed model may be changed to another model depending on the situation.

There are various types of demands in the market for different types of items. These demands may be linear, exponential, quadratic, and so on. In this model, a linear demand is proposed. The model is suitable for products that have finite life and ultimately causes the products to decay. The model is explained in the Figure 1 below.

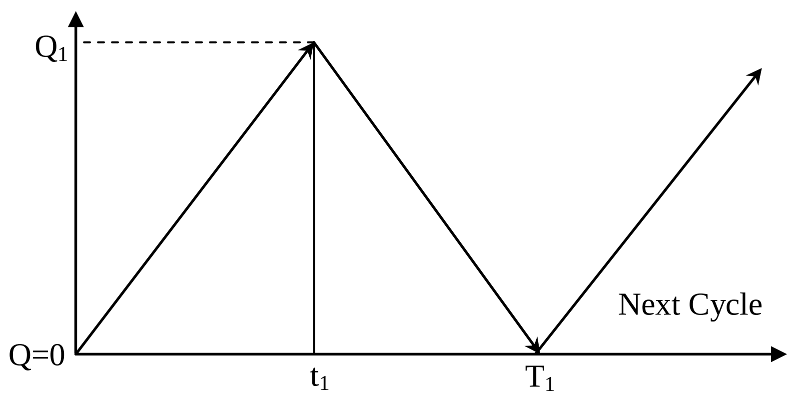

Figure 1: Inventory model with linear

Here, it is considered that at time $T = 0$, the production starts with zero inventories, where the production rate λ is constant for entire production cycle. During $T = 0$ to t_1, the inventory increases at the rate of $\lambda - a\theta - \mu I(\theta)$, as $a\theta$ is the demand in the market and $\mu I(\theta)$ is the decay of $I(\theta)$ inventories at any instant θ, where μ is the decay of a unit of inventory. Under these conditions, we get the following equations,

$$I(\theta + d\theta) = I(\theta) + (\lambda - a\theta)d\theta - \mu I(\theta)d\theta$$

$$or,\ I(\theta + d\theta) - I(\theta) = \{(\lambda - a\theta) - \mu I(\theta)\}d\theta$$

$$or,\ \underset{d\theta\to 0}{Lim}\frac{I(\theta+d\theta)-I(\theta)}{d\theta}=(\lambda-a\theta)-\mu I(\theta)$$

$$or,\ \frac{d}{d\theta}I(\theta)+\mu I(\theta)=\lambda-a\theta$$

The general solution of the differential equation is, $I(\theta)=\frac{\lambda}{\mu}-\frac{a\theta}{\mu}+\frac{a}{\mu^2}+Ae^{-\mu\theta}$

Applying the boundary condition, i.e. at $\theta=0$, we get, $I(\theta)=0$

By solving we get, $A=-\frac{\lambda}{\mu}-\frac{a}{\mu^2}$

Therefore,

$$I(\theta)=\frac{\lambda}{\mu}-\frac{a\theta}{\mu}+\frac{a}{\mu^2}-\left(\frac{\lambda}{\mu}+\frac{a}{\mu^2}\right)e^{-\mu\theta} \tag{1}$$

Putting the other boundary condition, i.e. at $\theta=t_1, I(\theta)=Q_1$, taking up to first order of μ we get the following equation,

$$Q_1=\frac{\lambda}{\mu}-\frac{at_1}{\mu}+\frac{a}{\mu^2}-\left(\frac{\lambda}{\mu}+\frac{a}{\mu^2}\right)e^{-\mu t_1}$$

$$=\frac{\lambda}{\mu}-\frac{at_1}{\mu}+\frac{a}{\mu^2}-\left(\frac{\lambda}{\mu}+\frac{a}{\mu^2}\right)(1-\mu t_1)$$

$$=\lambda t_1$$

$$\text{i.e. } Q_1=\lambda t_1 \tag{2}$$

Now considering μ up to first order, the total un-decayed inventory during $\theta=0$ to t_1 gets as below,

$$I_1=\int_0^{t_1}I(\theta)d\theta=\int_0^{t_1}\left[\frac{\lambda}{\mu}-\frac{a\theta}{\mu}+\frac{a}{\mu^2}-\left(\frac{\lambda}{\mu}+\frac{a}{\mu^2}\right)e^{-\mu\theta}\right]d\theta$$

$$=\frac{\lambda}{\mu}t_1-\frac{at_1^{\,2}}{2\mu}+\frac{a}{\mu^2}t_1-\left(\frac{\lambda}{\mu}+\frac{a}{\mu^2}\right)\left[\frac{e^{-\mu\theta}}{-\mu}\right]_0^{t_1}$$

$$=\frac{1}{2}\lambda t_1^{\,2} \tag{3}$$

Considering the decay of the items, we calculate the deteriorating items during the period as below:

$$D_1 = \int_0^{t_1} \mu I(\theta) d\theta = \mu \frac{\lambda}{\mu} t_1 - \mu \frac{a t_1^{\,2}}{2\mu} + \mu \frac{a}{\mu^2} t_1 - \mu \left(\frac{\lambda}{\mu} + \frac{a}{\mu^2} \right) \left[\frac{e^{-\mu\theta}}{-\mu} \right]_0^{t_1}$$

$$= \frac{1}{2} \lambda \mu t_1^{\,2} \tag{4}$$

Again during $T = t_1$ to T_1, the inventory decreases at the rate of $a\theta$, as there is no production after time t_1 and inventory reduces due to market demand only. Applying the similar condition as used before, we get the differential equation as mentioned below:

$$\frac{d}{d\theta} I(\theta) + \mu I(\theta) = -a\theta$$

The general solution of the differential equation is defined below:

$$I(\theta) = \frac{-a\theta}{\mu} + \frac{a}{\mu^2} + B e^{-\mu\theta}$$

Applying the boundary condition at $\theta = T_1$, we get, $I(\theta) = 0$

By solving we get, $B = \left(\frac{aT}{\mu} - \frac{a}{\mu^2} \right) e^{\mu T_1}$

Therefore,

$$I(\theta) = -\frac{a\theta}{\mu} + \frac{a}{\mu^2} - \left(\frac{aT}{\mu} - \frac{a}{\mu^2} \right) e^{\mu(T-\theta)} \tag{5}$$

Putting on another boundary condition, i.e. at $\theta = t_1$, $I(\theta) = Q_1$, taking up to first order of μ ,we get the following equation,

$$Q_1 = -\frac{a t_1}{\mu} + \frac{a}{\mu^2} + \left(\frac{aT}{\mu} - \frac{a}{\mu^2} \right) e^{\mu(T_1 - t_1)})$$

$$= -\frac{a t_1}{\mu} + \frac{a}{\mu^2} + \left(\frac{aT}{\mu} - \frac{a}{\mu^2} \right) \{1 + \mu(T_1 - t_1)\}$$

$$= aT(T_1 - t_1) \tag{6}$$

The value of time t_1 is a part of total time cycle T_1. Hence, for an arbitrary decimal value of ν we assume the following relation,

$$t_1 = \nu T \tag{7}$$

For any arbitrary value of ν the optimum length of time t_1 and time cycle T will change.

From the equations (2), (6) and (7), we get the following relation,

$$\lambda t_1 = aT(T_1 - t_1)$$

Or,

$$(T - t_1)^2 = \frac{\lambda^2 \nu^2}{a} \tag{8}$$

From the equations (7) and (8), we get the relation as given below.

Now, considering up to the first degree of μ we get the un-decayed inventory during $T = t_1$ to T_1 as:

$$I_2 = \int_{t_1}^{T_1} I(\theta)d\theta = \int_{t_1}^{T_1}\left[-\frac{a\theta}{\mu} + \frac{a}{\mu^2} + \left(\frac{aT}{\mu} - \frac{a}{\mu^2}\right)e^{\mu(T-\theta)}\right]d\theta$$

$$= -\frac{a}{2\mu}\left(T^2 - t_1^{\,2}\right) + \frac{a}{\mu^2}(T - t_1) + \left(\frac{aT}{\mu} - \frac{a}{\mu^2}\right)\left(\frac{1}{\mu}\right)\{\mu(T - t_1)\}$$

$$= \frac{\lambda^2 \nu^2}{2a\mu} \tag{9}$$

Considering the decay of the items, we calculate the deteriorating items during the period as below:

$$D_2 = \int_{t_1}^{T_1} \mu I(\theta)d\theta = \int_{t_1}^{T_1} \mu\left[-\frac{a\theta}{\mu} + \frac{a}{\mu^2} + \left(\frac{aT}{\mu} - \frac{a}{\mu^2}\right)e^{\mu(T-\theta)}\right]d\theta$$

$$= \frac{\lambda^2 \nu^2}{2a} \tag{10}$$

Total Cost Function: Total cost function can be written as below,

$$TC(T_1) = \frac{K_0 + h(I_1 + I_2) + \eta(D_1 + D_2)}{T_1})$$

By using the equation no (3), (4), (9) and (10), we get the following result,

$$TC(Q_1)=\frac{1}{T_1}\left[K_0+h\left(\frac{\lambda t_1^{\ 2}}{2}+\frac{\lambda^2\nu^2}{2a\mu}\right)+\eta\left(\frac{\lambda\mu t_1^{\ 2}}{2}+\frac{\lambda^2\nu^2}{2a}\right)\right]$$

$$=\frac{1}{T_1}\left[K_0+\left(h+\eta\mu\right)\left(\frac{\lambda t_1^{\ 2}}{2}\right)+\left(\frac{h}{\mu}+\eta\right)\left(\frac{\lambda^2\nu^2}{2a}\right)\right]$$

$$=\frac{1}{T_1}\left\{K_0+\frac{(h+\eta\mu)\lambda^2\nu^2}{2a\mu}\right\}+\frac{(h+\eta\mu)\lambda\nu^2 T_1}{2} \tag{11}$$

Hence, the objective is to find out the total time cycle T_1 that minimizes the total cost for the inventory system which depicts the equation no (10). With a view to determine the optimum time cycle $T_1^{\ *}$ and to verify that the equation no (10) is convex in T_1, we must show that the following two convex properties are satisfied,

(i) $\frac{d}{dT_1}TC(T_1)=0$ and

(ii) $\frac{d^2}{dT_1}TC(T_1)>0$

From the convex property (i) i.e. $\frac{d}{dT_1}TC(T_1)=0$ we get the equation as below:

$$-\frac{1}{T_1^{\ 2}}\left\{K_0+\frac{(h+\eta\mu)\lambda^2\nu^2}{2a\mu}\right\}+\frac{(h+\eta\mu)\lambda\nu^2}{2}=0$$

or $\frac{1}{T_1^{\ 2}}\left\{K_0+\frac{(h+\eta\mu)\lambda^2\nu^2}{2a\mu}\right\}=\frac{(h+\eta\mu)\lambda\nu^2}{2}$

Therefore, the optimum time cycle,

$$T_1^{\ *}=\sqrt{\frac{2K_0 a\mu+(h+\eta\mu)\lambda^2\nu^2}{(h+\eta\mu)a\mu\lambda\nu^2}} \tag{12}$$

Now differentiating the equation no (11) twice with respect to T_1, we get,

$$\frac{d^2}{dT_1^{\ 2}}TC(T_1)=\frac{2}{T_1^{\ 3}}\left\{K_0+\frac{(h+\eta\mu)\lambda^2\nu^2}{2a\mu}\right\} \tag{13}$$

From equation no (13) we can conclude that the convex property (ii) $\frac{d^2}{dQ_1^2}TC(Q_1)>0$, as $K_0, a, \lambda, \mu, \nu, h.T_1$ all are positive.

Finally, we can conclude that the total cost function (11) is convex in T_1. Hence, there is an optimal solution in T_1 for which the total cost function is going to be minimal.

3 Numerical Illustration

Here we have provided a numerical illustration to justify the proposed model in this section. Let the inventory system has the values of the parameters as 1 $K_0 = 100, h = 1, a = 2, \lambda = 20, \nu = 0.51$ and $\mu = 0.1$. With the help of Mathematica 10.0,in this model we can easily find out the various optimum values as of Q_1^*, t_1^*, T_1^* and TC^*. Instead of a traditional algorithm, the software Mathematica 10.0 is used for better accuracy. After putting these values in equation no (6),(7), (11) and (12) we get the following optimum result respectively,

(i) Optimum order quantity Q_1^* = 135.61units,

(ii) Optimum time t_1^* = 5.83 units,

(iii) Optimum total time cycle T_1^* = 11.68 units,

(iv) Total optimum cost $TC^*(T_1)$ = 64.23units.

Table 1: Total Time Cycle (T_1) verses Total Cost (TC)

Total Time Cycle (T_1)	7.5	8.5	9.5	10.5	**11.68**	12.5	13.5	14.5	15.5
Total Cost (TC)	70.63	67.49	65.60	64.59	**64.23**	64.38	64.90	65.74	66.82

For total time cycle $T_1^* = 11.68$, the total cost TC gives a minimum cost. The total cost gets more, if the total time cycle becomes more or less than the total time $T_1^* = 11.68$. Hence, optimum total time is justified, which is shown in the Figure 2 below.

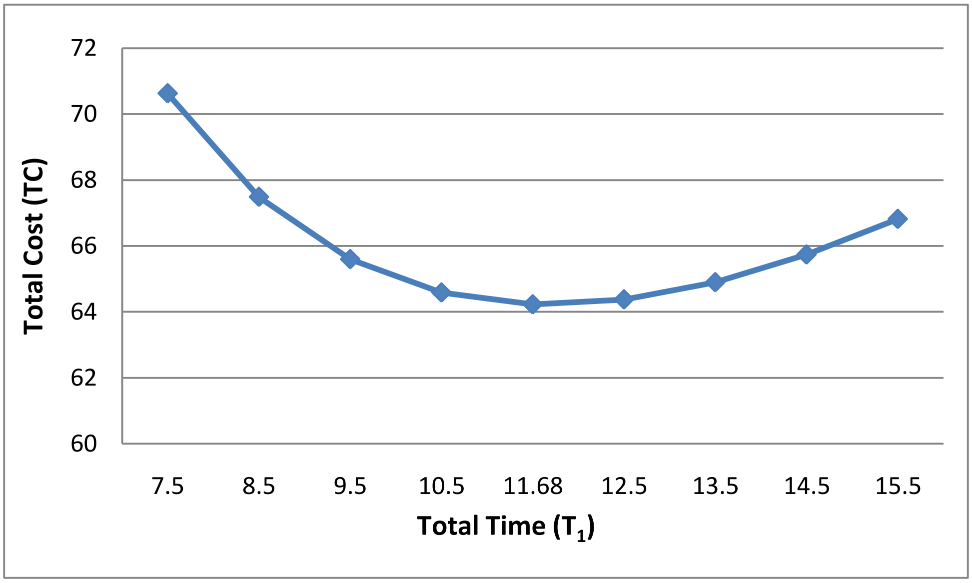

Figure 2: Time Verses Total Cost

Here, in this case, the optimum result will be as below:

(i) Optimum order quantity Q_1^* = 135.61units,

(ii) Optimum total time cycle T_1^* = 11.68 units,

(iii) Total optimum cost $TC^*(T_1)$ = 64.23 units.

Table 2: Time (t_1) verses Total Cost (TC)

Time (t_1)	5.00	5.20	5.40	5.60	**5.83**	6.00	6.20	6.40	6.60
Total Cost (TC)	62.50	62.16	61.91	61.76	**61.70**	61.65	61.78	61.92	62.11

For time $t_1^* = 5.83$, the total cost TC gives a minimum cost. The total cost increases if the time become more or less than the total time $t_1^* = 5.83$. Hence, the optimum time is justified, which is shown in the Figure 3 below.

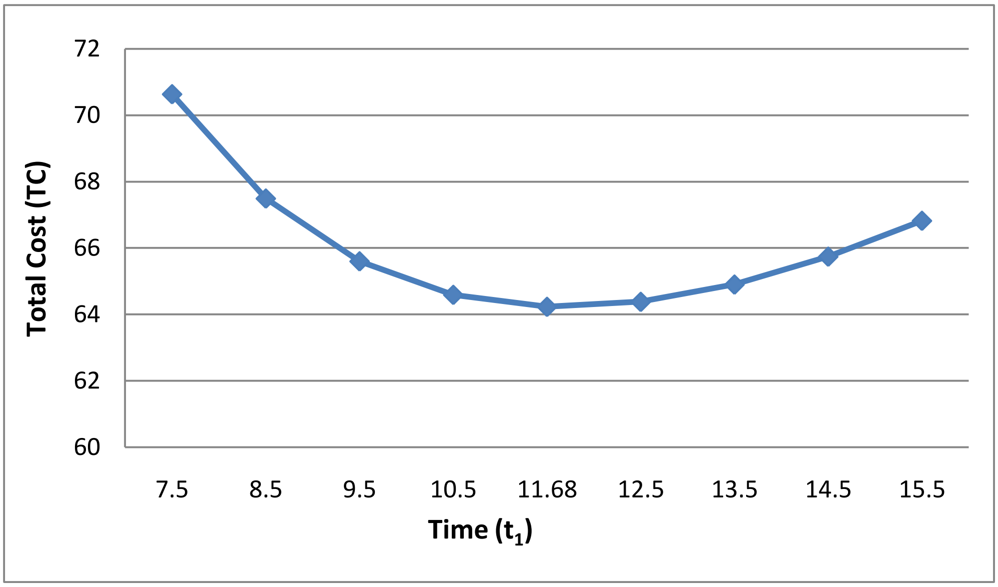

Figure 3: Ordering Time Verses Total Cost

The enterprises are likely to go for calculating the time and put importance on when and how much will be ordered. This will dictate the firm to take decision on the basis of production plan. Considering the results in the table 1 and 2 as well as figure 2 and 3, we can calculate that the total optimum cost gives better result based on the time t_1 than that of total time T_1. Hence, the enterprises will give emphasize on the ordering time t_1 than the total time cycle T_1.

4 Sensitivity Analysis

4.1 Case I (arbitrary value $\nu = 0.5$)

In this case, we have studied the effect of changes of parameters $K_0 = 100, h = 1, a = 2, \lambda = 20, \nu = 0.5$ and $\mu = 0.1$ on the optimal length of ordering cycle t_1, optimal time cycle T, optimal ordering quantity Q_1, and the optimum total cost TC per unit time in the model. We have performed the sensitivity analysis by changing each of the parameters by +50%, +25%, +10%, -10%, -25% and -50% taking one parameter at a time while keeping other parameters unchanged. The detail results are shown in the Table 3 below.

Table 3: Sensitivity Analysis for $\nu = 0.5$

Parameters	Change in %	Value of			
		t_1^*	T_1^*	Q_1^*	TC^*
K_0	+50	5.84	12.43	135.61	68.51
	+25	5.84	12.00	135.61	66.37
	+10	5.84	11.83	135.61	65.08
	-10	5.84	11.52	135.61	63.37
	-25	5.84	11.28	135.61	62.09
	-50	5.84	10.87	135.61	59.95
λ	+50	5.84	13.20	135.61	109.72
	+25	5.84	12.41	135.61	85.50
	+10	5.84	11.96	135.61	72.38
	-10	5.84	11.42	135.61	56.54
	-25	5.84	11.11	135.61	45.90
	-50	5.84	11.08	135.61	30.51
a	+50	5.84	10.15	204.98	56.38
	+25	5.84	10.79	170.82	59.52
	+10	5.84	11.28	150.32	62.09
	-10	5.84	12.14	122.99	66.84
	-25	5.84	13.03	102.49	72.07
	-50	5.84	15.37	68.33	87.77
h	+50	5.84	13.23	135.61	89.53
	+25	5.84	12.44	135.61	76.88
	+10	5.84	11.97	135.61	69.29
	-10	5.84	11.40	135.61	59.17
	-25	5.84	11.05	135.61	51.58
	-50	5.84	10.80	135.61	38.92
μ	+50	5.84	10.07	135.61	58.55
	+25	5.84	10.75	135.61	60.68
	+10	5.84	11.27	135.61	62.57
	-10	5.84	12.16	135.61	66.31
	-25	5.84	13.06	135.61	70.63
	-50	5.84	15.43	135.61	84.17
ν	+50	5.84	11.60	135.61	66.76
	+25	5.84	11.6 4	135.61	65.49
	+10	5.84	11.66	135.61	64.73
	-10	5.84	11.69	135.61	63.72
	-25	5.84	1 1.71	135.61	62.96
	-50	5.84	11.75	135.61	61.70

4.2 Case II (arbitrary value $\nu = 0.75$)

Here we have studied the effect of changes of parameters $K_0 = 100, h = 1, a = 2, \lambda = 20, \nu = 0.75$ and $\mu = 0.1$ on the optimal length of ordering cycle t_1, optimal time cycle T, optimal ordering quantity Q_1,and the optimum total cost TC per unit time in the model. We have performed the sensitivity analysis by changing each of the parameters by +50%, +25%, +10%, -10%, -25% and -50% taking one parameter at a time while keeping other parameters unchanged. The detail results are shown in the Table 4 below.

4.3 Case III (arbitrary value $\nu = 1$)

Similarly we have studied the effect of changes of parameters $K_0 = 100, h = 1, a = 2, \lambda = 20, \nu = 1$ and $\mu = 0.1$ on the optimal length of ordering cycle t_1, optimal time cycle T, optimal ordering quantity Q_1,and the optimum total cost TC per unit time in the model. We have performed the sensitivity analysis by changing each of the parameters by +50%, +25%, +10%, -10%, -25% and -50% taking one parameter at a time while keeping other parameters unchanged. The detail results are shown in Table 5 below.

A detail analysis of the results given in Tables 3, 4 and 5, we have observed that for all three values of ν, the model has same kind of sensitivity for any value of ν. we have summarized the analysis given below.

1. With the increase in the value of the parameters K_0, λ and h., T_1^* and TC^* increase while t_1^* and Q_1^* remain unchanged. Here TC^* is highly sensitive to λ and h, moderately sensitive to K_0, whereas T_1^* is moderately sensitive to the changes in K_0, λ and h. However, both t_1^* and Q_1^* are moderately sensitive to the changes in K_0, λ and h.

Table 4: Sensitivity Analysis for $\nu = 0.75$

Parameters	Change in %	Value of t_1^*	T_1^*	Q_1^*	TC^*
K_0	+50	5.84	11.20	135.61	132.40
	+25	5.84	11.00	135.61	130.25
	+10	5.84	10.89	135.61	128.97
	-10	5.84	10.73	135.61	127.26
	-25	5.84	10.62	135.61	125.97
	-50	5.84	10.41	135.61	123.83
λ	+50	5.84	12.70	135.61	225.82
	+25	5.84	11.77	135.61	173.80
	+10	5.84	11.20	135.61	145.63
	-10	5.84	10.43	135.61	111.68
	-25	5.84	9.88	135.61	88.74
	-50	5.84	9.16	135.61	55.70
a	+50	5.84	9.14	204.98	11126
	+25	5.84	9.85	170.82	118.00
	+10	5.84	10.38	150.32	123.52
	-10	5.84	11.32	122.99	133.73
	-25	5.84	12.26	102.49	144.97
	-50	5.84	14.73	68.33	178.61
h	+50	5.84	12.71	135.61	185.04
	+25	5.84	11.78	135.61	156.58
	+10	5.84	11.20	135.61	139.50
	-10	5.84	10.43	135.61	116.73
	-25	5.84	9.86	135.61	99.65
	-50	5.84	9.07	135.61	71.78
μ	+50	5.84	9.12	135.61	113.70
	+25	5.84	9.84	135.61	119.30
	+10	5.84	10.38	135.61	124.06
	-10	5.84	11.32	135.61	133.14
	-25	5.84	12.27	135.61	143.75
	-50	5.84	14.74	135.61	174.63
η	+50	5.84	11.76	135.61	65.79
	+25	5.84	11.79	135.61	64.48
	+10	5.84	11.85	135.61	63.72
	-10	5.84	11.93	135.61	62.71
	-25	5.84	11.99	135.61	62.96
	-50	5.84	12.00	135.61	60.45

Table 5: Sensitivity Analysis for $\nu = 1$

Parameters	Change in %	Value of			
		t_1^*	T_1^*	Q_1^*	TC^*
K_0	+50	5.84	10.69	135.61	225.38
	+25	5.84	10.58	135.61	223.24
	+10	5.84	10.51	135.61	221.96
	-10	5.84	10.42	135.61	220.24
	-25	5.84	10.35	135.61	218.96
	-50	5.84	10.24	135.61	216.82
λ	+50	5.84	12.50	135.61	394.79
	+25	5.84	11.52	135.61	302.33
	+10	5.84	10.89	135.61	252.24
	-10	5.84	10.03	135.61	191.75
	-25	5.84	9.36	135.61	151.11
	-50	5.84	8.31	135.61	92.36
a	+50	5.84	8.73	204.98	191.13
	+25	5.84	9.46	170.82	203.12
	+10	5.84	10.02	150.32	212.93
	-10	5.84	10.98	122.99	231.09
	-25	5.84	11.95	102.49	251.07
	-50	5.84	14.47	68.33	310.99
h	+50	5.84	12.51	135.61	322.31
	+25	5.84	11.52	135.61	271.70
	+10	5.84	10.89	135.61	241.34
	-10	5.84	10.03	135.61	200.85
	-25	5.84	9.35	135.61	170.50
	-50	5.84	8.26	135.61	119.89
μ	+50	5.84	8.72	135.61	195.48
	+25	5.84	9.46	135.61	205.44
	+10	5.84	10.02	135.61	213.90
	-10	5.84	10.99	135.61	230.02
	-25	5.84	11.96	135.61	248.18
	-50	5.84	14.48	135.61	303.78
η	+50	5.84	12.77	135.61	66.90
	+25	5.84	12.39	135.61	65.87
	+10	5.84	12.82	135.61	64.73
	-10	5.84	11.76	135.61	63.55
	-25	5.84	11.01	135.61	63.02
	-50	5.84	10.37	135.61	62.67

2. T_1^* and TC^* decrease, Q_1^* increases, while t_1^* remains unchanged with the increase in the value of the parameter a. Here Q_1^* and TC^* are highly sensitive to the changes in a while the other values are moderately sensitive to the changes in a.

3. T_1^* and TC^* decrease and t_1^* and Q_1^* remain unchanged with the increase in the value of the parameter μ. Here TC^* is highly sensitive to the changes in μ, while the other values are moderately sensitive to the changes in μ.

4. t_1^* and TC^* increase, T_1^* decreases, and Q_1^* remains unchanged with the increase in the value of the parameter ν. Here t_1^* and TC^* are highly sensitive to the changes in ν, while the other parameters are moderately sensitive to changes in ν.

5. T_1^* decreases, TC^* increases, and t_1^* and Q_1^* remain unchanged with the increase in the value of the parameter η. Here T_1^* and TC^* are highly sensitive to the changes in η.

Comparing the sensitivity analysis for different values of ν we found that the nature of the optimum values of t_1, T_1, Q_1 and TC^* remained unchanged for whatever be the arbitrary value of ν, which showed the accuracy of assumption of the arbitrary value ν. All three tables 3-5 of sensitivity analysis provided the same kind of result. This result has justified that ν could be any arbitrary value and it is depending on the total time cycle T_1. The optimum time T_1 will dictate what will be the arbitrary value ν. The following graphs depicted the optimum state of various parameter verses total cost.

Here, it is observed that with the increase of the value of setup cost, holding cost, production rate, deterioration cost and decay rate, the optimum total cost increases whatever may be the values of ν.

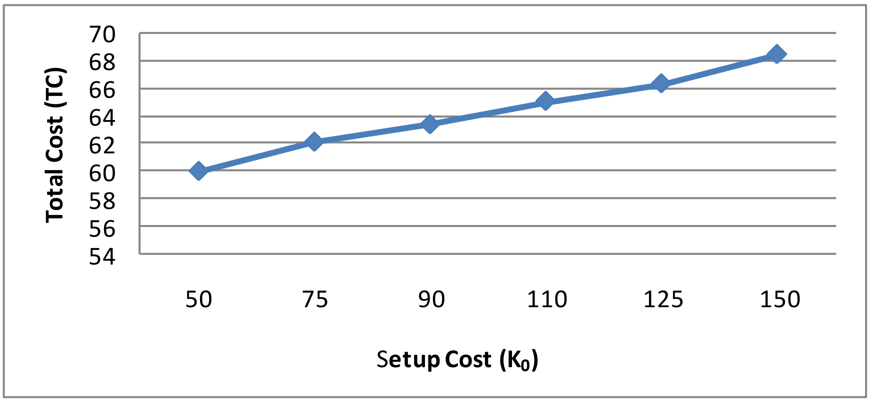

Figure 4: Setup Cost (K_0) verses Total cost (TC)

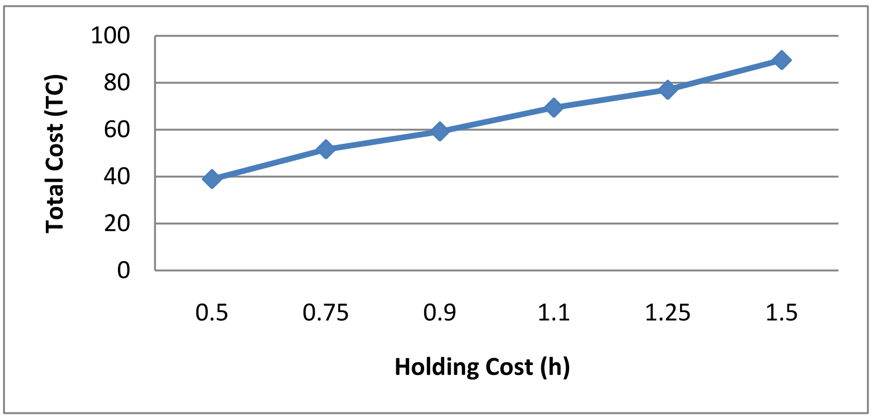

Figure 5: Holding Cost (h) verses Total Cost (TC)

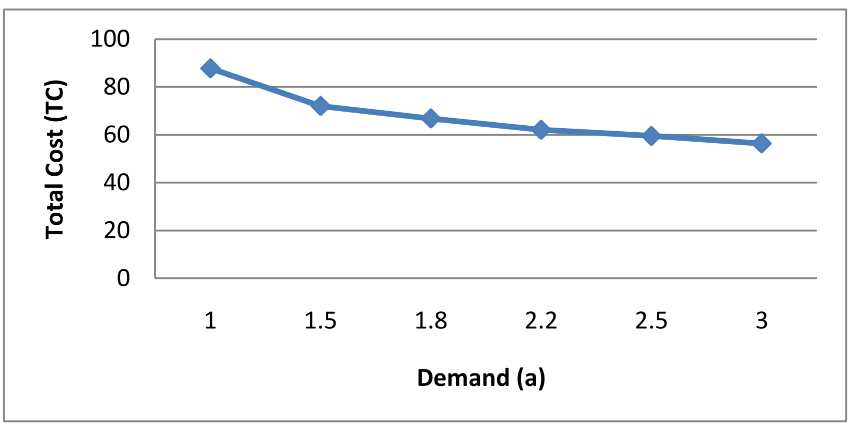

Figure 6: Demand (a) verses Total Cost (TC)

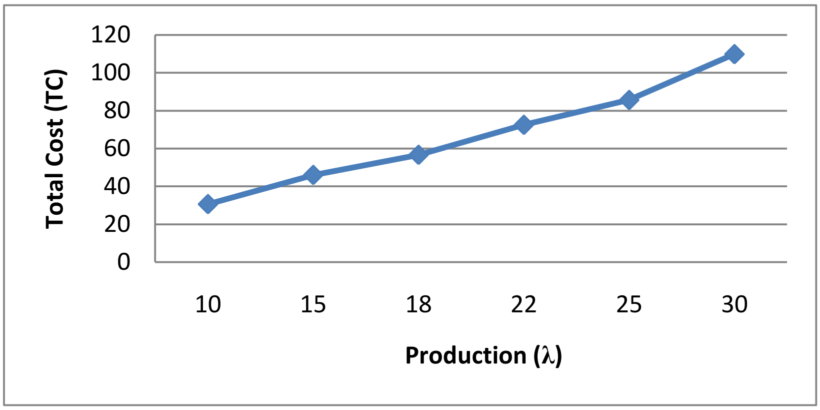

Figure 7: Production (λ) verses Total Cost (TC)

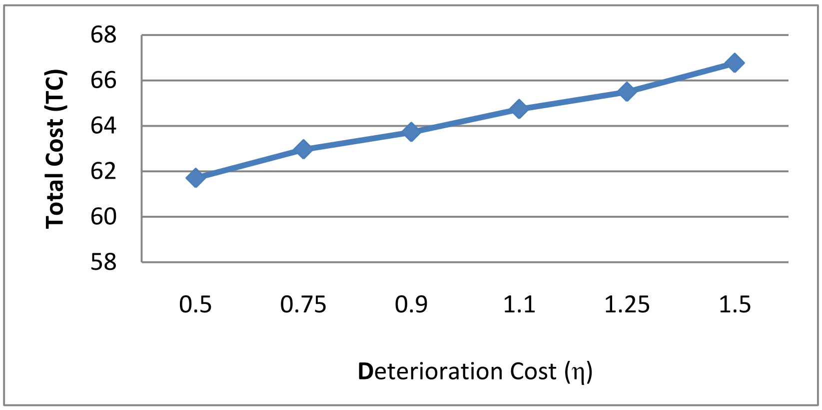

Figure 8: Deterioration Cost (η) verses Total Cost (TC)

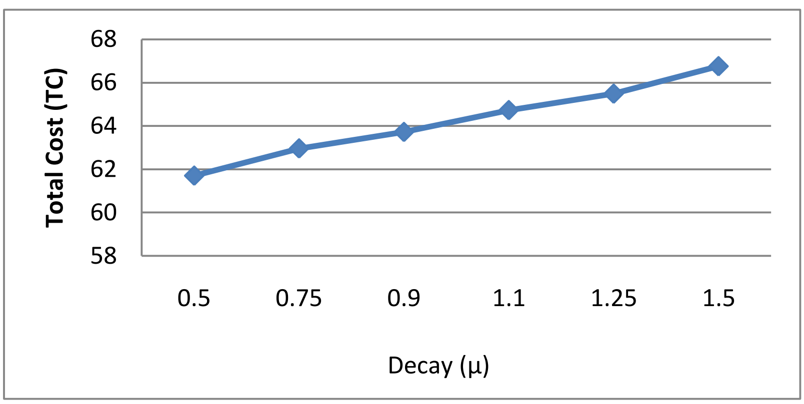

Figure 9: Decay (μ) versus Total Cost (TC)

On the other hand, with the increases of demand, total cost decreases for any value of ν. In this case, it is also mentioned that for the increases of demand, total cost decreases. So the firms may take initiative for increasing the demand in the market.

5 Conclusion

Without controlling the cost of inventory, it is difficult for a business to be profitable in the present competitive world of modern business. An appropriate and effective inventory estimation model could reduce inventory cost and maintain production. It is expected that an efficient management and appropriate inventory model is going to reduce firms' production and inventory cost. However, there are various types of market demand that influence the firms' decision of how to develop the model and what would be the production pattern. The model is generally developed considering the market demand. The proposed production and inventory model is suitable for the items having limited life, constant production rate and linear type of demand. As the products have finite life in this model, it is very practical and suitable for particular situations in our daily lives. The model develops with the help of Mathematica 10.0 to determine the optimum order quantity Q_1^* = 135.61units, optimum order interval t_1^* = 5.83 units, optimum time cycle T_1^* = 11.68 units and total optimum inventory cost $TC^*(T_1)$ = 64.23 units, before and after this point all the cost increases sharply. For this type of model, the firm can take decision to increase the demand in the market to decrease the total cost at an optimum level.

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