Journal of Statistical and Econometric Methods, vol.1, no.2, 2012, 65-70
ISSN: 2241-0384 (print), 2241-0376 (online)
Scienpress Ltd, 2012

A Specification Test for Linear Dynamic Stochastic General Equilibrium Models

Kenichi Tamegawa[1]

Abstract

In this paper, we introduce the procedure of a specification test for linear dynamic stochastic general equilibrium (DSGE) models. Given a parameterized DSGE model, we can empirically find omitted variables and check whether the model's structure is correct.

Mathematics Subject Classification: 62J07
Keywords: DSGE model, Specification test

1 Introduction

In this paper, we introduce a simple model-specification test for linear dynamic stochastic general equilibrium (DSGE) models. Given a parameterized DSGE model, one can empirically find omitted variables that should have been included in the model and check whether the model's structure is correct. We add another approach to the series of methods that evaluate DSGE models, for example, DeJong et al. [1], Schorfheide [3], Smets and Wouters [4], and Fernández-Villaverde and Rubio-Ramirez [2].

The first step of our test is to obtain reduced shocks from a given parameterized DSGE model. Theoretically, these shocks are spanned by structural shocks. The current structural shocks should not include any information that is available up to

---

[1] School of Commerce, Meiji University, e-mail: tamegawa@kisc.meiji.ac.jp

Article Info: *Received* : April 4, 2012. *Revised* : May 26, 2012
Published online : July 30, 2012

the previous period. If the current reduced shocks are correlated with that information, they are spanned not only by the current structural shocks but also by the previous information, as explained in Section 3. In this case, we can conclude that a given DSGE model may be miss-specified. Concretely, the null hypothesis is that a given DSGE model is correctly specified. This hypothesis can be tested by regressing reduced shocks on some lagged variables that are excluded from the model and lagged endogenous variables.

This paper is organized as follows. Section 2 shows how reduced shocks are obtained from a given DSGE model. Section 3 introduces our test procedure. Section 4 presents an example of our test. Finally, we conclude our paper in Section 5.

2 Obtaining Reduced Shocks

In this section, we setup a linear DSGE model and get reduced shocks from the model and actual data.

Consider the following model:

$$AE_t[y_{t+1}] + By_t + Cy_{t-1} + D\varepsilon_t = 0, \tag{1}$$

where y_t denotes a vector of endogenous variables and ε_t denotes a vector of structural shocks. Coefficient matrices A, B, C, and D are conformable and are parameterized by a user in advance of our test. If model (1) has a unique solution, its reduced form is written as

$$y_t = Py_{t-1} + S\varepsilon_t. \tag{2}$$

The reduced shocks denoted by e_t can be recovered with P and the actual data as follows:

$$e_t = y_t - Py_{t-1}. \tag{3}$$

The structural shocks ε_t might be serially correlated and then the model's structure might explicitly include $E_t[\varepsilon_{t+1}]$, but even if this is the case, obtaining P (and therefore e_t) is independent of this serial correlation.

3 Test Procedure

In the previous section, we could recover the reduced shocks e_t by (3). Under the null hypothesis that a given DSGE model is correctly specified, e_t should not correlate with any economic variables available up to the previous period, which is denoted by Ω_{t-1}. If the model is incorrectly specified, e_t correlate with Ω_{t-1} as follows. First, assume that a given DSGE model is misspecified and the true model is described by y_t plus the omitted variables x_t as follows:

$$\begin{bmatrix} y_t \\ x_t \end{bmatrix} = \widetilde{P}\begin{bmatrix} y_{t-1} \\ x_{t-1} \end{bmatrix} + \widetilde{S}\varepsilon_t ,$$

where $\widetilde{P}$ and $\widetilde{S}$ are the true coefficient matrices. In this case,

$$\begin{aligned} y_t &= \widetilde{P}_{11} y_{t-1} + \widetilde{P}_{12} x_{t-1} + \widetilde{S}_1 \varepsilon_t \\ &= P y_{t-1} + \left(\widetilde{P}_{11} - P\right) y_{t-1} + \widetilde{P}_{12} x_{t-1} + \widetilde{S}_1 \varepsilon_t \end{aligned}$$

Therefore, the reduced shocks (e_t) in the false model can be expressed as follows:

$$e_t = \left(\widetilde{P}_{11} - P\right) y_{t-1} + \widetilde{P}_{12} x_{t-1} + \widetilde{S}_1 \varepsilon_t . \tag{4}$$

Thus, if the underlying model is misspecified, its reduced shocks correlate with the omitted variables. Furthermore, if the model cannot correctly capture the partial effects of y_{t-1} on y_t, that is, $\widetilde{P}_{11} - P \neq 0$, the reduced shocks would correlate with y_{t-1} (This means that the model should explicitly include lagged variables). Therefore, in our test, we can find two types of misspecification: the omitted variable if $\widetilde{P}_{12} \neq 0$ and the modeling error if $\widetilde{P}_{11} - P \neq 0$.

In our test, we simply regress e_t on y_{t-1} and x_{t-1}. This can be done using ordinary least squares (OLS) estimation under the regularity conditions such as that $\{e_t, y_{t-1}, x_{t-1}\}$ is jointly stationary and ergodic. In this situation, standard test statistics such as the *t* - test can be applied. If some variables of the OLS estimator are statistically significant, the null hypothesis is rejected, and we can conclude that a given DSGE model is misspecified. Although of course in practice, the true variables are not known, researchers have a hypothesis that certain variable is important. Then, using our procedure, they can detect whether the variable is really important.

4 Example

Consider the following real business cycle model with the production function

$$Y_t = e^{\varepsilon_t} K_{t-1}^{\alpha} L_t^{1-\alpha} ,$$

where Y_t, K_t, L_t and ε_t are output, capital stock, labor, and an i.i.d. random variable with a mean of 0, respectively. A temporal-utility function is $\log(C_t)$ where C_t denotes consumption, and capital accumulation function

$$K_t = (1-\delta)K_{t-1} + Y_t - C_t .$$

The problem to be solved by a social planner is

$$\text{Max} \quad E_0\left[\sum_{i=0}^{\infty}\beta^i \log(C_{t+i})\right],$$
$$\text{s.t.} \quad K_t = (1-\delta)K_{t-1} + Y_t - C_t,$$
$$Y_t = e^{\varepsilon_t}K_{t-1}^{\alpha}L_t^{1-\alpha}.$$

Setting $L_t = 1$ for simplicity and linearizing the model around the non-stochastic steady states, we have

$$\hat{K}_t = (1-\delta)\hat{K}_{t-1} + (\varepsilon_t + \alpha\hat{K}_{t-1})Y/K - \hat{C}_t C/K,$$
$$E_t[\hat{C}_{t+1}] - \hat{C}_t = (\alpha-1)\hat{K}_t(1-\beta(1-\delta)),$$

where "^" denotes the deviation from the steady states and Y, K, and C denote the steady-state values. We calibrate the parameter as follows:

$$\begin{bmatrix} Y/K & C/K & \delta & \beta & \alpha \end{bmatrix} = \begin{bmatrix} 0.080 & 0.057 & 0.023 & 0.994 & 0.362 \end{bmatrix}.$$

Under this parameterization, the linearized model above has the following reduced form:

$$\begin{bmatrix} \hat{K}_t \\ \hat{C}_t \end{bmatrix} = \begin{bmatrix} 0.971 & 0 \\ 0.614 & 0 \end{bmatrix}\begin{bmatrix} \hat{K}_{t-1} \\ \hat{C}_{t-1} \end{bmatrix} + \begin{bmatrix} 0.078 \\ 0.049 \end{bmatrix}\varepsilon_t.$$

Table 1: Result of model specification test

	Parameter estimates	Stanndard error
Reduced shock for capital		
Own lag	0.0283	0.0950
K_t	-0.2250***	0.0617
C_{t-1}	0.0180	0.0521
G_{t-1}	-0.0242**	0.0109
Residual for consumption		
Own lag	0.4275***	0.0808
K_t	-0.5474***	0.0964
C_{t-1}	0.1415	0.0894
G_{t-1}	0.0073	0.0157

"***" and "**" indicate significance at the 1% and 5% levels, respectively.

Using the procedure shown in the previous section, we can test whether the above model is correctly specified. In particular, we consider government investment denoted by G_t as an omitted variable. Our test is done by regressing the residual shock e_t from (3) on $\hat{K}_{t-1}$, $\hat{C}_{t-1}$, and $\hat{G}_{t-1}$, following the previous section.[2] Furthermore, to capture the serial correlation of structural shocks, we add the own lag of reduced shocks e_{t-1} to the regressors. The result is shown in Table 1. In the regression for residuals of capital, the coefficients of capital lag and government investment lag are significant at the 5% and 1% critical levels, respectively. In the regression for residuals of consumption, the coefficients of both the own lag and capital lag are significant at the 1% level. Therefore, our test indicates that the above model's structure should be corrected and that it incorporates government investment.

5 Concluding Remarks

In this paper, we introduce the model specification test for a parameterized DSGE model and apply it to a simple real business cycle model as an example.

Our test procedure is constructed on the basis of a simple idea: if reduced shocks are spanned by structural shocks, as the theory requires, the current reduced shocks do not correlate with any information up to the previous period. Therefore, the correlation between the current reduced shocks and the previous economic information is a sign of misspecification. Using a simple OLS estimation, we can easily check this correlation.

ACKNOWLEDGEMENTS. I am grateful to an anonymous referee and Shin Fukuda for their helpful comments.

---

[2] The data for $\{\hat{K}_t\}, \{\hat{C}_t\}$, and $\{\hat{G}_t\}$ are constructed from the Japanese System of National Accounts quarterly data (from 1980:Q1 to 2010:Q4) and its HP-filtered data.

References

[1] D. DeJong, B. Ingram and C. Whiteman, A Bayesian approach to dynamic macroeconomics, *Journal of Econometrics*, **98**, (2000), 203-223.

[2] J. Fernández-Villaverde and J. Rubio-Ramírez, Comparing dynamic equilibrium models to data: A Bayesian approach, *Journal of Econometrics*, **123**, (2004), 153-187.

[3] F. Schorfheide, Loss function-based evaluation of DSGE models, *Journal of Applied Econometrics*, **15**, (2000), 645-670.

[4] F. Smets and R. Wouters, An estimated dynamic stochastic general equilibrium model of the Euro area, *Journal of European Economic Association*, **20**, (2003), 1123-1175.