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A Class of Time Series Analysis Model with Grey Item and Its Application

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Abstract

In this paper, we use the grey system theory and time series analysis, put forward the grey time series model, analyze the kind of model, and give model-building and forecasting methods. It is applied in agriculture with total output value and grain output value. The forecasting test has proved the model's correctness. This method can be extended to various fields, including educational statistics.

Keywords: Time series analysis, AR model, Grey system, Forecasting, Agriculture economy, Educational.

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1. Introduction

In time series analysis, we often resolve the original time series into some parts: the definite trend, random trend, etc.[1][2]

In this paper, we resolve the original time series into a grey trend and a random trend. Grey trend satisfied the grey model and random trend satisfied the time series AR model. Hence we build a grey time series model.

We can resolve time series $W(t)$ into[3]:

$$W(t) = X(t) + Y(t) \tag{1}$$

Where, series $X(t)$ satisfied grey model, series $Y(t)$ satisfied AR model[4].

We must obtain expressions of $X(t)$ and $Y(t)$, we give out methods as following.

2. Grey Trend GM(1,1) Model

For time series $X(t)$, after one accumulative generating, we obtain

$$X^{(1)}(t) = \sum_{k=1}^{t} X(k), (t = 1,2,\cdots,M) \tag{2}$$

series $X^{(1)}(t)$ satisfied **GM**(1,1) model[5],

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b \tag{3}$$

We know

$$x(t) = x^{(1)}(t) - x^{(1)}(t-1), t = 1,2,\cdots,M.$$

Suppose $\hat{B} = (a,b)^T$, then

$$\hat{B} = (X^T X)^{-1} X^T Y,$$

Where

$$X = \begin{pmatrix} -\frac{1}{2}\left(x^{(1)}(2) + x^{(1)}(1)\right) & 1 \\ -\frac{1}{2}\left(x^{(1)}(3) + x^{(1)}(2)\right) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}\left(x^{(1)}(M) + x^{(1)}(M-1)\right) & 1 \end{pmatrix},$$

$Y = (x(2), x(3),..., X(M))^T$,

hence, solve (3), we obtain

$$X^{(1)}(t) = \left(X^{(1)}(0) - \frac{b}{a} \right) e^{-at} + \frac{b}{a} \tag{5}$$

(5) be forecasting formula.

3. AR Model-building

3.1 Estimation of Model parameters

Suppose series $Y(t)$ satisfied

$$\Phi(B)y(t) = a_t , \tag{6}$$

$$\Phi(B) = 1 + \varphi_1 B + \cdots + \varphi_p B^p ,$$

Where

B be shift operator $B^n y_t = y_{t-n}$ (sign: $Y(t) = Y_t$)

Suppose $Da_t = \sigma_a^2, \xi(t) = (-y_{t-1}, -y_{t-2}, \cdots, -y_{t-p})^T$,

$\Phi = (\varphi_1, \varphi_2, \cdots, \varphi_P)^T$

Expression (6) into

$$y_t = \xi^T(t)\Phi + a_t , \tag{7}$$

in (7), estimation of Φ be

$$\Phi = \arg\min_{\Phi} \left\| \left[\sum_{t=1}^{N} Z_m(t)\xi^T(t) \right] \Phi - \left[\sum_{t=1}^{N} Z_m(t) y_t \right] \right\|_Q^2 \tag{8}$$

Where,

N be the length of the sample.
To vector D, we have $\|D\|_Q^2 = D^T Q D$,

Where,

Q is a right matrix,
$Z_m(t) = G(\mathbf{B})(y_{t-q-1}, \cdots, y_{t-q-m})^T$

is aid variable vector, $G(B)$ is a filter, and $G(0) = 1$.
Hence, expression (8) changed into

$$\Phi = \left[\frac{1}{N} \sum_{t=1}^{N} Z_m(t)\xi^T(t) \right]^T \left[Q(\frac{1}{N} \sum_{t=1}^{N} Z_m(t)\xi^T(t) \right]^{-1} \times$$

$$\left[(\frac{1}{N} \sum_{t=1}^{N} Z_m(t)\xi^T(t))^T Q \frac{1}{N} \sum_{t=1}^{N} Z_m(t) y_t \right], \tag{9}$$

Where

$$Q = (\sigma_a^2 S)^{-1}, \tag{10}$$

$$S = E\left[Z_m(t) Z_m^T(t) \right] = \frac{1}{\sigma_a^2} \hat{r}_a(0) \hat{R}_2(0),$$

$$\hat{r}_a(0) = \frac{1}{N} \sum_{t=0}^{N} a_t^2 ,$$

$$\hat{R}_2(0) = \frac{1}{N} \sum_{t=0}^{N} Z_m(t) Z_m^T(t). \tag{11}$$

Because $a_t = \Phi(B) y_t$,

Then

$$\hat{r}_a(0) = \sum_{i=0}^{P}\sum_{j=0}^{P}\hat{\varphi}_i\hat{\varphi}_j r_y(i-j) \tag{12}$$

Estimate algorithm as follows:

(1) Choose $Q = I, G(B) = 1, m \geq p$, use (9), estimate $\hat{\Phi}$, obtain the first step estimation.

(2) According to the first step estimation, use (10), (11), (12), we obtain Q value, turn into (9), then, obtain the second step estimation.

(3) According to two steps as above, again and again, may obtain the most estimation

$$\hat{\Phi} = (\varphi_1, \varphi_2, \cdots, \varphi_p)^T$$

3.2 Determine of AR model order

According to AIC theory, define

$$\text{AIC}(k) = \ln(\sigma_k^1) + \frac{2k}{N}, k = 0,1,\cdots, L_p,$$

Where

$$L_p = \lfloor\sqrt{p}\rfloor + 1,$$

Then

$$\text{AIC}(P) = \min_{0\leq k\leq L_p} AIC(k)$$

4. Model Test

We use a forecasting method for testing model, τ - step expression forecasting (1).

$$\hat{W}(t+\tau) = \hat{x}(t+\tau) + \hat{y}(t+\tau), \tag{13}$$

$$x(t+\tau) = \hat{x}^{(1)}(t+\tau) - \hat{x}^{(1)}(t+\tau-1),$$

$$\hat{y}(t+\tau) = \frac{1}{\sigma_a}\sum_{j=0}^{p+1}\bar{\beta}_j(\tau)y(t-j),$$

Where [6]

$$\begin{bmatrix} \bar{\beta}_0(\tau) \\ \bar{\beta}_1(\tau) \\ \vdots \\ \bar{\beta}_{p-1}(\tau) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \\ \varphi_1 & 1 & & \\ \vdots & \ddots & \ddots & \\ \vdots & & & \\ \varphi_{p-1} & \cdots & \varphi_1 & 1 \end{bmatrix} \begin{bmatrix} c_\tau \\ c_{\tau+1} \\ \vdots \\ c_{\tau+p-1} \end{bmatrix},$$

$$c_s = \begin{cases} \sigma_a, & s = 0, \\ -\sum_{t=1}^{s} \varphi_t c_{s-t}, & 1 \le s \le p, \\ -\sum_{t=0}^{m} \varphi_t c_{s-t}, & p > s. \end{cases}$$

5. The Analysis of Agriculture Economic Forecasting Model

According to relative statistical material[7], we take these data from 1976 year to 1985 year, use these methods as above and use the electronic computer to process data, and use models to forecasting from 1984 year to 1992 year, obtain results as follows.

(1) Agriculture total output value model

$$W(t) = 12603.96e^{0.1257713t} - 11206.96 - 0.00646\,y(t-1) + 0.00434\,y(t-2)$$
$$+ 0.00178\,y(t-3) + 0.00929\,y(t-4) - 0.00853\,y(t-5) - 0.00868\,y(t-6) + a_t$$

Model forecasting test are shown in Table 1.

Table 1: Forecasting test

Year No.	1986	1987	1988	1989	1990	1991	1992
$W(t)$	4013	4676	5865	6535	2662	8157	9085
$\hat{W}(t)$	4018	4684	5873	6542	2677	8169	9097
Error	5	8	8	7	15	12	12

(2) The index model of agriculture total output value

$$W(t) = 27836.96e^{0.0037955055t} - 27728.86 + 0.08385\,y(t-1) - 0.03644\,y(t-2)$$
$$- 0.03072\,y(t-3) + 0.03754\,y(t-4) - 0.01154\,y(t-5) - 0.03433\,y(t-6) + a_t$$

Model forecasting test are shown in Table 2.

Table 2: Forecasting test

Year No.	1985	1986	1987	1988	1989	1990	1991
$W(t)$	103.4	105.8	103.9	103.1	107.6	103.7	106.4
$\hat{W}(t)$	103.9	106.7	104.4	104.1	107.8	103.2	104.8
Error	0.5	0.9	0.5	1.0	0.2	−0.5	−1.6

(3) The model of average grain output value

$$W(t) = 14745e^{0.02241063t} - 14427.26 + 0.01111y(t-1) + 0.01409y(t-2)$$
$$- 0.02319y(t-3) + 0.04968y(t-4) + a_t$$

Model forecasting test are shown in Table 3.

Table 3: Forecasting test

Year No.	1984	1985	1986	1987	1988	1989	1990	1991
$W(t)$	361	367	372	358	364	393	378	380
$\hat{W}(t)$	367	374	384	363	372	388	392	395
Error	6	7	12	5	8	−5	14	15

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