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Conditional VaR using GARCH-EVT approach: Forecasting Volatility in Tunisian Financial Market

Héla Ben Soltane[1], Adel Karaa[2] and Makram Bellalah[3]

Abstract

In this paper Extreme Value Theory (EVT) and GARCH model are combined to estimate conditional quantile (VaR) and conditional expected shortfall (the expected size of a return exceeding VaR) so as to estimate risk of assets more accurately. This hybrid model provides a robust risk measure for the Tunisian Stock Market by combining two well known facts about security return time series: dynamic volatility resulting in the well-recognized phenomenon of volatility clustering, and non-normality giving rise to fat tails of the return distribution. We fit GARCH models to return data using pseudo maximum likelihood to estimate the current volatility and use a GPD-approximation proposed by EVT to model the tail of the innovation distribution of the GARCH model. This methodology was compared to the performances of other well-known modeling techniques. Results indicate that GARCH-EVT-based VaR approach

[1] CRIISEA, University of Picardie Jules Verne, Amiens, France,
e-mail: hela.bensoltane@etud.u-picardie.fr
[2] Higher Institute of Management of Tunis, e-mail: adel.karaa@isg.rnu.tn
[3] CRIISEA, University of Picardie Jules Verne, Amiens, France,
e-mail: makram.bellalah@u-picardie.fr

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appears more effective and realistic than Historical Simulation, static EVT, static normal and GARCH forecasts. The GARCH-EVT forecasts responded quickly to changing volatility, enhancing their practical applicability for Tunisian market risk forecasts.

Mathematics Subject Classification: C.2, C.5, G.1, G.2

Keywords: Risk, Dynamic volatility, Value at Risk, Regulation, Extreme Value Theory, GARCH estimation, Backtesting

1 Introduction

The improvement of management risk systems has become one of the top priorities for financial institutions. Indeed, recent financial disasters triggered the need of regulator market to risk measures which segregate extreme events. That is why a huge effort was invested into developing statistical risk measurement methods such as Value at Risk (VaR), which is widely employed in decisions investment and external regulation.

According to [1], VaR summarizes the worst loss over a target horizon that will not be exceeded with a given level of confidence. In other words, it answers the question about how much we can lose with a given probability over a certain time horizon. VaR has become extremely popular and an industry standard for measuring downside risk due to its conceptual simplicity: It summarizes in a single number the overall market risk faced by an institution.

The first approach using to compute VaR was the Variance-Covariance method. It measures VaR analytically by assuming that returns are normally distributed, and using variances and covariances across portfolios risks. However, the use of normal return distribution results in underestimated tails and disgreads excess kurtosis and skewness displayed by the empirical distributions. To overcome the shortcomings of distributional return, historical simulations represent the simplest way of estimating VaR by running hypothetical portfolios from historical data. VaR is determined by the actual price movement without assuming any distribution of asset return. However, this approach is based on the assumption of history repeating itself, in other words on the assumption of constant volatility of stock return over time. This is inconsistent with empirical evidence which find that asset returns exhibit certain patterns such as volatility clustering.

The assumption of constant volatility is untenable as the phenomenon of volatility clustering is well documented in the finance literature. It has been observed across financial markets that asset returns movement's exhibit periods of extreme volatility followed by periods of relative calm. In periods of extreme volatility, a static VaR would underestimate risk whereas it would be needlessly conservative during calm periods. The Autoregressive Conditional Heteroskedastic (ARCH) model of [2] and the subsequent Generalized Autoregressive Conditional Heteroskedastic (GARCH) model proposed by [3] are introduced to resolve such problems of clustering in financial data. These models manage changing volatility with the assumption of conditional normality. They do yield VaR estimates which reflect the current volatility background. The main weakness of this approach is that the assumption of conditional normality does not seem to hold for real data. As shown for in [4], models based on conditional normality are therefore not well-suited for analyzing large risk.

Since VaR estimations are only related to the tails of a probability distribution and Extreme value theory (EVT) focuses directly on the tails, use EVT in calculating VaR could give better forecasts of risk. EVT has been widely used in diverse fields, such as hydrology, engineering, physics, and insurance. It provides a solid framework to formally study the behavior of extreme observations. But applying EVT to the return series is inappropriate as they are not independently and identically distributed and the current volatility background is not taken account. To overcome this shortcoming, [5] followed by [6] propose a hybrid method, where a GARCH model is first estimated and EVT is applied to the estimated residuals, providing a robust VaR estimate which need market regulators.

A major concern for regulators is catastrophic market risk and the adequacy of capital to meet such risk. In fact, periods of extreme volatility impinge upon efficient price discovery and possible breakdown of the market mechanism itself due to heavy defaults. Regulatory margins are setting based on VaR estimates in order to safeguard the stock exchange system breaking down in such periods. In the absence of a robust risk measure, the regulators would be burdened with the unsavory task of 'managing' volatility. However, the suspension of quotations and the close of Tunis Stock Exchange in January 2011 could have been avoided if there is a robust dynamic VaR estimate. During period Tunisian revolution, the index of Stock Exchange of Tunis recorded a historic loss. Facing to a significant fall in prices to which brokers were unprepared, the decision to close the doors of Tunis Stock Exchange was taken by The Financial Market Council to save market. Hence, we require now a dynamic VaR model that is solid during extreme events which will build a dynamic margin system for regulator and protect the stock exchange from default crisis.

The rest of the paper is divided into five sections. The model and econometric methodologies used are presented in section 2. Section 3 tests and analyses the hybrid methodology which provides a robust VaR measure for the Tunisian Stock Market. Section 4 discusses the backtesting results using multiple VaR methods. The expected shortfall estimation is discussed in Section 5, and section 6 concludes the paper.

2 Risk Measurement Using GARCH-EVT approach

2.1 Modeling Dynamic Volatility Using GARCH

Financial time series have typical non-normal characters, such as leptokurtosis, fat tails, volatility clustering and leverage effect. To describe these features, many different models have been proposed in the econometric literature including models from the ARCH/GARCH (Autoregressive Conditionally Heteroskedastic /Generalized Autoregressive Conditionally Heteroskedastic) family. The standard ARCH model was developed by [2] describing volatility dynamics. When the lag of ARCH models became too large, [3] proposed adopting the generalized ARCH, known as the GARCH model. GARCH models have found extraordinarily wide use since they incorporate the two main stylized facts about financial returns, volatility clustering and unconditional non-normality. The most common form of the GARCH model is the GARCH (1,1), employed in this paper :

$$\begin{cases} X_t = \mu + \varepsilon_t = \mu + \sigma_t Z_t \\ \sigma_t^2 = w + a\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \end{cases} \tag{1}$$

with $w > 0$; $a > 0$; $\beta > 0$; $a + \beta < 1$, X_t is the actual return, μ is the expected return ; σ_t is the volatility of the returns on day *t* ;

Hence, the conditional volatility today depends on the yesterday's innovations ($\varepsilon_{t-1} = X_{t-1} - \mu_{t-1}$), the yesterday's conditional volatility (σ_{t-1}) and the unconditional volatility (*w*).

The randomness in the model comes through the stochastic variables Z_t, which are the residuals or the innovations of the process. These residuals are conventionally assumed to be independently and identically distributed and to follow a normal distribution. X_t are dependent and identically distributed.

Verifying that the error series has constant mean and variance, and that there is no autocorrelation among various lags can test the validity of the model. This

GARCH model with normal innovations is fitted using the pseudo Maximum Likelihood procedure.

2.2 Modeling Tails Using EVT

The GARCH model assumption of conditional normality does not seem to hold for real data. Indeed, the conditional distribution of GARCH models has been shown to have a heavier tail than that of a normal distribution. Although, the extreme movements are related by their very nature to the tails of the distribution of the underlying data, VaR based on such model has difficulties in predicting extreme events. Extreme Value Theory (EVT) appears to be an appropriate approach for modeling the tail behavior since it models the extrema (maxima or minima) of stochastic variable.

For financial time series, the Peak Over Threshold (POT) method is employed to modeling extreme events. Observations that exceed a given threshold u constitute extreme events. Considering the excess distribution above the threshold u given by :

$$F_u(y) = P\{X - u \le y \mid X > u\} = \frac{F(y+u) - F(u)}{1 - F(u)} \tag{2}$$

For some underlying distribution F describing the entire time series Xt. y are the excesses of X over the threshold u. $0 \le y < x_F - u$, where $x_F \le \infty$ is the right endpoint of F. We are interested in estimating the extremes, that is, F_u.

[8] and [9] showed that for a large class of underlying distribution functions the conditional excess distribution function $F_u(y)$ is well approximated, for a large value of u, by the Generalized Pareto Distribution (GPD) which describes the limit distribution of scaled excesses over high thresholds :

$$F_u(y) \cong G_{\xi,\beta}(y); \quad u \to \infty \tag{3}$$

where :

$$G_{\xi,\beta}(y) = \begin{cases} 1-(1+\frac{\xi\beta}{y})^{-1/\xi}, & \text{if } \xi \neq 0 \\ 1-e^{-y/\beta}, & \text{if } \xi = 0 \end{cases} \tag{4}$$

For $y \in [0, x_F - u]$ if $\xi \geq 0$, and $y \in [0, -\frac{\beta}{\xi}]$ if $\xi < 0$. $y = (X - u)$. $G_{\xi,\beta}$ is the generalized Pareto distribution. Parameter ξ (the tail index) accounts for the shape of the distribution. It takes a negative, a positive and a zero value. β is the parameter of scale, it is kept equal to one. In general, one cannot fix an upper bound on financial losses. Therefore, the only relevant value of ξ for financial data is greater than zero. The method of estimation of u, ξ and β would be discussed in Section 3 when we present our data analysis.

According to (2) and (3), the expression for underlying distribution function F(x) thus becomes

$$F(x) = F(u) + [(1 - F(u))G_{\xi,\beta}(x-u)] \tag{5}$$

for $x > u$. In order to construct a tail estimator for underlying distribution $F(x)$, we require an estimate of $F(u)$. This can be done from the empirical distribution function $\widehat{F}(u) = \frac{n - N_u}{n}$, where n is the total number of observations and N_u is the number of observations above the threshold, using the method of historical simulation (HS). We denote the estimates of ξ and β as $\widehat{\xi}$, $\widehat{\beta}$. The tail estimator of $F(x)$ is given by :

$$\widehat{F}(x) = 1 - \frac{N_u}{n}(1 + \widehat{\xi}\ \frac{x-u}{\widehat{\beta}})^{-1/\widehat{\xi}} \tag{6}$$

For $x > u$. For a given probability, $q > F(u)$, the VaR estimate is calculated by inverting the tail estimation formula above to get ([10])

$$VaR_q = u + \frac{\widehat{\beta}}{\widehat{\xi}}\left[(\frac{n}{N_u}(1-q))^{-\widehat{\xi}} - 1\right] \tag{7}$$

2.3 Combining the Two Models

As daily returns often exhibit heteroskedasticidy and autocorrelation, we introduce in this section dynamic (time-varying) volatility into VaR calculations. We follow the approach introduced in [5], [6] and [7].

We are interested in the conditional return distribution $F_{t+1+\cdots+t+k|F_t(x)}(x)$, where $F_t(x)$ represents the history of the process X_t up to day t. This is the distribution of forecasted return over the next day and we want to come up with an estimate for the quantiles in the tails of this unconditional distribution $F(x)$.

The dynamic nature of this VaR model is reflected by the notation VaR_q^t, where the subscript t indicates that it is a dynamic measure to be calculated at the close of day t; q is the quantile at which VaR is being calculated. Here, we study one-day horizons.

To calculate daily VaR estimates, it is considered necessary to take into account the current volatility of the equity security. Many researchers (i.e. [11]) have emphasized the need to scale the VaR estimates by some measure of current volatility and not an unconditional volatility for the entire period. The GARCH family of models seems to be appropriate for such modeling as described in an earlier section. The one- day VaR measure for the dynamic volatility model (GARCH) described earlier can be formulated as:

$$VaR_q^t = \mu_{t+1} + \sigma_{t+1} VaR(Z)_q \tag{8}$$

$VaR(Z)_q$ denotes the qth quantile of the noise variable Z_t . μ_{t+1} and σ_{t+1} are the mean and volatility return forecasts using GARCH model.

This VaR measure incorporates volatility clustering. A correct specification of the model makes the error terms iid, guaranteeing the theoretical soundness of $VaR(Z)_q$ calculation. As described earlier, the assumption of normal standard distribution of Z_t, on which this model is based, underestimates the conditional quantile. Therefore, applying EVT to the noise variable Z_t seems to be ideally suited for modeling tails, without assuming any functional form for F(z). Applying EVT to the random variable X_t is an appropriate as X_t is not independently and identically distributed.

Hence, the estimation procedure for calculating a dynamic VaR at the end of day t using the return data of the last n days can be summarized as:

- Fit an AR model with GARCH errors to the return data using a pseudo maximum likelihood approach. The standardized residuals of this model are extracted. If the model were correct, the residuals series Zt would be realizations of the unobserved iid noise variables. Estimate then using the fitted model μ_{t+1} and σ_{t+1} calculate the implied model residuals.
- Use extreme value theory (EVT) to model the tail behavior. Calculate $VaR(Z)_q$ using the GPD tail estimation procedure.
- VaR_q^t is calculated using the expression described in Eq. (8).

We go into these stages in more detail in the next sections and illustrate them by means of an example using daily returns on the Tunisian Market index.

3 Empirical Analyses and Discussion

To test the utility and the performance of the improved dynamic VaR model, we choose the context of Tunisian stock market as the base of analysis. The data employed was the 1235 daily observations of prices of the Tunisian stock market index (Tunindex) covering the period from November 1st 2006 to November 1st 2011. The daily data are obtained from Tunis Stock Exchange. The volatility clustering effect was easily identified at the Figure 1. There are periods of high turbulence with many picks clustering followed by periods of quiescence where volatility stays low.

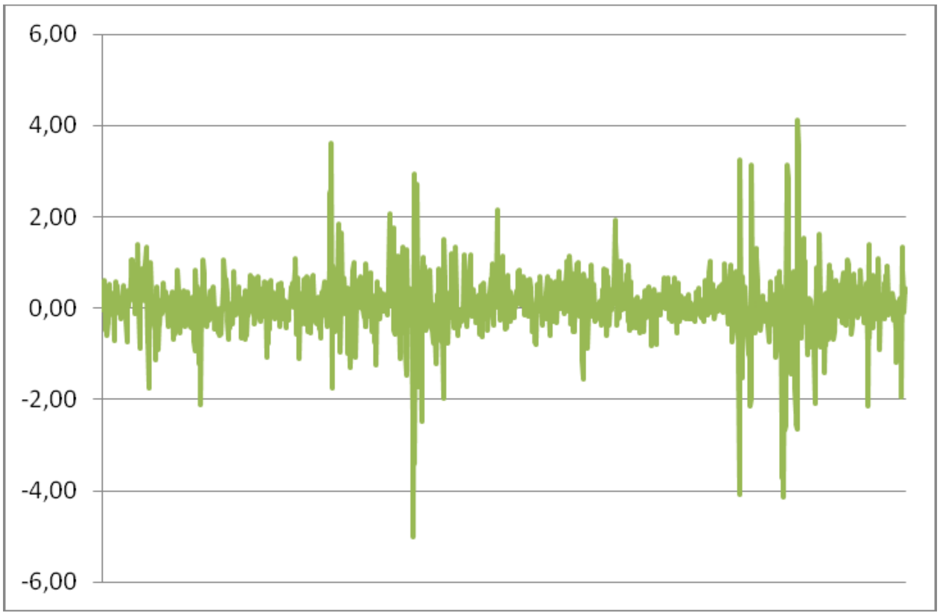

Figure 1: Daily returns of Tunindex index (2006.11.1 – 2011.11.1)

Recognizing the heteroskedasticity and volatility clustering nature of this time series, we first use AR(1) model with GARCH(1,1) errors to fit the Tunindex returns and forecast volatility and expected return. Specifically, the model is:

$$\begin{cases} X_t = \mu_t + \sigma_t Z_t \\ \sigma_t^2 = w + a(X_{t-1} - u)^2 + \beta\sigma_{t-1}^2 \end{cases} \tag{9}$$

The model parameters were estimated using Gauss program. The result of GARCH estimation is given in Table 1.

The coefficients of the volatility equation are all found to be significant. The Durbin-Watson Statistic shows that the residuals are free from auto-correlation. Thus, the specification is tenable. The validity of the AR equation is verified from the correlogram. Correlations at all lags have been found to be insignificant, implying that the return series are stationary, a necessary condition to use the GARCH model.

Table 1: GARCH estimation results

	Coefficient	Std.error	z-statistics	Prob.
Return equation				
μ	0.0580	0.0135	4.300	0.0000
Variance equation				
w	0.0923	0.0205	4.503	0.0000
α	0.4410	0.0688	6.414	0.0000
β	0.3324	0.1001	3.319	0.0005

The GARCH specification has been shown to be appropriate. This takes care of volatility clustering. However, as argued in the previous sections, this is not enough as the descriptive statistics of the standard residuals clearly show that the conditional distribution has a heavier tail than that of a normal distribution.

Table 2: Residual statistics

		Statistic	Std.error
Standardized residuals	Mean	-0,01712721	0,028462767
	Median	-0,01391622	
	Variance	1,001	
	Std. dev	1,000254690	
	Minimum	-5,486991	
	Maximum	5,020543	
	Skewness	-0,298	0,070
	Kurtosis	3,152	0.139

As can be seen in Table 2, the residual serie is found to have significant excess kurtosis and negative skewness. The Jarque-Bera Statistic is significant even at very low level. The results can be summarized in the followings: Neither the return series nor the residual series can be considered to be normally distributed, since both the series have a leptokurtic distribution with a fat left tail. Therefore, the assumption of conditional normality is unrealistic.

We begin the stage 2 of the estimation procedure of dynamic VaR by estimating the tail distribution using EVT. As described in earlier section, to do that we would employ the POT method using GPD for modeling extreme events. The first step in this modeling is to choose the proper threshold for identifying the relevant tail region. The choice of the optimal threshold can be tricky because there is a tradeoff between high precision and low variance. A very high threshold leaves us with too few extreme data for estimation. On the other hand, very low threshold generates biased estimates because the limit theorems do not apply any more. Many researches choose the exceedances to be the "high" enough percentile of the sample.

We employ a more systematic approach as described by [6] $e(u) = E(X - u \mid X > u)$ defines the mean-excess function of X over the threshold u. For heavy tailed distributions, this function tends to infinity. The mean-excess function can be modeled as the expected value of a random variable following GPD :

$$e(u) = \frac{\sigma + \xi u}{1 - \xi} \tag{10}$$

ξ takes a value greater than zero for financial data. The choice of u is given by the value above which the observed mean excess function is approximately linear. From Figure 2, we get value of threshold as 1.6. we get N_u (the number of points above the threshold) as 58, which is large enough to facilitate a good estimation.

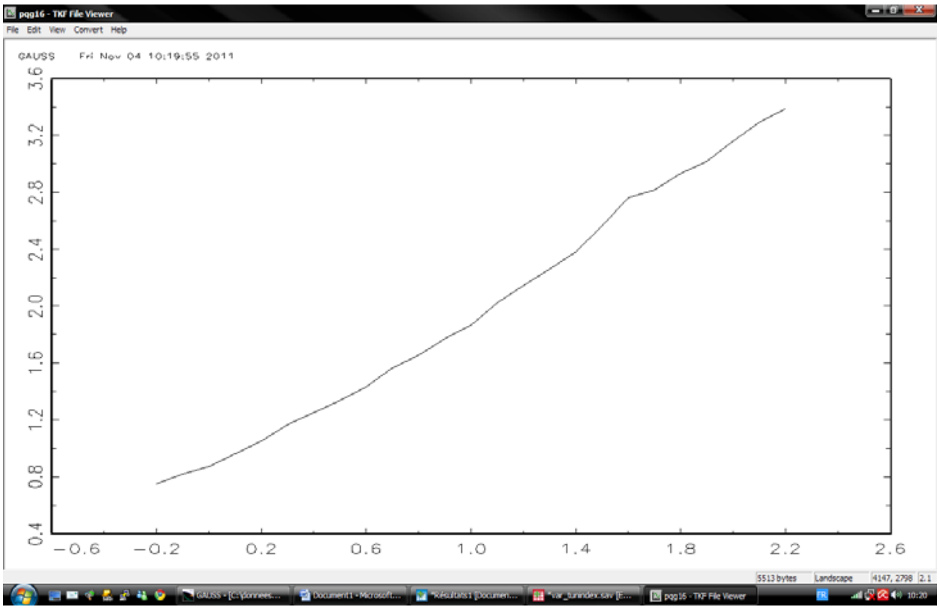

Figure 2: Excess Mean Function

The next step is the estimation of parameters ξ and β of the GPD, which can be obtained using the method of maximum likelihood. We follow [6] in estimating tail index using the Technique of Hill because of their wide acceptability and they are shown to have better performance. [12] shows that the estimator of the tail index is given by:

$$\hat{\xi} = \frac{1}{N_u} \sum_{i=0}^{N_u - 1} \log \frac{X_{n-i+1}}{X_{n-N_u}} \tag{11}$$

Here, we use a correction to the Hill estimator based on the methodology applied by the National Institute of Standards and Technology, U.S. and followed by [6]. The steps for estimating the parameters are:

- Compute the quantities
- $M^{(r)} = \frac{1}{N_u} \sum_{i=0}^{N_u - 1} \left[\log \frac{X_{n-i+1}}{X_{n-N_u}} \right]^r \qquad \text{for } r = 1, 2$

- $\xi = M^{(1)} + 1 - \dfrac{1}{2\left[\dfrac{1-(M^{(1)})^2}{M^{(2)}}\right]}$
- $\beta = u\,M^{(1)}$

According to the above steps, we obtain the estimates $\hat{\xi} = 0.4570$ and $\hat{\beta} = 0.2865$. The model is now completely specified. Hence, we can get the robust dynamic VaR estimates by using Eq (7). We report the 97.5 percentile VaR. the value of $VaR(Z)_{0.975}$ is found out be 1.809661. Basing on Eq (8), the dynamic VaR specification for Tunindex returns is:

$$VaR^{t}_{0.975} = \mu_{t+1} + 1.809661\sigma_{t+1} \qquad (12)$$

μ_{t+1} and σ_{t+1} are conditional GARCH estimates of mean and volatility.

Figure 3 shows the efficacy of our procedure. The VaR value changes dynamically to reflect market conditions. In periods of extreme volatility, the VaR value also increases and market safety is taken care of. We formally test the superiority of our model versus the other static and dynamic formulations of VaR through a back testing procedure.

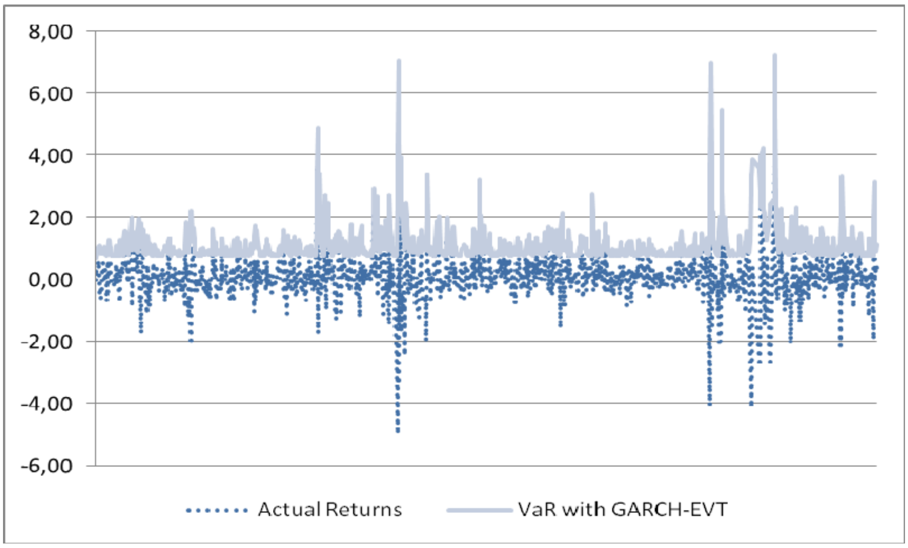

Figure 3: Robust dynamic VaR

4 Backtesting the Model

To test the reliability of our VaR methodology, we compare the daily VaR estimates with actual realized loss in the next day. A violation is said to occur when the realized loss exceeds the estimated VaR.

We backtest the method from September 28, 2010 to March 28, 2011. We choose this period of six month because it witnessed extreme volatility in the Tunisian Stock Exchange due to politically uncertain environment and fears about the policy of the new Government. This period includes the so called “Black October” for Tunisian Stock Exchange in 2010, and January 2011 during the political revolution – the period when Tunindex witnessed his largest fall.

Tunisian Stock Market underwent a phase of diminution during the last quarter of 2010 with a decline of 10% of the index price “Tunindex” in October, never seen since the creation of the Tunisian Stock Market. This decrease was in connection with the new law of taxation on capital gains on the stock market in the short term. The fall of Tunindex was accentuated at the beginning of 2011 due to political and social crisis occurred resulting in the collapse of the stock market in January and leading the authorities to suspend the market quotations. This decision was taken in order to protect the savings invested in securities and to preserve market integrity and equality among investors. However, the suspension of the market quotations and reassuring speeches companies have failed to restore stakeholder trust. The fall continued for the following months.

This period was also characterized by the lack of market liquidity: A decrease of 23.23% of market capitalization was performed, from 16 653.960 MD in September 2010 to 13 514.55 MD in Mars 2011. The part of foreign investments in market capitalization has fallen to 20.15% in the end of December 2010. The output of these investors has led to a negative net balance (acquisitions-sales) of 19MD for March 2011. The suspension of trading and investor mistrust induced a low volume of trade in the first quarter of 2011. Trading volume dropped to 247 822 shares at the end of March 2011 after it reached 10 400 847 shares in July 2010.

We show in Figure 4 the VaR violations in historical simulation method (a), in static normal method (b), in dynamic model assuming residual normal distribution (c), in static model with tail modeling by EVT (d) and in dynamic model with tail modeling by EVT (e).

Figure 4 shows part of the backtest for Tunindex. The dynamic VaR with tail modeling by EVT estimate clearly responds quickly to the high volatility. The VaR estimate with historical Simulation cannot respond quickly to changing volatility and tends to be violated several times in a row in stress periods.

Various dynamic and static methods of VaR estimation are compared by counting violations. Tests of the violation counts based on the binomial distribution can show when a systematic underestimation or overestimation of VaR is taking place.

Table 3 reports the number of VaR violations that occurred during the testing period when estimating VaR with different methods. The p-values indicate the success of the estimation method based on hypothesis tests for the number of violations observed as compared to the expected number of violations.

Table 3: Number of violations by different techniques

Length of series		Expected no. of violations	
110		6	
Method	No. of violations	Violation ratio	p-value
Static normal	26	0,2364	0.0000
Static HS	87	0,7909	0.0000
Static EVT	9	0,0818	0,0510
Dynamic normal	8	0,0727	0,0855
Dynamic EVT	6	0.0545	0,1614

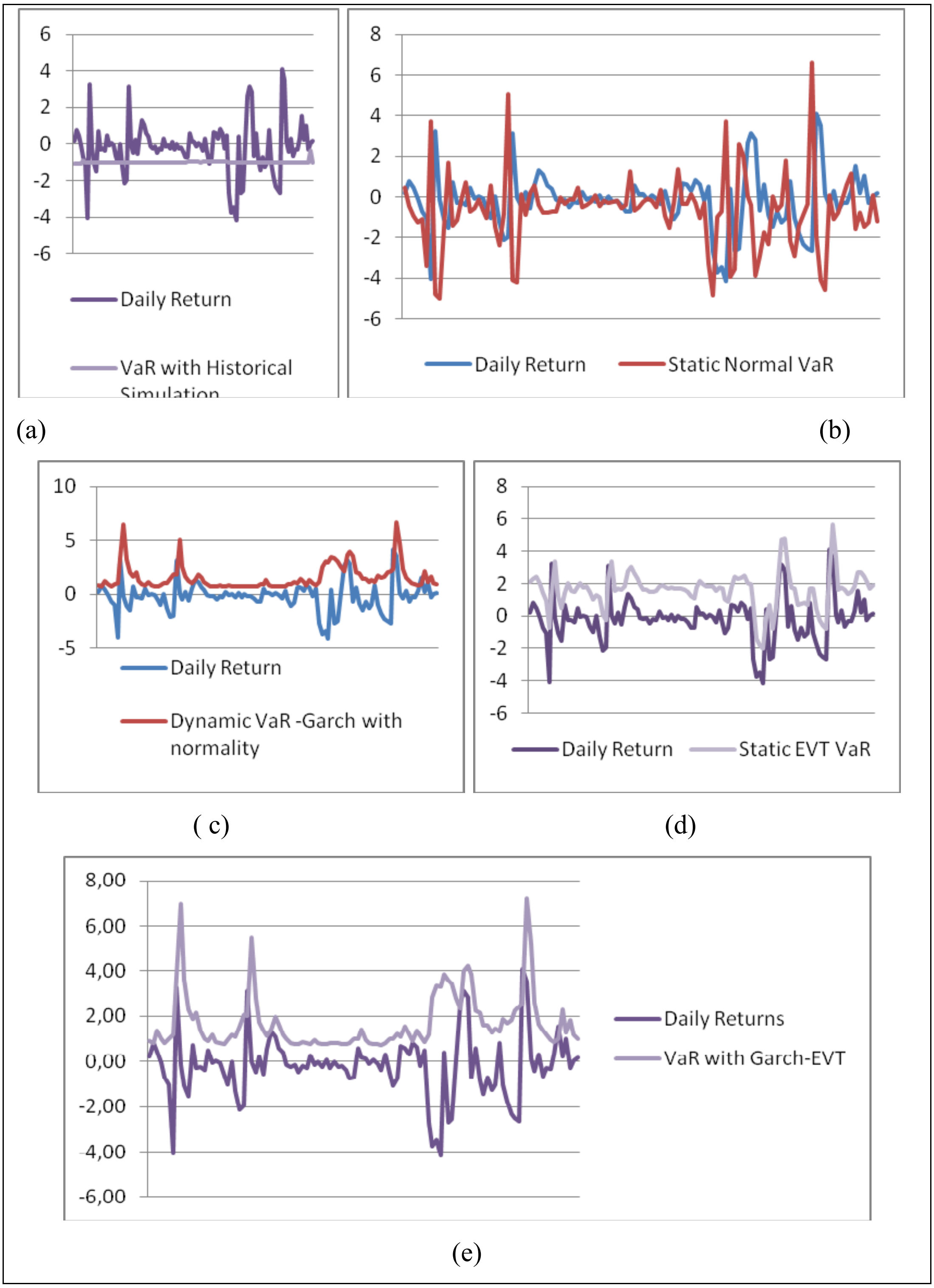

Figure 4: VaR Violations

The expected number of violations is assumed to follow binomial distribution. A p-value more than 0.05 implies success of the method at 5% significance level.

As shown graphically, the results clearly indicate that estimates based on historical simulation and exclusively on normality assumption fare badly. This evidently underlines the importance of accounting for fat tails and time varying volatility while modeling financial time series.

5 Expected Shortfall estimation

To take into account the severity of an incurred damage event, we consider an alternative measure of risk for the tail of a distribution known as the expected shortfall (also called Conditional Value at Risk-CVaR). As proposed in [13], expected shortfall measures the expected loss given that the loss exceeds VaR. Formally, the expected shortfall once the VaR limit is breached is given by:

$$ES_q = VaR_q + E[X - VaR_q \mid X > VaR_q] \tag{13}$$

To estimate this risk measure, we need an estimate of the second term which can be considered as the mean of the excess distribution $F_{VaR_q}(y)$ over the threshold VaR_q. The EVT model for excess distribution above a given threshold is stable. With a higher threshold, the excess distribution above the higher threshold is also a GPD with the same shape parameter but a different scaling parameter. Hence, to estimate characteristics of the losses beyond VaR, we can use this corollary as in [6]:

$$F_{VaR_q}(y) = G_{\xi,\, \beta+\xi\, VaR_{q-u}}(y) \tag{14}$$

The mean of the excess distribution is given by $\dfrac{\beta + \xi\, VaR_{q-u}}{1-\xi}$. This gives us the estimate of the expected shortfall:

$$\mathrm{ES}_q = \frac{\mathrm{VaR}_q}{1-\xi} + \frac{\beta - \xi u}{1-\xi} \tag{15}$$

Using the above equation $\mathrm{ES}_{0.975}(Z)$ is found to be equal to 2.51374. This value seems to be large but this is to be expected in such heavy tailed data. Hence, the dynamic ES specification for Tunindex returns is:

$$\mathrm{ES}^t_{0.975} = \mu_{t+1} + 2.51374\sigma_{t+1} \tag{16}$$

μ_{t+1} and σ_{t+1} are conditional GARCH estimates of mean and volatility.

We report the actual losses and the shortfalls estimated by the model on the days the VaR limit was violated. Table 4 shows that the model gives conservative estimates of the losses. Risk management becomes more efficient now, as the regulators are aware of the magnitude of uncertainty about extreme events.

Table 4: Actual vs. estimated losses

Actual losses	Expected losses
3,126877	3,031424
1,324189	1,596343
3,137004	3,140889
4,108560	3,571728
3,521306	3,794589
1,536422	1,197811

6 Conclusion

The Tunisian financial industry has become increasingly aware of the impact of risk in assets due to recent political instability. Hence, market risk measurement and management has become thrust into the forefront of issues facing market regulators. In response of this, we were concerned in this paper with VaR

methodology which has gained fast acceptance and popularity in risk management, also the expected shortfall which has better theoretical properties.
Using both GARCH model and EVT to respectively describe volatility dynamics and track extreme losses, we provide a robust risk measure with much enhanced predictive abilities for the Tunisian stock market. Our results show that Tunisian market data are well suited for this hybrid model. GARCH-EVT estimate clearly responds quickly to changing volatility.

Comparing this methodology to other modeling techniques for tail estimation, we find that dynamic method with residual normal distribution provide a good estimates, as well as the static EVT method. But, both tend to be violated more often than EVT-GARCH methodology because they do not take into account the leptokurtosis of the residuals. The historical simulation method and static normal method were woefully inadequate in times of extreme volatility. They cannot respond quickly to changing volatility and tend to be violated several times in a row in stress periods.

This paper shows that this dynamic VaR model addresses the twin concerns of safety and efficiency of stock exchange market. During periods of large volatility, the dynamic nature of the model would lead to appropriate increases in VaR measures to ensure market safety. In practice, VaR estimation is often concerned with multivariate series. This analysis can be extended to an n-dimensional asset allocation problem for estimating risk of a multi-asset portfolio.

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