Theoretical Mathematics & Applications, Vol. 15, No. 1, 2025, 1-10
ISSN: 1792-9687(print), 1792- 9709(online)
https://doi.org/10.47260/tma/1511
Scientific Press International Limited

On 'Shipping the Good Apples Out'

Mitch Kunce[1]

Abstract

Alchian and Allen's (1964) economic theorem, 'shipping the good apples out', forwards that standard or lower quality goods are more heavily consumed within the vicinity of production as opposed to much farther away. The better apples are shipped away because the fixed cost of transportation lowers the relative price. This paper reconciles two determining theoretical views supporting this substitution theorem. Borcherding and Silberberg (1978) defend the theorem for substitute goods (standard versus quality apples) making the case that close substitutes would interact with all other goods modeled in a like manner. Bauman (2004) generalized Borcherding and Silberberg's work and developed a broader set of sufficient conditions that recognize limited complementarity. Situational numerical simulations are used to dissect model differences.

JEL classification numbers: C02, D11, L15.
Keywords: Constrained minimization, consumer theory, transportation cost.

[1] DouglasMitchell Econometric Consulting, Laramie, WY USA.

Article Info: *Received:* November 19, 2025. *Revised*: December 9, 2025.
Published online: December 10, 2025.

1. Theoretical modeling

Alchian and Allen's hypothesis suggests that a fixed charge, t, (e.g. a transportation charge) added to the price of two similar goods (high (x_1) and standard or lower quality (x_2) apples, where higher quality carries a premium price) leads to a relative increase in consumption of the high quality good. This proposition has additional caveats. First, transportation is essential for access and has no effect on the two goods (e.g., no spoilage in transport). Second, t is not considered a good, meaning it has no separable value (Umbeck, 1980). Lastly, the proposition focuses on individual choice behavior not the collective.

We begin with the dual of the standard consumer problem to avoid 'unusual' income effect issues (Gould and Segall, 1969, p. 131). The consumer's goal is to minimize expenditure on quantities, $x_1, x_2, \ldots. x_n$,

$$E = p_1(t)x_1 + p_2(t)x_2 + \sum_{i=3}^{n} p_i x_i \ , \tag{1}$$

with prices of x_1 and x_2 defined as,

$$p_1(t) = p_1 + t \ > \ p_2(t) = p_2 + t \, , \tag{2}$$

where, t is a per unit transportation cost. We constrain the problem to a constant level of Edgeworth utility represented by the 'well behaved' function, [2]

$$\overline{U} = U(x_1, x_2, \ldots. x_n) \, . \tag{3}$$

First order conditions from this constrained minimization problem will yield implicit optimal choices for all x_i, more specifically,

$$x_1 = x_1^*(p_1, p_2, \ldots.. p_n, \overline{U}) \, , \tag{4}$$

$$x_2 = x_2^*(p_1, p_2, \ldots.. p_n, \overline{U}) \, , \tag{5}$$

where the prices of x_1 and x_2 are still functions of t following Equation (2). Equations (4) and (5) are Hicksian compensated demand functions (Hicks, 1946). Under the assertion that only relative prices matter, Hicksian demands are homogeneous of degree zero in prices only. Homogeneity of compensated demand allows for the restatement of Equations (4) and (5) using Euler's theorem,

---

[2] Good behavior requires the desired configuration of strict quasiconcavity.

$$\frac{\partial x_1^*}{\partial p_1} p_1 + \frac{\partial x_1^*}{\partial p_2} p_2 + \sum_{i=3}^{n} \frac{\partial x_1^*}{\partial p_i} p_i = 0\,, \tag{6}$$

$$\frac{\partial x_2^*}{\partial p_1} p_1 + \frac{\partial x_2^*}{\partial p_2} p_2 + \sum_{i=3}^{n} \frac{\partial x_2^*}{\partial p_i} p_i = 0\,, \tag{7}$$

where the goods x_1 and x_2 are not forced to be net substitutes.[3]

To confirm 'shipping the good apples out', the ratio of the consumption of the high quality good to the lower quality good must increase with an increase in transportation cost (Alchian and Allen, 1964; Borcherding and Silberberg, 1978),

$$\frac{\partial \left(\frac{x_1^*}{x_2^*} \right)}{\partial t} > 0\,, \tag{8}$$

holding real income (utility) constant. We can expand Equation (8) using the quotient rule,

$$\frac{\partial \left(\frac{x_1^*}{x_2^*} \right)}{\partial t} = \frac{x_2^* \frac{\partial x_1^*}{\partial t} - x_1^* \frac{\partial x_2^*}{\partial t}}{\left(x_2^* \right)^2}\,. \tag{9}$$

By differentiating compensated demand Equations (4) and (5), via the chain rule, with respect to t we find,

$$\frac{\partial x_1^*}{\partial t} = \frac{\partial x_1^*}{\partial p_1} + \frac{\partial x_1^*}{\partial p_2}\,, \tag{10}$$

$$\frac{\partial x_2^*}{\partial t} = \frac{\partial x_2^*}{\partial p_1} + \frac{\partial x_2^*}{\partial p_2}\,. \tag{11}$$

Substituting Equations (10) and (11) into the right-hand-side of (9) yields,

[3] For reference, the own price response is of course negative. The cross price responses, if positive, denote net substitutes – if negative, net complements.

$$\frac{x_2^*}{\left(x_2^*\right)^2}\left(\frac{\partial x_1^*}{\partial p_1}+\frac{\partial x_1^*}{\partial p_2}\right)-\frac{x_1^*}{\left(x_2^*\right)^2}\left(\frac{\partial x_2^*}{\partial p_1}+\frac{\partial x_2^*}{\partial p_2}\right), \tag{12}$$

where the signs of each partial derivative stem from demand theory or postulate. Assigning a priori magnitudes to any of these terms, however, originates more from one's imagination. Regardless, Equation (12) is going to require a bit of manipulation in order to definitively sign. One initial obstacle in the way of signing Equation (12) is the presence of the optimal quantity choices of the two focus goods. Borcherding and Silberberg (1978) recognized this and offered a suitable rearrangement that focused on demand responses (elasticities) and prices. [4] Consider this restatement of Equation (12),

$$\begin{aligned}&\left(\frac{1}{x_2^*}\frac{\partial x_1^*}{\partial p_1}\frac{x_1^*}{x_1^*}\frac{p_1}{p_1}\right)+\left(\frac{1}{x_2^*}\frac{\partial x_1^*}{\partial p_2}\frac{x_1^*}{x_1^*}\frac{p_2}{p_2}\right)-\left(\frac{1}{x_2^*}\frac{x_1^*}{x_2^*}\frac{\partial x_2^*}{\partial p_1}\frac{p_1}{p_1}\right)-\left(\frac{1}{x_2^*}\frac{x_1^*}{x_2^*}\frac{\partial x_2^*}{\partial p_2}\frac{p_2}{p_2}\right)\\&=\frac{x_1^*}{x_2^*}\left(\frac{\varepsilon_{11}^*}{p_1}+\frac{\varepsilon_{12}^*}{p_2}-\frac{\varepsilon_{21}^*}{p_1}-\frac{\varepsilon_{22}^*}{p_2}\right)\end{aligned}, \tag{13}$$

where ε^* indicates the optimal compensated own and cross price elasticities. Explicit positive quantity choices are now factored out of the meaningful (bracketed) part of the lower expression in Equation (13).

To place emphasis on the requisite explicit demand interactions of x_1^*, x_2^* and all other goods, consider the following restatement of Euler demand Equations (6) and (7). First, divide both sides of Equation (6) by x_1^* yielding,

$$\varepsilon_{11}^*+\varepsilon_{12}^*+\sum_{i=3}^{n}\varepsilon_{1i}^*=0, \tag{14}$$

then divide both sides of Equation (7) by x_2^* yielding,

$$\varepsilon_{21}^*+\varepsilon_{22}^*+\sum_{i=3}^{n}\varepsilon_{2i}^*=0. \tag{15}$$

---

[4] A price elasticity represents the ratio: percentage change in quantity demanded / percentage change in price.

Following, first, Borcherding and Silberberg (1978) solve Equation (14) for ε^*_{12}, solve Equation (15) for ε^*_{22} then substitute both results into the bracketed part of the lower Equation (13) expression.[5] With suitable rearrangement we find,

$$\underbrace{\left(\varepsilon^*_{11}-\varepsilon^*_{21}\right)\left(\frac{1}{p_1}-\frac{1}{p_2}\right)}_{\text{left-side}}+\underbrace{\frac{1}{p_2}\left(\sum_{i=3}^{n}\varepsilon^*_{2i}-\sum_{i=3}^{n}\varepsilon^*_{1i}\right)}_{\text{right-side}}, \tag{16}$$

which is found on page 135 of Borcherding and Silberberg's paper. Given the initial assumptions regarding prices and that the two apples are substitutes, the product of the two terms on the left of the summation is positive. The right-side expression is indeterminate. Borcherding and Silberberg maintain that for close substitutes, demand interactions with all other goods would not be "widely disparate" (apples are indeed apples) therefore all other goods response differences are sufficiently small or perhaps vanish. The Alchian and Allen theorem is then supported with Equation (8) signed positive.

Bauman (2004) initiates with a different substitution. Equations (14) and (15) are solved for ε^*_{11} and ε^*_{21}, respectively, then substituted into (13). The denominator for these new terms is now p_1 not p_2. Accordingly,

$$\underbrace{\left(\varepsilon^*_{12}-\varepsilon^*_{22}\right)\left(\frac{1}{p_2}-\frac{1}{p_1}\right)}_{\text{(3a)}}+\underbrace{\frac{1}{p_1}\left(\sum_{i=3}^{n}\varepsilon^*_{2i}-\sum_{i=3}^{n}\varepsilon^*_{1i}\right)}_{\text{(3b)}}. \tag{17}$$

Term notation, (3a) and (3b), is Bauman's. His necessary condition, $\varepsilon^*_{12} > \varepsilon^*_{22}$, allows for limited ε^*_{12} complementarity. The Alchian and Allen result will hold as long as (3b) is either positive or sufficiently small relative to (3a). Moreover, Bauman forwards an unexplored set of sufficient conditions: if $\varepsilon^*_{12} > \varepsilon^*_{22}$ and the two focus goods are not close in price, $p_1 >> p_2$, Equation (17) is positive. His line of reasoning for the later condition is that (3b) will be close to zero if p_1 is large relative to p_2. He provides a French wine example where p_1 is 500 dollars and p_2 is 5 dollars (1/500 = 0.002). This later condition, however, is a merely consequence of his substitution into Equation (13).

[5] Borcherding and Silberberg (1978) represent the summation of all other goods with a Hicksian composite commodity, i = 3 (Hicks, 1946).

2. Simulations

Interestingly, the simplicity of Equations (16) and (17) provide a means to simulate how varying price and elasticity differences affect the chance that these expressions are positive. Numerically simulating unit prices combined with unit free elasticity measures, however, is problematic. For example, when prices are assumed sufficiently large, both sides of Equations (16) and (17) will tend toward zero. Meaningful simulations then require a relevant range for inputs, especially prices. To start, both papers agree that the left-side expressions will be positive due to demand theory[6] and that quality goods are priced at a premium. Accordingly, we will hold the elasticity difference modeled in both left-sides constant and equal in absolute value. Regarding the right-side, only cases where this expression is negative are relevant.

Figure 1 below depicts the graphic results of our first case scenario. In this simulation, we highlight price distortions introduced by using Bauman's French wine example, p_1 = \$500 and p_2 = \$5. The elasticity differences reflected in the left-sides are first held constant at 0.5 for Bauman[7] and -0.5 for Borcherding and Silberberg. We vary the summation elasticity differences in the right-sides from zero to -5. This range is displayed on the zero axis in Figure 1. As shown, the Bauman result is positive across the entire scale, essentially parallel to the axis. By setting the price of good one so high, other goods demand interactions are zeroed out. In contrast, Borcherding and Silberberg's expression is only positive when the sum difference,

$$\left(\sum_{i=3}^{n}\varepsilon_{2i}^{*}-\sum_{i=3}^{n}\varepsilon_{1i}^{*}\right), \tag{18}$$

approaches zero. These divergent results are purely a function of how Equations (14) and (15) were substituted into (13). Figure 2 below reflects changing the elasticity differences in the left-sides to 1.5 for Bauman and -1.5 for Borcherding and Silberberg. This change predictably shifts the modeled results upward.

[6] Providing context, Bittschi et al., (2019) estimated French wine own price elasticities ranging from -0.49 to -0.57. Durham and Eales (2010) estimated Oregon fresh apple own price elasticities ranging from -1.13 to -1.19.

[7] This value represents, for example, ε_{12} = -0.1, ε_{22} = -0.6 in Bauman's expression.

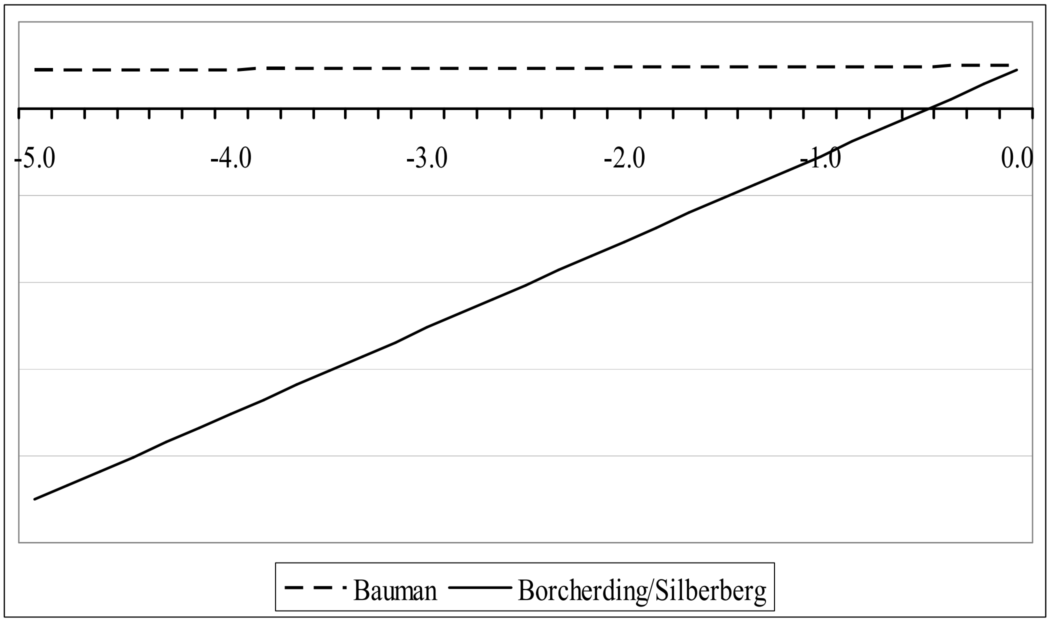

Figure 1: Case 1 simulation p_1 = 500, p_2 = 5

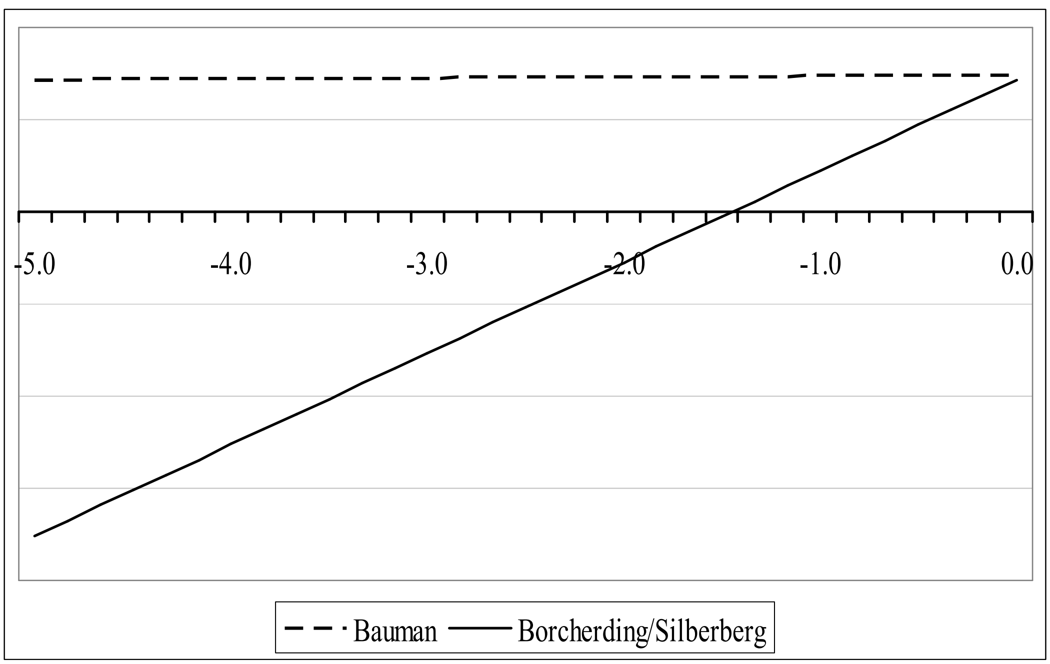

Figure 2: Case 2 simulation p_1 = 500, p_2 = 5

Figure 3 represents the Alchian and Allen classic apple example. Here we set p_1 = 10 cents and p_2 = 5 cents. The elasticity differences reflected in the left-sides are again held constant at 0.5 for Bauman and -0.5 for Borcherding and Silberberg. Now, the chance either expression is positive relies on Equation (18) being closer to zero. As shown in Case 2, changes in the absolute value of the left-side elasticity differences will shift the modeled results vertically – upward for an increase, downward for a decrease. With price levels markedly tempered, the two models tend toward predictable similarity.

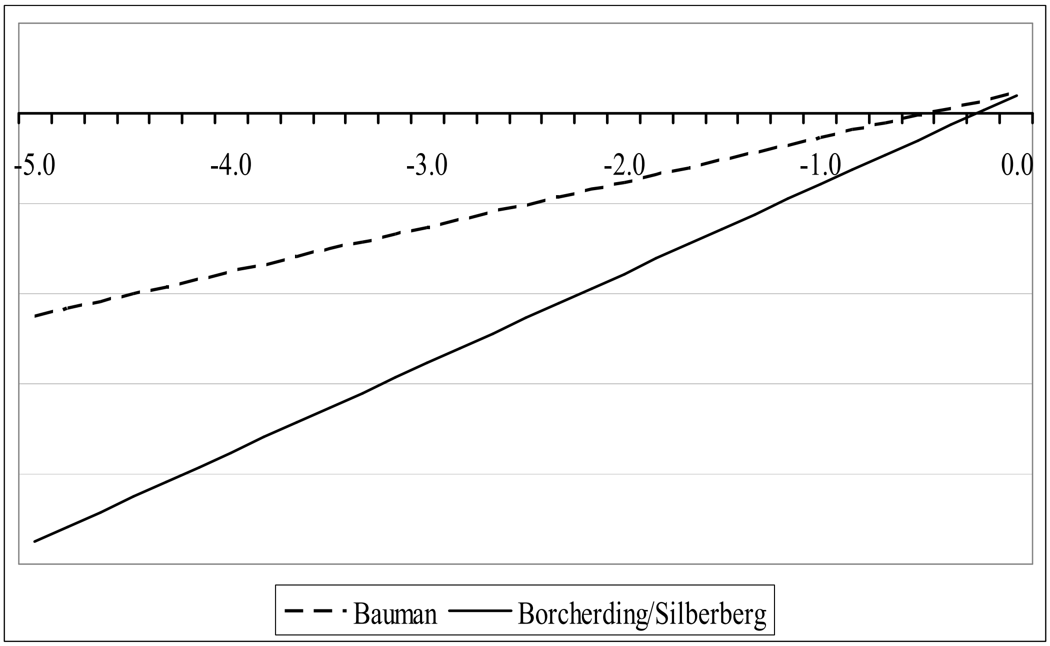

Figure 3: Case 3 simulation p_1 = 0.10, p_2 = 0.05

To understand how limited Bauman's complementarity condition is, we numerically simulate Equation (17) holding some parameters constant. This simulation is intended only to provide perspective, not empirical certainty. First, we simulate prices from Alchian and Allen's apple example. The own price elasticity is fixed to the upper estimate found in Durham and Eales (2010), $\varepsilon_{22}^{*} = -1.19$. Of course, the goal is to find Equation (17) positive so we set (3a) + (3b) = 0.50, just slightly above zero. Following the simulations above, we vary the (3b) summation elasticity differences, in this case from -0.5 to -1.5. Table 1 below provides the simulated values for ε_{12}^{*}. Complementarity emerges as the (3b) differences tend toward zero.

Table 2 below depicts results for a ten-fold increase in both prices. All other parameters are kept to the same values. Now, weak complementarity only appears as (3b) moves closer to vanishing. If we raise (3a) + (3b) to 0.70, the two focus goods cross-over to being substitutes at any (3b) summation elasticity difference of -0.5 or lower.

Table 1: Complementarity simulation $p_1 = 0.10$, $p_2 = 0.05$

ε_{12}^*	$\left(\sum_{i=3}^{n}\varepsilon_{2i}^* - \sum_{i=3}^{n}\varepsilon_{1i}^*\right)$
-0.64	-0.5
-0.14	-1.0
0.36	-1.5

Table 2: Complementarity simulation $p_1 = 1.00$, $p_2 = 0.50$

ε_{12}^*	$\left(\sum_{i=3}^{n}\varepsilon_{2i}^* - \sum_{i=3}^{n}\varepsilon_{1i}^*\right)$
-0.19	-0.5
0.31	-1.0
0.81	-1.5

3. Concluding remarks

Bauman's (2004) contribution builds on Alchian and Allen's (1964) substitution theorem by recognizing the potential for limited complementarity between the two focus goods. It falls short on showing how the two potential compliments would interact with all other goods modeled. His previously unexplored price condition is shown to be inconsequential. The key to a positive sign for Equation (8) above is Equation (18). If this term is negative, we end up playing the magnitude game. Borcherding and Silberberg's (1978) contention that an apple, premium or standard, will interact (in demand response) with all other goods in a like manner is forceful. It is hard to match this cogent line of reasoning when the two goods are weak complements.

ACKNOWLEDGEMENTS: To a great mentor, John T. Tschirhart.

References

[1] Alchian, A. and Allen, W. (1964). University Economics. Wadsworth Publishing.
[2] Borcherding, T. and Silberberg, E. (1978). Shipping the good apples out: The Alchian and Allen theorem reconsidered. Journal of Political Economy, 86(1), pp. 131-138.
[3] Bauman, Y. (2004). Shipping the good apples out: A new perspective. Economic Inquiry, 42(3), pp. 534-536.
[4] Umbeck, J. (1980). Shipping the good apples out: Some ambiguities in the interpretation of 'fixed charge'. Journal of Political Economy, 88(1), pp. 199-208.
[5] Gould, J. and Segall, J. (1969). The substitution effects of transportation costs. Journal of Political Economy, 77(1), pp. 130-137.
[6] Hicks, J. R. (1946). Value and Capital. Oxford University Press.
[7] Bittschi, B., Fortin, I., Koch, S., Sellner, R., Loretz, S. and Zwirn, G. (2019). Price elasticities and implied tax revenue for alcoholic beverages: Evidence from Poland, France and Spain. WIFO Working Paper # 579.
[8] Durham, C. and Eales, J. (2010). Demand elasticities for fresh fruit at the retail level. Applied Economics, 42(11), pp. 1345-1354.