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Portfolio Selection Using Genetic Algorithm

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Abstract

The selection of optimal portfolios is the central problem of financial investment decisions. Mathematically speaking, portfolio selection refers to the formulation of an objective function that determines the weights of the portfolio invested in each asset as to maximize return and minimize risk. This paper applies the method of genetic algorithm (GA) to obtain an optimal portfolio selection. However, the GA parameters are of great importance in the procedure of convergence of this algorithm towards the optimal solution such as crossover. While, a five stock portfolio example is used in this paper to illustrate the applicability and efficiency of genetic algorithm method, GA method can also be used however for a larger number of portfolio compositions. The results obtained confirm previous research studies about the validity and efficiency of genetic algorithm in selecting optimal portfolios.

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1 Introduction

Portfolio optimization is one of the most challenging problems in the field of finance. Selecting the weights of assets to invest in a portfolio as to meet the

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expectations about risk and return makes this problem more crucial. In dealing with this problem, Harry Markowitz 1959 developed a quantitative model, also called mean-variance model. The mean-variance model has been usually considered as either the minimization of an objective function representing the portfolio variance (risk) for a given level of return or the maximization of an objective function representing the portfolio return for a given level of risk. However, due to cardinality and bounding constraints, the applicability of the mean-variance model of Markowitz is limited (Fernandez and Gomez, 2007). To satisfy the limitations imposed, some constrained optimization algorithms such as: Constrained Optimization (CO), Quadratic programming, Linear Programming (LP), and Second-Order Cone Programming (SOCP) have been developed and used (Davidson, 2011). However, these constrained optimization algorithms have some drawbacks in portfolio optimization as are based on linear assumption and are therefore good for quadratic objective functions (deterministic) with a single objective (Roudier, 2007). But the important question that this paper is trying to answer is what if the objective function is not quadratic and has more than one objective: Maximisation of return and minimisation of risk simultaneously? .

Recently, some methods based on artificial intelligence such as Genetic algorithm have been applied to overcome this problem. GAs are stochastic, heuristic techniques based on the natural selection principles, and they can deal with nonlinear optimization problems with non-smooth and even non-continuous objective, and continuous and/or integer variables (Lin et al; 2005). However, the choice of GA parameters such as the mutation and crossover methods can influence the GA performance (Bakhtyar et al., 2012).

For the application of GA, three crossover procedures which are: Single point, two points, and arithmetic have been applied, while other procedures such as mutation and selection could be applied instead. The procedures of cross over are applied in order to know their impact on the convergence time of GA towards the optimal solution. GAs derives most of their power from cross over. Cross over, in combination with survival of the fittest structures, allows the best components of differing solutions to combine to form even better solutions (Mahfoud and Mani, 1996).

Although the use of GAs has progressed well in different fields like health, engineering, electronics, robotic and so, such progress however, is still not well advanced in the field of finance, especially in portfolio optimization problems. As such, this paper will shed more light on the contribution that GA can make in solving portfolio optimization problems.

2 Presentation of Genetic Algorithms and their Applications in Finance

Genetic algorithms (GAs) are stochastic optimization algorithms based on the

mechanisms of natural selection and Genetics (Holland, 1975) GA is now applied in many diverse applications such as simulation parameterization, real time control and optimization problem (Sawati Binti, 2005). Gas have been applied successfully to real world problems and exhibited; in many cases; better search efficiency compared with traditional optimization tools (Petridist et al, 1998).

According to (Vallée and Yildizoglu, 2003), the applications of genetic algorithm In the field of finance have been booming in recent years and begin to integrate in finance books. (Pereira, 2000), argues that Genetic algorithms are a valid approach to many practical problems in finance which can be complex and thus require the use of an efficient and robust optimization technique. Some applications of genetic algorithms to complex problems in financial markets include: forecasting returns, portfolio optimization, trading rule discovery, and optimization of trading rules.

Genetic algorithm has been successfully applied to different portfolio optimization. For example, (Laraschi et al., 1996) used the GAs to select an optimal portfolio. The GA was used to find the weights of a portfolio stocks that minimize a certain level of risk for an expected level of return. The study concluded on the effectiveness of the method including notably with regards to the possibility of existing multiple equilibrium. (Xia Lau Yang, 2006), applied GA method along with a dynamic portfolio optimized system to improve the efficiency of the stock portfolio. The findings of the study showed that the GA is of higher return compared to the other methods used in the study and simultaneously of less risk. In their study (Lin and Gen, 2007), used Markowitz model as a basic math model, looked for maximizing the return and minimizing the investing risk. Their findings proved the reliability and efficiency of the genetic algorithm in optimizing the stock portfolio. (Aranda and Iba, 2009) introduced a tree genetic algorithm that was used for the optimization of the stock portfolio. The smaller stock portfolios were obtained here. In a study done on 146 companies at Tehran Stock Exchange, (Garkaz, 2011) applied GA to select the optimal stock portfolio. The findings of the study proved the efficiency of the GA in optimizing of the stock portfolio.

3 Optimization using genetic algorithm

A genetic algorithm is an iterative method for searching the optimum solution; it manipulates a population with the constant size. This population consists of candidate points called chromosomes. This algorithm leads to a competition phenomenon between the chromosomes. Each chromosome is the encoding of a potential solution for the problem to be solved, it made up of a set of elements called genes, which can take several values. At each iteration (generation) a new population is created with the same size. This generation consists of the better chromosomes "adapted" to their environment as represented by the selective function. Gradually, the chromosomes will tend towards the optimum of the

selective function. The conception of the new population is made by applying the genetic operators which are selection, crossover and mutation.

- *Selection*: The new individuals selection is made as follows: Calculate the reproduction probability for each individual

$$p_i = \frac{f_i}{\sum_{i=1}^{n} f_i}$$

where f_i is the Fitness of the individual i. (a fitness function is needed to evaluate the quality of each candidate solution with regard to the task to be performed).
n is the size of the population. Each time a single chromosome is selected for the new population. This is achieved by generating a random number r from the interval [0, 1]. If $r \prec p_1$, then select the first chromosome, otherwise select the ith chromosome such as $p_{i-1} \prec r \leq p_i$.

- *Crossover*: The crossover operator follows:

Population resulting from selection is divided into two parts. Each pair formed will undergo the crossover with a certain probability P_c. Many different types of crossover exist in the literature for example: single point crossover, two point crossover, and arithmetic crossover.

- *Mutation*:

The individuals in the population after crossover will then undergo a process of mutation; this process is to randomly change some bits, with a certain probability P_m.

Genetic algorithms are more flexible than most search methods because they require only information concerning the quality of the solution produced by each parameter set (objective function values) and not like many optimization methods which require derivative information, or even more, complete knowledge of the problem structure and parameters (Bouktir et al, 2004).
There are some difference between Gas and traditional searching algorithms (Augusto et al, 2006). They could be summarized as follows:

- they work with a coding of the parameter set and not the parameters themselves;
- they search from a population of points and not a single point;
- they use information concerning of (payoff) and not derivatives or other auxiliary knowledge;
- they use probabilistic transition rules and not deterministic rules.

4 The mathematical formulation of the objective function

In a GA application, evaluation is performed by means of the fitness function which depends on the specific problem and the optimization objective of the GA (Petridis et al, 1998).

The aim is to select weights of the portfolio invested in each asset in order to maximize the portfolio return and minimize the portfolio risk. The crossover procedure in this regard, plays the role of exchanging weights of the securities of two chosen parents in such a manner that the offspring produced by the crossover represents (Lin and Gen, 2007).

The objective function (fitness function) is modeled to find the solution that scores less on the fitness scale, hence in this application, the crossover procedure with the least objective function should lead to better solution.

The expected return of the individual assets i is presented as a polynomial of first degree:

$$E(w_i) = w_i \cdot r_i \tag{1}$$

where

w_i denotes the weight of the individual asset i.

r_i denotes the expected return of asset i.

Thus the total expected return of portfolio P can be written as: $F = \sum_{i=1}^{n} E(w_i)$, and the objective function of the portfolio return to be maximized can be written as follows:

$$\max\left\{ F = \sum_{i=1}^{n} E(w_i) \right\} \tag{2}$$

where n is the number of assets.

The objective function of the Portfolio variance is presented as a polynomial of second degree:

$$\sigma^2(w_i) = \sum_{i=1}^{n} (w_i^2 \sigma^2(r_i)) + \sum_{i=1}^{n} \sum_{j=i+1}^{n} 2 w_i w_j \operatorname{cov}(r_i, r_j) \tag{3}$$

$\sigma^2(r_i)$: Variance of asset i

$\operatorname{cov}(r_i, r_j)$: Covariance between asset i and asset j

And the multi objective function to minimize is illustrated as:

$$H(w_i) = F(w_i) - \sigma^2(w_i) \tag{4}$$

Under the following constraints:

$$\sum_{i=1}^{n} w_i = 1 \tag{5}$$

$$W_i^{\min} \le w_i \le W_i^{\max} \tag{6}$$

To reach a positive portfolio return (what so ever are the weights values), let:

$$\sum_{i=1}^{n} r_i w_i \geq 0 \tag{7}$$

where

$W_i^{\max}$ and $W_i^{\min}$: maximum and minimum weights of asset i.

For the genetic algorithm application, the method of minimization under constraints has been used which is the penalty method.

$$g_i(w_i) \geq 0 \qquad i = 0, \ldots, n \tag{8}$$

These are the inequality constraints type

$$h_j(w_i) = 0 \qquad j = 0, \ldots, n \tag{9}$$

The problem is transformed into a penalty function, which is presented as follows:

$$F(w_i, r_k) = H(w_i) + \frac{1}{\mu_k} \sum_{j=1}^{n} [h_j(w_i)]^2 + \mu_k \sum_{j=1}^{m} [\frac{1}{g_i(w_i)}]^2 \tag{10}$$

where μ_k is the penalty coefficient.

5 Main Results

The main objective of this paper is to illustrate via a five (05) asset portfolio example the efficiency of the GA in solving portfolio optimisation problems. In order to achieve this goal, the objective of the fitness function in the GA method is set as to maximize the return and minimize the risk of the portfolio, and consequently the value that scores less on the fitness scale should lead to the best solution.

5.1 The data

For simplicity reasons, let's suppose the following historical returns from a five (05) stocks portfolio for a period of five years. The portfolio mean return and the portfolio variance are estimated using these historical data (Table 1).

Table 1: The data

Year	Stock 1	Stock 2	Stock 3	Stock 4	Stock 5
2007	-0.15	0.29	0.38	0.18	-0.10
2008	0.05	0.18	0.63	-0.12	0.15
2009	-0.43	0.24	0.46	0.42	0.15
2010	0.79	0.25	0.36	0.24	0.10
2011	0.32	0.17	-0.57	0.30	0.25

The mean return for each asset and the covariance matrix are given in the Tables 2, 3 below.

Table 2: The mean returns for each asset

	Stock 1	Stock 2	Stock 3	Stock 4	Stock 5
Mean return r_i	0,116	0,226	0,252	0,204	0,11

Table 3: The covariance matrix

	Stock 1	stock 2	stock 3	stock 4	stock 5
Stock 1	0,21728	-0,003376	-0,053492	-0,009264	0,01064
Stock 2	-0,003376	0,00253	0,008468	0,002376	-0,00456
Stock 3	-0,053492	0,008468	0,22247	-0,031128	-0,02392
Stock 4	-0,009264	0,002376	-0,031128	0,04068	0,00276
Stock 5	0,01064	-0,00456	-0,02392	0,00276	0,01675

5.2 Figures and Tables

The shaded cells represent the variance of asset i $(\sigma^2(r_i))$.

The MATLAB software was used to design the algorithm, and the results for each cross over procedure are presented below. Figures 1,2,3 and Tables 4,5,6 below illustrate the functions of genetic algorithm results obtained via the crossover procedures.

5.2.1 The single point procedure

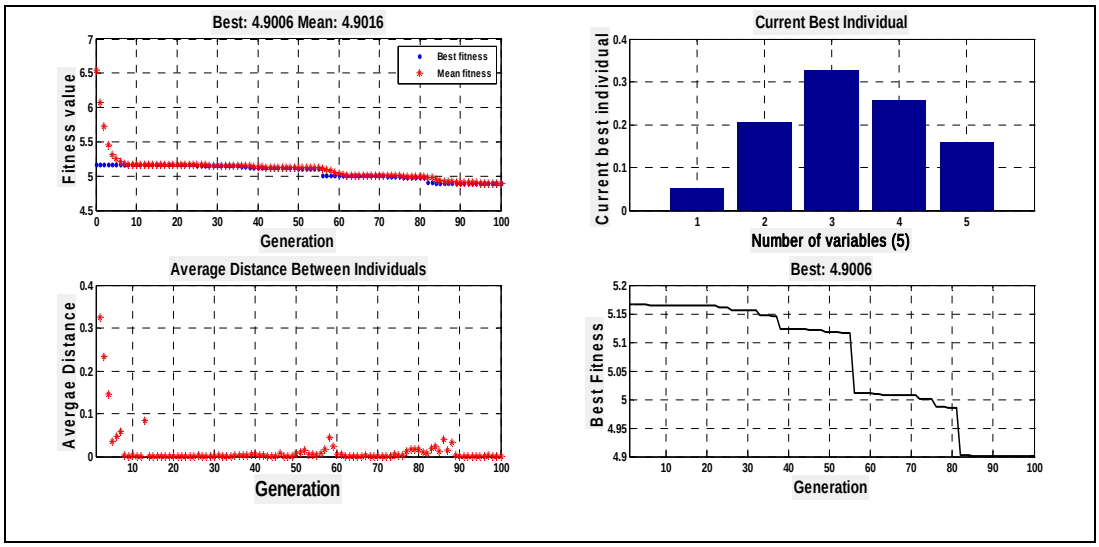

Figure 1: The variations of the GA functions according to generation under the single point cross over procedure

Table 4: Portfolio weights

W_1=	W_2=	W_3=	W_4=	W_5=
0.05128123839899	0.2051014436759	0.328635273473	0.255234628126	0.160744250453

Objective function value: 4.900620341135611
Variance of Portfolio = 0.0194
Average Return of Portfolio = 0.2049
Computing time = 3.5710 seconds

5.2.2 The two point procedure

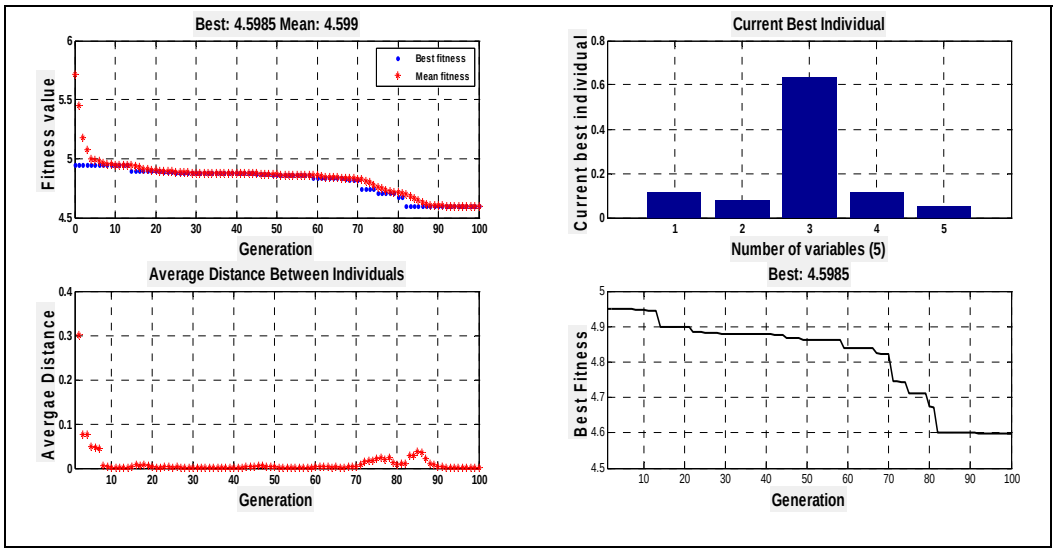

Figure 2: The variation of the GA functions according to generation under the two points cross over procedure

Table 5: Portfolio weights

W_1=	W_2=	W_3=	W_4=	W_5=
0.1169468063846	0.0793431661045	0.636040222078	0.1169054764309	0.05174945814081

Objective function value: 4.598464598013125
Variance of Portfolio = 0.0801
Average Return of Portfolio = 0.2213
Computing time = 4.0470 seconds

5.2.3 The arithmetic Procedure

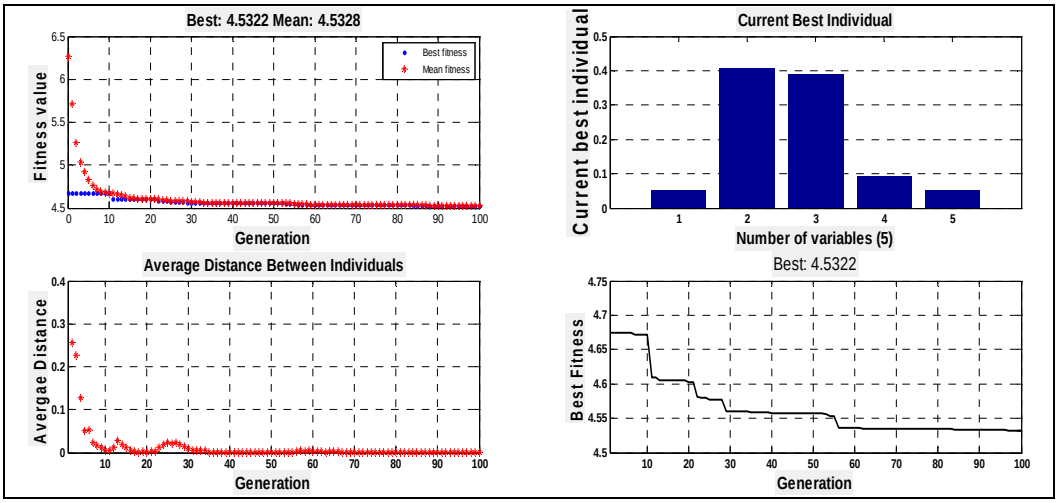

Figure 3: The variation of the GA functions according to generation under the arithmetic cross over procedure

Table 6: Portfolio weights

W_1=	W_2=	W_3=	W_4=	W_5=
0.0536704759282	0.407565381875	0.391342665955	0.0952988354731	0.0530909558293

Objective function value: 4.532249532470961
Variance of Portfolio = 0.0325
Average Return of Portfolio = 0.2222
Computing time = 3.5690 seconds

6 Discussion

For the application of the GA, an objective function (fitness function) was formulated to evaluate which among the three cross over procedures scores less on the fitness scale, and consequently should lead to the optimal portfolio.
The results show that the arithmetic cross over procedure gives better results than the two other procedures (i.e. single point and two points). The difference in the value of fitness function is clear. The arithmetic cross over procedure scores less on the fitness scale with 4.532249532470961, whereas the two point procedure scores 4.598464598013125, and the single point procedure scores 4.900620341135611. With regards to the fitness function value, the arithmetic

procedure should lead to the best choice of weights (w1: 0.0536704759282, w2:0.407565381875; and w3: 0.391342665955, w4: 0.0952988354731, w5: 0.0530909558293) and thus the optimal portfolio with a highest return of: 0.2222 and a lowest risk of: 0.0325.

As illustrated above, the GA can converge towards the optimal solution in a very little time: 3.5710 seconds under the single point, 4.0470 seconds under the two points, and 3.5990 seconds under the arithmetic procedure.

7 Conclusion

In this paper, a genetic algorithm method was applied to solve the optimal portfolio selection. The method was applied on a simple example of five (05) asset portfolio; the results obtained are interesting and confirm the efficiency of the genetic algorithm for its fast convergence towards the better solution and its interesting computing time.

A further research is needed to compare the results of GAs methods with regards to the mutation and selection procedures.

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