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Using a Multivariate Markov Chain Model for Estimating Credit Risk: Evidence from Taiwan

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Abstract

Dependence on credit ratings is an important issue for managing credit risk. This paper assesses credit risk using a multivariate Markov chain model that calculates dependence on credit ratings. We demonstrate the practical implementation of the proposed model using the rating data of 15 industries in Taiwan. Finally, our model is able to consider dependence structure to estimate credit risk.

JEL classification numbers: G21, G10
Keywords: credit risk, Basel Capital Accord, multivariate Markov chain model, Credit Value-at-Risk

1 Introduction

Credit risk management is a major issue of concern for bank industries and other financial intermediaries. During the past 10 years, developments in the financial markets have led to increasingly sophisticated approaches to credit risk management. This was confirmed by the Basel Committee on Banking Supervision when they formalized a universal approach to credit risk for financial

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institutions in July 1988. In 2001, the Bank for International Settlements (BIS) issued a consultative document (Basel II) that updated the credit risk assessment methods first proposed under a 1993 agreement (Basel I). In September 2010, Basel III was developed to address the regulative deficiencies revealed by the global financial crisis. The new rules will be phased in from January 2013 to January 2019.

Markov chain models are commonly used for various tasks, including queuing systems (Sharma, 1995; Ching, 2001) and inventory systems (Ching et al., 2008). Jarrow and Turnbull (1995) were the first to use the Markov chain model to assess credit risk. Jarrow et al. (1997), Lu and Kuo (2006), and Lu (2009) apply the firm's credit rating as a model for estimating credit risk.

However, dependency on rating the individual entities in a portfolio is a crucial aspect for modeling credit risks. There is considerable interest in modeling the dependency of individual credit risks in a portfolio. Some models for determining credit risks are mixture models, such as the Poisson mixture model of CreditRisk+ developed in 1997 by Credit Suisse Financial Products, or the infectious default model for Binomial Expansion Technique (BET) developed by Davis and Lo (2001). Another approach is to use copulas to estimate credit risk (Li, 2000).

Kijima et al. (2002) were the first to propose the multivariate Markov chain model to simulate the evolution of the dependent ratings for several credit risks. Their approach is an extension of the credit risk model developed by Jarrow et al. (1997). Siu et al. (2005) constructed different parameters for the multivariate Markov chain to model dependent credit risks, as well as proposing the use of credibility theory in actuarial science to calibrate the model. Despite the significant efforts and activities of banks and financial institutions to measure and manage credit risk, this issue is new in Taiwan and a multivariate Markov chain model has never been applied.

In addition to traditional credit risk measures such as credit spread and expected default probability, recent developments in credit risk management have highlighted the use of credit value at risk (CVaR) as a useful approach to measure credit risk. CVaR is derived from the framework of value at risk (VaR), which measures market risk. CVaR assigns probability to credit losses that could occur in a portfolio over a given period.

This paper uses a multivariate Markov chain model that includes a dependence structure of ratings to assess the CVaR of 15 industries in Taiwan. The main contribution of this paper is to apply the proposed model to credit risk measurement, a topic rarely discussed in Taiwan. This paper could also be used to generate suggestions for improving the current Basel regulatory standards.

The remainder of the paper is structured as follows: Section 2 presents the proposed model; Section 3 introduces a discussion on the data and an analysis of the empirical results; and lastly, Section 4 offers a conclusion.

2 Methodology

First, let a loan portfolio of *n* borrowers with each having *m* possible states. For each j=1,2,…,n and time t, we define a state probability vector of *j*th borrower, denote $X_t^{(j)}$ as follows

$$X_t^{(j)} := \left(x_t^{(j)}(1), x_t^{(j)}(2), \cdots, x_t^{(j)}(m)\right)$$

Then, we assume that the state probability distribution of the *j*th borrower at time *t+1* depends on the state probabilities of all the borrowers (including itself) at time *t*. Ching et al. (2002) propose the multivariate Markov chain model assumes the following relationship:

$$x_{t+1}^{(j)} = \sum_{k=1}^{n} \lambda_{jk} P^{(jk)} x_t^{(k)} \quad \text{for} \quad j = 1,2,...,n \tag{1}$$

where $\lambda_{jk} \geq 0,\ 1 \leq j,k \leq n$ and $\sum_{k=1}^{n} \lambda_{jk} = 1,$ for $j = 1,2,...,n.$ $P^{(jk)}$ be a transition matrix from the states in the *k*th sequence to the states in the *j*th sequence.

Equation (1) denotes that the state probability distribution of the *j*th borrower, $x_{t+1}^{(j)}$, at time *t*+1, depends on the weighted average of $P^{(jk)} x_t^{(k)}$ at time *t*. We can rewrite Equation (1) as

$$X_{t+1} \equiv \begin{pmatrix} x_{t+1}^{(1)} \\ x_{t+1}^{(2)} \\ \vdots \\ x_{t+1}^{(n)} \end{pmatrix} = \begin{pmatrix} \lambda_{11} P^{(11)} & \lambda_{12} P^{(12)} & \cdots & \lambda_{1n} P^{(1n)} \\ \lambda_{21} P^{(21)} & \lambda_{22} P^{(22)} & \cdots & \lambda_{2n} P^{(2n)} \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{n1} P^{(n1)} & \lambda_{n2} P^{(n2)} & \cdots & \lambda_{nn} P^{(nn)} \end{pmatrix} \times \begin{pmatrix} x_t^{(1)} \\ x_t^{(2)} \\ \vdots \\ x_t^{(n)} \end{pmatrix} \tag{2}$$

Then, we estimate $P^{(ij)}$ and λ_{ij} according to Ching et al. (2002). The first step is count transition frequency matrix as follows

$$F^{(jk)} = \begin{pmatrix} f_{11}^{(jk)} & \cdots & \cdots & f_{m1}^{(jk)} \\ f_{12}^{(jk)} & \cdots & \cdots & f_{m2}^{(jk)} \\ \vdots & \vdots & \vdots & \vdots \\ f_{1m}^{(jk)} & \cdots & \cdots & f_{mm}^{(jk)} \end{pmatrix} \tag{3}$$

where $f_{rs}^{(jk)}$ is transition frequency from the state s in the sequence $\{x_t^{(k)}\}$ to the state r in the sequence $\{x_t^{(j)}\}$. Then, we get the estimates for $P^{(jk)}$ as follows:

$$\hat{P}^{(jk)}=\begin{pmatrix}\hat{p}_{11}^{(jk)} & \cdots & \cdots & \hat{p}_{m1}^{(jk)}\\ \hat{p}_{12}^{(jk)} & \cdots & \cdots & \hat{p}_{m2}^{(jk)}\\ \vdots & \vdots & \vdots & \vdots\\ \hat{p}_{1m}^{(jk)} & \cdots & \cdots & \hat{p}_{mm}^{(jk)}\end{pmatrix} \tag{4}$$

where

$$\hat{P}_{rs}^{(jk)}=\begin{cases}\dfrac{f_{rs}^{(jk)}}{\sum_{s=1}^{m} f_{rs}^{(jk)}}, & \text{if} \quad \sum_{s=1}^{m} f_{rs}^{(jk)}\neq 0,\\ 0, & \text{otherwise}\end{cases}$$

For estimating λ_{ij}, we let the multivariate Markov chain has a stationary vector $\hat{x}$ and we would expect:

$$\begin{pmatrix}\lambda_{11}P^{(11)} & \lambda_{12}P^{(12)} & \cdots & \lambda_{1n}P^{(1n)}\\ \lambda_{21}P^{(21)} & \lambda_{22}P^{(22)} & \cdots & \lambda_{2n}P^{(2n)}\\ \vdots & \vdots & \vdots & \vdots\\ \lambda_{n1}P^{(n1)} & \lambda_{n2}P^{(n2)} & \cdots & \lambda_{nn}P^{(nn)}\end{pmatrix}\hat{x}\approx\hat{x} \tag{5}$$

Therefore, we can solve the optimization problem as follows:

$$\begin{cases}\min\limits_{\lambda}\sum_{i=1}^{m}\left[\sum_{k=1}^{n}\lambda_{jk}\hat{P}^{(jk)}\hat{x}^{(k)}-\hat{x}^{(j)}\right]_i^2\\ \textit{subject to}\sum_{k=1}^{n}\lambda_{jk}=1, \quad \textit{and} \quad \lambda_{jk}\geq 0, \quad \forall k\end{cases} \tag{6}$$

For estimating CVaR, we follow the method of Siu et al. (2005). First, we assume L_t^j represent the losses from the *j*th borrower at time t. The aggregate loss L_{t+1} of bank loans at time *t*+1 is given by $L_{t+1}=L_{t+1}^{(1)}+\cdots+L_{t+1}^{(j)}$. For each $\tilde{k}$ and K^* denote a positive integer in $\{1,2,...,M\}$, the conditional aggregate loss L_{t+1} equals $L_{t+1}(\tilde{k})$ is given by:

$$\begin{aligned}&P\left(L_{t+1}=L_{t+1}(\tilde{k})\right)\\ &\approx\sum_{(i_1,i_2,\dots,i_n)\in I_{t+1,\tilde{k}}}\left\{\prod_{j=1}^{n}\left[\sum_{k=1}^{n}(\lambda_{jk}^{-1}Q^{(jk)}+\lambda_{jk}^{-2}\hat{P}^{(jk)})X_t^{(k)}\Big|_{X_t=(e_{i1},e_{i2},\dots,e_{in})}\right]^{ij}\right\}\end{aligned} \tag{7}$$

where $Q^{(jk)}$ is transition probability matrix. The credit risk drivers in the portfolio model in the loan portfolio is determined based on the rating history and used in the calculation of CVaR as follows:

$$\mathrm{P}(\mathrm{L}_{t+1} \geq \mathrm{L}_{t+1}(\mathrm{K}^*)) = \sum_{\tilde{k}=\mathrm{K}^*}^{\mathrm{M}} \mathrm{P}(\mathrm{L}_{t+1} = \mathrm{L}_{t+1}(\tilde{k})) \leq \alpha \tag{8}$$

and

$$P(L_{t+1} \geq L_{t+1}(K^*)+1) = \sum_{\tilde{k}=K^*+1}^{M} P(L_{t+1} = L_{t+1}(\tilde{k})) > \alpha \tag{9}$$

where the probability level $\alpha \in (0,1)$ is usually chosen to be 1% or 5% according to different purposes and practices of risk measurement. From equation (8) and (9), we have

$$CVaR_\alpha(L_{t+1}) = L_{t+1}(K^*) \tag{10}$$

3 Data and empirical results

3.1 Data

The sample data in this study were collected from the TCRI database of the Taiwan Economic Journal (TEJ). The TEJ applies a numerical definition ranging from 1 to 9 for each rating classification. These categories are defined by default risk and the likelihood of payment for each borrower. Credit ratings with investment grades of 1-4 have the lowest default risks. Ratings graded from 5 to 6 have significant speculative characteristics. Rates graded from 7 to 9 belong to the most risky borrowers. We combined grades 1-4, 5-6, and 7-9 into new rating grades, denoted as 1^*, 2^*, and 3^*. The sample period was 1998-2008 for 396 companies in 15 industries.

3.2 Empirical results

To calculate credit risks, including dependency ratings, we estimated the parameter λ_{jk} using Equation (6). The results of estimating λ_{jk} are shown in Table 1. Table 1 shows the strength of the dependency among the three ratings. The transition matrices of the Markov chain model are diagonally dominant, meaning that a heavy concentration is around the diagonal. The diagonal numbers were higher than the non-diagonal ones, as shown in Table 1. This means that the dependence of switching in original ratings is higher; that is, it stays in the original ratings.

Thereafter, we estimated the CVaR, including the inter-rating dependence, according to the method developed by Siu et al. (2005). The CVaR gives the value of extreme loss at a given probability level of α. This paper used two risk measures to evaluate the probability levels α =5% and α =1%. Table 2 and

Figure 1 show the CVaR of 15 industries with 95% and 99% levels of confidence (α =5% and α =%). CVaR is an estimate of the portfolio's losses in the worst case scenario with a relatively high level of confidence. The portfolio loss of a CVaR with 99% confidence (α =1%) is higher than one that has 95% confidence (α =5%).

Table 1: Estimations of λ

Industry	*Ratings*	1^*	2^*	3^*
Building and Cons	1^*	0.7739	0.0871	0.1390
	2^*	0.2133	0.5344	0.2523
	3^*	0.1047	0.2241	0.6713
Cement	1^*	0.3424	0.1548	0.5028
	2^*	0.2224	0.4682	0.3094
	3^*	0.1929	0.2747	0.5324
Chemical, Biotech	1^*	0.7964	0.1704	0.0332
	2^*	0.2178	0.5646	0.2176
	3^*	0.0200	0.0819	0.8981
Electronics	1^*	0.7790	0.0633	0.1578
	2^*	0.2214	0.3332	0.4454
	3^*	0.1574	0.2004	0.6422
Electrical and Cable	1^*	0.2347	0.6838	0.0815
	2^*	0.7388	0.8021	0.4591
	3^*	0.1137	0.4044	0.4818
Electric. Machinery	1^*	0.6973	0.0905	0.2122
	2^*	0.2547	0.4737	0.2717
	3^*	0.2328	0.2687	0.4985
Foods	1^*	0.8241	0.1543	0.0216
	2^*	0.1620	0.2045	0.6335
	3^*	0.2670	0.1357	0.5973
Iron and Steel	1^*	0.7488	0.1105	0.1406
	2^*	0.0563	0.6456	0.2982
	3^*	0.1401	0.1240	0.7359
Others	1^*	0.6609	0.1598	0.1793
	2^*	0.3646	0.4438	0.1916
	3^*	0.2461	0.1915	0.5624
Paper and Pulp	1^*	0.7109	0.1135	0.1757
	2^*	0.3696	0.6304	0.0000
	3^*	0.2396	0.1598	0.6006
Plastics	1^*	0.7140	0.1319	0.1541
	2^*	0.1693	0.7385	0.0922
	3^*	0.0928	0.1554	0.7518
Rubber	1^*	0.8644	0.0018	0.1338

	*2**	0.0682	0.7951	0.1368
	*3**	0.0000	0.0766	0.9234
Shipping and Trans	*1**	0.8447	0.0646	0.0907
	*2**	0.1537	0.4489	0.3973
	*3**	0.2447	0.1390	0.6163
Textiles	*1**	0.6753	0.1410	0.1837
	*2**	0.2243	0.5110	0.2647
	*3**	0.1697	0.2113	0.6190
Trading and Consumer	*1**	0.6543	0.2474	0.0983
	*2**	0.3180	0.5682	0.1139
	*3**	0.0252	0.0241	0.9508

Table 2: Estimated CVaR for 15 industries

Industry	*CVaR*	
	1%	*5%*
Building and Cons	2.1478	1.4888
Cement	1.8232	1.1535
Chemical, Biotech	1.8062	1.4916
Electronics	1.8587	0.8406
Electrical and Cable	1.8492	1.8202
Electric. Machinery	1.8486	1.4946
Foods	1.5088	0.8163
Iron and Steel	2.1552	1.8252
Others	2.4937	1.8476
Paper and Pulp	1.5098	1.1849
Plastics	0.8156	0.4761
Rubber	1.1999	0.8510
Shipping and Trans	2.4915	2.1471
Textiles	2.5283	1.8334
Trading and Consumer	1.8352	0.8534

To provide an example, the estimated CVaR of the plastics industry is 0.8156 (0.4761) with 99% (95%) confidence. Therefore, this industry could lose 0.8156 (0.4761) per dollar in the worst case scenario with a 99% (95%) confidence. In 2006, although firms in the plastics industry were influenced by the volatility of the economy and the cost of raw materials, they were profitable because of investments made in China’s electronics market. Therefore, because the performance of the plastics industry was strong, its estimated CVaR was lower than that of other industries.

To provide another example, the estimated CVaR of the textile industry was 2.5283 and 1.8334 with 99% confidence and 95% confidence, respectively. This means that the industry could lose 2.5283 (1.8334) per dollar in the worst case scenario with a 99% (95%) confidence. In Taiwan, many firms in the textile industry have moved their factories to Vietnam, such as Tainan Spinning-the largest Taiwanese textile firm. However, in 2008, Vietnam's currency depreciated by approximately 50%, resulting in decreased profits for the textile industry. Therefore, the estimated CVaR of the textile industry was higher than that of other industries.

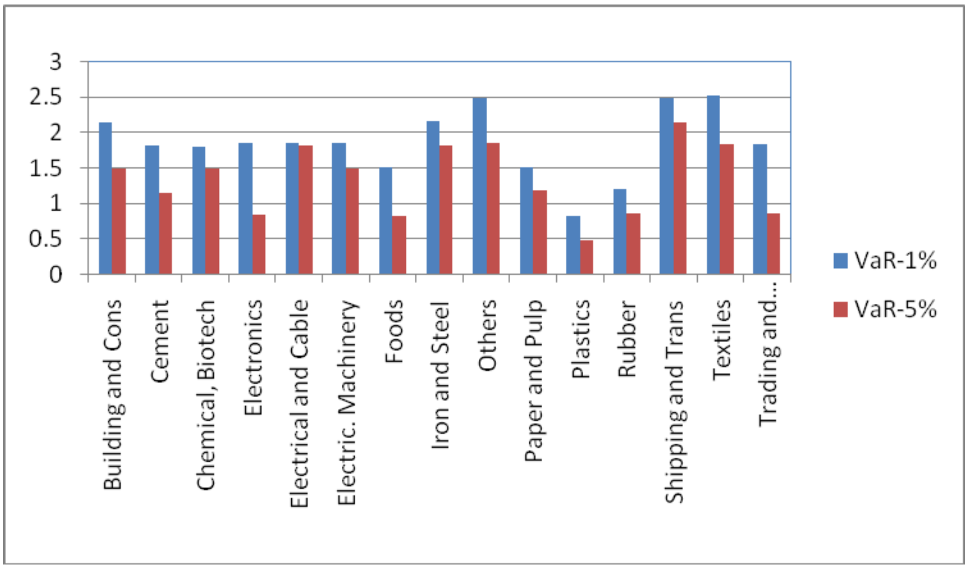

Figure 1: Estimated CVaR for 15 industries

4 Conclusion

Many of the financial decisions made by householders, business firms, governments, and financial institutions focus on the management of credit risk. Although dependency rating is an important aspect for measuring credit risk, it has not been explored using the estimation of CVaR in Taiwan. This paper assessed credit risk and CVaR, using a multivariate Markov chain model that considered dependency rating. The proposed model was used to examine 396 firms in 15 industries. The proposed model provides more reliable estimated results for financial institutions in Taiwan.

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