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Generalized Near Fields and (m, n)Bi-ideals over Noetherian Regular δ-Near-Rings (GNF-BI-NR-δ--NR)

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Abstract

In this paper we generalize the notion of Near Fields and obtain equivalent conditions, main results for generalized near-fields and generalized (m; n)/ bi-ideals over Noetherian Regular delta Near Rings.

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1 Preliminaries

In this section we give the preliminary definitions, examples and the required literature to this paper.

Definition 1.1 A Near–Ring is a set N together with two binary operations "+" and "." Such that

(i) (N, +) is a Group not necessarily abelian

(ii) (N, ·) is a semi Group and

(iii) For all $n_1, n_2, n_3 \in N$, $(n_1 + n_2). n_3 = (n_1. n_3 + n_2 . n_3)$ i.e. right distributive law.

Example 1.2 Let $M_{2x2} = \{(a_{ij}) / Z$; Z is treated as a near-ring$\}$. M_{2x2} under the operation of matrix addition '+' and matrix multiplication '.'

Example 1.3 Z be the set of positive and negative integers with 0. (Z, +) is a group. Define '.' on Z by $a . b = a \ \forall \ a, b \in Z$. Clearly (Z, +, .) is a near-ring.

Example 1.4 Let $Z_{12} = \{0, 1, 2 ,\ldots, 11\}$. $(Z_{12}, +)$ is a group under '+' modulo 12.Define '.' on Z_{12} by $a . b = a \ \forall \ a \in Z_{12}$. Clearly $(Z_{12}, +, .)$ is a near-ring.

Definition 1.5 A near-ring N is Regular Near-Ring if each element $a \in N$ then there exists an element x in N such that a = axa.

Definition 1.6 A Commutative ring N with identity is a Noetherian Regular δ-Near Ring if it is Semi Prime in which every non-unit is a zero divisor and the

Zero ideal is Product of a finite number of principle ideals generated by semi prime elements and N is left simple which has $N_0 = N$, $N_e = N$.

Definition 1.7 A Noetherian Regular delta Near Ring (is commutative ring) N with identity, the zero-divisor graph of N, denoted Γ(N), is the graph whose vertices are the non-zero , zero-divisors of N with two distinct vertices joined by an edge when the product of the vertices is zero.

Note 1.8 : We will generalize this notion by replacing elements whose product is zero with elements whose product lies in some ideal I of N. Also, we determine (up to isomorphism) all Noetherian Regular δ- near rings N_i of N such that Γ(N) is the graph on five vertices.

Definition 1.9 A near-ring N is called a δ-Near - Ring if it is left simple and N_0 is the smallest non-zero ideal of N and a δ-Near-Ring is a non-constant near ring.

Definition 1.10 A δ-Near-Ring N is isomorphic to δ-Near-Ring and is called a Regular δ-Near-Ring if every δ-Near-Ring N can be expressed as sub-direct product of near-rings {Ni}, Ni is a non-constant near-ring or a δ-Near-Ring N is sub-directly irreducible δ-Near-Rings Ni.

Definition 1.11 Let N be a Commutative Ring. Let N be a Noetherian Regular δ-Near-Ring if each P ∈ A (N_N) is strongly prime i.e., P is a δ-Near – Ring of N.

Example: 1.12 Let N = $\begin{bmatrix} F & F \\ 0 & F \end{bmatrix}$ where F is a field. Then P (N) = $\begin{bmatrix} 0 & F \\ 0 & 0 \end{bmatrix}$

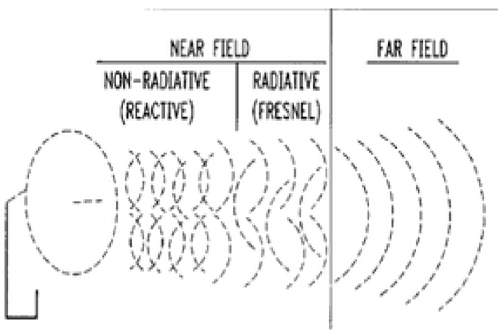
Let σ: N → N be defined by, σ $\left(\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & c \end{bmatrix}$

It can be seen that a σ endomorphism of N and N is a σ(*)-Ring or Noetherian Regular δ-Near– Ring.

Definition 1.13 Let (N, +, •) be a near-ring. A subset L of N is called a ideal of N provided that (1). (N, +) is a normal subgroup of (N, +), and (2). m.(n + i) - m.n $\in$ L $\forall$,i $\in$L and m, n $\in$ N.

2 Introduction

By analogy with the concept of an "Inverse Semi-Group" in semi group theory, in this paper the concept of "Generalized Near Fields over Noetherian Regular Delta Near Rings (GNF-NR-Delta-NR)" introduced by NVNagendram, T Radha Rani, Dr T V Pradeep Kumar and Dr Y V Reddy in Noetherian Regular Delta Near Rings of Near Rings. A Near-Ring N is called a Generalized Near Field (GNF) if $\forall$ a $\in$ N $\exists$ a unique b $\in$ N such that a=aba and b=bab i.e., (N , •) is an inverse Semi-Group.



Surprisingly, this concept in Rings coincides with that of Strong Regularity. But this is not true in the case of Near Rings.

Every generalized near Field (GNF) is strongly regular, but in general converse is not true. The aim of this paper to show that for any Noetherian Regular Delta near Ring is having following conditions are Equivalent:-
(i) N is GNF, (ii) N is Regular and each idempotent is central and (iii) N is regular and Sub commutative.

Also we prove that if Noetherian Regular Delta Near Ring (NR-Delta-NR) is a Near Ring with d c c (descending chain condition) on Ideals, then (i) N is a GNF if and only if it is direct sum of finitely many Near Fields (ii) is equivalent to (N, .) is a Clifford Semi-Group and Semi Properties of inverse Semi- Groups. See [4] for the properties of inverse semi-groups.

Throughout this paper, N stands for Right Noetherian Regular Delta near Ring (RNR-δ-NR). For the basic terminology and notations we refer to Gunter Pilz [24].

Definition 2.1 A Noetherian near Ring N is called Regular if $\forall$ a $\in$ N $\exists$ a unique b $\in$ N such that a = aba, $\forall$ b $\in$ N.

Lemma 2.2 If a Noetherian regular delta near ring N is a GNF, then N is zero-symmetric.

Proof. Since N is a a GNF, for each n $\in$ N there is a unique x $\in$ N such that n0 = n0 x n0, x = xn0x. Both 0 and n0 satisfy the above equations. So by uniqueness 0 = n0. Thus N is zero-symmetric.

By [2, Theorem 1.2 p.130] N is a GNF if and only if N is regular and idempotent commute. Recall that N is called strongly regular delta near ring if for each a $\in$ N there exists b $\in$ N such that a = ba^2. For a brief discussion of these near – rings, see [6], [2] and [1]. In [2], a Noetherian regular delta near ring N is called sub-commutative if aN = Na for all a $\in$ N. □

Lemma 2.3 If a Noetherain regular delta near-ring N is a GNF, then N has no non-zero nilpotent elements.

Proof. Let $a \in N$, $a^2 = 0$, and let a have inverse b. Then b^2 = babbab = 0. Since ab, ba are idempotent elements and hence commute. Also, ba(ba + b) is an inverse for a, so ba(ba + b) by uniqueness. Thus 0 = b^2 = ba(ba + b)b = babab = bab = b. So a must be 0. □

Theorem 2.4 To prove the following are equivalent:

(i) N is a GNF

(ii) N is regular and each idempotent is central and

(iii) N is regular and sub-commutative.

Proof. Given a Noetherian regular δ-near ring N is a GNF. We prove by cyclic.

To prove (i) ⇒(ii) : Let $e = e^2 \in N$ and a, b ∈ N. since, $e^2 = e$, (a – ae)e =0 by [24,chapter 9a & 9b], since N has no non zero nilpotent elements by lemma 2, (a – ae) be =0, so abe = aebe. But, (eb – ebe)e = 0 for the same reason, eb(eb-ebe) = 0, ebe(eb – ebe) = 0 so $(eb - ebe)^2$ =0 and eb = ebe. Thus ebe = aeb. Since N is noetherian regular δ-near ring, a = fa where f is a suitable idempotent. So ae = fae = fea = ea as idempotent commutes. So (ii) holds good. Hence (i) ⇒(ii) proved.

To prove (ii) ⇒(iii) : Let a ∈ N. since N is a Noetherian regular δ-near ring, a = axa for some x ∈ N. since ax and xa are idempotent elements , by (ii) we have aN = axaN = aNxa ⊆ Na = Naxa ⊆ aN. Thus aN = Na ∀ a ∈ N. hence proved (ii) ⇒(iii).

To prove (iii) ⇒(i) : Let e , f be idempotent elements Then Ne = eN. So ∃ x , y ∈ N such that fe = ex and ef = ye. Hence efe = fe = ef. So ef = fe and Noetherian regular δ-near ring N is a GNF. Proved (iii) ⇒(i).

Hence proved the theorem by cyclic method. □

Corollary 2.5 Every GNF is a strongly noetherian regular δ-near ring.

Proof. by (ii) a =aba = ba^2 since ba is an idempotent where b is the inverse of a. In [26], R Raphael showed that in a strongly Noetherian regular near ring N, for each $0 \neq$ a in N there exists a unique b in N such that a =aba and b = bab, now the converse follows from cor. 1. Thus in the case of Rings the notions 'strongly regularity' and 'GNF' are equivalent. In general the converse of cor. 1 does not hold in near rings. □

Example 2.6 Let (N, +) be any group. Define multiplication on N as follows ab = a for all a and $0 \neq$ b in N such that a0 = 0 for all a in N. Then clearly, N is strongly regular delta near ring but not GNF.

Corollary 2.7 Every homomorphic image of a GF is again a GNF.
Proof. The definition of GNF shows that the properties are preserved under homomorphism's. By the combinations of Theorem 1 and result of Leigh [27] we have the following.

Corollary 2.8 Every GNF is isomorphic to a sub-direct product of near fields and hence (N, +) is abelian.

Theorem 2.9 N is a GNF and integral if an only if N is a near field.
Proof. Refer [7,Theorem 2]. □

Corollary 2.10 Suppose N is sub-directly irreducible. Then N is a GNF if and only if N is a near field. In general every GNF is not a near filed.

Example 2.11 Take a near field N. then the direct sum of N with itself is a GNF, but not a near field.

Corollary 2.12 Suppose for each $0 \neq a$ in N there exists a unique $b \in N$ such that $a= aba$. Then N is a near field.

Corollary 2.13 (S Leigh [27]): Let N be a Noetherian regular delta near ring with more than one element. Then N is a division ring if and only if for each $0 \neq a$ in N, $Na = N$ there exists a unique b in N such that $a = aba$.

In [2] A Near-ring N is called left simple if $\forall\ 0 \neq a \in N$, $Na = N$. clearly left simple near ring contains no zero-divisors.

Theorem 2.14 Suppose a Noetherian regular δ-near ring N has dcc on ideals. Then N is a GNF if and only if $N = N_1 \oplus N_2 \oplus N_3 \oplus N_4 \oplus \ldots\ldots \oplus N_k$ where each N_i is a near field.

Proof. Refer [5, theorem 3.2], [24, theorem 2.50, p.57]. □

Corollary 2.15 Suppose Noetherian regular δ-near-ring N is a GNF and satisfies dcc on ideals. Then (a) N has the identity (b) $a(-b) = (-a)b = (-ab)$ for all a, b in N.

3 Some concepts on Generalized (m, n) bi-ideals of Noetherian Regular δ- near-Rings

For basic definitions of near rings one can refer to Gunter Pilz [24] and Tamizh Chelvam & Ganesan [29] introduced the notion of bi-ideals in near-rings. Further, Tamizh Chelvam [30] introduced B-regular near-rings and obtained equivalent conditions for regularity in terms of Bi-ideals. In this section, we generalize the notion of bi-ideals and obtain conditions equivalent to generalized near-fields over Noetherian Regular δ-Near Rings in terms of generalized (m, n)/ bi-ideals.

Definition 3.1 Let A and B be two subsets of a Noetherian regular δ-near-ring N. Then $AB = \{ab/ a \in A; b \in B\}$ and $A^* B = \{a_1.(a_2 + b) - a_1a_2 / a_1, a_2 \in A; b\in B\}$.

Definition 3.2 A subgroup B of a Noetherian Regular δ-Near Ring (N, +) is said to be a bi-ideal of N if $BN B \cap (BN) * B \subseteq B$ [30]. In the case of a zero-symmetric near-ring, a subgroup B of (N, +) is a bi-ideal if $BNB \subseteq B$. A bi-ideal B of (N, +) is a generalized (m, n) bi-ideal if $B^m N B^n \subseteq B$, where m and n are prime integers.

Definition 3.3 A subgroup A of Noetherian Regular δ-Near Ring (N, +)/ is said to be a left (right) N -subgroup of N if $N A \subseteq A(AN \subseteq A)$. N is said to be a two sided near-ring if every left (right) N-subgroup is a right (left) N - subgroup of N.

Definition 3.4 A Noetherian Regular δ-Near Ring N is called B-regular near-ring if $a \in (a)_r N(a)_l$ for every $a \in N$ where $(a)_r((a)_l)$ is the right (left) N -subgroup generated by $a\in N$.

Definition 3.5 A Noetherian Regular δ-Near Ring N is said to have property (α), if xN is a subgroup of (N, +) for every $x\in N$.

Definition 3.6 A Noetherian Regular δ-Near Ring N is said to be sub-commutative if $xN = N x$ for every $x \in N$.

Note 3.7: Let every sub-commutative Noetherian Regular δ-Near Ring is a Noetherian Regular δ-Near Ring with property (α). A Noetherian Regular δ-Near Ring N is said to be a S-Noetherian Regular δ-Near Ring if $x \in Nx$ for all A S-Noetherian Regular δ-Near Ring N is said to be S^N-Noetherian Regular δ-Near

Ring $x \in xN$ for $x \in N$.

Definition 3.8 N is called regular if for each $a \in N$ $\exists$ $a = aba$ for some $b \in N$ and N is called strongly regular if for each $a \in N$, there exists $b \in N$ such that $a = ba^2$.

Note 3.9: Every strongly regular near-ring is always a regular near-ring [6]. N is reduced if it has no non-zero nilpotent elements. N is said to have IFP(Insertion of Finite Property) if $ab = 0 \Rightarrow axb = 0$ $\forall$ $x \in N$.

Definition 3.10 A Noetherian Regular δ-Near Ring is called left bi-potent if $Na = N a^2$ for $a \in N$.

Definition 3.11 A Noetherian Regular δ-Near Ring N is called a generalized near-field (GNF) if for each $a \in N$, there exists a unique $b \in N$ such that $a = aba$ and $b = bab$ [7].

Definition 3.12 Let E denotes the set of all idempotents of N. $C(N) = \{n \in N$ such that $nx = xn$ for all $x \in N\}$ is called the center of N . N is said to be a S_k (S'_k) Noetherian Regular δ-Near Ring is $x \in N x^k(x^k N)$ $\forall x \in N$. One can see that if N is a S_k Noetherian Regular δ-Near Ring, then N is S_j Noetherian Regular δ-Near Ring $\forall$ $j \leq k$.

Definition 3.13 Let A Noetherian Regular δ-Near Ring N is said to be P(r, m) Noetherian Regular δ-Near Ring if $a^r N = N a^m$ for each $a \in N$ where r, m are positive integers.

Definition 3.14 A Noetherian Regular δ-Near Ring N is said to be $P_k(P'_k)$ Noetherian Regular δ-Near Ring if $x^k N = xN x(N x^k = xN x)$ $\forall x \in N$ and P_k (r, m)(P'_k (r, m)) Noetherian Regular δ-Near Ring if $x^k N = x^r N x^m$($N x^k = x^r N x^m$) $\forall x \in N$.

Definition 3.15 A Noetherian Regular δ-Near Ring N is said to be of Type I or Type II according as $(xy)z = (yx)z$ or $x(yz) = x(zy)$ $\forall x, y, z \in N$.

Definition 3.16 A sub-commutative Noetherian Regular δ-Near Ring N is said to be stable if $xN = xNx$ for all $x \in N$.

4 Main Results

In this section we obtain certain results which will be useful in subsequent sections.

Lemma 4.1 If N has the condition $eN = eNe = N e$ for $e \in E$ and $n \in N$, then $E \subseteq C(N)$.

Proof. Let us assume that $eN = eN e = N e$ for $e \in E$ and $n \in N$. Then there exists $p; q \in N$ such that $ne = epe$ and $en = eqe$.

$\Rightarrow ene = e(ne) = e(epe) = epe = ne$ and $ene = (en)e = (eqe)e = eqe = en$.

Thus $en = ene = ne$ for all $n \in N$. Therefore $E \subseteq C(N)$. □

Proposition 4.2 Let N be aS^N-Noetherian regular δ-near-ring. Then N is a sub commutative Noetherian regular δ-near-ring P=PNP for every bi-ideal P of N if and only if N is a P(1,2) Noetherian regular δ-near-ring.

Proof. If N is sub-commutative S- Noetherian regular δ-near-ring with P = PN P for every bi-ideal P of N , then N is left bi-potent, i.e., $N a = Na^2$ (by Proposition 2.5 [30]). Hence $aN = Na = Na^2$. Thus N is a P (1,2) Noetherian regular δ-near-ring.

Conversely, let N be P (1, 2) Noetherian regular δ-near-ring. Now for $e \in E$; $e N = N e^2 = N e$ and so $e N e = e(N e) = e(e N) = e N$. Hence, $e N = e N e = N e$ for all $e \in E$ and $n \in N$. Thus by known lemma $E \subseteq C(N)$. Since, N is a S^N- Noetherian regular δ-near-ring $a \in aN = N a^2$ and this shows that N is strongly regular. So N becomes Regular. Let $a \in N$, $a = aba$. Then ab and ba are clearly idempotent elements in N. Thus $aN = (aba)N = aN(ba) \subseteq Na = n(aba) = (ab) Na \subseteq aN \Rightarrow aN = Na$ $\forall a \in N$. Therefore, N is a sub-commutative Noetherian regular δ-near-ring and hence N is left bi-potent Noetherian regular δ-near-ring. If P is a bi-ideal of N, then P = PNP follows from (by Proposition 2.5 [30]). Hence proved the proposition. □

Theorem 4.3 Let N be a sub-commutative S-near-ring. Then the following conditions are equivalent:

(i) $P = P^r N P^m$ for every generalized (r, m) bi-ideal P of N.

(ii) N is regular.

(iii) N is strongly Noetherian regular delta near ring.

(iv) N is left bi-potent.

(v) $aN a = N a = N a^2$ for every $a \in N$.

(vi) P = PNP for every bi-ideal P of N.

Proof. To prove (i)) ⇒ (ii) :

If $P = P^r N P^m$ for every generalized (r, m) bi-ideal P of N , then $P = P^r N P^m \subseteq PNP$. Hence P = PN P for every bi-ideal P of N. By Corollary 2.3 [30],N becomes Noetherian regular δ- near- ring.

To prove (ii)) ⇒ (iii) : Proof is trivial.

To prove (iii)) ⇒ (iv) : Trivially, $N a^2 \subseteq N a$. On the other hand, since N is strongly regular, $N a \subseteq N a^2$ and so $N a = N a^2$.

To prove (iv) ⇒ (v) : Since N is sub-commutative Noetherian regular delta Near-

Ring and S^N-Noetherian regular δ- Near-Ring, $a \in Na = Na^2 = (Na)a = aNa$

Therefore, $aN\,a = N\,a = N\,a^2$ for every $a \in N$.

To prove (v) $\Rightarrow$ (vi) : Let us assume that $aNa = Na = Na^2$ $\forall$, $a \in N$.

Since N is a S^N- Noetherian regular delta near-ring $a \in Na = aN\,a$ $\forall$, $a \in N$.

Thus N is regular. Therefore, P = PNP for every bi-ideal P of N.

To prove (vi) $\Rightarrow$ (i) : Let us assume that P = PNP for every bi-ideal P of N. Since N is a S^N- Noetherian regular δ- Near-Ring with the property (α) by corollary 2.3[30] N is regular. Since, N is sub-commutative Noetherian regular δ- Near-Ring , from the above N is a generalized near field (Theorem 1[7]). This implies that N is regular and idempotent elements lie in center.

Let P be a generalized (r, m) bi-ideal of N and $x \in P$.

Now, $x = xyx = (xax)y(xax) = xa(xyx)ax = (xa)^r(xyx)(ax)^m$. Since, xa and ax are idempotent elements ,we get $(xa)^2 = x^2a^2$ and so $(xa)^r = x^ra^r$.

From $x = x^ra^r(xyx)a^mx^m \in x^rNx^m \in P^r\,N\,P^m$ for all positive integers r and m. .

Hence, $P \subseteq P^r\,N\,P^m$ and consequently $P = P^r\,N\,P^m$ for every generalized (r, m) bi-ideal P of N. Hence proved the Theorem. □

Theorem 4.4 Let N be S_k –Noetherian regular δ- near-ring for $k \geq 2$. Then the following are equivalent:

$Q = Q^r\,N\,Q^m$ for every generalized (r, m) bi-ideal Q of N and N is sub-commutative.

(i) N is a $P_k^{'}$ (1,1) Noetherian regular δ- near-ring.

(ii) N is a P_k (r, m) Noetherian regular δ- near-ring for all r, m positive integers.

(iii) N is left bi-potent and $E \subseteq C(N)$.

(iv) N is a S' and P (1, 2) Noetherian regular δ- near-ring.

(v) N is a GNF.

(viii) N is a stable near-ring N is a P (r, m) near-ring for all positive integers r and m and regular.

(ix) N is S', B-regular, 2- sided Noetherian regular δ- near-ring with property (α)

(x) Let Q,S be two N-subgroups of N. Then (a) $Q \cap S = QS$; (b) $S^2 = S$;

(c) $S \cap N Q = SQ$ and N is sub-commutative Noetherian regular δ- near-r.

Proof. We prove by cyclic method**.**

Let us prove (i) ⇒ (ii): Assume that $P = P^r N P^m$ for every generalized (r, m) bi-ideal P of N. Then trivially P = PNP for every bi-ideal P of N. Since N is a S_k - Noetherian regular δ-near-ring, N becomes a S^N- Noetherian regular δ-near-ring.

Again by the assumption of sub-commutative, N is with property (α)and so by Corollary 2.3 [30], N is regular. By Theorem 1 [7] N is regular and idempotents lie in center.

Now for all n; x in N; $nx^k = (nx)x^{k-1} = (nxbx)x^{k-1}$. Since xb and bx are idempotent elementss, we have $nx^k = (xbnx)x^{k-1} \in xN x$. From this we get that $N x^k \subseteq xN x$.

On the other hand, $xnx = (xbx)nx = xnbx^2 = (xbx)(nbx^{2)} = xnb^2x^3$. Repeating this process, we get $xnx = n'x^k \in N x^k$ for all positive integers k.

Thus, $N x^k = xN x$ for all x in N.

To prove (ii) ⇒ (iii) : By assumption $N x^k = xN x$ for all x in N . Since N is a S_k – Noetherian Regular δ-near-ring, N becomes strongly regular and so N has no non-zero nilpotent elements.

Let $e \in E$ and $n \in N$. Then (en – ene)e = 0 and so (en – ene)ene = 0. Since N is a IFP near-ring we get e(en – ene) = 0 and ene(en – ene) = 0. From this en = ene. Now,eN = eN e = N e for all $e \in E$: By known Lemma r, m idempotents lie in center.

For $x \in N$; $x^r N x^m \subseteq xN x = N x^k$. That is, $x^r N x^m \subseteq N x^k$.

Now let $z \in N x^k$ (= xN x). Then therer exists $y \in N$ such that z = xyx = (xax)y(xax) = xa(xyx)ax= $(xa)^r (xyx)(ax)^r$ as xa, ax in E. Thus $z = x^r a^r xyx a^m x^m \in x^r N x^m$.

That is, $N x^k \subseteq x^r N x^m \Rightarrow$ N is $P_k(r, m)$ Noetherian regular δ-near-ring.

To prove (iii) $\Rightarrow$ (iv) : Since N is a S_k –Noetherian Regular δ-near-ring, $x \in N$ $x^k = x^r N x^m = x(x^{r-1} N x^{m-1})x \subseteq xN x$.

That is, N is regular. Since N is a S_k and $P'_k(r, m)$- Noetherian Regular δ-near-ring, N is strongly regular and hence N has no non-zero nilpotent element. This implies that N has IFP and so en = ene for e in E. Thus we get eN = eN e = N e. By Lemma 2.1, $E \subseteq C(N)$ and so by Theorem 1 [7] N is regular and sub-commutative. Thus $N x^2 = xN x \subseteq N x$.

On the other hand $N x = N xax = xN ax \subseteq x N x$. Hence N is left bi-potent and $E \subseteq C(N)$.

To Prove (iv) $\Rightarrow$ (v): Assume N is left bi-potent and $E \subseteq C(N)$. Since N is S-Noetherian Regular δ-near-ring, $x \in N x = N x^2$; N is strongly regular and so N is regular. By Theorem 1 [7], N is regular and sub-commutative Noetherian Regular δ-near-ring. From this we get that $xN = N x^2$. That is, N is a P(1, 2) Noetherian Regular δ-near-ring. Since N is sub-commutative Noetherian Regular δ-near-ring, so N is S'- Noetherian Regular δ-near-ring.

To Prove (v) $\Rightarrow$ (vi) : By assumption, $x \in xN = Nx^2$. That is, N^2 is strongly regular and so N is regular. Since N is P (1, 2) Noetherian regular δ-near-ring.

For $e \in E$; eN = N e = N e = eN e. By Lemma 2.1[7], $E \subseteq C(N)$.

By Theorem 1 [7] N is a GNF.

To Prove (vi) $\Rightarrow$ (vii): By Theorem 1 [7], N is regular and sub-commutative. Now let $y \in xN$. Then $y = xa = xbxa = xabx \in xN x \subseteq xN$. Thus xN = xN x = N x. That is, N is stable.

To Prove (vii) $\Rightarrow$ (viii): Let N be stable. Then eN = eN e = N e for $e \in E$. By Lemma 2.1[7], $E \subseteq C(N)$.

Since N is a S_k - Noetherian regular δ-near-ring, N becomes a S- Noetherian regular δ-near-ring. Since N is stable, N is regular.

Let r and m be two positive integers

Let $a= x^r N = (xyx)^r n = x^r(yx)^r n = x^r(yxn) = x^r n (yx) = x^r n (yx)^m = x^r n y^m x^m =$

$(x^r n y^m)x \in Nx^m$. that is $x^r N \subseteq N x^m$. similarly, $N x^m \subseteq x^r N$. So, $x^r N = N x^m$.

To prove (viii) $\Rightarrow$ (ix) : If N is a P (r, m) Noetherian Regular δ-near-ring, then $e^r N = N e^m$. That is, $eN = N e$ and so $eN e = e(N e) = eeN = eN$. Therefore, by Lemma 2.1 [7], $E \subseteq C(N)$. Since N is regular, by Theorem 1[7], N is sub-commutative Noetherian Regular δ-near-ring and so N has property (α) In the case of a S-Noetherian Regular δ-near-ring with property (α); $(x)_r = xN = N x = (x)_l$ and so N is two sided. Also every Noetherian Regular δ-near-ring is B-regular.

To prove (ix) $\Rightarrow$ (x): By assumption and so by Proposition 3.5 [30], N is regular. If N is two sided, then $xN = (x)_r = (x)_l = N x$, and hence N is sub-commutative. Let Q; S be two left N -subgroups of N . To prove (a) let $x \in Q \cap S$. Since N is regular, $x = xax = (xa)x \in QNS \subseteq QS$, i.e., $Q \cap S \subseteq QS \subseteq Q \cap S$.

(a) By taking $S = Q$ in the above, we get $Q^2 = Q$.

(b) $Q \cap N S \subseteq Q \cap S = QS \subseteq Q \cap N S$.

To prove (x) $\Rightarrow$ (i): Since N is a S- Noetherian Regular δ-near-ring, $a^2 N a = N a \cap N a = N aN a \subseteq N a^2$. Therefore, N is strongly regular and so N is regular. Since N is sub-commutative, Noetherian Regular δ-near-ring by Theorem 1 [7],

$E \subseteq C(N)$. Let $x \in S$ since N is regular, $x = xyx = (xax)y(xax) = xa(xyx)ax = (xa)^r(xyx)(ax)^m = x^r a^r (xyx) a^m x^m \in S^r N S^m$.

By the definition of a generalized (r, m) bi-ideal, $S \subseteq S^r N S^m$. Hence

$S^r N S^m \subseteq S$. Hence, $S = S^r N S^m$ for every generalized (r, m) bi-ideal S of N .

By cyclic method of proof complete the theorem. Hence the Theorem. □

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