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Minimizing Total Completion Time in a Flow-shop Scheduling Problems with a Single Server

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Abstract

We consider the problem of two-machine flow-shop scheduling with a single server and equal processing times, we show that this problem is NP-hard in the strong sense and present a busy schedule for it with worst-case bound $7/6$.

Mathematics Subject Classification: 90B35

Keywords: flow-shop scheduling problem, total completion time, worst-case

1 Introduction

We consider the two-machine flow-shop scheduling problem with minimizing total completion time and equal processing times, that

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is $F2\left|p_{1,j}=p\right|\sum C_j$. Complexity results for $F2\left|\ \right|\sum C_j$ problem obtained by Garey, et al [1], J.A. Hoogereen [2] studied some special cases for two-machine flow-shop problems with minimizing total completion times, and proved that the problem with equal processing times on first machine, that is $F2\left|p_{1,j}=p\right|\sum C_j$,is NP-hard in the strong sense, and present an $O(n\log n)$ approximation algorithm for it with worst-case bound $4/3$.Complexity results for flow-shop problems with a single server was obtained by Brucher, et al [3]. In this paper, we derive some new complexity results for two-machine problem with a single server, introduce an improved algorithm, and prove that its worst case is $7/6$, the bound is tight.

2 Complexity of the $F2,S1\left|p_{i,j}=p\right|\sum C_j$ problem

Let $C_{i,j}$ denote the completion times of job J_j on machine M_i. If there are no idle times on M_1 and M_2, we have $C_{1,1}=s_{1,1}+p_{1,1}$, $C_{2,1}=s_{1,1}+p_{1,1}+s_{2,1}+p_{2,1}$, $C_{1,j}=C_{1,j-1}+s_{1,j}+p_{1,j}$, $C_{2,j}=\max\{C_{2,j-1},C_{1,j}\}+s_{2,j}+p_{2,j}$, for $j=2,\ldots,n$.

Theorem 1 The problem of deciding whether for a given instance of the $F2,S1\left|p_{i,j}=p\right|\sum C_j$ problem there exists a schedule with cost no more than a given threshold value y is NP-hard in the strong sense.

Proof. Our proof is based upon a reduction from the problem *Numerical Matching with Target Sums* or, in short, *T S,* which is known to be NP-hard in the strong sense.

T S Given two multisets $X=\{x_1,\ldots,x_n\}$ and $Y=\{y_1,\ldots,y_n\}$ of positive integers and an target vector $\{z_1,\ldots,z_n\}$,where $\sum_{j=1}^{n}(x_j+y_j)=\sum_{j=1}^{n}z_j$,is there a position of the set $X\cup Y$ into n disjoint set $Z_1,\ldots,Z_n$,each containing exactly one element from each of X and Y, such that the sum of the numbers in Z_j equal z_j, for $1,\ldots,n$?

(1) P **-jobs:** $s_{1,i} = b, p_{1,i} = b; s_{2,i} = b + x_i, p_{2,i} = b(i = 1,\ldots,n)$,

(2) Q **-jobs:** $s_{1,i} = 0, p_{1,i} = b; s_{2,i} = b + y_i, p_{2,i} = b(i = 1,\ldots,n)$,

(3) R **-jobs:** $s_{1,i} = 0, p_{1,i} = b; s_{2,i} = b - z_i, p_{2,i} = b(i = 1,\ldots,n)$,

(4) U **-jobs:** $s_{1,i} = 0, p_{1,i} = b; s_{2,i} = 0, p_{2,i} = b(i = 1,\ldots,n)$,

(5) V **-jobs:** $s_{1,i} = 0, p_{1,i} = b; s_{2,i} = 0, p_{2,i} = b(i = 1,\ldots,n)$,

(6) W **-jobs:** $s_{1,i} = 0, p_{1,i} = b; s_{2,i} = 0, p_{2,i} = b(i = 1,\ldots,n)$,

(7) L **-jobs:** $s_{1,i} = 4b, p_{1,i} = b; s_{2,i} = b, p_{2,i} = b(i = 1,\ldots,n)$.

Observe that all processing times are equal to b .To prove the theorem we show that in this constructed if the $F2, S1\left|p_{i,j} = p\right|\sum C_j$ problem a schedule S_0 satisfying $\sum C_j(S_0) \le y = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n}(x_i + y_i) + (77n^2 - 13n - 4)b/2$ exists if and only if $T\ S$ has a solution. Suppose that $T\ S$ has a solution. The desired schedule S_0 exists and can be described as follows. No machine has intermediate idle time. M_1 process the jobs in order of the sequence σ , i.e., in the sequence

$$\sigma = \{ \sigma_{P_{1,1}}, \sigma_{Q_{1,1}}, \sigma_{R_{1,1}}, \sigma_{U_{1,1}}, \sigma_{V_{1,1}}, \sigma_{W_{1,1}}, \sigma_{L_{1,1}}, \ldots, \sigma_{P_{1,n}}, \sigma_{Q_{1,n}}, \sigma_{R_{1,n}}, \sigma_{U_{1,n}}, \sigma_{V_{1,n}}, \sigma_{W_{1,n}},$$

While M_2 process the jobs in the sequence

$$\tau = \{ \tau_{P_{2,1}}, \tau_{Q_{2,1}}, \tau_{R_{2,1}}, \tau_{U_{2,1}}, \tau_{V_{2,1}}, \tau_{W_{2,1}}, \tau_{L_{2,1}}, \ldots, \tau_{P_{2,n}}, \tau_{Q_{2,n}}, \tau_{R_{2,n}}, \tau_{U_{2,n}}, \tau_{V_{2,n}}, \tau_{W_{2,n}}$$

as indicated in Figure 1.

Then we define the sequence σ and τ shown in Figure 1. Obviously, these sequence σ and τ fulfills $C(S) = C(\sigma, \tau) \le y$. Conversely, assume that the flow-shop scheduling problem has a solution σ and τ with $C(S) \le y$.

Considering the path composed of M_1 operations of jobs $\{P_{1,1}, Q_{1,1}, R_{1,1}, U_{1,1}, V_{1,1}, W_{1,1}\}$, M_2 operations of jobs

$\{R_{2,1}, U_{2,1} V_{2,1}, W_{2,1}, L_{2,1}, \ldots, R_{2,n}, U_{2,n}, V_{2,n}, W_{2,n}, L_{2,n}\}$, we obtain that

$$\begin{aligned} C(S) \geq\ & 3b + x_1 + 5b + x_1 + y_1 + 7b + x_1 + y_1 - z_1 + 8b + 9b + 10b + \ldots + \\ & (3 + (n-1)11)b + x_+ + (5 + (n-1)11)b + x_n + y_n + (7 + (n-1)11)b \\ & + \ldots + (11n+1)b \\ =\ & \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} (x_i + y_i) + (77n^2 - 13n - 4)b/2 = y. \end{aligned}$$

So we have $C(S) = y$.

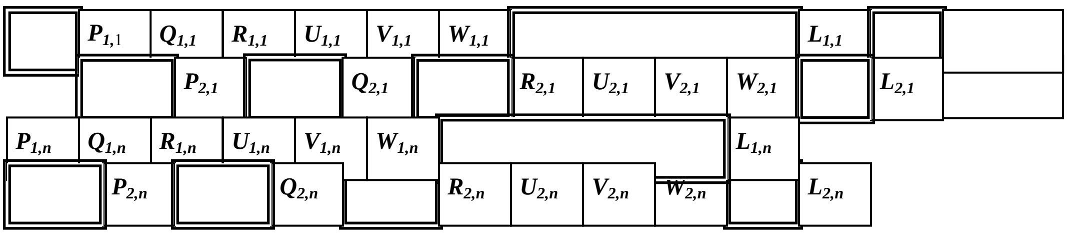

Figure 1: Gant chart for the $F2, S1 | p_{i,j} = p | \sum C_j$ problem

(a) If S has a partition μ, then there is a schedule with finish times y. One such schedule is shown in Figure 1.

(b) If S has no partition, then all schedule must have a finish times $> y$. Since S has no partition, then $x_i + y_i \neq z_i (i = 1, \ldots, n)$. Let $\xi_i = x_i + y_i - z_i (i = 1, \ldots, n)$, so

$$\begin{aligned} \sum C_j(S) =\ & \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} (x_i + y_i) + (77n^2 - 13n - 4)b/2 + 5\sum_{i=1}^{n} \xi_i \\ & + 10\sum_{i=1}^{n-1} \xi_i + \ldots + 5n\xi_1 > y \end{aligned}$$

3 Worst-case for the $F2, S1 | p_{i,j} = p | \sum C_j$ problem

In examining "worst" schedule, we restrict ourselves to busy schedule. A busy schedule is a schedule in which at all times from start to finish at least one server is processing a task.

Theorem 2 The $F2, S1\left|p_{i,j}=p\right|\sum C_j$ problem**,** let S_0 be a busy schedule for this problem, S^* be the optimal solution for the $F2, S1\left|p_{i,j}=p\right|\sum C_j$ problem, then

$\sum C_j(S_0)/\sum C_j(S^*) \le 7/6$, the bound is tight.

Proof. For a schedule S, let $I_{i,j}(S)(i=1,2;\ j=1,\dots,n)$ denote the total idle times of job J_j on M_i .Considering the path composed of M_1 operations of jobs $1,\dots,j$, M_2 operation of job j, we obtain that

$$C_j=\sum_{i=1}^{j}(s_{1,i}+p_{1,i})+I_{1,j}+s_{2,j}+p_{2,j} \tag{1}$$

Considering the path composed of M_1 operations of jobs 1, M_2 operation of job $1,2,\dots,j$, we obtain that

$$C_j=s_{1,1}+p_{1,1}+\sum_{i=1}^{j}(s_{2,i}+p_{2,i})+I_{2,j} \tag{2}$$

Considering the path composed of M_1 operations of jobs $1,\dots,l$, M_2 operation of job $l,\dots,j$, we obtain that

$$C_j=\sum_{i=1}^{l}(s_{1,i}+p_{1,i})+I_{1,i}+\sum_{i=l}^{j}(s_{2,i}+p_{2,i})+I_{2,j} \tag{3}$$

So we have

$$\begin{aligned}6\sum C_j(S_0)&=(2(\sum_{i=1}^{j}(s_{1,i}+p_{1,i})+I_{1,j}))+(2(\sum_{i=1}^{j}(s_{2,i}+p_{2,i})+I_{2,j}))+\\&\quad(2(\sum_{i=1}^{l}(s_{1,i}+p_{1,i})+I_{1,j}+\sum_{i=1}^{j}(s_{2,i}+p_{2,i}+(2(s_{1,1}+p_{1,1})+2(s_{2,j}+p_{2,j}))\\&\le 7\sum C_j(S^*)\end{aligned}$$

$\sum C_j(S_0)/\sum C_j(S^*)\le 7/6$.

To prove the bound is tight, introduce the following example as follows and show in Figure 2 and Figure 3.

(1) P -jobs: $s_{1,i}=2b, p_{1,i}=b, s_{2,i}=2b, p_{2,i}=b(i=1,2)$;

(2) Q -jobs: $s_{1,i}=0, p_{1,i}=b, s_{2,i}=0, p_{2,i}=b(i=3,4)$.

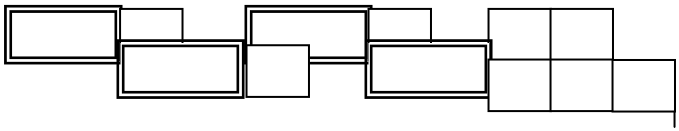

Figure 2: $\sum C_j(S_0) = 35b$

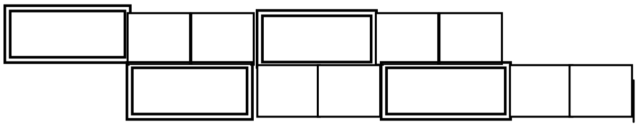

Figure 3: $\sum C_j(S^*) = 30b$

So we have $\sum C_j(S_0)/\sum C_j(S^*) = 35b/30b = 7/6$, the bound is tight.

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