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The Marlet wavelet density degree of the two-order polynomial stochastic processes

Xuewen Xia[1]

Abstract

In this paper, we use the wavelet transform to the two-order polynomial processes by Marlet wavelet, we obtain some statistical properties about the transform and density degree and wavelet express.

Mathematics Subject Classification: 60H15

Keywords: two-order polynomial processes, wavelet analysis, wavelet transform, marlet wavelet, density degree.

1 Introduction

With the rapid development of computerized scientific instruments comes a wide variety of interesting problems for data analysis and signal processing. In

[1] Hunan Institute of Engineering, Xiangtan, 411104, China.
Hunan Normal University, Changsha, 410082, China, e-mail:xxw1234567@163.com

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fields ranging from Extragalactic Astronomy to Molecular Spectroscopy to Medical Imaging to computer vision. One must recover a signal, curve, image, spectrum, or density from incomplete, indirect, and noisy data .Wavelets have contributed to this already intensely developed and rapidly advancing field .

Wavelet analysis is a remarkable tool for analyzing function of one or several variables that appear in mathematics or in signal and image processing. With hindsight the wavelet transform can be viewed as diverse as mathematics, physics and electrical engineering. The basic idea is to use a family of building blocks to represent the object at hand in an efficient and insightful way, the building blocks come in different sizes, and are suitable for describing features with a resolution commensurate with their sizes. Some persons have studied wavelet problems of stochastic process or stochastic system (see[1]-[12]). In this paper, we study a class of random processes using wavelet analysis methods, and study its energy, the energy express by density degree.

2 Basic Definition

Definition 1 If stochastic processes(see[14])

$$x(t)=At^2+Bt+C \quad (1)$$

where, A and B and C are independent random variable; E(A)=a, E(B)=b, E(C)=c, D(A)= δ_1^2, D(B)= δ_2^2, D(C)= δ_3^2; We call x(t) as Two-order polynomial processes.

Definition 2 If x(t) is a stochastic processes on probability space (Ω, p, ϑ), and

$E|x(t)|^2 < +\infty$,

$$\text{W(s,x)}=\frac{1}{s}\int_R x(t)\psi(\frac{x-t}{s})dt \tag{2}$$

We call W(s,x) as wavelet alteration of x(t),Where $\psi(t)$ is mather wavelet.(see[5])

Definition 3 Let $\varphi(t)=e^{-u^2}, u\in R,$ We call $\varphi(t)$ as Marlet wavelet.

3 Some properties

We have relational function of x(t)

$$\text{E[x(t)x(s)]=Rx}\,(s,t)=s^2t^2(\delta_1^2+a^2)+abs^2t+acs^2+abst^2+$$

$$(\delta_2^2+a^2)st+bcs+cat^2+cbt+\delta_3^2+c^2$$

Let $\varphi(t)=e^{-u^2}, u\in R,$

then we have the relational function of w(s,x)

$$R(\tau)=E[w(s,x)w(s,x+\tau)]=E\left[\frac{1}{s}\int_R x(t)\varphi(\frac{x-t}{s})dt\cdot\frac{1}{s}\int_R x(t_1)\varphi(\frac{x+\tau-t_1}{s})dt_1\right]$$

$$=\frac{1}{s^2}\iint_{R^2}E[x(t)x\ \ (t_1)]\ \varphi(\frac{x-t}{s})\varphi(\frac{x+\tau-t_1}{s})dt_1$$

$$=\frac{1}{s^2}\iint_{R^2}E[x(t)x_1(t)]\ e^{-(\frac{x-t}{s})^2}e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1$$

$$=\frac{1}{s^2}\iint_{R^2}\Big[t^2t_1^2(\delta_1^2+a^2)+abtt_1^2+act_1^2+abt_1t^2+(\delta_2^2+b^2)tt_1$$

$$+bct_1+act^2+cbt+\delta_3^2+c^2\Big]e^{-(\frac{x-t}{s})^2}e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1$$

$$=\frac{1}{s^2}\iint_{R^2}t^2t_1^2(\delta_1^2+a^2)e^{-(\frac{x-t}{s})^2}e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1+$$

$$=\frac{1}{s^2}\iint_{R^2} t^2t_1^2(\delta_1^2+a^2)e^{-(\frac{x-t}{s})^2}e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1+$$

$$+\frac{1}{s^2}\iint_{R^2} abtt_1^2e^{-(\frac{x-t}{s})^2}e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1+\frac{1}{s^2}\iint_{R^2} act_1^2e^{-(\frac{x-t}{s})^2}e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1+$$

$$+\frac{1}{s^2}\iint_{R^2} abt_1t^2e^{-(\frac{x-t}{s})^2}e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1+\frac{1}{s^2}\iint_{R^2} (\delta_2^2+b^2)tt_1e^{-(\frac{x-t}{s})^2}e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1+$$

$$+\frac{1}{s^2}\iint_{R^2} bct_1e^{-(\frac{x-t}{s})^2}e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1+\frac{1}{s^2}\iint_{R^2} act^2e^{-(\frac{x-t}{s})^2}e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1+$$

$$+\frac{1}{s^2}\iint_{R^2} cbte^{-(\frac{x-t}{s})^2}e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1+\frac{1}{s^2}\iint_{R^2} (\delta_3^2+c^2)e^{-(\frac{x-t}{s})^2}e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1$$

$=I_1+I_2+\ldots+I_9$

where

$$I_1=\frac{1}{s^2}bc\int_{-\infty}^{+\infty}e^{-u^2}(-s)du\left[\int_{-\infty}^{+\infty}(x+t)e^{-u^2}-\int_{-\infty}^{+\infty}sue^{-u^2}du\right](-s)$$

$$=\frac{1}{s^2}\int_{-\infty}^{+\infty}t^2(\delta_1^2+a^2)e^{-(\frac{x-t}{s})^2}dt\int_{-\infty}^{+\infty}t_1^2e^{-(\frac{x+\tau-t_1}{s})^2}dt_1$$

$$=\frac{1}{s^2}(\delta_1^2+a^2)\int_{-\infty}^{+\infty}(x-su)^2e^{-u^2}(-s)du\int_{-\infty}^{+\infty}(x+\tau-su)^2e^{-u^2}(-s)du$$

$$=(\delta_1^2+a^2)\int_{-\infty}^{+\infty}(su-x)^2e^{-u^2}du\int_{-\infty}^{+\infty}(su-x+\tau)^2e^{-u^2}du$$

$$=(\delta_1^2+a^2)\int_{-\infty}^{+\infty}(s^2u^2-2sxu+x^2)e^{-u^2}du\int_{-\infty}^{+\infty}[s^2u^2-2(x+\tau)su+(x+\tau)^2]e^{-u^2}du$$

$$=(\delta_1^2+a^2)\left[\int_{-\infty}^{+\infty}s^2u^2e^{-u^2}du-\int_{-\infty}^{+\infty}2sxue^{-u^2}du+\int_{-\infty}^{+\infty}x^2e^{-u^2}du\right]\cdot$$

$$\left[\int_{-\infty}^{+\infty}s^2u^2e^{-u^2}du-2(x+\tau)s\int_{-\infty}^{+\infty}ue^{-u^2}du+\int_{-\infty}^{+\infty}(x+\tau)^2e^{-u^2}du\right]$$

$$=(\delta_1^2+a^2)\left[s^2\int_{-\infty}^{+\infty}-\frac{1}{2}ude^{-u^2}-2sx\int_{-\infty}^{+\infty}ue^{-u^2}du+x^2\int_{-\infty}^{+\infty}x^2e^{-u^2}du\right]\cdot$$

$$\left[s^2\int_{-\infty}^{+\infty}u^2e^{-u^2}du-2s(x+\tau)\int_{-\infty}^{+\infty}ue^{-u^2}du+(x+\tau)^2\int_{-\infty}^{+\infty}e^{-u^2}du\right]$$

$$=(\delta_1^2+a^2)\left[s^2\frac{\sqrt{\pi}}{2}-0+\sqrt{\pi}x^2\right]\cdot\left[\frac{\sqrt{\pi}}{2}s^2-0+\sqrt{\pi}(x+\tau)^2\right]$$

$$=(\delta_1^2+a^2)\pi(\frac{1}{2}s^2+x^2)\left[\frac{1}{2}s^2+(x+\tau)^2\right]$$

$$\mathrm{I}_2=\frac{1}{s^2}\iint_{R^2} abtt_1^2 e^{-(\frac{x-t}{s})^2} e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1$$

$$=\frac{1}{s^2}\int_{-\infty}^{+\infty} abte^{-(\frac{x-t}{s})^2}dt\int_{-\infty}^{+\infty} t_1^2 e^{-(\frac{x+\tau-t_1}{s})^2}dt_1$$

$$=\frac{1}{s^2}\int_{-\infty}^{+\infty} ab(x-su)e^{-u^2}(-s)du\int_{-\infty}^{+\infty}(x+\tau-su)^2 e^{-u^2}(-s)du$$

$$=ab[\int_{-\infty}^{+\infty}(x-su)e^{-u^2}du][\int_{-\infty}^{+\infty}(x+\tau-su)^2e^{-u^2}du]$$

$$=ab\left[\int_{-\infty}^{+\infty}xe^{-u^2}du-\int_{-\infty}^{+\infty}sue^{-u^2}du\right]\left[\int_{-\infty}^{+\infty}(x+\tau)^2e^{-u^2}du-\right.$$

$$\left.-\int_{-\infty}^{+\infty}2s(x+\tau)ue^{-u^2}du+\int_{-\infty}^{+\infty}s^2u^2e^{-u^2}du\right]$$

$$=ab[\sqrt{\pi}x-0]\left[\sqrt{\pi}(x+\tau)^2-0+\frac{\sqrt{\pi}}{2}s^2\right]$$

$$=ab\pi x\left[(x+\tau)^2+\frac{s^2}{2}\right]$$

$$\mathrm{I}_3=\frac{1}{s^2}\iint_{R^2} act_1^2 e^{-(\frac{x-t}{s})^2} e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1$$

$$=\frac{1}{s^2}\int_{-\infty}^{+\infty} ace^{-(\frac{x-t}{s})^2}dt\int_{-\infty}^{+\infty} t_1^2 e^{-(\frac{x+\tau-t_1}{s})^2}dt_1$$

$$=\frac{1}{s^2}ac\int_{-\infty}^{+\infty} e^{-u^2}(-s)du\int_{-\infty}^{+\infty}(x+\tau-su)^2 e^{-u^2}(-s)du$$

$$=\sqrt{\pi}ac\left[(x+\tau)^2\sqrt{\pi}+s^2\frac{\sqrt{\pi}}{2}\right]=\pi ac\left[(x+\tau)^2\sqrt{\pi}+\frac{s^2}{2}\right]$$

$$\mathrm{I}_4=\frac{1}{s^2}\iint_{R^2} abt_1t^2 e^{-(\frac{x-t}{s})^2} e^{-(\frac{x+\tau-t_1}{s})^2}dtdt_1$$

$$= \frac{1}{s^2}\int_{-\infty}^{+\infty} abt^2 e^{-(\frac{x-t}{s})^2} dt \int_{-\infty}^{+\infty} t_1 e^{-(\frac{x+\tau-t_1}{s})^2} dt_1$$

$$= \frac{1}{s^2} ab\int_{-\infty}^{+\infty} (x-su)^2 e^{-u^2}(-s)du \int_{-\infty}^{+\infty}\left[(x+\tau)-su\right] e^{-u^2}(-s)du$$

$$= ab\int_{-\infty}^{+\infty}(x-su)^2 e^{-u^2} du \int_{-\infty}^{+\infty}(x+\tau-su)e^{-u^2} du$$

$$= ab\left[\int_{-\infty}^{+\infty} x^2 e^{-u^2} du - 2xs\int_{-\infty}^{+\infty} ue^{-u^2} du + s^2\int_{-\infty}^{+\infty} u^2 e^{-u^2} du\right]\cdot$$

$$\left[\int_{-\infty}^{+\infty}(x+\tau)^2 e^{-u^2} du - \int_{-\infty}^{+\infty} sue^{-u^2} du\right]$$

$$= ab\left[\sqrt{\pi}x - 0 + \frac{\sqrt{\pi}}{2}s^2\right][\sqrt{\pi}(x+\tau)-0]$$

$$= ab\pi(x+\frac{1}{2}s^2)(x+\tau)$$

$$I_5 = \frac{1}{s^2}\iint_{R^2}(\delta_2^2+b^2)tt_1 e^{-(\frac{x-t}{s})^2} e^{-(\frac{x+\tau-t_1}{s})^2} dtdt_1$$

$$= \frac{1}{s^2}(\delta_2^2+b^2)\int_{-\infty}^{+\infty} te^{-(\frac{x-t}{s})^2} dt \int_{-\infty}^{+\infty} t_1 e^{-(\frac{x+\tau-t_1}{s})^2} dt_1$$

$$= \frac{1}{s^2}(\delta_2^2+b^2)\int_{-\infty}^{+\infty}(x-su)e^{-u^2}(-s)du\int_{-\infty}^{+\infty}(x+\tau-su)e^{-u^2}(-s)du$$

$$= (\delta_2^2+b^2)\left[\int_{-\infty}^{+\infty} xe^{-u^2} du - \int_{-\infty}^{+\infty} sue^{-u^2} du\right]\left[\int_{-\infty}^{+\infty}(x+\tau)e^{-u^2} du - \int_{-\infty}^{+\infty} sue^{-u^2} du\right]$$

$$= (\delta_2^2+b^2)[\sqrt{\pi}x-0][(x+\tau)\sqrt{\pi}-0]$$

$$= (\delta_2^2+b^2)\pi x(x+\tau)$$

$$I_6 = \frac{1}{s^2}\iint_{R^2} bct_1 e^{-(\frac{x-t}{s})^2} e^{-(\frac{x+\tau-t_1}{s})^2} dtdt_1$$

$$= \frac{1}{s^2}\int_{-\infty}^{+\infty} bce^{-(\frac{x-t}{s})^2} dt \int_{-\infty}^{+\infty} t_1 e^{-(\frac{x+\tau-t_1}{s})^2} dt_1$$

$$= \frac{1}{s^2} bc \int_{-\infty}^{+\infty} e^{-(\frac{x-t}{s})} dt \int_{-\infty}^{+\infty} (x+\tau - su) e^{-u^2} (-s) du$$

$$= \frac{1}{s^2} bc \int_{-\infty}^{+\infty} e^{-u^2} (-s) du \left[\int_{-\infty}^{+\infty} (x+t) e^{-u^2} du - \int_{-\infty}^{+\infty} sue^{-u^2} du \right] (-s)$$

$$= bc\sqrt{\pi}[(x+t)\sqrt{\pi} - 0] = bc\pi(x+\tau)$$

$$I_7 = \frac{1}{s^2} \iint_{R^2} act^2 e^{-(\frac{x-t}{s})^2} e^{-(\frac{x+t-t_1}{s})^2} dt dt_1$$

$$= \frac{1}{s^2} \int_{-\infty}^{+\infty} act^2 e^{-(\frac{x-t}{s})^2} dt \int_{-\infty}^{+\infty} e^{-(\frac{x+t-t_1}{s})^2} dt_1$$

$$= \frac{1}{s^2} ac \int_{-\infty}^{+\infty} (x-su)^2 e^{-u^2} (-s) du \int_{-\infty}^{+\infty} e^{-u^2} (-s) du$$

$$= ac \int_{-\infty}^{+\infty} (x-su)^2 e^{-u^2} du \int_{-\infty}^{+\infty} e^{-u^2} du$$

$$= ac \left[\int_{-\infty}^{+\infty} x^2 e^{-u^2} du - 2sx \int_{-\infty}^{+\infty} ue^{-u^2} du + s^2 \int_{-\infty}^{+\infty} u^2 e^{-u^2} du \right] \cdot \int_{-\infty}^{+\infty} e^{-u^2} du$$

$$= ac \left[\sqrt{\pi} x^2 - o + \frac{\sqrt{\pi}}{2} s^2 \right] \sqrt{\pi} = ac\pi (x^2 - \frac{1}{2} s^2)$$

$$I_8 = \frac{1}{s^2} \iint_{R^2} cbte^{-(\frac{x-t}{s})^2} e^{-(\frac{x+t-t_1}{s})^2} dt dt_1$$

$$= \frac{1}{s^2} \int_{-\infty}^{+\infty} cbte^{-(\frac{x-t}{s})^2} dt \int_{-\infty}^{+\infty} e^{-(\frac{x+t-t_1}{s})^2} dt_1$$

$$= \frac{1}{s^2} cb \int_{-\infty}^{+\infty} (x-su) e^{-u^2} (-s) du \int_{-\infty}^{+\infty} e^{-u^2} (-s) du$$

$$= cb \left[\int_{-\infty}^{+\infty} xe^{-u^2} du - s \int_{-\infty}^{+\infty} ue^{-u^2} du \right] \sqrt{\pi}$$

$$= cb[\ \sqrt{\pi} x - o\ ] \sqrt{\pi} = cb\pi x$$

$$I_9 = \frac{1}{s^2} \iint_{R^2} (\delta_3^2 + c^2) e^{-(\frac{x-t}{s})^2} e^{-(\frac{x+t-t_1}{s})^2} dt dt_1$$

$$= \frac{1}{s^2}(\delta_3^2 + c^2)\int_{-\infty}^{+\infty} e^{-(\frac{x-t}{s})^2} dt \int_{-\infty}^{+\infty} e^{-(\frac{x+t-t_1}{s})^2} dt_1$$

$$= \frac{1}{s^2}(\delta_3^2 + c^2)\int_{-\infty}^{+\infty} e^{-u^2}(-s)du \int_{-\infty}^{+\infty} e^{-u^2}(-s)du$$

$$= (\delta_3^2 + c^2)\pi$$

Then, $R(\tau) = (\delta_1^2 + a^2)\pi(\frac{1}{2}s^2 + x^2)\left[\frac{1}{2}s^2 + (x+\tau)^2\right] +$

$$ab\pi x\left[(x+\tau)^2 + \frac{s^2}{2}\right] + \pi ac\left[(x+\tau)^2 + \frac{s^2}{2}\right] +$$

$$ab\pi(x + \frac{1}{2}s^2)(x+\tau) + (\delta_2^2 + b^2)\pi x(x+\tau) + bc\pi(x+\tau)$$

$$+ ac\pi(x^2 - \frac{1}{2}s^2) + cb\pi x + (\delta_3^2 + c^2)\pi$$

Then , $R'(\tau) = (\delta_1^2 + a^2)\pi(\frac{1}{2}s^2 + x^2)2(x+\tau) + 2ab\pi x(x+\tau)$

$$+ 2\pi ac(x+\tau) + ab\pi(x + \frac{1}{2}s^2) + (\delta_2^2 + b^2)\pi x + bc\pi$$

$$R''(\tau) = 2(\delta_1^2 + a^2)\pi(\frac{1}{2}s^2 + x^2) + 2ab\pi x + 2\pi ac$$

$$R'''(\tau) = R^{(4)}(\tau) = 0$$

Then,

$$R(0) = (\delta_1^2 + a^2)\pi(\frac{1}{2}s^2 + x^2)\left[\frac{1}{2}s^2 + x^2\right] + ab\pi x\left[x^2 + \frac{1}{2}s^2\right]$$

$$+ \pi ac\left[x^2 + \frac{1}{2}s^2\right] + ab\pi(x + \frac{1}{2}s^2)x + (\delta_2^2 + b^2)\pi xx + bc\pi x + ac\pi(x^2 - \frac{1}{2}s^2)$$

$$+ cb\pi x + (\delta_3^2 + c^2)\pi$$

$$= (\delta_1^2 + a^2)(\frac{1}{2}s^2 + x^2)^2\pi + 2ab\pi x(x + \frac{1}{2}s^2) + 2\pi acx^2$$

$$+ (\delta_2^2 + b^2)\pi x^2 + 2bc\pi x + (\delta_3^2 + c^2)\pi$$

$$R''(0) = 2\pi(\delta_1^2 + a^2)(\frac{1}{2}s^2 + x^2) + 2\pi a(bx + c)$$

The use above, we can obtain the zero density of w(s,x): $\sqrt{\left|\frac{R''(0)}{\pi^2 R(0)}\right|}$

The same time, we have:

Theorem The averge density of w(s, x) is zero.

Because(see[6])

$$\sqrt{\left|\frac{R^{(4)}(0)}{\pi^2 R^{(2)}(0)}\right|} = 0$$

4 Wavelet representation

Let real function φ is standard orthogonal element of multiresolution analysis $\{V_j\} j \in Z$ (see [13]), then exist $h_k \in l^2$, have

$$\varphi(t) = \sqrt{2}\sum_k \varphi(2t - k)$$

Let $\psi(t) = \sqrt{2}\sum_k (-1)^k h_{1-k}\varphi(2t - k)$

Then wavelet express of $X(t)$ in mean square is

$$X(t) = 2^{-\frac{J}{2}}\sum_K C_n^J \varphi(2^{-J}t - n) + \sum_{j \le J} 2^{-\frac{j}{2}} \sum_{n \in Z} d_n^j \psi(2^{-j}t - n)$$

where, $C_n^j = 2^{-\frac{j}{2}}\int_R X(t)\varphi(2^{-j}t - n)dt$, $d_n^j = 2^{-\frac{j}{2}}\int_R X(t)\psi(2^{-j}t - n)dt$

Let

$$\varphi(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{other} \end{cases}$$

Then have

$$E[C_n^j]=2^{-\frac{j}{2}}\int_R E[x(t)]\varphi(2^{-j}t-n)dt=2^{-\frac{j}{2}}\int_{n2^{-j}}^{(n+1)2^{-j}}(a^2t^2+bt+c)dt$$
$$=2^{-\frac{3j}{2}}\left\{a^2/3[(n+1)^3 2^{-2j}-n^3 2^{-2j}]+\frac{b}{2}2^{-j}(2n+1)+c\right\}$$

$$E(d_n^j)=2^{-\frac{j}{2}}\int_R E[x(t)]\psi(2^{-j}t-n)dt$$

$$E\left[d_n^j d_m^k\right]=2^{-\frac{j+k}{2}}\iint_{R^2}E\left[X(t)X(s)\right]\psi(2^{-j}t-n)\psi(2^{-k}s-m)dsdt$$

We can obtain the value on above. We may also can analyse C_n^j as above.

Now we consider function $\psi(t)$ that exist compact support set on $[-k_1,k_2],k_1,k_2\geq 0$, and exist enough large M, have

$$\int_R t^m\psi(t)dt=0,0\leq m\leq M-1,$$

then φ exist compact support set on $[-k_3,k_4]$ satisfy $k_1+k_2=k_3+k_4,k_3,k_4\geq 0$.

Let

$$b(j,k)=< X(t),\psi_{jk}>,\quad a(j,k)=< X(t),\varphi_{jk}>$$

Let J is a constant, then

$$\left\{2^{\frac{J}{2}}\varphi(2^J x-k),k\in Z\right\}\cup\left\{2^{\frac{j}{2}}\psi(2^j t-k),k\in Z\right\}_{j\geq J}$$

are a standard orthonormal basis of space $L^2(R)$, then have

$$X(t)=2^{\frac{J}{2}}\sum_{K\in Z}a(J,K)\varphi(2^J t-K)+\sum_{j\geq J}\sum_{K\in Z}2^{\frac{j}{2}}b(j,K)\psi(2^j t-K)$$

Therefore, the self-correlation function of $b(j,m)$

$$R_b(j,K;m,n)=E[b(j,m)b(k,n)]$$
$$=2^{-\frac{j+K}{2}}\iint_{R^2}E[X(t)X(s)]\psi(2^j t-m)\psi(2^K s-n)dtds$$

Let

$$F(2^{j-K},t)=\int_R \psi(2^{j-K}s-t)\psi(s)ds\ .$$

If $\psi(t)$ have $(M-1)$ -order waning moments, then $F(2^{j-K},t)$ have $(2M-1)$ order waning moments. In actual,

$$\iint_{R^2}t^m\psi(2^{j-K}s-t)\psi(s)dsdt=-\iint_{R^2}(2^{j-K}s-t)^m\psi(t)\psi(s)dsdt$$
$$=-\iint_{R^2}\sum_n C_m^n(2^{j-K}s)^{m-n}(-t)^n\psi(t)\psi(s)dsdt=0,\ \ m<2M$$

Therefore we have:

Theorem Let $X(t)$ is solution process of system (1), $\psi(t)$ have compact supported set on $[-K_1,K_2]$, $K_1,K_2>0$, and $\psi(t)$ have $(M-1)$-order waning moments, and $\psi(t)$ is standard orthonormal wavelet function. Then stochastic process $b(J,m)$ are stationary process.

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