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Modelling of Nigeria gross domestic product using seasonal and bilinear autoregressive integrated moving average models

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Abstract

This paper compares Seasonal Autoregressive Integrated Moving Average Model with Bilinear Autoregressive Integrated Moving Average Model in the analysis of Nigeria Gross Domestic Product. The analysis makes use of quarterly data (Table 7) of Nigeria Gross Domestic Product at 1990 Constant Basic Prices from first quarter 1993 to fourth quarter 2012, Statistical Bulletin (p. 144-148, 2012). SARIMA $(0,1,2)(0,1,1)_4$ and BARIMA (0,2,1,0,1) Models were fitted to original GDP and seasonally adjusted data, and the results do not change the values of the parameter estimates from both series. The autocorrelation functions in Figures 7 & 8 describe pure white noise process for the residual values of the two models. Furthermore, Ljung Box Chi-Square test Statistic confirms non-existence of autocorrelations in the residual values, while Analysis of Variance test suggests overall fitness of the Bilinear Regression Model. The Akaike Information Criterion shows a better performance of BARIMA model than SARIMA model in

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fitting Nigeria Gross Domestic Product (NGDP). The estimates from the two models compare favourably with the actual values of the Nigeria Gross Domestic Product (see Figures 9, 10). This paper recommends Bilinear Autoregressive Moving Average Model as a class of non-linear model for modelling Nigeria Gross Domestic Product.

Mathematics Subject Classification: 62K05

Keywords: Gross Domestic Product; Autocorrelation Function; Partial Autocorrelation Function; SARIMA model; BARIMA Model and Trend Analysis

1 Introduction

There is no gainsaying the fact that many people are developing interest in the growth of the Nigerian economy as well as studying the behaviour of the nation's Gross Domestic Product. Gross Domestic Product of a nation is essentially a measure of the health and size of its economy. As an indicator of the economic health, it is the value of a country's overall output of goods and services at market prices excluding net income abroad. Nigerian Gross Domestic Product is aggregation of factors that have significant contributions to the growth of the nation's economy. These factors include; agriculture, industries, building & construction, whole sales/retail trade and services, CBN Statistical Bulletin (2012). Gradual change in the growth of the Nigeria Gross Domestic Product over time has triggered many interests in this research area. For about three decades now (1981-1990, 1991-2000 and 2001-2012), there have been variations in the pattern of growth of the Nigeria Gross Domestic Product. Growth in the gross domestic product of a nation is synonymous with changes in its economy. Sometimes, economy of a country may experience either rapid or slow growth as a result of the activities in different sectors of the economy which characterised the country's gross domestic product. In Nigeria today, some sectors of the economy are suffering set back, and these tantamount to reduction in the gross domestic product of the country. From economic point of view and with the reality about the growth, Nigerian gross domestic product is an economic variable that does not have a predetermined pattern of growth, and requires constant study of its behaviour with respect to changes in time. In view of this fact, it is pertinent to consider different classical models from time to time for the analysis of the country's Gross Domestic Product. This is true because changes in the distribution should be studied with suitable models to provide good and useful forecasts that would guide the government of the country in its economic policies. As a tool used in measuring the monetary value of all the finished goods and services produced within a country's borders in a specific time period, consistent study has to be carried out so that as the growth of the GDP changes with time, suitable models are provided to give better estimates and forecasts.

Different authors have carried out time series analysis and modelling of the Nigeria Gross Domestic Product using different classes of Seasonal and Non-Seasonal Autoregressive Moving Average Models. The paper intends to introduce Bilinear Time Series Model and compare with the popular Seasonal Autoregressive Moving Average Models adopted by different schools of thought. The motivation behind the introduction and comparison of bilinear time series model with the seasonal time model in this research work is that the behaviour in the data series of Nigeria Gross Domestic Product is not far at variance with the assertion of Maravall (1983), that most of the economic and financial time series exhibit non-linearity behaviour. Maravall (1983) studied the application of bilinear models to modelling and forecasting non-linear processes and demonstrated improvement over ARIMA model having worked with economic data. Granger and Anderson (1978), Subba Rao and Gabr (1984), Terdik (1999) asserted bilinear process as a class of processes that can (partially) capture possible non-linearity in economic series of data. The non-linearity behaviour of Nigerian gross domestic product explains the reason for introduction of bilinear time series models in analysis of the data. This is because, the country's GDP exhibits neither a complete linear nor non-linear characteristics in its behaviour.

2 Review of Related Literature

This paper has mentioned the use of Seasonal Autoregressive Integrated Moving Average Model for the analysis of the Gross Domestic Product. Apart from Gross Domestic Product, SARIMA models have been widely applied in time series analysis. The review in this paper may not be limited to the use of SARIMA or bilinear models to modelling Nigeria Gross Domestic Product. Etuk (2011) applied SARIMA model to modelling of daily Nigeria-British pound Exchange rate. He estimated parameters of SARIMA $(0,1,1)x(0,1,1)_7$ for the daily exchange rate. The period of seasonality was 7(seven) because the data were on weekly basis. Amadi and Aboko(2013) fitted ARIMA (2,1,2) model to Nigeria Gross Domestic Product. AIC criterion was used to arrive at ARIMA (2,1,2) as the best among the family of ARIMA models selected. Eke et al(2015) fitted three time series trend models, namely; linear trend model, quadratic trend model and exponential trend model to annual data on Gross Domestic Product from 1982 to 2012. The analysis carried out revealed that exponential trend model has the least mean absolute percentage error. Forecasts were based on the exponential trend model. Ismail and Maphol (2005) modelled and forecasted Malaysian electricity generated using SARIMA model. Iwueze et al (2013) adopted ARIMA model to modelling and forecasting of Nigeria External Reserves. Iwueze et al (2013) initially fitted ARMA(1,0,0) to the non-stationary data series. After differencing, ARIMA (2,1,0) was fitted to the Nigeria External Reserves. From the results, ARIMA (2,1,0) provided better estimates than the initial ARMA (1,0,0) which was fitted to the data series. Etuk (2012) Fitted Seasonal Autoregressive Moving Average Model to Nigeria Gross Domestic Product. The data used were from 1980-2007. From Etuk's work, both seasonal and non-seasonal differencing were carried out. The fitted model was $X_t = 0.235X_{t-4} - 0.9043e_{t-1}$ (Seasonal part of Autoregressive process and Non-Seasonal part of Moving Average process). Okereke and Bernard (2014) fitted SARIMA $(2,1,2)x(1,0,1)_4$ to the quarterly Nigeria Gross Domestic Product. Okereke and Bernard applied log transformation and obtained first order regular difference for the series. The model stated did not show presence of seasonal differencing. This is similar to Buckman and Enock (2013) who fitted SARIMA $(1,1,2)(1,0,1)_{12}$ to Ghana's Inflation rate (1985-2011). This implied only non-seasonal differencing was carried out by Okereke and Bernard (2014) and Buckman and Enock (2013). Apart from the assumption of linearity in time series by many researchers, there are occasions where data suggest that linear models are unsatisfactory. In this case, it is desirable to look at non-linear alternatives, Jan and Kuldeep(1992). Usoro and Omekara (2008) fitted Bilinear Autoregressive Vector (BARV) models to Local Government Revenue data. Usoro (2015) compared the performances of linear and bilinear time series models in fitting revenue data from a Local Government in Nigeria. The results of the analysis confirmed the fact that bilinear model was more suitable in fitting the revenue series. Amadi and Aboko (2013) only considered different classes of ARIMA and considered ARIMA (2, 1, 2) to be the best. The works of Etuk (2012), Okereke and Bernard (2014) have already contradicted the work of Amadi and Aboko (2013). This is because GDP is an economic variable, and does not exhibit linearity in nature to justify the conclusion of ARIMA (2, 1, 2) as the best. ARIMA model is a linear model, and the non-linearity in the behaviour of the GDP justified the use of SARIMA model as a multiplicative time series model by Etuk (2012) and Okereke and Bernard (2014). The fact has been established as agreed by Etuk, Okereke and Bernard that different classes of non-linear models are suitable in fitting economic data. The motivation behind the introduction of bilinear time series in this paper is that Usoro and Omekara (2008) fitted bilinear time series to revenue data and observed better performance of bilinear model over ARIMA in modelling revenue data. Apart from the fact that some non-linear time series models outperform ordinary linear time series models, the adoption of SARIMA model is on the basis that the data were recorded on quarterly (with period s=4). This is done to compare the performance SARIMA model of Okereke and Bernard with the bilinear time series model in modelling Nigeria gross domestic product. So far, no authors have tested the performance of bilinear time series model with GDP. The contribution in this paper is the introduction of bilinear time series model and comparison of its performance with the popular SARIMA model in the analysis of unadjusted quarterly Nigeria GDP.

3 Method of Analysis

As already mentioned, the two models for the analysis of the Nigeria Gross Domestic Product are Seasonal and Bilinear Autoregressive Moving Average Models.

3.1 SARIMA Model

The general form of SARIMA as proposed by Box and Jenkins (1976) and Kendall and Ord(1990) is given by
SARIMA (p, d, q)(P, D, Q)$_s$. The expression of the process is in the form,

$$\phi_p(B)\psi_P(B^s)\nabla^d \nabla_s^D X_t = \theta_q(B)\Theta_Q(B)e_t \qquad (1)$$

This is a mixed seasonal model of period *s* with regular and seasonal components of order *p, P* for *AR*, regular and seasonal components of order *q, Q* for *MA*, regular and seasonal differencing of order *d, D* respectively. From the model, each of the autoregressive and moving average processes has both the seasonal and non-seasonal parts. However, the distribution of the correlogram is the determining factor to the choice of the class of ARIMA or SARIMA model. The reason for the use of SARIMA (p, d, q)(P, D, Q)$_4$ in this research work is that the

GDP data are recorded on quarterly basis. That is the seasonality period is 4. This agrees with Etuk (2012), Okereke and Bernard (2014).

3.2 BARIMA Model

A bilinear time series model is made up of two different parts; the linear and the non-linear parts. The linear part is the sum of the autoregressive and moving average processes, while the non-linear part is the product of the two processes. A general bilinear autoregressive integrated moving average time series process is given by BARIMA(p, q, d, P, Q). This expression can be written in the form

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + \sum_{j=1}^{q} \theta_j e_{t-j} + \sum_{k=1}^{P} \sum_{l=1}^{Q} \beta_{kl} X_{t-k} e_{t-l} \qquad (2)$$

Where, X_t is the time series process, φ_i and θ_j are the parameters of the linear part of the autoregressive and moving average processes with p and q as the orders respectively, β_{kl} is the parameter of the nonlinear part of the model with P and Q as the orders respectively, d is the differencing (see Granger and Anderson, 1978). The explanation for the introduction of BARIMA model is that many schools of thought have proposed classes of non-linear models for the analysis of economic and financial data. Besides, BARIMA model is an extension of ARIMA model with the nonlinear part. Therefore, the condition for the application of ARIMA applies with BARIMA model.

3.3 Choice of SARIMA and BARIMA Models

According to Box and Jenkins (1976), the non-seasonal seasonal difference operators are respectively

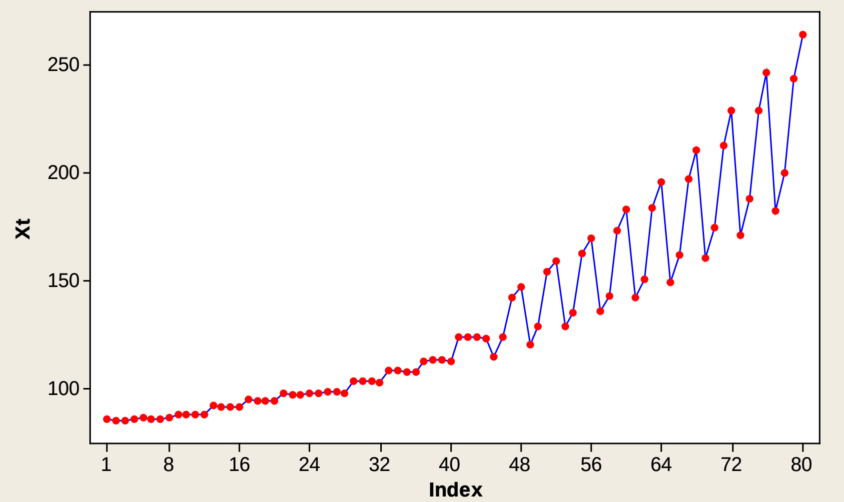
$$\nabla^d = (1 - B)^d \qquad (3)$$

and

$$\nabla_s^D = (1 - B^s)^d \quad (4)$$

The set of data for the analysis are quarterly series of Nigerian Gross Domestic Product from the first quarter of 1992 to the last quarter of 2012 CBN Statistical Bulletin (2012). The time plots of the original GDP and Autocorrelation function are in Figures 1 and 2.

FIGURE1: Time Series Plot of original series Xt



The time plot in Figure1 exhibits evidence of non-stationarity in the series.

FIGURE2: Autocorrelation Function for Xt
(with 5% significance limits for the autocorrelations)

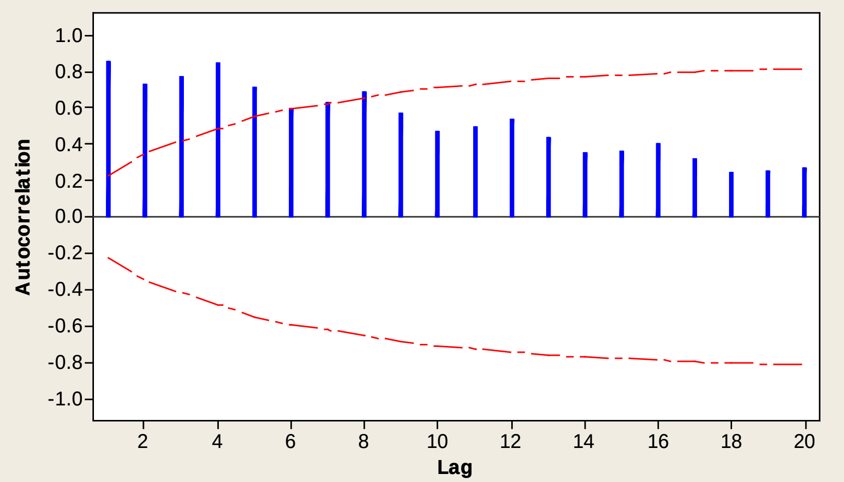


Figure 2 is the autocorrelation function of the original series. The autocorrelation function shows gradual decay in the autocorrelations. On top of the gradual decay, there is some evidence of sine-wave pattern. This explains periodic characteristics of the data series. This is because the data are on quarterly basis. On this note, both regular and seasonal differencing is suggested as adopted by Etuk (2012) to Gross Domestic Product.

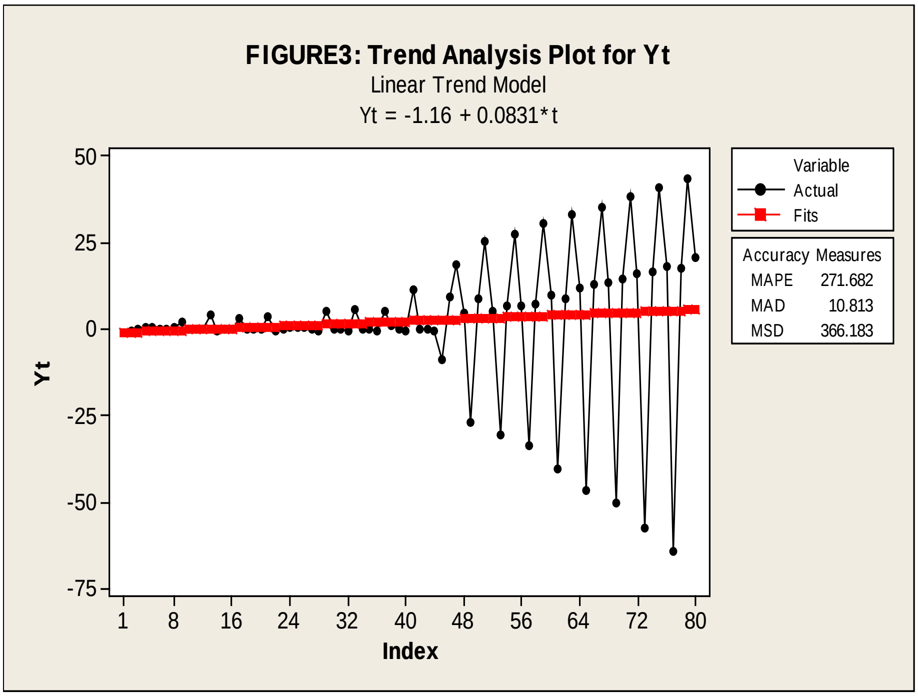


Figure 3 is the trend analysis plot of the regular differenced series, Y_t.

Figure 4 is the trend analysis plot of both regular and seasonal differenced series, Z_t. This confirms more stability than the first differenced series. Hence, this paper applies the second differenced series for the analysis.

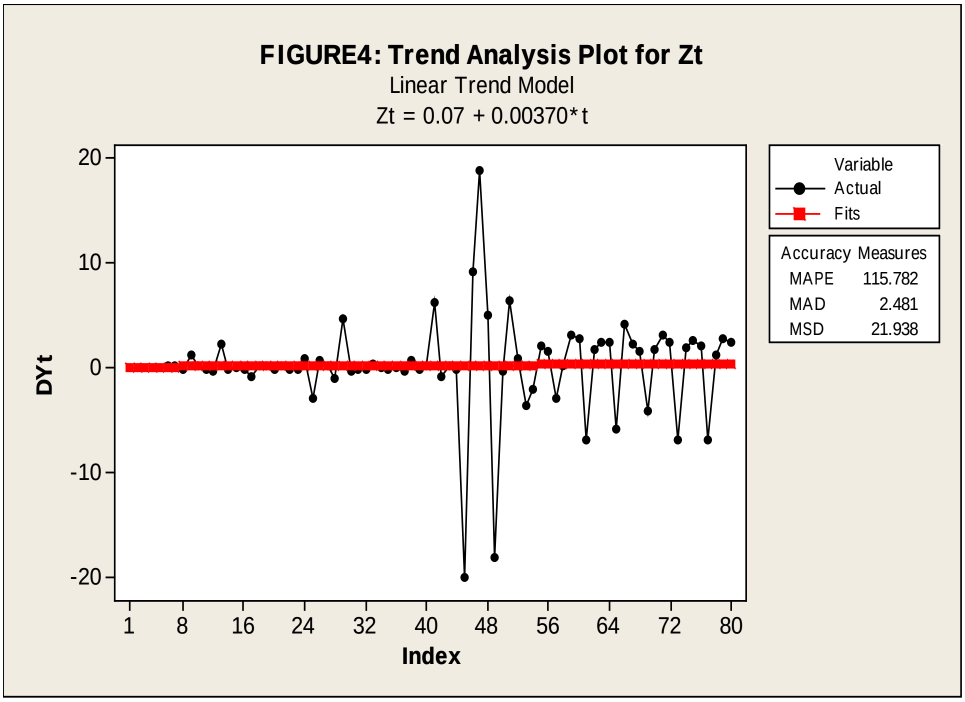


FIGURE4: Trend Analysis Plot for Zt
Linear Trend Model
Zt = 0.07 + 0.00370*t
DYt
Index
Variable
Actual
Fits
Accuracy Measures
MAPE 115.782
MAD 2.481
MSD 21.938

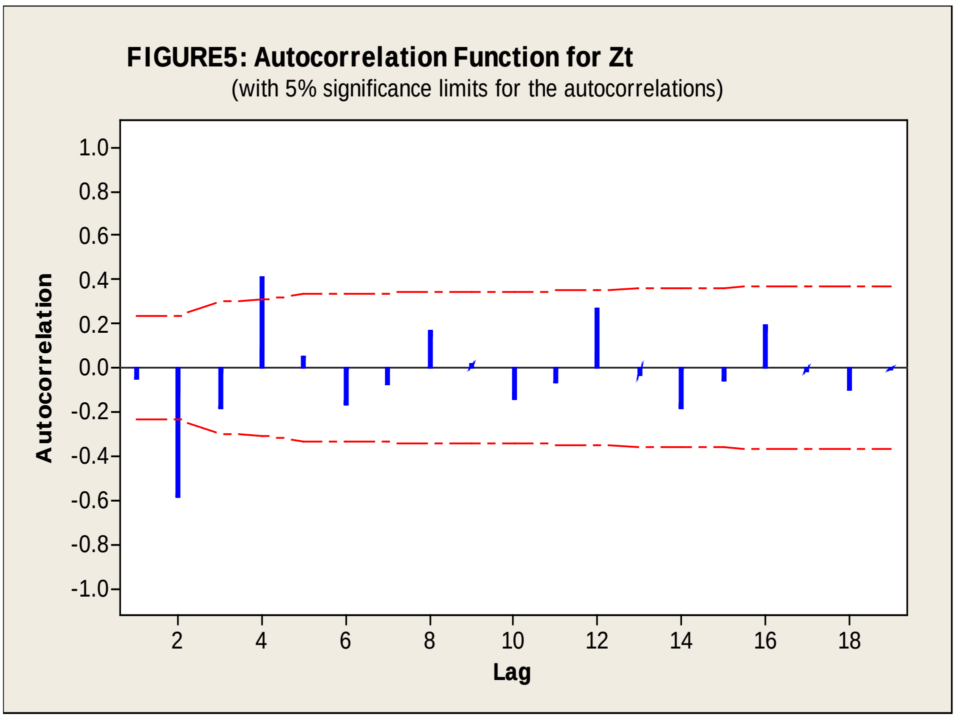
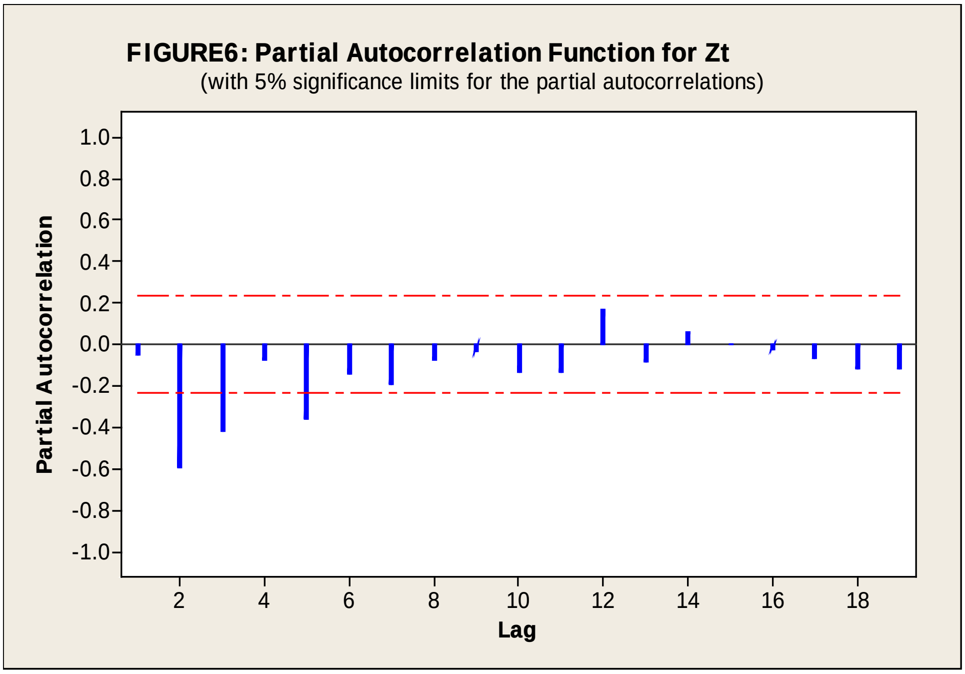


FIGURE5: Autocorrelation Function for Zt
(with 5% significance limits for the autocorrelations)
Autocorrelation
Lag



Figures 5 and 6 are the autocorrelation and partial autocorrelation functions of the regular and seasonal differenced series, Z_t. The distributions in the autocorrelation and partial autocorrelation functions suggest SARIMA $(0,1,2)(0,1,1)_4$. There are spikes at the second non-seasonal lag and first seasonal lag(s=4) of the autocorrelation function, while the partial autocorrelation function exhibits gradual decay at the first set of the negative lags.

The chosen SARIMA is

$$(1-B)(1-B^4)X_t = (1-\theta_1 B - \theta_2 B^2)(1-\phi_4 B^4)\epsilon_t \quad (5)$$

By expansion, the above model becomes

$$X_t - X_{t-1} - X_{t-4} + X_{t-5} = \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - \phi_4\epsilon_{t-4} + \theta_1\phi_4\epsilon_{t-5} + \theta_2\phi_4\epsilon_{t-6}$$

$$X_t = X_{t-1} + X_{t-4} - X_{t-5} + \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - \phi_4\epsilon_{t-4} + \theta_1\phi_4\epsilon_{t-5} + \theta_2\phi_4\epsilon_{t-6} \quad (6)$$

The bilinear form of ARIMA (0, 1, 2) is

$$X_t = \sum_{i=0}^{0} \phi_0 X_{t-0} + \sum_{j=1}^{2} \theta_j e_{j-j} + \sum_{k=0}^{0} \sum_{l=1}^{2} \beta_{0l} X_{t-0} e_{t-l} \quad (7)$$

Given that i=k=0, P=R=0, j,l=1,2, Q=S=2, the above model reduces to

$$X_t = \sum_{j=1}^{2} \theta_j e_{j-j} + \sum_{l=1}^{2} \beta_{0l} X_{t-0} e_{t-l} \quad (8)$$

By expansion (8) becomes

$$X_t = \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \beta_{01} X_t \epsilon_{t-1} + \beta_{02} X_t \epsilon_{t-2} \quad (9)$$

The model excludes the seasonal part of the SARIMA model because bilinear time series model is the aggregation of the linear and non-linear part of the autoregressive and moving average process. The seasonal component does not form part of bilinear time series model. In this case, the parameters are estimated with the regression of X_t on ϵ_{t-1}, ϵ_{t-2}, $X_t\epsilon_{t-1}$ and $X_t\epsilon_{t-2}$.

4.0 Analysis and Results

This section provides estimates of the parameters of Seasonal Autoregressive Integrated Moving Average and Bilinear Autoregressive Integrated Moving Average Models.

4.1 Estimation

Table 1: SARIMA Estimates of the Parameters

TYPE	COEFFICIENT	SE. COEFFICIENT	T	P
MA(1)	0.3185	0.1040	3.01	0.003
MA(2)	0.5205	0.1069	4.87	0.000
SMA(4)	-0.4070	0.1132	-3.60	0.001

The estimated model is

$$(1\text{-}B)(1\text{-}B^4)X_t = (1\text{-}0.3185B - 0.5205B^2)(1+0.4070B^4)\epsilon_t \quad (10)$$

$$\hat{X}_t = X_{t-1} + X_{t-4} - X_{t-5} - 0.3185\epsilon_{t-1} - 0.5205\epsilon_{t-2} + 0.4070\,\epsilon_{t-4} - 0.1296\epsilon_{t-5} - 0.2118\epsilon_{t-5} \quad (11)$$

Table 2: BARIMA estimates of the parameters

TYPE	Coeff	Se. Coeff	T	P
ϵ_{t-1}	-0.5272	0.1204	-4.38	0.000
ϵ_{t-2}	-0.4172	0.1408	-2.96	0.004
$X_t\epsilon_{t-1}$	0.05771	0.01591	3.63	0.001
$X_t\epsilon_{t-2}$	-0.01459	0.01047	-1.39	0.168

The model is

$$X_t = -\,0.5272\epsilon_{t-1} - 0.4172\epsilon_{t-2} + 0.05771X_{t-0}\epsilon_{t-1} - 0.01459X_{t-0}\epsilon_{t-2} \quad (12)$$

The results in Table 2 are the estimates of the parameters obtained from the regression analysis. The fourth parameter is not significant. Therefore, the component $X_t\epsilon_{t-2}$ is removed, since its non-linearity has no significant effect in the model. The final estimates of the parameters are in Table 3.

Table 3: BARIMA final estimates of the parameters

TYPE	Coeff	Se. Coeff	T	P
ϵ_{t-1}	-0.4525	0.1085	-4.17	0.000
ϵ_{t-2}	-0.5124	0.1239	-4.14	0.000
$X_t\epsilon_{t-1}$	0.04704	0.01404	3.35	0.001

Model from the final estimates is

$$\hat{X}_t = -\,0.4525\epsilon_{t-1} - 0.5124\epsilon_{t-2} + 0.04704X_{t-0}\epsilon_{t-1} \quad (13)$$

4.2 Check for Adequacy of Models

FIGURE7: Autocorrelation Function for the Residual of SARIMA model
(with 5% significance limits for the autocorrelations)

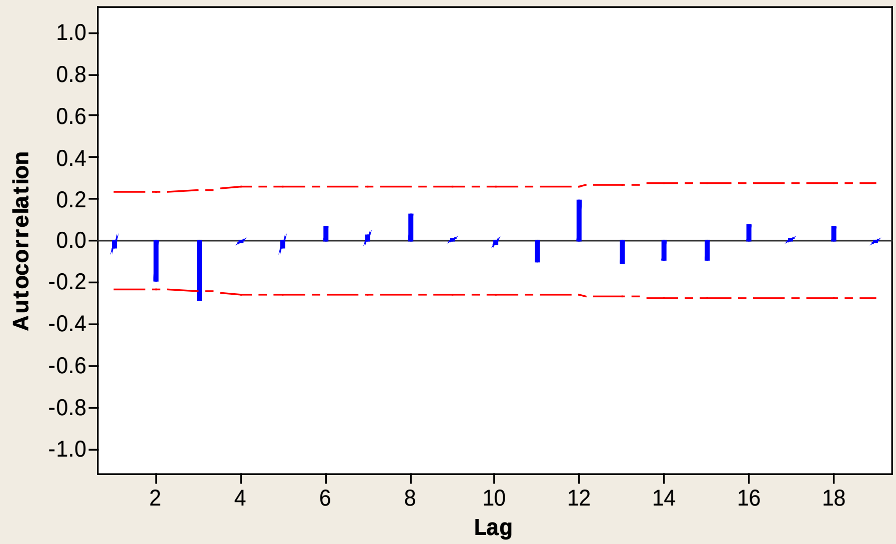
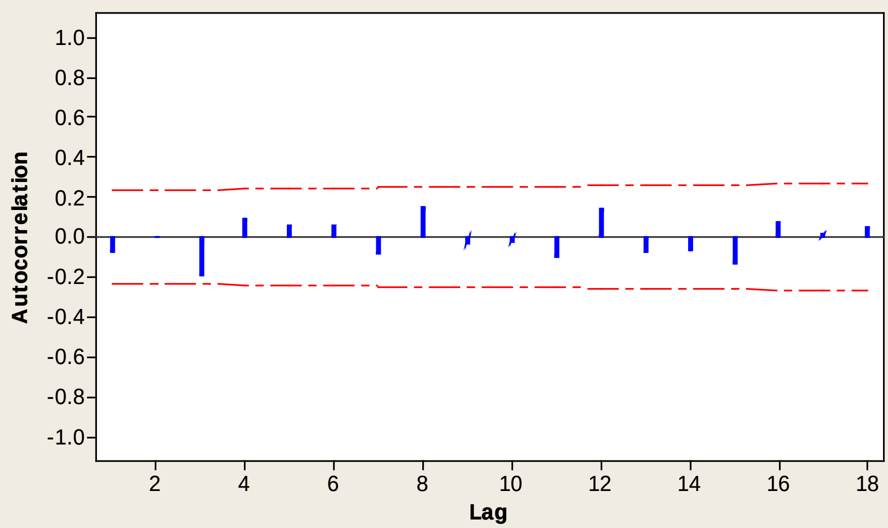


FIGURE8: Autocorrelation Function for the Residual of BARIMA model
(with 5% significance limits for the autocorrelations)



4.2.1: Test for adequacy of SARIMA $(0,1,2)(0,1,1)_4$ model

Table 4: MODIFIED BOX-PIERCE(Ljung-Box) Chi-Square Statistic

Lag	12	24	36	48
Chi-Square	15.9	25.2	31.3	32.1
DF	9	21	33	45
P-Value	0.069	0.237	0.553	0.925

The above hypothesis has shown that autocorrelation coefficients of the residual term of the SARIMA $(0,1,2)(0,1,1)_4$ model are zeros. This confirms the model adequacy (see Johnston and DiNardo,1997, p219).

4.2.2: Test for adequacy of BARIMA (0,2,1,0,1,) model

Table 5: Analysis of variance of the regression

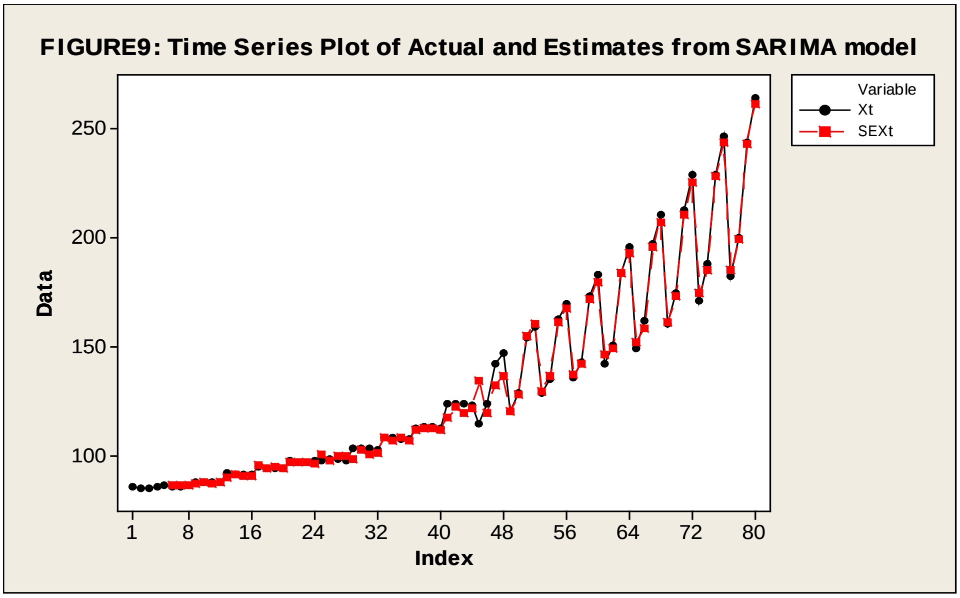
SOURCE	DF	SS	MS	F	P
Regression	3	813.39	271.13	22.70	0.000
Residual Error	70	836.13	11.94		
Total	73	1649.52			

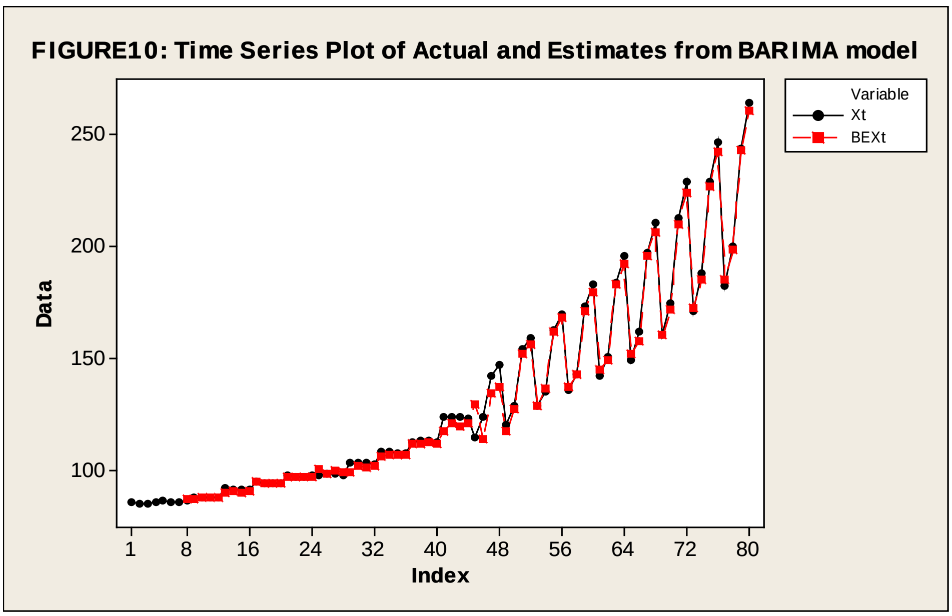
The above F and P values show overall fitness of the model to the data set. Hence, the bilinear model is adequate.

Table 6 : Akaike Information Criterion for the Two Models

Model	Specification	AIC
SARIMA	$(0,1,2)(0,1,1)_4$	199.2888
BARIMA	(0,2,1,0,1)	193.7408

Table 6 has shown a smaller value of AIC in the Bilinear Autoregressive Integrated Moving Average Model. Hence, the Bilinear Model has a comparative advantage over the Seasonal Model.





5 Summary and conclusion

The Seasonal and Bilinear Autoregressive Integrated Moving Average Models are useful in modelling Nigerian Gross Domestic Product. The comparison as to which of the models performs better in modelling Nigeria Gross Domestic Product was the primary motivation behind this research paper. The data used for the analysis are Nigeria Gross Domestic Product at 1990 constant basic price, from first quarter 1992 to fourth quarter 2012($x10^6$). Regular and seasonal differencing of both the original and seasonally adjusted GDP data were obtained. The distribution of the autocorrelation and partial autocorrelation functions suggested same SARIMA $(0,1,2)(0,1,1)_4$ and BARIMA(0,2,1,0,1,). This implies the original GDP data and seasonally adjusted GDP data do not vary in the choice of orders of both SARIMA and BARIMA models. The use of the original or seasonally adjusted series has not changed the parameters of the models. The condition for the use of BARIMA model is not at variance with the ARIMA model. The difference between ARIMA and BARIMA models is the non-linear part of the BARIMA model, which is the extension from ARIMA model. Adequacy of bilinear model is verified in the significant test of its parameters. From the analysis, autocorrelation functions of the residual values from estimated seasonal autoregressive integrated moving average model and bilinear autoregressive integrated moving average model are plotted (Figures 7 & 8). Ljung Box Chi-Square statistic has been used to confirm that the residual values of the seasonal model exhibit pure white noise process. Analysis of Variance test reveals the overall fitness of the bilinear model. Estimates obtained from the Seasonal and Bilinear models are plotted with the actual values. These are in Figures 9 & 10 respectively. These imply that the estimated values from the two models compare favourably with the actual values of GDP. The use of Akaike Information Criterion, as shown in Table5 to compare the performances of the two models has revealed a better performance of Bilinear Autoregressive Integrated Moving Average Model than Seasonal Autoregressive Integrated Moving Average Model. This agrees with the assertions from previous researches on the use of bilinear models to fit economic and financial data. Sometimes the GDP exhibits exponential growth. For instance, the growth of GDP from 2004 to 2012 is more of exponential than from 1998 to 2003. This has been verified by Eke *et al*(2015). Eke *et al*(2015) recommended exponential trend model for annual data of Nigeria GDP. So far, the contribution offered by this paper is the introduction of Bilinear Time Series Models in fitting Nigeria Gross Domestic Product as a class of non-linear models. This paper recommends Bilinear Time Series Model as an extension of ARIMA model for modelling Nigeria Gross Domestic Product. For further study, the use of BARIMA is not sacrosanct. The situation may change subsequently to warrant another class of times series model. It calls for future investigation as the GDP changes with time.

Table 7: Original GDP(X_t) and Seasonally Adjusted GDP (SX_t)

S/N	X_t	SX_t	S/N	X_t	SX_t	S/N	X_t	SX_t
1	85.9	86.158	28	98.1	97.477	55	162.5	162.395
2	85.4	85.870	29	103.2	103.458	56	169.3	168.677
3	85.3	85.195	30	103.2	103.670	57	135.8	136.058
4	85.8	85.177	31	103.2	103.095	58	142.8	143.270
5	86.4	86.658	32	102.7	102.077	59	173.1	172.995
6	86.1	86.570	33	108.1	108.358	60	182.6	181.977
7	86.2	86.095	34	108.1	108.570	61	142.6	142.358
8	86.5	85.877	35	108.0	107.895	62	150.9	151.370
9	88.3	88.558	36	107.5	106.877	63	183.7	183.595
10	88.1	88.570	37	112.6	112.858	64	195.6	194.977
11	88.1	87.995	38	113.3	113.770	65	149.2	149.458
12	88.1	87.477	39	113.1	112.995	66	162.1	162.570
13	92.1	92.358	40	112.7	112.077	67	197.1	196.995
14	91.7	92.170	41	124.0	124.258	68	210.6	209.977
15	91.7	91.595	42	123.9	124.370	69	160.1	160.358
16	91.6	90.977	43	123.8	123.695	70	174.7	175.170

17	94.7	94.958	44	123.3	122.677	71	212.8	212.695
18	94.4	94.870	45	114.6	114.858	72	228.7	228.077
19	94.5	94.395	46	123.7	124.170	73	171.3	171.558
20	94.2	93.577	47	142.4	142.295	74	187.8	188.270
21	97.5	97.758	48	146.9	146.277	75	228.5	228.395
22	97.1	97.570	49	120.0	120.258	76	246.4	245.777
23	97.1	96.995	50	128.8	129.270	77	182.1	182.358
24	97.7	97.077	51	153.9	153.795	78	199.8	200.270
25	98.1	98.358	52	159.2	158.577	79	243.3	243.195
26	98.4	98.870	53	128.6	128.858	80	263.7	263.077
27	98.5	98.359	54	135.4	135.870			

Data Source: Central Bank of Nigeria Statistical Bulletin 2012 (First quarter 1993 to Fourth quarter 2012, pages 144 - 148).

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