

The Forecasting Performances of Volatility Models in Emerging Stock Markets: Is a Generalization Really Possible?

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Abstract

In almost all stages of forecasting volatility, certain subjective decisions need to be made. Despite of an enormous literature in the area, these subjectivities are hindrances to reaching an overall conclusion on the performances of the models. In order to find out outperforming model in general not just in the contexts of studies, volatility models should be evaluated in many markets with the same methodology consisting both simple and complex models at different forecast horizon. With this motivation, the purpose of the paper is to search for the possibility of the generalization that one of the competing model outperforms no matter what the market is by analyzing 19 emerging stock market volatilities at 8 different forecast horizons with models grouped into three main categories: Simple models (Random Walk, Historical Mean, Moving Average, EWMA), GARCH family models (GARCH, GRJ-GARCH, GARCH, APARCH, NAGARCH, FIGARCH) and Stochastic Volatility model. The evaluation of the forecasts based on the recent developments in statistics, i.e. Reality Check (RC), Superior Predictive Ability (SPA) and Model Confidence Set (MCS), not only the rank of the error statistics. The scope and the methodology of the study enable us to reach a general conclusion on model performances and their over prediction and under prediction tendencies.

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1 Introduction

Varied subjective decisions in different dimensions need to be made during the process of forecasting volatility and the comparison of forecasts. [1] is a very good source to see these subjectivities and other issues in forecasting volatility and to gain insight what kind of questions arise when forecasting volatility in financial markets such as which approach will be used for the proxy of observed volatility, which competing models will be included, what the forecast horizons will be, which error statistics will be used for the comparison, and how error statistics will be evaluated to reach the conclusion on the performance of the models. The decisions based on these questions naturally affect the results of researches, which eventually is a handicap in the area of forecasting volatility to compare and evaluate the results of the previous studies. As it is pointed out by [2], even for a certain stock market, different conclusions are drawn due to the different observations and forecasting methodology. Hence, the performance of volatility models is evaluated in myriad number of studies; the results of the studies are relevant only in their own context. To find out whether there really is a model that performs better than the alternatives, they need to be evaluated all together with the same methodology in order to eliminate the effect of these subjective decisions on the forecasting performances. To be able to accomplish this, the models included in the analysis needs to be as comprehensive as possible, the number of markets needs to be as large as possible and the comparison of the error statistics should depend on some statistical analysis not just the rank of the error statistics. The rest of this section briefly explains these subjectivities in forecasting volatility in order to see the reason behind the motivation to perform such an analysis.

Firstly, there are two different approaches to measure volatility in the literature: variance and standard deviation. [3], [4], [5] and [6] used variance as a volatility measure while [7], [8] preferred standard deviation. Secondly, the researcher has to decide how to measure observed volatility since it is a latent variable. General approaches are daily squared returns [9], [10], mean adjusted daily squared returns [11], [12], daily squared return adjusted for serial correlation [4], [13], the absolute change in returns [14], [15]³. The existence of different approaches is mainly based on the question of whether the returns are adjusted for mean (conditional or constant) or not. The advocates of the use of squared returns adjusted for mean and serial dependence put forward that empirically proved high autocorrelation in the return should be controlled while the opponents claim that the statistical properties of the sample mean make it very inaccurate estimate of true mean, therefore taking deviations around zero instead of sample mean increases the forecast accuracy.

Another difference in studies in the area of forecasting volatility is that how the sample is used for parameter estimation and forecasts. One of the approaches is to apply a rolling scheme to estimate parameters of the models as in [5], [16] etc. The other approach, which is more commonly preferred, is that the division of data as in-sample and out-of-sample just once as in [17], [2]. Since in-sample data, therefore, the parameter estimates, is updated for every forecast in the rolling scheme approach, this approach may provide a better reflection of the structural changes in the economy to the parameters of the model and may prevent biases which depend on fixing in-sample for every forecast on

³Since there is an enormous literature on forecasting volatility, it is not just practical to list every reference to make the point. Only a few of them have been given here.

the model performances. From this point of view, the rolling scheme approach is preferred in the paper.

The comparison of model performances is quite important in the process of evaluation of the models performances. Although this stage is as important as forecasting; the comparison of error statistics has been limited to the evaluation of the rank of some error statistics, which are the subjective choice of researchers, so far. The conclusions are basically drawn from the rank obtained by the error statistics. However, error statistics of the competing models are most of the time so close that the question that the performances of the models are really distinguished from one another arises. The recent developments in econometrics, namely Reality Check (RC), Superior Predictive Ability (SPA) and Model Confidence Set (MCS), provide a solution to this problem. These procedures help the evaluation of the error statistics in a way that researches can be sure about the statistical significance of the ranks implied by the error statics, which in fact put the comparison in a more sound ground. The choice of error statistic is another issue for the comparisons. As it is stated in [18], it directly affects the evaluation of model performances. Most commonly used error statistics in the area of forecasting volatility are those that have symmetric property. Later, asymmetric error statistics has started to be used in order to address different exposure of risks coming from the positions (long/short) of investors in markets. In the paper, both symmetric and asymmetric error statistics have been included in the evaluation process of models.

As for the forecast horizon, the relevant forecast horizon varies by the purpose of the agents. Short forecast horizons are relevant for trading purposes and VaR estimations of financial institutions. For derivative markets, longer horizons are also relevant. Besides this, while a certain model performs very well in a specific forecast horizon, it may not be the same for the other horizons. Therefore, the evaluation of model performances in different horizons would provide some insight about the forecasting ability of the models in different horizons. Since the purpose of the paper is to evaluate the model performances in a general context, the results are evaluated for eight different forecast horizons varying between 1-day and 240-day.

Finally, which models should be included in the analysis is also critical since the performances are relatively evaluated. Therefore, the comprehensiveness of an analysis in terms of models that are included increases the generalizability of the results. With this perspective, the study covers Random Walk, Historical Mean, Moving Average, and EWMA as simple nonparametric models and GARCH family models and Stochastic volatility models as parametric models. Lately, the researches that apply and/or develop Stochastic Volatility model is more popular and mostly focus on the parameter estimation methods, however, GARCH family models are still dominant in the literature. Many GARCH models are developed by the different researchers with different approaches in order to incorporate the empirically proved patterns in volatility in the stock market returns, i.e. leverage effect, nonlinearity, long-memory. Therefore the inclusion of all developed GARCH family models is simply not practical. To address this issue, a representative set of GARCH family models are formed in order to cover the models addressing at least one of the above mentioned volatility patterns.

In summary, all of the points mentioned above complicate any attempt to compare and generalize the results in volatility forecasting literature. This paper aims to complement the literature in two ways. Firstly, to the best of our knowledge, this is the most comprehensive study for emerging stock markets in terms of the number of countries (i.e.

19 stock exchanges), the variety of the forecast horizons (short-, mid- and long-term) and the number of models (11 models). The comprehensiveness in different dimensions provides one to draw general conclusions in forecasting performances of the models for emerging stock markets. Secondly, this study is distinguished from the others by its comparison methodology.

2 Volatility Models and Forecasting Methodology

This section briefly introduces the data, volatility models and the methodology. Argentina, Brazil, Chile, Mexico, Peru, Venezuela, Czech Republic, Hungary, Poland, Russia, Turkey, China, India, Korea, Malaysia, Philippines, Srilanka, Taiwan, Thailand emerging stock market indices have been chosen based on SP/IFC classification and daily data are obtained from Bloomberg databases⁴.

Daily observed volatilities are estimated as mean adjusted daily squared return, i.e. $\sigma_t^2 = (R_t - \mu)^2$ and since the data is in daily frequency, h-day observed volatilities are estimated as the sum of the daily volatilities for the relevant period, which is $\sigma_t^2 = \sum_{i=t}^{i=t+h-1} (R_t - \mu)^2$ where $i = 1, 1 + h, 1 + 2h, \dots$ and R_t is the logarithmic return, μ is the sample mean, and $h = 1, 5, 10, 20, 60, 120, 240$ days are the forecast horizons. The data is divided into two parts since the focus is to compare the out-of-sample forecasts. The rolling scheme in which the sample size and forward shifting step was fixed at $w=2000$ and $s=20$ respectively is applied for the estimations. Below is the brief explanation of the models and the forecasting procedure:

Random Walk (RW): The best forecast of the tomorrow volatility is today volatility:

$$\hat{\sigma}_{t+1}^2 = \sigma_t^2 \quad (1)$$

where $t = w, w + s, w + 2s, \dots$ $\hat{\sigma}_{t+1}^2$ is the volatility forecast, σ_t^2 is the observed volatility, w is the sample size, s the forward shifting step in the rolling scheme.

Historical Mean (HM): This is basically the mean of all observations before the relevant forecast is performed. That is, the sample size grows as additional observations are added.

$$\hat{\sigma}_{t+1}^2 = \frac{1}{t} \sum_{i=1}^t \sigma_i^2 \quad (2)$$

where $t = w, w + s, w + 2s, \dots$

Moving Average (MA): According to HM, all past observation is used for the forecast. However, MA only takes into account past n observations, which is a subjective choice. MA can be considered as a recent historical mean of the variable. In the paper n is chosen as 240, which can be considered as one-year historical mean:

$$\hat{\sigma}_{t+1}^2 = \frac{1}{n} \sum_{i=k}^{k+n-1} \sigma_i^2 \quad (3)$$

where $t = w, w + s, w + 2s, \dots$ and $k = w - n + 1, w - n + s + 1, w - n + 2s + 1, \dots$

⁴The Bloomberg tickers are respectively Merval, IBOV, IPSA, MEXBOL, IGBVL, IBVC, PX, BUX, WIG20, RTSI, XU100, SHCOMP, SENSEX, KOSPI, FBMKLCI, PCOMP, CSEALL, TWSE, SET. The data period for all indices is the same and between 2nd January 1995 and 23th April 2010 except Russia for which it starts on 2nd April 1995.

Exponentially Weighted Moving Average (EWMA): As opposed to MA, EWMA gives exponentially decreasing weights to past observations as past observations gets older. The intuition behind incorporation of decay in weights is that recent observations have much more importance in forecasting future volatility than older observations.

$$\hat{\sigma}_{t+1}^2 = \frac{\sum_{i=0}^{n-1} \lambda^i \sigma_{t-i}^2}{\sum_{i=1}^n \lambda^i} \quad (4)$$

where $t = w, w + s, w + 2s, \dots, n = 240$ and λ is the smoothing constant and estimated by minimizing the sum of in-sample squared errors in the study.

h-day volatility forecasts of the above nonparametric models are estimated by simple scaling rule, which is $\hat{\sigma}_{t+1} \sqrt{h}$.

The GARCH family models: It is not wrong to say that the current interest in volatility modeling and forecasting started with the seminal papers [19] and [20] in which GARCH and ARCH models were proposed respectively. After these seminal papers, variety of versions taking into account different characteristics of financial time series such as leverage effect, long memory, nonlinearity have been developed from GARCH modeling perspective. Therefore, the literature on conditional volatility models is enormous. Although the entire GARCH model universe is not included in the analysis, selected models, namely GARCH, GJR-GARCH, EGARCH, APARCH, NAGARCH and FIGARCH, can be considered as a representative set of GARCH family models since the model set includes those focusing on different patterns in volatility such as asymmetry, nonlinearity and long memory.

Let define R_t as the return process of the stock market.

$$\begin{aligned} R_t &= \mu + \varepsilon_t \\ \varepsilon_t &= \sigma_t z_t, \quad z_t \sim N(0,1) \end{aligned} \quad (5)$$

GARCH(1,1) model [19]:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (6)$$

GJR-GARCH(1,1) model [21]:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1} \quad (7)$$

where $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$, $I_{t-1} = 0$ otherwise.

EGARCH(1,1) model [22]:

$$\log \sigma_t^2 = \omega + \beta \log \sigma_{t-1}^2 + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \alpha \left[\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - \frac{2}{\pi} \right] \quad (8)$$

Asymmetric Power ARCH - APARCH(1,1) model [23]:

$$\sigma_t^\delta = \omega + \alpha (|\varepsilon_{t-1}| - \gamma \varepsilon_{t-1})^\delta + \beta \sigma_{t-1}^\delta \quad (9)$$

Nonlinear Asymmetric GARCH - NAGARCH(1,1) model [24]:

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \alpha (\varepsilon_{t-1} + \gamma \sigma_{t-1})^2 \quad (10)$$

Fractionally Integrated GARCH -FIGARCH(1,d,1) model [25]⁵:

⁵For the parameter estimation of FIGARCH, NAGARCH and APARCH models Prof. Kevin Sheppard's matlab codes, which are provided in his website

$$\sigma_t^2 = \omega + [1 - BL - (1 - \phi L)(1 - L)^d] \varepsilon_t^2 + \beta \sigma_{t-1}^2 \quad (11)$$

where L is the back shift operator, i.e. $Lx_t = x_{t-1}$.

One lag delay in both past innovations and past conditional volatilities is presumably enough for the elimination of the heteroscedasticity in the return series because of the following reasons: there is a general notion of that (1, 1) lag structure is the most parsimonious lag structure for GARCH family models in the literature [3], [26], [27]. This is especially supported by the extensive study of [28] in which they evaluated 330 different GARCH family models. They reported that (2, 2) lag structure rarely performs better than the same model with fewer lags. Secondly, tradeoff between the number of parameters estimated by the use of in-sample data and out-of-sample performances of the models makes (1,1) lag structure very reasonable to use in forecasting. Despite of these favorable supports, to check the validity of this assumption, Lagrange Multiplier test is performed after the estimation of parameters in every model. The test results show that (1, 1) lag structure is enough to eliminate the heteroscedasticity in the time series with very few exceptions⁶.

Stochastic Volatility (SV): Consider the univariate stochastic model:

$$\begin{aligned} r_t &= e^{0.5\sigma_t} \varepsilon_t & \sigma_t &= \gamma + \phi \sigma_{t-1} + \eta_t \\ \eta_t &\sim NID(0, \sigma_\eta^2), & E(\varepsilon_t \eta_t) &= 0, & \varepsilon_t &\sim NID(0, 1) \end{aligned} \quad (12)$$

where $r_t = R_t - \mu$ is the mean adjusted return. Since working in logarithms ensures that σ_t^2 is always positive and provides linearity, by taking logarithms of the squared mean adjusted returns one obtains:

$$\begin{aligned} \ln(r_t^2) &= \sigma_t + \zeta_t & \sigma_t &= \gamma + \phi \sigma_{t-1} + \eta_t \\ \zeta_t &= \ln(\varepsilon_t^2), & \eta_t &\sim NID(0, \sigma_\eta^2), & E(\zeta_t \eta_t) &= 0 \end{aligned} \quad (13)$$

If the ε_t is standard normal then ζ_t follows the $\ln(X^2)$ distribution whose mean and variance are known to be -1.27 and $\pi^2/2$, respectively. In recent years, many parameter estimation techniques for SV models have been developed. Quasi-Maximum Likelihood (QML) method based on the Kalman Filter is chosen for the estimation of the parameters since this method is relatively faster than the other methods [29]. The state space form of the model and Kalman filter for parameter estimations and prediction equations can be found in the appendix.

h-day forecasts of the parametric models, namely GARCH family models and SV model, are obtained as follows: The rolling scheme in which the sample size was fixed at 2000 is used for the parameters estimations. For the first forecast of h-day volatility, these parameter estimates are used to make one-step-ahead to h-step-ahead forecasts for the next h days in a recursive manner. The sum of these h forecasts gives h-day volatility forecast of the corresponding model. By shifting the sample forward by 20 observations,

(<http://www.kevinshppard.com/wiki/Category:MFE>), have been used by modifying the codes according to the needs of the analysis.

⁶ Only 104 out of 9618 estimated models can not eliminate the heteroscedasticity in the time series. In detail, only 88 out of 1603 estimated APARCH(1,1) model can not eliminate heteroscedasticity according to Engle's LM test with 0.05 significant level, which implies that APARCH(1,1) is not enough for Poland, India and Thailand for some periods. The test results are not reported here, they can be provided upon request.

the new parameter estimates are obtained. One-step-ahead to h-step-ahead forecasts for the next h days are performed in a recursive manner with these new parameters for the second forecast of h-day volatility. It continues in the same way until the end of the sample.

3 Comparison Of Forecast Performances

A sound comparison of model performances is as important as performing the forecasts. Both symmetric and asymmetric error statistics are relevant for the evaluation of the volatility forecasts in stock markets. Asymmetry in the error statistics can be especially important for participants of derivative market. For example, the major parameter that determines the value of an option contract is volatility of the underlying, the investors who take long/short position may prefer to penalize over/under predictions more heavily to reduce to exposure to volatility modeling risk. However, it should be noted that the symmetric error statistic are more suitable to evaluate a model overall success in terms of fitting to observed data. Hence, performance results of the models are primarily deduced from the symmetric error statistics, while asymmetric error statistics are used to determine the tendencies of models in general in making over/under predictions with the purpose of addressing the different needs of the investors. The following error statistics are used in the study⁷.

Symmetric error statistics⁸:

$$\begin{aligned} \text{MSE} &= \frac{1}{n} \sum_{i=1}^n (\sigma_i - \hat{\sigma}_i)^2 & \text{RMSE} &= \sqrt{\frac{1}{n} \sum_{i=1}^n (\sigma_i - \hat{\sigma}_i)^2} \\ \text{MAE} &= \frac{1}{n} \sum_{i=1}^n |\sigma_i - \hat{\sigma}_i| & \text{MAPE} &= \frac{1}{n} \sum_{i=1}^n \frac{|\sigma_i - \hat{\sigma}_i|}{\sigma_i} \end{aligned} \quad (14)$$

Asymmetric error statistics⁹:

$$\begin{aligned} \text{MME} - U &= \frac{1}{n} \left[\sum_{i=1}^k |\sigma_i - \hat{\sigma}_i| + \sum_{i=1}^l |\sigma_i - \hat{\sigma}_i|^{0.5} \right] \\ \text{MME} - O &= \frac{1}{n} \left[\sum_{i=1}^l |\sigma_i - \hat{\sigma}_i| + \sum_{i=1}^k |\sigma_i - \hat{\sigma}_i|^{0.5} \right] \end{aligned} \quad (15)$$

where k denotes the number of over predictions and l the number of under predictions among the out-of-sample forecasts, which is $k + l = n$.

$$\begin{aligned} \text{MLAE} &= \frac{1}{n} \sum_{i=1}^n \ln |\sigma_i - \hat{\sigma}_i| \\ \text{LINEX} &= \frac{1}{n} \sum_{i=1}^n \left[e^{-a(\sigma_i - \hat{\sigma}_i)} + a(\sigma_i - \hat{\sigma}_i) - 1 \right] \end{aligned} \quad (16)$$

where the choice of the value of the parameter a is subjective, which allows different weights to over- and under-predictions. When $a > 0$, it punishes heavily the under

⁷Error Statistic of the models for the markets are not presented in the paper. Upon request, the file that contains the tables can be provided.

⁸MSE, RMSE, MAE, MAPE stand for Mean Square Error, Root Mean Square Error, Mean Absolute Error, Mean Absolute Percentage Error, respectively.

⁹MME-U, MME-O, MLAE stand for Mean Mixed Error-Under, Mean Mixed Error-Over, Mean Logarithm of Absolute Error, respectively.

predictions. In the study $\alpha = 5$.

To evaluate by just looking at the rank implied by the error statistics does not provide a sound comparison. Fortunately, in the last decade, some important statistical techniques have been developed to check whether the rank of the performances of the models deduced from a certain error statistic is statistically significant or not. Reality Check (RC) in [30] and Superior Predictive Ability (SPA) in [31] tests allow us to determine whether the differences obtained from error statistics are significant or not. The null hypothesis of both RC and SPA are that the models included in the analysis do not have superior performance relative to the benchmark, while the alternative hypothesis is that at least one of the models has superior forecasting performance relative to the benchmark. This implies that, the performance of the model that has the best error statistic is superior to the benchmark even though one would not tell anything about the comparison between the best model and models other than the benchmark. During the empirical analysis, it has been seen that the best model does not always show significantly superior performance than the benchmark according to certain error statistics, while it does according to some other error statistics. Therefore, these tests can also be used to determine which error statistics can really distinguish the performances of models. The difference between these two tests is that RC is quite sensitive to the set of the models included in the analysis. That is, if the comparison involves irrelevant or poor alternatives, then RC is not able to reject the null hypothesis even though it is the case. When the model set comprises reasonable alternative both RC and SPA produce quite similar results. In this study, when RC and SPA test are performed the simplest model RW is chosen as the benchmark model. The other technique used to distinguish the performances of the models is the Model Confidence Set (MCS) procedure of [32]. The MCS method characterizes the entire set of models as those that are/are not significantly outperformed by other models, on the other hand, RC and SPA tests only provide evidence about the relative performance according to the benchmark model. [32] illustrates the difference between RC/SPA and MCS with analogy of the difference between confidence interval of a parameter and point estimate of a parameter. The significance of the performance of the best model relative to the benchmark can be determined with RC and SPA tests, however, RC and SPA tell nothing about the case in which the performances of other models are very close to the model that shows the best performance. At that point, The MCS helps one determine whether the performances of other models performances are close to the best model or not by grouping the models into two categories (sets), namely inferior and superior models sets, by assigning probability values to each model. If p-value of a model is greater than a subjectively determined p-value, then it is accepted as in the superior set. The critical p-value for the study is chosen as 0.9. In the study, all of the three techniques are used for the evaluation of the forecasting performances. First step is to determine which error statistics give significant results by applying SPA and RC tests. At this step, the best performing model can be confidently declared as the best performing model according to corresponding error statistic, however, to what extent that the best performing model is significantly superior than the other models can only be determined by the MCS procedure, which is the second step of the evaluation process¹⁰.

¹⁰For SPA, RC and MCS estimations, Prof. Kevin Sheppard's matlab codes, which are provided in his website (<http://www.kevinsheppard.com/wiki/Category:MFE>), have been used. SPA/RC test results and MCS p-values can be provided upon request.

4 Empirical Analysis and Results

In this section, out-of-sample forecasts of 11 models for short-term (1-day, 5-day, 10-day), medium-term (20-day, 60-day and 120-day) and long-term (180-day and 240-day) forecast horizons are performed and compared. Table 1 to Table 8 presents the results. Since the results in the table are based on the error statistics and RC, SPA, and MCS test results, the following points should be taken into account in order to be able understand how conclusions are drawn from the tables. First of all, the best performing model according to each error statistic is reported in the tables as the first input of the cells. When the best performing model is significantly different from the benchmark based on RC/SPA tests results, the model is superscripted by “*” and is called “the significant best model”. Hence, if there is not any model superscripted by “*” in a cell than the performances of models are not significantly different from each other according to corresponding error statistic. This is the first step of the evaluation of the results of the error statistic. It in fact provides one to determine which error statistic results should be taken into account for the rest of the comparison process.

Let consider 1-day volatility forecast results for PERU in Table 1. According to MSE, NAGARCH is the best model while EWMA is the best model according to RMSE. However, RC/SPA test results show that MSE cannot distinguish the model performances while RMSE does. Therefore, the best model according to MSE is not taken into account for the rest of the analysis, and the result of RMSE, i.e. EWMA, is superscripted to show that it will be taken into account for the rest of the comparison process. That is, only superscripted models and corresponding error statistics are evaluated after that point. As explained in section III, even though a model is determined as the significant best model with the help of RC/SPA tests, this doesn't tell anything about the difference between the best model and the second best model, third best model and so on. The second step of the evaluation process addresses to this issue by determining MCS set of superiors of the significant best model if there is one. If there exists a MCS set of superiors for a significant best model, then the models in the MCS are added to place where the significant best model is in the table. Therefore, where the cells include more than one model, they report the set of models whose performances are the same as that of the significant best model, which is superscripted by *. This set of models will be called “MCS set of superiors” of the corresponding significant best model. If we back to the example of PERU in Table 1, MCS set of superiors of EWMA is NAGARCH and GRJ-GARCH. Lastly, the success of the models are evaluated based on the symmetric error statistic, and the asymmetric error statistic are used to determine the tendency of the models in making under/over predictions, which is considered as beneficial for those who have preferences over under/over prediction in their decision processes.

First thing that should be noticed in Table 1 is the outperformance of SV model on 1-day volatility forecasts. For 14 out of 19 stock markets, SV is the significant best model according to more than one error statistics for most of the markets. For a few of these markets, namely Argentina, Czech and China, the MCS set of superiors of SV comprises GARCH family models, which means that SV is sharing the same performance level as GARCH family models for 3 out of 14 markets. Even though the performance of GARCH family models is close to that of SV for these three market SV is successful in 14 markets. Therefore, it is not wrong to make the generalization that SV model is the best model to forecast stock market volatility in emerging markets for 1-day volatility forecasts. On the

other hand, this outstanding success of SV is completely vanished for 5-day volatility forecasts since it does not show the best performance even at one market as it can be seen from Table 2. This is a quite strong indication of that the smaller the forecast horizon, the better the performance of SV gets. For 5-day volatility forecasts, GARCH family models have dominance over the others by outperforming in 11 out of 19 emerging markets. EWMA is the second model by outperforming in 7 out of 19 markets. When the MCS set of superiors of the models are examined, the MCS set of superiors of GARCH family models includes EWMA only in 2 out of 11 markets, and the MCS set of superiors of EWMA includes GARCH family models in 3 out of 7 markets. This implies that there is not an important intersection in which these two models show the same out-performance in the same markets. Over all, when MCS results are taken into account, GARCH family models outperform in **15** markets (**11** as the best model + **4** as the MCS set of superiors of other models), while EWMA outperforms in **9** markets (**7** as the best model + **2** as the MCS set of superiors of other models). Table 9 provides quick overlook this whole **11+4** and **7+2** summation and generalization process by reporting the number of cases (markets) in which a certain model is the best model and the number of cases (markets) in which the model is the MCS set of superiors of other models¹¹. So, GARCH family model are considerably successful for 5-day volatility forecasts. Therefore, it would be more reasonable to choose GARCH family models for 5-day volatility forecasts in the case that one has to choose a volatility model without performing any forecasting analysis. As for the results of 10-day volatility forecasts in Table 3, GARCH family models and EWMA show outperformance in almost equal number of markets, and again the MCS set of superiors of either one include each other in the same number of markets. RW is another model found as the significant best model for 5 out of 19 markets for 10-day volatility forecasts. However, the MCS set of superiors of RW involves GARCH family models and EWMA for 4 out of these 5 markets. This implies that RW just shares the same performance level as those of EWMA and GARCH family models in those 4 markets, which eventually strengthens the generalization of outperformance of GARCH family models and EWMA for 10-day volatility forecasts. Furthermore, if it is remembered that EWMA is a special case of Integrated GARCH model, this is a very strong support for the choice of GARCH family models for 10-day volatility forecasts. For 20-day volatility forecasts, the out-performance of GARCH family models is noteworthy in Table 4. GARCH family models are the significant best model for 9 out of 19 markets. Also, they are MCS set of superiors of both EWMA, in 4 out of 8 markets, and RW, in 2 out of 2 markets. This implies that GARCH family models are the significant best model for 9 markets and they show the same performance level as EWMA and RW for additional 6 markets, that is, in total, for 15 out of 19 markets, GARCH family models have superior performance. For both 60-day and 120-day volatility forecasts in Table 5 and Table 6 respectively, EWMA and GARCH family models outperform in almost equal number of markets. However, it should be noted that, as the forecast horizon increases, the number of markets where EWMA is the best models is getting closer to that of GARCH family models, And MA also starts to outperform in some markets. For 60-day volatility forecasts, MA is the significant best model in 3 markets; however, it shares the same performance level as those of GARCH family

¹¹While Table 1 to Table 8 reports the results for each horizon, Table 9 and Table 10 facilitate to see the generalizations deduced from these tables.

models and EWMA in these markets. On the other hand, for 120-day volatility forecast, MA is the significant best model in 5 markets, and it shares the same performance level only in 1 out of these 5 markets as those of GARCH family models and EWMA, which is the first sign of that how MA gets stronger as the forecast horizon increases.

For 180-day volatility forecasts, MA is the significant best model for almost half of the markets in Table 7, while the rest of the markets are shared by GARCH family models and EWMA. As for 240-day volatility forecasts in Table 8, MA has dominance over the other models by outperforming 12 out of 19 markets. It is not wrong to make the generalization that MA is the best choice to forecast stock market volatility in emerging markets for 240-day volatility forecasts.

When the results of asymmetric error statistics are evaluated, the first striking result is that SV model consistently under predicts, and GARCH family models over predict for almost all forecast horizons. Table 10 provides the generalization of the over prediction and under prediction patterns of the models for different forecast horizon. The choice of over prediction or under prediction depends on investors' preferences. Generally, some investors may find it beneficial to choose models that over predict for the sake of being in the safe side. However, it should be noted that GARCH family models over predict at the ordinary times. It implies that investors, who use GARCH family models for prediction, are being too cautious in ordinary times, and may not be ready enough for the high risk periods. Therefore it is recommended that the investors should ask themselves the question of how this over prediction (or under prediction) pattern in ordinary times affects their positions and pricing decisions. Especially in option markets, investors can take positions on the volatility of the underlying. For instance, investors who use straddle/strangle are exposed to different risks inherent in forecasting of volatility of the underlying. Let think about an investor who applies straddle strategy on an underlying places his bids based on the volatility forecast of the underlying for the relevant horizon. There is a possibility that he prices the options contracts higher than they should be in case that he uses GARCH family models. Or let think that he writes straddles on a certain underlying. In this case if he forms his volatility expectation of the underlying by the forecast of SV model, he is exposed to risk of predicting the volatility lower than it should be. Therefore he increases the possibility of loss in his position without having been sufficiently compensated for the risk that he carries due to lower ask price. At that point, there are a few things to be mentioned about the use of symmetric and asymmetric error statistics. If a model is the best model according to both symmetric and asymmetric error statistics, this model is what investors look for if they have preferences over under prediction or over prediction. If a GARCH family model is the significant best model according to both symmetric and asymmetric error statistics, it should be interpreted as the model produces the closest prediction to the observed volatility but usually the predictions are higher than the observed one, which is very suitable for those who apply straddle if we back to the example above.

Beside the performance of the models, there are a few points needed to be mentioned. First of all, as the forecast horizon increases, the significant best models uniquely outperform in the relevant markets. That is, while more than one models share the same performance levels for the most of the markets for 1-day volatility forecasts, as the horizon increases difference in the model performances gets bigger, and eventually for 240-day forecast horizon, the MCS set of superiors of the significant best models are empty sets for the all markets. Secondly, as a side result of this study, it is found that

RMSE is the only symmetric error statistic that can always distinguish the model performances no matter what the forecast horizon is. If the scope of this study is taking into account, the success of RMSE is so consistent that it is not wrong to say that RMSE is the strongest symmetric error statistic in terms of the power of distinguishing differences among the models.

The results provide a very good reference for the choice of volatility model for different forecast horizon even though it is not the main motivation of the study. Table 11 presents the best volatility model for each emerging market at different forecast horizon. General tendency both in academia and in practice is to use GARCH family models to estimate and forecast volatility. This tendency is so strong that GARCH family models are almost default choice. However, Table 11 tells that this widespread use of GARCH family models is not that appropriate in every case. From table 11, one can find that the simple models like EWMA and MA are the best model for many forecast horizons. The results are not commented here country by country, the reader can make inferences easily. However, there are a few pattern that needs to be mentioned specifically. For three emerging markets in Europe, namely Turkey, Poland and Russia, EWMA is the best model in most forecast horizons. Hence, for the actors in these markets, the best choices for volatility model is EWMA not GARCH family models. Many institution use GARCH family models to calculate their market risk as a part of their capital adequacy ratio. However, Table 10 in which over prediction and under prediction tendencies of the models in general are reported implies that GARCH family models usually overpredict, which means that these institutions may have unnecessarily low capital adequacy ratios. On the other hand, GARCH family models are the best volatility models at all forecast horizon for the stock market in Czech Republic. Another pattern which is quite strong for the stock market in Thailand is that FIGARCH model is quite successful at almost all forecast horizon.

5 Conclusion

In the paper, the forecast evaluations of the volatilities of the 19 emerging stock market indices for forecast horizons from 1 day to 240 days are performed with the purpose of examining whether there really is a certain model superior to the alternatives for the majority of the emerging markets. The most general results can be listed as follows: First of all, SV is the best performing model for 1-day volatility forecasts for majority of the emerging market. For 10-day, 20-day, 60-day and 120-day volatility forecasts, GARCH family models and EWMA show superior performance in almost equal number of countries, and, EWMA outnumbers GARCH family models as the forecast horizon increases. For 240-day volatility forecasts, MA outperforms for most of the countries. That is, as the forecast horizon increases, there is a movement from the sophisticated models to more naive models. When the results of asymmetric error statistics are taken into account, it is found that SV consistently underpredicts while GARCH family models overpredict.

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Appendix

Table 1: Best performing models for 1-day volatility forecasts

	Symmetric Error Statistics				Asymmetric Error Statistics			
	MSE	RMSE	MAE	MAPE	MME-U	MME-O	MLAE	LINEX
Argentina	NAGARCH*	SV*, GARCH, GRJ-GARCH, EGARCH, NAGARCH, FIGARCH, APARCH	SV*, GARCH, GRJ-GARCH, EGARCH, NAGARCH, FIGARCH, APARCH	SV	SV*	HM*	SV*	NAGARCH*
Brazil	EWMA*	SV*	SV*	SV	SV*	HM*	SV*, FIGARCH, NAGARCH, APARCH	EWMA*
Chile	NAGARCH*	SV*	SV*	RW	SV*	HM*	SV	NAGARCH*
Mexico	EWMA*, NAGARCH, GJR-GARCH, EGARCH, APARCH, FIGARCH, GARCH,	EWMA*	EWMA*	RW	SV*	HM*	SV	EWMA*, NAGARCH, GARCH, GRJ-GARCH, EGARCH, APARCH, FIGARCH
Peru	NAGARCH	EWMA*, NAGARCH, GRJ-GARCH	EWMA	SV	SV*	FIGARCH*	GRJ-GARCH	NAGARCH
Venezuela	APARCH	SV*	SV	SV	SV*	GARCH*, HM, EGARCH, GRJ-GARCH, NAGARCH	SV	APARCH
Czech	GRJ-GARCH*	SV*, GRJ-GARCH, EWMA, EGARCH, NAGARCH	SV	SV*	SV*	FIGARCH*	SV	GRJ-GARCH*
Hungary	EGARCH	SV*	SV*	SV*	SV*	EGARCH*, NAGARCH, GJR-GARCH	SV*	GRJ-GARCH
Poland	FIGARCH	EWMA*	EWMA*	SV*	SV*	HM*	SV*	FIGARCH*
Russia	EWMA	SV*	SV	RW	SV*	HM*	SV	RW
Turkey	SV*	SV*	SV*	SV*	SV*	HM*	SV*	SV*, MA, EWMA, GARCH, GRJ-GARCH, EGARCH, NAGARCH, APARCH, FIGARCH
China	APARCH*	SV*, EWMA, NAGARCH, EGARCH, APARCH	SV*, EWMA, NAGARCH, EGARCH, APARCH	SV	SV*	HM*, EWMA, GRJ-GARCH, EGARCH, FIGARCH, GARCH	SV	APARCH*
India	GRJ-GARCH*, EWMA, APARCH, EGARCH	EWMA*, GRJ-GARCH, NAGARCH, APARCH, EGARCH	EWMA*, GRJ-GARCH, NAGARCH, APARCH, EGARCH	SV	SV*	HM*	EGARCH	GRJ-GARCH*, EWMA, EGARCH, APARCH
Korea	NAGARCH*, SV, EGARCH	SV*	SV*	SV*	SV*	HM*	SV	NAGARCH*
Malaysia	SV*	SV*	SV*	SV	SV*	HM*	SV	SV*
Philippines	GRJ-GARCH*	GRJ-GARCH*, EWMA, GARCH, NAGARCH, FIGARCH, APARCH, EGARCH, SV	GRJ-GARCH*, SV, EGARCH, NAGARCH, APARCH, FIGARCH	SV*	SV*	HM*, EWMA, GARCH	APARCH	GRJ-GARCH*
Srilanka	EWMA	SV*	SV	SV	SV*	EWMA*, HM, FIGARCH	SV	EWMA
Taiwan	APARCH*, EGARCH, NAGARCH, FIGARCH, GRJ-GARCH, SV	SV*	SV*	SV*	SV*	HM*	SV*	APARCH*, EGARCH, NAGARCH, FIGARCH
Thailand	FIGARCH*, SV, EGARCH, NAGARCH, APARCH, EWMA	SV*	SV*	SV	SV*	NAGARCH*, HM, GRJ-GARCH, GARCH, EWMA	SV	NAGARCH*, SV, EGARCH, FIGARCH, APARCH, EWMA

Note: First model in a cell of the table is the best model according to relevant error statistic. When it is superscripted with *, this implies that it is the significant best model due to RC/SPA results. The cells including more than one model confidence set of the corresponding significant best model ,please read section IV for more detailed explanations about reqading of the tables

Table 2: Best performing models for 5-day volatility forecasts

	Symmetric Error Statistic				Asymmetric Error Statistic			
	MSE	RMSE	MAE	MAPE	MME-U	MME-O	MLAE	LINEX
ARGENTINA	GRJ-GARCH*	EGARCH*	EGARCH*	EGARCH	SV*	HM	EGARCH*	GRJ-GARCH*
BRAZIL	EWMA	EWMA*	EWMA	SV	SV*	GRJ-GARCH*	APARCH	EWMA
CHILE	APARCH*, GRJ-GARCH EWMA, NAGARCH GARCH, EGARCH	APARCH* EWMA	APARCH* EWMA	EWMA	SV*	GARCH*	APARCH	APARCH*, GRJ-GARCH EWMA, NAGARCH GARCH, EGARCH
MEXICO	NAGARCH	EWMA*, EGARCH, APARCH, RW NAGARCH, GRJ-GARCH,	EWMA	RW	SV*	GARCH*	EWMA	NAGARCH
PERU	RW	RW*, GRJ-GARCH, NAGARCH, EWMA, EGARCH, APARCH FIGARCH	RW	APARCH	SV*	FIGARCH*	APARCH	RW
VENEZUELLA	APARCH	APARCH*	APARCH*	SV*	SV*	FIGARCH*, EGARCH GRJ-GARCH, HM	APARCH*	APARCH
CZECH	GRJ-GARCH	GRJ-GARCH*, EWMA NAGARCH, GARCH FIGARCH, APARCH EGARCH, RW	GRJ-GARCH	APARCH	SV*	FIGARCH*, GARCH	APARCH	FIGARCH
HUNGARY	FIGARCH	FIGARCH*	FIGARCH	APARCH	SV*	FIGARCH*, HM	EWMA	FIGARCH
POLAND	EWMA	EWMA*, APARCH	EWMA	EWMA	SV*	HM*, GARCH EGARCH	APARCH	GARCH
RUSSIA	EWMA	EWMA*	EWMA	SV	SV*	GARCH*	APARCH	EWMA
TURKEY	APARCH*	EWMA*	EWMA*	SV*	SV*	GARCH*	EWMA	APARCH
CHINA	EGARCH	APARCH*	APARCH	APARCH	SV*	GRJ-GARCH*	APARCH*	EGARCH
INDIA	GRJ-GARCH	EGARCH*	EGARCH*	APARCH	SV*	GARCH*	APARCH*, RW EGARCH	GRJ-GARCH
KOREA	EGARCH	NAGARCH*	NAGARCH	SV	SV*	HM*	RW	EGARCH
MALAYSIA	EGARCH	EGARCH*	EGARCH*	SV	SV*	GRJ-GARCH	EWMA	EGARCH*
PHILIPPINES	EGARCH	APARCH*, EGARCH, RW NAGARCH, GRJ-GARCH	APARCH	RW	SV*	HM*	APARCH	EGARCH
SRI LANKA	EWMA	EGARCH*, APARCH FIGARCH, RW	EGARCH	SV	SV*	GARCH*	RW	EWMA
TAIWAN	EWMA*	EWMA*	EWMA*	RW	RW	FIGARCH*, APARCH EGARCH, HM, NAGARCH	EWMA*	EWMA*
THAILAND	EWMA	EWMA*	EWMA	EWMA	RW	APARCH*, HM, SV FIGARCH	EWMA	EWMA

Note: As in Table 1.

Table 3: Best performing models for 10-day volatility forecasts

	Symmetric Error Statistic				Asymmetric Error Statistic			
	MSE	RMSE	MAE	MAPE	MME-U	MME-O	MLAE	LINEX
ARGENTINA	GRJ-GARCH*	EGARCH*	EGARCH*	EGARCH*	SV*	HM*, GRJ-GARCH, MA GARCH, EGARCH	EGARCH*	GRJ-GARCH
BRAZIL	EWMA	EWMA*	EWMA	EWMA	SV*	GARCH*	EWMA	EWMA
CHILE	GRJ-GARCH	APARCH*, EWMA, EGARCH, NAGARCH, GRJ-GARCH, RW, FIGARCH, GARCH	APARCH	RW	SV*	GARCH*, HM FIGARCH	RW	GRJ-GARCH
MEXICO	EWMA	RW*, EWMA	RW	RW	SV*	GARCH*	RW	EWMA
PERU	RW	RW*, GRJ-GARCH, GARCH, EWMA	RW	RW	SV*	MA*	RW	RW
VENEZUELLA	APARCH	APARCH*	APARCH	SV	SV*	EGARCH*	APARCH	APARCH
CZECH	GRJ-GARCH	FIGARCH*, APARCH, GARCH, EGARCH, EWMA, GRJ-GARCH	FIGARCH	APARCH	SV*	FIGARCH*	APARCH	GRJ-GARCH
HUNGARY	FIGARCH	FIGARCH,*	FIGARCH	EWMA	SV*	FIGARCH*	EWMA	FIGARCH
POLAND	EWMA	EWMA*	EWMA	EWMA	SV*	GARCH*	APARCH	EWMA
RUSSIA	EWMA	EWMA*	EWMA	APARCH	SV*	GARCH*	APARCH	EWMA
TURKEY	EWMA	RW*	RW	RW	SV*	GRJ-GARCH*	RW	EWMA
CHINA	EGARCH	APARCH*	APARCH	APARCH	SV*	GRJ-GARCH*, GARCH	APARCH	EGARCH
INDIA	GARCH	EWMA*, GARCH	EWMA	EWMA	SV*	GARCH*	RW	GARCH
KOREA	APARCH	RW*, NAGARCH, FIGARCH, EGARCH, APARCH, EWMA	RW	RW	SV*	HM*	RW	GRJ-GARCH
MALAYSIA	MA	RW*	RW	RW	SV*	GARCH*, MA	RW	EGARCH
PHILIPPINES	EGARCH	EGARCH*	EGARCH	APARCH	SV*	HM*	EGARCH	EGARCH
SRILANKA	EWMA	EWMA*, EGARCH, RW, APARCH, FIGARCH	EWMA	SV	SV*	GRJ-GARCH*	EGARCH	EWMA
TAIWAN	EWMA*	EWMA*	EWMA	RW	RW	SV*	EWMA*	EWMA*
THAILAND	GRJ-GARCH	FIGARCH*, RW, GARCH, GJR-GARCH, EGARCH, NAGARCH	FIGARCH	RW	SV*	HM*	EWMA	GRJ-GARCH

Note: As in Table 1.

Table 4: Best performing models for 20-day volatility forecasts

	Symmetric Error Statistic				Asymmetric Error Statistic			
	MSE	RMSE	MAE	MAPE	MME-U	MME-O	MLAE	LINEX
ARGENTINA	NAGARCH*	NAGARCH*	NAGARCH*	NAGARCH*	SV*	GARCH*	NAGARCH*	NAGARCH*
BRAZIL	GRJ-GARCH	EWMA*, RW, EGARCH	EWMA	EWMA	SV*	GRJ-GARCH*, GARCH, NAGARCH, EGARCH	RW	GRJ-GARCH
CHILE	GRJ-GARCH	EGARCH*	EGARCH	EGARCH	SV*	GARCH*, HM	FIGARCH	GRJ-GARCH
MEXICO	GRJ-GARCH	RW*, EGARCH, FIGARCH	RW	RW	SV*	GARCH*	RW	GRJ-GARCH
PERU	GRJ-GARCH	EGARCH*	EGARCH	EGARCH	APARCH*	EWMA*	EGARCH	RW
VENEZUELLA	EWMA	EWMA*	EWMA	APARCH	SV*	EGARCH*	EWMA	RW
CZECH	GRJ-GARCH	GRJ-GARCH*, GARCH, APARCH, EGARCH, FIGARCH	GRJ-GARCH	APARCH	SV*	FIGARCH*	FIGARCH	GARCH
HUNGARY	FIGARCH	FIGARCH*, EWMA, RW, APARCH	FIGARCH	APARCH	SV*	FIGARCH*	RW	FIGARCH
POLAND	FIGARCH	EWMA*, APARCH, RW, NAGARCH, FIGARCH	EWMA	EWMA	SV*	GARCH*	APARCH	FIGARCH
RUSSIA	RW	EWMA*	EWMA	EWMA	SV*	GARCH*	APARCH	RW
TURKEY	EWMA	EWMA*	EWMA	EWMA	SV*	GARCH*, GRJ-GARCH	EWMA	EWMA
CHINA	EWMA	EWMA*, APARCH, EGARCH	EWMA	APARCH	SV*	GARCH*, GRJ-GARCH	EWMA	EWMA
INDIA	GARCH	GARCH*	GARCH	FIGARCH	SV*	GARCH*	FIGARCH	GARCH
KOREA	GRJ-GARCH	APARCH*, GRJ-GARCH, RW, NAGARCH, EGARCH, FIGARCH	APARCH	RW	SV*	EGARCH*	NAGARCH	GRJ-GARCH
MALAYSIA	EWMA	EWMA*	EWMA	EWMA	SV*	GRJ-GARCH*, GARCH	EWMA	EWMA
PHILIPPINES	FIGARCH	APARCH*, GJR-GARCH, FIGARCH, EGARCH	APARCH	APARCH	SV*	HM*, FIGARCH, EGARCH, GARCH	APARCH	FIGARCH
SRI LANKA	EGARCH	EWMA*, EGARCH, RW	EWMA	SV	SV*	GARCH*, GRJ-GARCH	RW	EGARCH
TAIWAN	EGARCH	RW*, FIGARCH, EGARCH, EWMA, APARCH	RW	RW	SV*	GRJ-GARCH*, GARCH, FIGARCH	APARCH	EGARCH
THAILAND	FIGARCH	FIGARCH*	FIGARCH	RW	SV*	GARCH*	FIGARCH	GRJ-GARCH

Note: As in Table 1.

Table 5: Best performing models for 60-day volatility forecasts

	Symmetric Error Statistic				Asymmetric Error Statistic			
	MSE	RMSE	MAE	MAPE	MME-U	MME-O	MLAE	LINEX
ARGENTINA	NAGARCH	EWMA*, GRJ-GARCH NAGARCH, FIGARCH EGARCH, GARCH, MA	EWMA	EWMA	SV*	GARCH*	FIGARCH	NAGARCH
BRAZIL	GRJ-GARCH	EGARCH*	EGARCH	EWMA	SV*	GARCH*	EWMA	GRJ-GARCH
CHILE	GRJ-GARCH	EGARCH*	EGARCH	EGARCH	SV*	HM*	EWMA	MA
MEXICO	FIGARCH	FIGARCH*, RW APARCH, EWMA	FIGARCH	RW	SV*	GARCH*, FIGARCH GRJ-GARCH	NAGARCH	FIGARCH
PERU	GRJ-GARCH	GRJ-GARCH*	GRJ-GARCH	EGARCH	APARCH*	EWMA*	EGARCH	MA
VENEZUELLA	EWMA	EWMA*	EWMA	EWMA	SV*	EGARCH*	MA	EGARCH
CZECH	GARCH	GRJ-GARCH*	GRJ-GARCH*	EGARCH*	SV*, APARCH	GARCH*	EGARCH	GARCH
HUNGARY	MA	MA*, EWMA FIGARCH, RW	MA	EWMA	SV*	FIGARCH*	MA	FIGARCH
POLAND	EWMA	EWMA*	EWMA	EWMA	SV*	NAGARCH*	NAGARCH	GARCH
RUSSIA	RW	EWMA*	EWMA	EWMA	SV*	EGARCH*	EWMA	RW
TURKEY	EWMA	EWMA*	EWMA	EWMA	SV*	EGARCH*	EWMA	EWMA
CHINA	EWMA	MA*	MA	MA	APARCH*	FIGARCH*	MA	EWMA
INDIA	GARCH	GARCH *	GARCH	RW	SV*	GARCH*	RW	GARCH
KOREA	GRJ-GARCH	APARCH*, EWMA GRJ-GARCH, RW	APARCH	RW	SV*	GRJ-GARCH*, EGARCH	RW	GRJ-GARCH
MALAYSIA	EWMA	EWMA*	EWMA	EWMA	SV*	GRJ-GARCH	EWMA	EWMA
PHILIPPINES	FIGARCH	MA*, EWMA FIGARCH	MA	EWMA	SV*	HM*	EWMA	FIGARCH
SRI LANKA	HM	EWMA*	EWMA	RW	SV*	EGARCH*	EWMA	HM
TAIWAN	EGARCH	RW*	RW	RW	SV*	FIGARCH*, GARCH GRJ-GARCH	RW	EGARCH
THAILAND	FIGARCH	FIGARCH*	FIGARCH	FIGARCH	SV*	HM*	FIGARCH	FIGARCH

Note: As in Table 1.

Table 6: Best performing models for 120-day volatility forecasts

	Symmetric Error Statistic				Asymmetric Error Statistic			
	MSE	RMSE	MAE	MAPE	MME-U	MME-O	MLAE	LINEX
ARGENTINA	EGARCH*	EGARCH*	EGARCH*	GARCH*,EWMA EGARCH, MA	APARCH*	EGARCH*, GARCH	MA*, GARCH GRJ-GARCH	EGARCH*
BRAZIL	EGARCH	EGARCH*, EWMA	EGARCH	EWMA	SV	GARCH*, EGARCH GRJ-GARCH	MA	GRJ-GARCH
CHILE	MA	MA*, EGARCH, FIGARCH NAGARCH, EWMA	MA	EWMA	SV*	HM*	RW	MA
MEXICO	FIGARCH	FIGARCH*	FIGARCH	FIGARCH	SV	HM*	RW	FIGARCH
PERU	MA	HM*,MA, GRJ-GARCH	HM	EGARCH	EGARCH	MA*	MA	MA
VENEZUELLA	EGARCH	EWMA*	EWMA	EWMA	SV*	EGARCH*	EWMA*	EGARCH
CZECH	GARCH	GARCH*	GARCH*	MA*	APARCH*	GARCH*	HM*, EWMA FIGARCH, MA	FIGARCH
HUNGARY	MA	MA*	MA	MA	APARCH	FIGARCH*	MA	HM
POLAND	MA	MA*	MA	EWMA	SV	GARCH*	MA	MA
RUSSIA	EGARCH	EWMA*	EWMA	EWMA	SV*	FIGARCH*	EWMA	HM
TURKEY	MA*	EWMA*	EWMA*	EWMA*	SV*	FIGARCH*, MA, GARCH	EWMA	MA
CHINA	EWMA*	EWMA*	EWMA*	MA*, EWMA FIGARCH	EWMA	FIGARCH*	FIGARCH*	EWMA*
INDIA	FIGARCH	FIGARCH*	FIGARCH	EGARCH	SV	FIGARCH*	FIGARCH	FIGARCH
KOREA	GRJ-GARCH	RW*, GRJ-GARCH, MA EWMA, APARCH, FIGARCH	RW	RW	SV	GRJ-GARCH*, HM EGARCH	RW*, FIGARCH, MA APARCH, EWMA	GRJ-GARCH
MALAYSIA	EWMA	EWMA*	EWMA	EWMA	SV	MA*	EWMA*	EWMA
PHILIPPINES	MA	MA*	MA	MA	APARCH	HM*	MA	MA
SRI LANKA	HM	HM*	HM	HM	SV*	EGARCH*	EWMA	HM
TAIWAN	EWMA	EWMA*, EGARCH, RW FIGARCH, APARCH	EWMA	EWMA	SV*	GARCH*	RW, APARCH MA, EWMA	EGARCH
THAILAND	FIGARCH	FIGARCH*	FIGARCH	FIGARCH	SV*	HM*	FIGARCH	FIGARCH

Note: As in Table 1.

Table 7: Best performing models for 180-day volatility forecasts

	Symmetric Error Statistic				Asymmetric Error Statistic			
	MSE	RMSE	MAE	MAPE	MME-U	MME-O	MLAE	LINEX
ARGENTINA	EGARCH*	MA*	MA*	MA*	APARCH	EGARCH*	MA*	EGARCH*
BRAZIL	EGARCH	MA*, EWMA EGARCH	MA	EWMA	SV	EGARCH*	MA	MA
CHILE	MA	MA*	MA	MA	APARCH	HM*	FIGARCH	MA
MEXICO	MA	MA*	MA	MA	APARCH	HM*	MA	MA
PERU	MA	MA*	MA	MA	EGARCH	MA*	MA	MA
VENEZUELLA	EGARCH	EWMA*	EWMA	EWMA	SV *	EGARCH*	EWMA	EGARCH
CZECH	GARCH	GARCH*	GARCH*	HM*	APARCH*	GARCH*	EWMA*	FIGARCH
HUNGARY	MA	MA*	MA	MA	APARCH	HM*	FIGARCH	HM
POLAND	MA	MA*	MA	MA	APARCH	GARCH*, HM	EWMA	MA
RUSSIA	HM	EWMA*, MA	EWMA	EWMA	SV	EGARCH*, HM, MA EWMA, FIGARCH	MA	HM
TURKEY	MA*	EWMA*	EWMA*	EWMA*	SV *	FIGARCH*	EWMA	MA
CHINA	EWMA	EWMA*, FIGARCH	EWMA*, FIGARCH	FIGARCH*, EWMA	EWMA	FIGARCH*	FIGARCH	EWMA*
INDIA	FIGARCH	FIGARCH*	FIGARCH	FIGARCH	RW	FIGARCH*	FIGARCH	FIGARCH
KOREA	MA	MA*	MA	MA	SV	GRJ-GARCH*	MA	MA
MALAYSIA	EWMA	EWMA*	EWMA*	EWMA*	SV*	HM*, MA, EGARCH	EWMA	EWMA
PHILIPPINES	MA	MA*	MA*	MA*	MA*	HM*	MA	MA*
SRILANKA	HM*	HM*	HM*	HM	SV	HM*	HM	HM*
TAIWAN	EGARCH	APARCH*, EWMA EGARCH	APARCH	APARCH	SV	GARCH*	FIGARCH	EGARCH
THAILAND	FIGARCH	FIGARCH*	FIGARCH	FIGARCH	SV	HM*	FIGARCH	HM*

Note: As in Table 1.

Table 8: Best performing models for 240-day volatility forecasts

	Symmetric Error Statistic				Asymmetric Error Statistic			
	MSE	RMSE	MAE	MAPE	MME-U	MME-O	MLAE	LINEX
ARGENTINA	EGARCH*	MA*	MA*	MA*	APARCH	EGARCH*	MA	EGARCH*
BRAZIL	MA	MA*	MA	MA	EWMA	GRJ-GARCH*, EGARCH	MA	MA
CHILE	MA	MA*	MA*	MA*	APARCH	HM*	FIGARCH	MA
MEXICO	MA	MA	MA	MA*	APARCH	HM*	EWMA	MA
PERU	MA	MA*	MA*	MA*	MA	MA*	MA*	MA*
VENEZUELLA	EGARCH	EWMA*	EWMA	EWMA	SV*	EGARCH*	EWMA	EGARCH
CZECH	GARCH	GARCH*	GARCH*	HM*	APARCH*	GARCH*	MA*	FIGARCH*
HUNGARY	MA	MA*	MA	MA*	MA	HM*	MA*	HM
POLAND	MA	MA*	MA	MA	MA	GARCH*	MA	MA
RUSSIA	HM	MA*	MA	MA	SV	MA*, HM	APARCH	HM
TURKEY	MA*	MA*	MA*	MA*	SV*	MA*	EWMA	MA*
CHINA	MA*	MA*	MA*	MA*	MA*	FIGARCH*	FIGARCH*	MA*
INDIA	MA	HM*	HM	HM	SV	GARCH*	HM*	MA
KOREA	MA	MA*	MA	MA	SV*	HM*	MA	MA
MALAYSIA	EWMA	EWMA*	EWMA*	EWMA*	SV*	MA*	EWMA*	MA
PHILIPPINES	MA*	MA*	MA*	MA*	SV*	HM*	MA*	MA*
SRILANKA	HM*	HM*	HM*	HM*	SV*	EGARCH*	HM	HM*
TAIWAN	APARCH	APARCH*	APARCH	APARCH	APARCH*	GARCH *	APARCH	EGARCH
THAILAND	FIGARCH	FIGARCH*	FIGARCH*	FIGARCH	SV	HM*	FIGARCH	HM

Note: As in Table 1.

Table 9: Generalization of results based on symmetric error statistic

	The Significant Best Model	MCS set
1-Day	14 markets: SV	3 markets: GARCH family
	4 markets: EWMA	2 markets: GARCH family
	1 market: GARCH family	empty set
5-Day	11 markets : GARCH family	2 markets: EWMA
	7 markets: EWMA	3 markets: GARCH family
	1 markets: RW	1 market: GARCH family
10-Day	8 markets : GARCH family	2 markets: EWMA, 2 markets: RW
	6 markets: EWMA	2 markets: GARCH family, 1 market: RW
	5 markets: RW	2 markets: GARCH family, 2 markets: EWMA
20-Day	9 markets : GARCH family	1 market: EWMA, 1 market: RW
	8 markets: EWMA	4 market: GARCH family, 3 markets: RW
	2 markets: RW	2 markets: GARCH family, 1 market: EWMA
60-Day	8 markets : GARCH family	2 markets: EWMA, 2 markets: RW
	7 markets: EWMA	1 market: GARCH family
	3 markets: MA, 1 market: RW	2 market: EWMA, 2 market: GARCH, 1 market: RW
120-Day	6 markets : GARCH family	1 market: EWMA
	6 markets: EWMA	1 market: GARCH family
	5 markets: MA, 2 market: HM, 1 market: RW	3 market: GARCH family, 2 markets: EWMA, 1 market: MA
180-Day	9 markets : MA	1 market: GARCH family, 1 market: EWMA
	5 markets : EWMA, 1 market: HM	1 market: GARCH family
	4 markets : GARCH family	1 market: EWMA
240-Day	12 markets : MA	empty set
	3 markets : GARCH family	empty set
	2 markets : EWMA, 1 market: HM	empty set

Table 10: Generalization of results based on asymmetric error statistic

	UNDER PREDICTIONS	OVER PREDICTIONS
1-Day	19 markets : SV	12 markets: HM
		6 markets: GARCH family
		1 market : EWMA
5-Day	17 markets : SV	16 markets: GARCH family
	2 countries : RW	2 markets: HM
		1 market : MA
10-Day	18 markets : SV	14 markets: GARCH family
		3 markets: HM
		1 market: EWMA, 1 market: RW
20-Day	18 markets : SV	17 markets: GARCH family
	1 market : APARCH	1 market: HM
		1 market: EWMA
60-Day	17 markets : SV	15 markets: GARCH family
	2 markets : APARCH	2 markets : HM
		1 market: EWMA, 1 market: MA
120-Day	7 markets : SV	13 markets: GARCH family
	2 markets : APARCH	4 markets : HM
		2 markets : MA
180-Day	3 markets : SV	11 markets: GARCH family
	1 market: APARCH	6 markets : HM
	1 market: MA	2 markets: MA
240-Day	6 markets : SV	9 markets: GARCH family
	2 markets: APARCH	5 markets : HM
	1 market: MA	5 markets : MA

