Advances in Management & Applied Economics, vol.3, no.2, 2013, 1-23
ISSN: 1792-7544 (print version), 1792-7552 (online)
Scienpress Ltd

Identifying Taiwan real estate cycle turning points- An application of the multivariate Markov-switching autoregressive Model

Chun-Chang Lee[1], Chih-Min Liang[2] and Hsing-Jung Chou[3]

Abstract

The identification of real estate cycles has always been an important issue in the study of real estate. This paper selected as indicators the Composite Leading Index and Reference Cycle Index regarding the real estate cycles in Taiwan, as they incorporate real estate activities, such as investments, production, transactions and utilization. This paper applied the bivariate Markov-switching autoregressive model (MS-ARX) and the Markov-switching vector auto-regression model (MSVAR) to identify the turning points of real estate cycles. The empirical results indicate that both intercepts and variances were subject to the influence of

[1] Department of Real Estate Management Associate, National Pingtung Institute of Commerce, Taiwan. No. 51, Mingsheng East Road, Pingtung, Taiwan, e-mail: lcc@npic.edu.tw

[2] Department of Public Finance and Tax Administration, National Taipei Institute of College of Business, Taiwan No.321, Sec. 1, Jinan Rd., Zhongzheng District, Taipei City, Taiwan, e-mail: ljimmy.ljimmy@msa.hinet.net

[3] Department of Real Estate Management, National Pingtung Institute of Commerce, Taiwan No.51, Mingsheng East Road, Pingtung, Taiwan, e-mail: portwo@gmail.com

Article Info: *Received* : November 8, 2012. *Revised* : December 11, 2012
Published online : March 15, 2013

unobservable variables. Also, the models with the best fit are MSIAH (2)-VAR (8) with the lags being 8 and L (1) – MSIH (2)-AR (8) with the lags being 8 and the intercepts, coefficients and co-variances are subject to the influence of state variables. Both models showed that the real estate cycles in Taiwan are undergoing contraction rather than expansion. This is in line with the results published by Taiwan Real Estate Research Center. Generally speaking, L(1)-MSIH(2)-ARX(8) and MSIAH(2)-VAR(8) produced rather accurate results in terms of identifying the turning points of real estate cycles in Taiwan.

JEL classification numbers: E27, E32, R30
Keywords: Real estate cycle, Composite leading index, Turning point, Markov-switching vector auto-regression models

1 Introduction

The identification of real estate cycles has always been an important issue in the study of real estate. The turning points of business cycles are an important reference for the government and private-sectors in terms of their economic decisions. However, the Taiwanese government currently determines the dates of peaks and troughs of real estate cycles from ex-post perspectives. This paper constructed its own historical series of Reference Cycles in order to identify the turning points of real estate cycles and hence identify the expansion and contraction periods within Taiwanese real estate cycles.

In order to grasp the turning points of real estate cycles, both government agencies and academic institutions compile various indexes, such as the Real Estate Indicator, the Countermeasure Signal and the Consumer (Manufacturing Business) Confidence Index, hoping to gain an early insight into potential future changes in real estate cycles. Among the existing indicators, the Composite Leading Index has drawn the most attention. The Composite Leading Index is compiled by the Architecture and Building Research Institute under the Ministry of the Interior and Taiwan Real Estate Research Center at National Chengchi University by incorporating information pertaining to investments, production, transactions and utilization in the real estate market.

In the past, considerable literature has discussed the relevance of whether leading indicators help to determine business cycle changes. Diebold and Rudebusch (1991); Estrella and Mishkin (1998) used linear vector autoregression (VAR) models and Probit models and discovered that leading indicators were not helpful in terms of out-of-sample forecasting performance when it comes to the turning points of business cycles. However, Hamilton and Perez-Quiros (1996); Filardo (1994); Camacho and Perez-Quiros (2002) all suggested that if the leading indicators were to be applied to Markov-switching models (MS), this could prove beneficial to the determination of the turning points in business cycles. According to the empirical study by Camacho and Perez-Quiros (2002), the combination of MS models and non-parametric models derived the best out-of-sample forecasting performance. Birchenhall et al. (1999) utilized the logistic classification method and argued that leading indicators were able to accurately determine the turning points of business cycles. Therefore, leading indicators remain highly relevant to the determination of changes in future business cycles and model set-ups. In terms of the studies on the turning points of real estate cycles, Lahiri and Wang (1994) used the two-state MS model to estimate how leading indicators in commerce are used to forecast the turning points of business cycles, and they found the predictability of leading indicators to be quite good. Krystalogianni et al. (2004) used leading indicators as the dependent variable in the Probit model to predict the probabilities of declines or increases in UK capital value. These findings show leading indicators to be useful decision tools in terms of real estate investments. These studies also suggest that leading indicators have good predictability in terms of the turning points of business cycles.

Academics usually adopt quantitative methods and models to identify the status of business cycles. Frequently used quantitative models are the dynamic factor model by Stock and Watson (1989; 1991) and the MS model by Hamilton (1989; 1994). According to Diebold and Rudebusch (1996), in order for a model to be able to depict business cycles, it must be able to capture the co-volatility of the overall variables and the sustainability of the volatility state (such as expansion or contraction) in business cycles. However, the dynamic factor model only considers the co-volatility of variables, while the univariate MS model is only suitable to determine various states and capture the sustainability of states; it is not able to grasp the co-volatility of different variables. In contrast, the multivariate MS meets the above two requirements.

Traditionally, leading indicators have been used to determine the changes of business cycles going forward. They often consist of nothing more than rules of thumb based on past experience. If leading indicators started to descend from a peak, it was determined that the cycle might report a turning point in the following few months. This approach seems too rough when we consider that there is no validation made with any statistical methods. Thus, the main purpose of this study was to establish a model able to identify the changes in real estate cycles by recognizing the turning points and the lasting periods of cycle states. Firstly, this paper considers the bivariate MS model using leading indicators as explanatory variables so as to examine whether these leading indicators are beneficial to the recognition of turning points in real estate cycles and early identification of peaks and troughs within real estate cycles according to the ex-ante information revealed by the leading indicators. Secondly, this paper uses the Hamiltion (1989) model modified by Krolzig (1997) to construct a Markov-switching vector auto-regression model (MSVAR) for analysis purposes. The MSVAR model did not only carry the characteristics of the univariate MS model with its abilities to differentiate cycle states and capture the sustainability of states, but also reflected the co-movement between the time series and economy series, which previous univariate MS models failed to address.

The remainder of this paper is as follows. Section 2 discusses the Research Methodology with an introduction to MS-ARX and MS-VAR. Section 3 is on data sources and processing. Section 4 is on the analysis of the empirical findings, and the last section offers conclusions and recommendations.

2 Research Method

The MS model treats data as it is from different populations. It sets up an autoregression model and controls the switching of states with a Markov chain. The state of the current period subject to the influence of the previous period are taken into account so that the data cross periods have sustainability and relevance. This approach also solves the irregular jumps of states. The MS model proposed by Hamilton (1989) is able to effectively depict the non-linear and asymmetric characteristics emphasized in business cycle theories; therefore, it is widely valued by economists.[4] Various types of the MS models exist to cope with the form variances. Examples include the MS model that allows the switching of intercepts in tandem with the changes of states (Hamilton, 1994), the MS model whose variances switch along with the changes of states (Cai, 1994; Hamilton and Susmel, 1994; Gray, 1996), and multivariate Markov-switching vector auto-regression models (MSVAR) (Krolzig, 1997).

A brief explanation of the MS model is given here as a basis for constructing the empirical model later. The MS model assumes that business cycles are under the influence of a stochastic, non-observable state variable s_t , and $s_t \in \{1, \cdots, M\}$. That variable is generated by a discrete time and state within the Markov stochastic process (the usual assumption being that the Markov process is irreducible and ergodic).[5] Also, the state variable s_t is defined by its switching probability:

$$p_{ij} = \Pr(s_{t+1} = j \mid s_t = i), \quad \sum_{j=1}^{M} p_{ij} = 1 \quad \text{for all} \quad i, j \in \{1, \cdots M\} \tag{1}$$

In a two-state business cycle model, the switching probability of state variable s_t in the one-order Markov chain can be expressed as follows:

$$\begin{aligned} &\Pr ob[s_t = 2 \mid s_{t-1} = 2] = p, \quad \Pr ob[s_t = 1 \mid s_{t-1} = 2] = 1 - p, \\ &\Pr ob[s_t = 1 \mid s_{t-1} = 1] = q, \quad \Pr ob[s_t = 2 \mid s_{t-1} = 1] = 1 - q \end{aligned} \tag{2}$$

The MSM(M)-AR(p)) with P orders and M states and an adjusting form of

---

[4] The asymmetry refers to the inconsistency between the periods of expansion and the periods of contractions within business cycles. Sichel (1993) pointed out deepness and steepness as the two characteristics in the asymmetric volatility of business cycles. In his article, Sichel indicated that deepness refers to the furthest distance volatility jumps from the trend line at troughs as compared to peaks; whereas steepness means that the slope is steeper within troughs than at peaks within business cycles. In other words, when the cycle reaches a trough, the rebound back to a point above the trend line is faster.

[5] If all of the states of Markov chains can be reached from other states, they are called irreducible Markov chains. When the number of states in the Markov chain is limited, irreducible and non-periodical (meaning that the state only occurs at a particular point in time), it can be classified into an ergodic Markov chain. If a Markov chain is ergodic or irreducible under a limited state, the non-periodical chain can reach equilibrium (Chen et al., 1998).

intercepts can be expressed as follows:

$$\Delta y_t = \nu(s_t) + A_1(s_t)\Delta y_{t-1} + \cdots + A_p(s_t)\Delta y_{t-p} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma^2(s_t)) \tag{3}$$

Equation (3) represents that the intercepts are subject to the influence of state variables and therefore, able to reflect the differing growth states due to business fluctuations. Parameters $A_p(s_t)$ and $\sigma^2(s_t)$ signify that the coefficients in the regression and the variance of errors are subject to the influence of state variable s_t. The averages of the MSI (Markov-switching intercepts) model smoothly jump to new levels once the state is switched. The averages of the MSM (Markov-switching mean) model also immediately jump to new levels once the state is switched. MSM models emphasize the changes of the averages; whereas MSI models focus on the changes of intercepts.[6] Since we know that all forms of MS models are developed on the basis of MSI and MSM, if the time series data has more sustainability, an analysis with MSI models is preferred. When the data are more volatile, it is more appropriate to perform any analysis with MSM models. According to the Taiwan Real Estate Research Center, only three complete cycles have occurred over the past three-and-a-half decades (1971-2006) in the Taiwan real estate market. This indicated significantly sustainable data; therefore, the empirical analysis was performed using the MSI models.

$$\Delta y_t = \nu(s_t) + A_1(s_t)\Delta y_{t-1} + \cdots + A_p(s_t)\Delta y_{t-p} + \varepsilon_t, \varepsilon_t \sim N(0, \sigma^2(s_t)) \tag{4}$$

---

[6] When we compared these two models with a two-status lagging period of k, the MSM model becomes subject to the influence of the status variable of the lagging k period, making it necessary to consider 2^{k+1} situations. However, the MSI model can only be influenced by the status variable of the previous one period; therefore, it was only necessary to consider 2 situations. In order to facilitate discerning of which parameters in the model were subject to the influence of status variables, this paper used the following symbols below the MS items. For example, MSM means only averages were under the influence of the status variable for the following reasons: M Markov-switching mean— the averages change over the switching of status; able to reflect the various growth status amid business fluctuations. I Markov-switching intercept—intercepts change during the switching of status; Markov-switching autoregressive parameters—regression coefficients change over the switching of status; H Markov-switching heteroscedasticity—co-variances change over the switching of status; able to reflect the levels of business fluctuations. Please refer to Krolzig (1997) for detailed classifications.

2.1 Bivariate MS-ARX

Diebold and Rudebusch (1996) believed that a model able to depict business cycles must at the same time capture the co-volatility of the overall variables and the sustainability of volatility states exhibited within business cycles.

The univariate MS model can only analyze a single variable and cannot capture the co-volatility of different variables. In order to determine the optimal number of periods that the Composite Leading Index stayed ahead of the Reference Cycle Dates, a bivariate MS-ARX model was established. This allowed the annual growth rates of the Reference Cycle Index of the period to stay under the influence of the annual growth rates of the Reference Cycle Indexes of the previous period and annual growth rates of the Composite Leading Index, an exogenous variable, of the previous periods. We defined the relationship between the growth rate of the Composite Leading Index (ΔL_t) and the growth rate of the Reference Cycle Index (ΔR_t) as follows:

$$\Delta R_t = \beta_0(s_t) + \sum_{i=1}^{p} \beta_i(s_t)\Delta R_{t-i} + \sum_{j=1}^{p} \gamma_j(s_t)\Delta L_{t-j} + \varepsilon_t, \varepsilon_t \sim N(0, \Sigma(s_t)) \qquad (5)$$

The intercept, $\beta_0(s_t)$ of the above model is surely subject to the influence of the state variable s_t. Coefficients β_i and γ_j, as well as covariance Σ are only subject to the influence of state variable s_t under certain conditions. The models established included MSI-ARX, MSIA-ARX, MSIH-ARX and MSIAH-ARX.

2.2 Multivariate Markov-switching vector auto-regression models (MS-VAR)

The univariate MS model proposed by Hamilton (1989) is able to cope with asymmetric and non-linear characteristics of business cycles; however, it obviously fails to reflect the co-movement of time series variables. In order to gain an understanding into whether multiple variables share synchronization in business cycles, this paper used the MS-VAR model to conduct an analysis. As with other state-switching models, the MS-VAR model is a vector auto-repression process able to observe time series vector $\mathrm{y}_{\mathrm{t}} = (\mathrm{y}_{1\mathrm{t}}, \cdots, \mathrm{y}_{\mathrm{kt}})'$. The length of all series was the same, t. There are a total of k series. Some parameters are limited to the non-observable state variable $s_t \in \{1, \cdots, M\}$. There are a total of M states, which are not constant as they change over time. This switching model represents the common factor structure that is non-linear, state-switching and cyclical.

For the purpose of this paper, the number of states was 2, one expansion and the other contraction. This paper assumed that the growth rate of the Reference Cycle Index and the growth of the Composite Leading Index were subject to the same business cycle state, expressed as follows:

$$Y_t = v(s_t) + \sum_{p=1}^{k} A_j(s_t) Y_{t-p} + u_t, \text{ among which, } Y_t = \begin{bmatrix} \Delta R_t \\ \Delta L_t \end{bmatrix}, \ u_t \sim \mathrm{NID}(0, \textstyle\sum(s_t)) \quad (6)$$

The intercepts of (6) and estimation coefficients of lagging periods changed in accordance with states. Not only did estimation coefficients change along with the states, but variances also changed in tandem with states. The MS-VAR estimated parameters with the Expectation-Maximization (EM) algorithm. The calculation process consisted of two steps: (1) the expectation step: the state probability derived from the given initial value; and (2) the maximization step: the application of the derived state probability to compute parameters with the maximum likelihood method, repeated until the parameter estimates converged.

Kim and Nelson (1998); McConnell and Perez-Quiros (2000) divide a business cycle into an expansion state and a contraction state, while others such as Huang (1999); Smith (2000) divide the economy into high-growth, moderate-growth and low-growth states. This paper also attempted to estimate three states using the MS model but found that the changes between states were too frequent. Also, the lack of same state sustainability made it difficult to come up with empirical explanations. All these indicate that when there are more than two states in the MS model, the model cannot properly discern the data characteristics due to over complexity. Therefore, this paper adopted the bivariate, two-state MS model to perform the empirical analysis.

3 Data Collection and Processing

For the empirical analysis, this paper used the Composite Leading Index and the Reference Cycle Index obtained from the quarterly Real Estate Cycle Indicators, compiled by the Architecture and Building Research Institute, Ministry of the Interior and Taiwan Real Estate Research Center. The data period covered the Composite Leading Index and Reference Cycle Index for the real estate market in Taiwan from the first quarter of 1971 through to the fourth quarter of 2006.[7] Adherent to the growth cycle theory, this paper used the growth rates of the indexes to conduct its empirical analysis.

In theory, it is required that all series should be stationary during the construction of a structure undergoing multivariate analysis, and the MS model is no exception. In the application of multivariate MS models, if data are stationary, the adoption of MS-VAR is suggested; if the data are not stationary, it becomes necessary to perform a co-integration test to determine whether the MS-VAR model or the MS-VECM should be used. Figure 1 and Figure 2 illustrate the statistical charts of the processed data. These charts show that in each business cycle, the dates of peaks and troughs of the Composite Leading Index were largely ahead of the turning point dates of the Reference Cycle Index. Therefore, in practice, both government agencies and private institutions often refer to the Composite Leading Index as an important basis to determine the recovery or decline of the business cycle going forward.

As the Composite Leading Index and Reference Cycle Index used in this paper were adjusted from quarterly to annual and the adoption of growth rates also helps to eliminate the time trend, we expected the growth rates of the Composite Leading Index and Reference Cycle Index to be stationary data. This paper performed the unit-root test with an ADF test (augmented Dickey-Fuller). The results confirmed that the original data of these two indexes were stationary after the conversion into the annual growth rates and rejected the null hypothesis that the unit root existed (Table 1).

---

[7] Currently, the Composite Leading Index consists of GDP, M2, the Construction Index on the stock market, Changes in the Outstanding Balance of Loans to Constructions, and the Consumer Price Index. The Reference Cycle Index is comprised of the Raw Land Trading Index, the Construction License Area, the Housing Price Index and the Residential Utilization Rate.

Table 1: ADF Unit Root Test and Analysis

	Original Values of Reference Cycle Index	Annual Growth Rates of Reference Cycle Index	Original Value of Composite Leading Index	Annual Growth Rates of Composite Leading Index
Statistics of ADF test	-0.251881	-5.306137*	-0.369634	-4.746777*
Critical Value (5%)	-1.9421	-1.9422	-1.9421	-1.9422

Note 1: * indicates that the test statistics rejected the null hypothesis that unit roots exist among the variables at the 5% significance level.

Note 2: The variance between the annual growth rates and the critical values of the original data of the two indexes was due to differing numbers of samples.

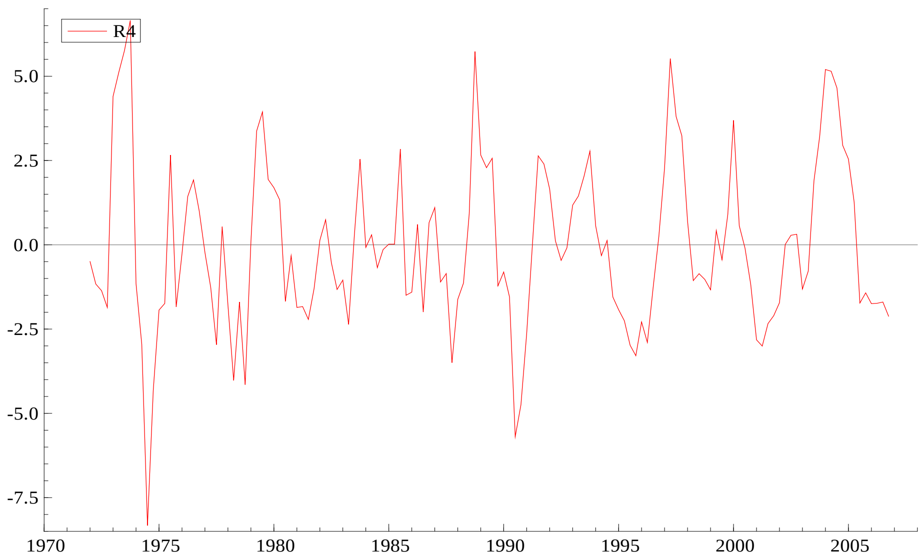

Figure 1: Annual Growth Rates of Reference Cycle Index: 1Q1971-4Q 2006

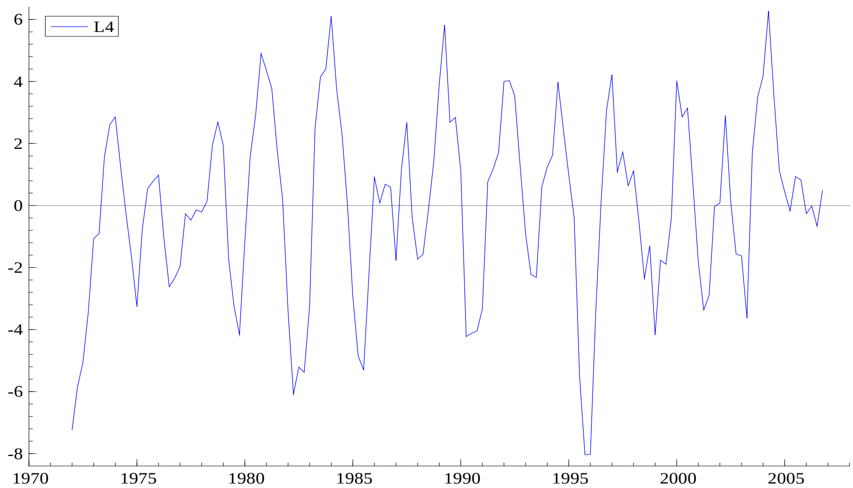

Figure 2 : Annual Growth Rates of Composite Leading Index: 1Q1971-4Q 2006

4 Empirical Analysis

The focus of this paper was not on the embedded economic implications of the estimation coefficients; rather, we hoped to identify the turning points of business cycles and estimate cycle lengths. First, we performed an analysis using the bivariate MS model, and then another using the multivariate MS-VAR model.

4.1 Bivarite MS-ARX

Firstly, in order to determine the optimal number of periods that the Composite Leading Index stayed ahead of the Reference Cycle Dates, we established a bivariate MS-ARX model for analysis, which allowed the annual growth rates of the Reference Cycle Index of the period to stay under the influence of the annual growth rates of the Reference Cycle Indexes for the previous period, and the annual growth rates of the Composite Leading Index, an exogenous variable, for the previous periods. The empirical results (Table 2) indicated that the MSI model chosen with the minimal AIC value lagged the growth rate of the Composite Leading Index by one period and the growth rate of the Reference Cycle Index by eight periods. We denote this as L(1)- MSI(2)-ARX(8).

Table 2 : AIC Values of the Bivariate MS-ARX Model

R(i)	*L* (j) =1	*L* (j) =2	*L* (j) =3	*L* (j) =4
1	4.0804	4.0708	4.0817	4.0869
2	3.9876	4.0775	4.0897	4.095
3	3.9535	3.9528	4.0369	4.0446
4	3.8457	3.8599	3.8706	3.8807
5	3.8293	3.8403	3.8393	3.8469
6	3.8225	3.8359	3.8397	3.8538
7	3.8344	3.8485	3.854	3.8685
8	3.6606**	3.6757*	3.6899*	3.7008*
R(i)	*L* (j) =5	*L* (j) =6	*L* (j) =7	*L* (j) =8
1	4.102	4.1162	4.1134	4.0961
2	4.1102	4.1239	4.1216	4.1083
3	4.0585	4.0735	4.0743	4.0642
4	3.8887	3.9004	3.8965	3.8842
5	3.8588	3.8735	3.8752	3.8595
6	3.8587	3.8656	3.8668	3.8573
7	3.8688	3.8734	3.8719	3.86
8	3.7024*	3.6896*	3.6856*	3.6652*

Note 1: *L* represents the Composite Leading Index and j represents its lagging periods; *R* represents the Reference Cycle Index and i represents its lagging periods.

Note 2: * indicates the MSI-ARX(i) model with the minimal AIC value under *L* (j), the growth rate of the Composite Leading Index of the same lagging period. i=1...8. ** represents the models with the minimal AIC values among all composite models.

We also performed an LR test to validate the linear assumptions of the model on the above chosen L(1)-MSI(2)-ARX(8) and its derivative models. The empirical results shown in Table 3 indicated that the LR statistics of these models were sufficient to reject the null hypotheses $\beta_0(s_1)=\beta_0(s_2)$ and $\sigma(s_1)=\sigma(s_2)$

at the 5% significance level.[8] Finally, we examined whether the residual of these models was compliant with white noise assumptions and found that only L(1)-MSIH(2)-ARX(8) was in compliance.

Table 3 : LR Values of Bivariate MS-ARX Model

model	L(1)-MSI(2) -ARX(8)	L(1)-MSIH(2) -ARX(8)	L(1)-MSIA(2) -ARX(8)	L(1)-MSIAH(2) -ARX(8)	ARX(8)
log-likelihood	-227.5967	-227.0632	-218.4539	-218.3797	-232.3524

According to Table 4, in the L(1)-MSIH(2)-ARX(8) model, the estimated state switching probability is P_{11}=0.7898 and P_{22}=0.7174, indicating that both expansion and contraction states within business cycles have certain sustainability. The probability for a business cycle to be in the expansion state (regime 2) was 42.66%, with an average lasting period of 3.54 quarters; for the contraction state (regime 1), the probability was 57.34%, with an average lasting period of 4.76 quarters. This means that the real estate market in Taiwan remains in a contraction period longer than in an expansion period. This also shows the asymmetric relationship between the contraction period and the expansion period within business cycles. This finding is in line with the results released by Taiwan Real

[8] It is worth noting that some reservations should be taken regarding the inference here. This concerns the same difficulty that the majority of empirical literature is confronted with because in these types of tests, it is impossible to determine the parameters relevant to state switching in the null hypotheses. Therefore, the distribution of the conventional LR statistics is subject to the influence of nuisance parameters and becomes a non-normal distribution. Although Hansen (1992; 1996) came up with solutions, those methods are highly complex and time-consuming, and few empirical studies adopt them. In addition, Garcia (1998) simplified Hansen's methods and simulated the critical values under the set-up of a few specific parameters. However, as the set-up of parameters in this paper differed from those of Garcia, it was not possible to directly apply this solution. It was only feasible to refer to some similar set-ups. In practice, this paper also estimated the AR(8) and MSIH(2)-AR(8) models, with an LR value of 21.0342, as being significantly higher than the critical value of 13.68 at the 5% significance level according to Garcia's (1998) Table 6A. Therefore, it seems reasonable to accept the inference regarding the two states after incorporation of the result of the residual tests, although this is not without reservations.

Estate Research Center.

The empirical findings show that the lengths of different business cycle states were also different. In terms of real GDP cycles, the expansion periods usually lasted longer than contraction periods. However, real estate is a durable good, so its contraction periods last longer than expansion periods. This also explains why it is not appropriate to explain the present business cycle fluctuations according to real GDP cycles.

Also, the L(1)-MSIH(2)-ARX(8) model exhibited that the growth rate of the Reference Cycle Index faced significant influence from the growth rate of the Composite Leading Index from the previous period (t=2.9420). Therefore, the changes to the Composite Leading Index shed light on the prediction of changes regarding the Reference Cycle Index. This empirical finding is consistent with the criteria that government agencies apply to the selection of the leading indicators: the dates of the leading indicator turning points have to stay ahead of the reference cycle dates by an average of 3 to 5 months. In this way, the leading indicators stay ahead of business cycle volatility.

Table 4: Estimates by L(1)-MSIH(2)-ARX(8) Model

	Switching Probability	State Probability	Average Lasting Period
Contraction	0.7898	0.5734	4.76
Expansion	0.7174	0.4266	3.54

Using the MS model, we were able to estimate parameters, the filtering probability and the smoothing probability, and thereby determine the probability of a certain point in time existing in the expansion or contract periods within a cycle. This helped to identify the turning points of business cycles. We referred to the state classification rules proposed by Hamilton (1989) and the smoothing probability as the basis for the determination. The classification rules indicate that in the simple two-state situation, when the smoothing probability $\Pr(s_t = 1 \mid Y_T) > 0.5$, we classified the observed value y_t at time t as Regime 1 (contraction). When $\Pr(s_t = 1 \mid Y_T) < 0.5$, we classified the observed value y_t at time t as Regime 2 (expansion). The peak was located one period before the emergence of the contraction, while trough was located within the last period of the contraction. However, it is worth noting that the smoothing probability is only a guideline, rather than a certain outcome. Therefore, there is no certain rule when it comes to the determination of a business cycle state. We used 0.5 as the watershed because this is a subjective and convenient choice. According to Figure 3, we referred to the smoothing probability to determine the states at different time points of real estate cycles in Taiwan. Although the real estate cycles depicted by L(1)- MSIH(2)-ARX(8) saw frequent switches between states, they retained certain sustainability while in either expansion or contraction periods.

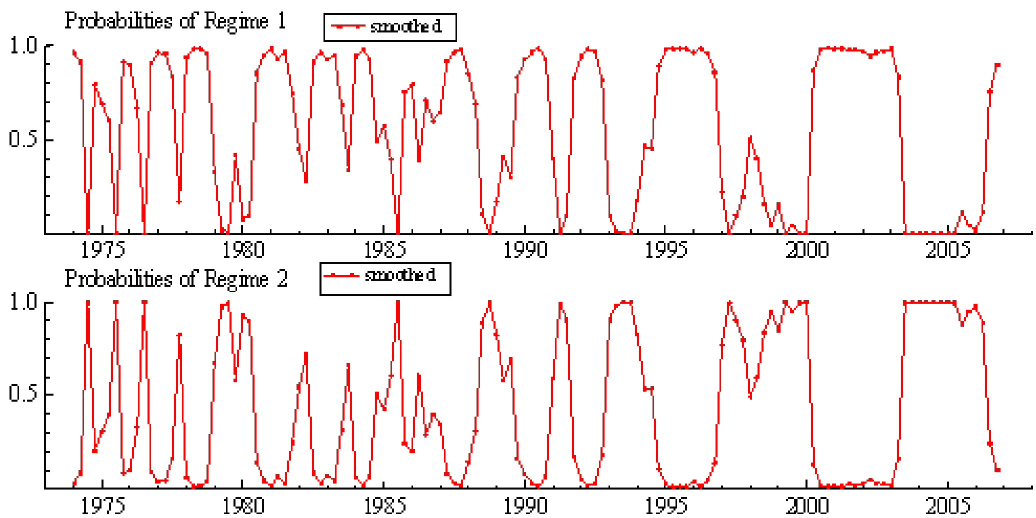

Figure 3: State Probabilities of L(1)- MSIH(2)-ARX(8)

After the determination of cycle states, this paper made comparisons with the peaks and troughs announced by the Taiwan Real Estate Research Center. [9]According to Table 5, the L(1)- MSIH(2)-ARX(8) model was one quarter ahead of the troughs in the fourth quarter of 1978 and the fourth quarter of 1992, and ahead of the peak in the third quarter of 1989, as identified by the Taiwan Real Estate Research Center. However, it was behind the Taiwan Real Estate Research

[9] Table 3.5 illustrates the turning points in L(1)-MSIH(2)-ARX(8). We eliminated the contraction or expansion periods that lasted than less five months by observing the procedures in the identification of turning points suggested by Bry and Boschan (1971).

Center by one and two quarters, respectively, when it came to the recognition of the trough in the second quarter of 1988 and the peak in the second quarter of 1982. It was in line with the Taiwan Real Estate Research Center in terms of the identification of the trough in the second quarter of 2003 and the peak in the third quarter of 1994. Generally speaking, the L(1)- MSIH(2)-ARX(8) model remained fairly close to the Taiwan Real Estate Research Center in terms of the identification of the four troughs and three peaks announced.

Table 5: Dates of Turning Points of Real Estate Cycles in L(1)- MSIH(2)-ARX(8) Model

Troughs			Peaks		
Taiwan Real Estate Research Center	L(1)- MSIH(2)-ARX(8)		Taiwan Real Estate Research Center	L(1)- MSIH(2)-ARX(8)	
1979:Q1	1978:Q4	*-1*	1981:Q4	1982:Q2	*+2*
1988:Q1	1988:Q2	*+1*	1989:Q4	1989:Q3	*1*
1993:Q1	1992:Q4	*-1*	1994:Q3	1994:Q3	*+0*
2003:Q2	2003:Q2	*+0*			

Note 1: "+" represents the lagging date of the Reference Cycle. "-" represents the dates of the leading Reference Cycles.

Note 2: ?: This paper cannot determine whether the one in the second quarter of 2006 is an erroneous judgment because the Taiwan Real Estate Research Center has not yet announced the newest Reference Cycle Dates since the second quarter of 2003.

4.2 MS-VAR

In the selection of models, we first referred to the AIC criteria by choosing from the MSI-VAR models with better fitness and with lag orders ranging from 1 to 8. Table 6 shows that MSI(2)-VAR(8) had the smallest AIC values.

Later, we examined whether MSI(2)-VAR(8) and its derivatives, MSIA(2)-VAR(8), MSIH(2)-VAR(8) and MSIAH(2)-VAR(8), were in compliance with the non-linear and white-noise assumptions. Finally, we chose MSIAH(2)-VAR(8), due to its intercepts, coefficients and co-variances all being subject to the influence of state variables and a lag of 8.

Table 6 : AIC Values of MSI-VAR Models

lag	AIC	lag	AIC
1	8.1731	5	7.543
2	8.0214	6	7.5134
3	7.7792	7	7.5349
4	7.5835	8	7.1556*

Note 1: * represent the models with the minimum AIC values.

According to Figure 4, MSIAH(2)-VAR(8) exhibited that real estate business cycles in Taiwan have certain sustainability. Also, it appeared that the cycles stay longer in the contraction periods as compared to expansion periods.

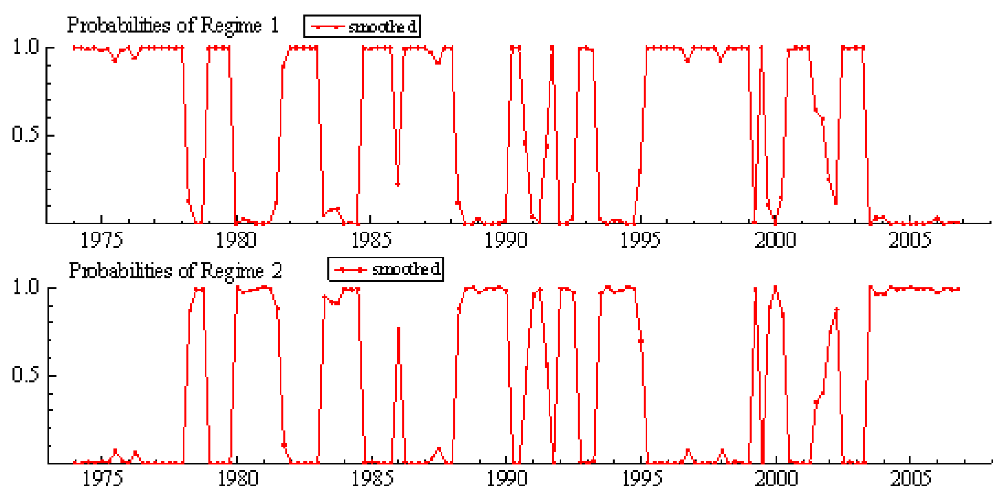

Figure 4 : State Probabilities of MSIAH(2)-VAR(8)

According to Table 7, in the MSIAH(2)-VAR(8) model, the probability of the current period being in contraction and the next period remaining in contraction was P_{11}=0.8340; whereas the probability of the current period being in expansion and the next period remaining in expansion was P_{22}=0.7923. This supports the argument that business cycles are sustained. The probability for the business cycle to be in the contraction status was 55.59%, with an average lasting period of 6.03 quarters; while the probability for the expansion status was 44.41%, with an average lasting period of 4.66 quarters. This means that the real estate market in Taiwan remains in contraction periods longer than in expansion periods, and also shows the asymmetric relationship between contraction periods and expansion periods within business cycles. This finding is in line with the results released by Taiwan Real Estate Research Center.

Table 7: Estimates by MSIAH(2)-ARX(8) Model

	Switching Probability	State Probability	Average Lasting Period
Contraction	0.8340	0.5559	6.03
Expansion	0.7923	0.4441	4.81

Table 8 illustrates the turning points in real estate business cycles identified by MSIAH(2)-VAR(8). We compared the results with the Reference Cycle dates for troughs and peaks announced by the Taiwan Real Estate Research Center. In terms of the identification of troughs, the MSIAH (2)-VAR(8) model was three quarters behind the Taiwan Real Estate Research Center in terms of recognizing the turning point in the fourth quarter of 1979 and one quarter behind in terms of the recognition of the turning point in the second quarter of 1993. Recognition of the turning points in the first quarter of 1988 and the second quarter of 2003 was in line with the Taiwan Real Estate Research Center. In terms of the identification of peaks, the MSIAH (2)-VAR(8) model was ahead of the Taiwan Real Estate Research Center by one in terms of the recognizing the turning point in the third quarter of 1981, but lagged behind by one and two quarters, respectively, in terms of the recognition of the turning points in the first quarter of 1990 and the first quarter of 1995. Except for the large gap in the identification of the trough from the date announced by the Taiwan Real Estate Research Center in the fourth quarter of 1979, the MSIAH (2)-VAR(8) model showed good performance in identifying the turning points of the real estate business cycles in Taiwan, generally speaking.

Table 8: Dates of Turning Points of Real Estate Cycles in MSIAH(2)-VAR(8)

Troughs			Peaks		
Taiwan Real Estate Research	MSIAH(2) -VAR(8)		Taiwan Real Estate Research	MSIAH(2) -VAR(8)	
1979:1	1979:4	+*3*	1981:4	1981:3	-*1*
1988:1	1988:1	+*0*	1989:4	1990:1	+*1*
1993:1	1993:2	+*1*	1994:3	1995:1	+*2*
2003:2	2003:2	+*0*			

Note 1: "+" represents the lagging date of Reference Cycle. "-" represents the dates of the leading Reference Cycles.

In order to compare the performance of MS-ARX and MSIAH(2)-VAR(8) in terms of the recognition of turning points in real estate cycles, we used TPE (turning point error) as the measurement, as follows:

$$TPE = T^{-1}\sum_{t=1}^{T}\left[P\left(s_t = 1 \mid y_T, y_{T-1}, \cdots, y_1\right) - d_t\right]^2 \tag{7}$$

when $d_t = 1$, this indicates that the timing t announced by Taiwan Real Estate Research Center was a contraction period within a business cycle. When $d_t = 0$, it was an expansion period. When the TPW is smaller, the error in forecasting business cycles was also smaller. As Table 9 illustrates, MSIAH(2)-VAR(8) reported a smaller TPE and therefore, it was a better choice for identifying the turning points of real estate business cycles.

Table 9 : Turning Point Error

model	L(1)-MSIH(2)-ARX(8)	MSIAH(2)-VAR(8)
TPE	0.228703	0.202408

5 Conclusions and Suggestions

The MS model applied in this paper is different from ARMA or VAR models commonly seen in past relevant studies. It is able to effectively capture non-linear and asymmetric characters emphasized by business cycle theories. Therefore, it proved suitable for the identification of turning points in business cycles. This model is able to monitor the changes of business cycles and identify the turning points in the real estate cycles accordingly. This helps avoid the pitfall of only predicting business cycle trends while failing to capture the changes in the real estate market. It also partially fills the gap caused by the scarce number of studies on forecasts of real estate "cycles" in Taiwan. It is able to provide pre-warnings, and amends the situation where quarterly ex-post announcements from "Real Estate Cycle Indicators" are not timely. It assists both government agencies and investors to respond before business status changes occur and thereby make correct decisions.

The empirical results indicate that the bivariate MS-ARX model is subject to the influence of the growth rate of the Composite Leading Index in the previous quarter and the growth rate of the Reference Cycle Index during the previous eight quarters.

In the best fit L(1)-MSIH(2)-ARX(8), intercepts and variances change along with the changes in cycle state. In addition, the model is able to discern the relevant characteristics when contraction periods last longer than expansion periods. In terms of the multivariate MS-VAR models, the model with best fit in this study is MSIAH(2)-VAR(8), whose lag is 8 and whose intercepts, variances and coefficients are all subject to the state variable s_t. MSIAH(2)-VAR(8) supports the argument that real estate business cycles in Taiwan are sustained, and that the real estate market in Taiwan remains in contraction periods for longer than expansion periods. The probability of a business cycle being in a contraction state was 55.59%, with an average lasting period of 6.03 quarters, while for an expansion state the probability was 44.41%, with an average lasting period of 4.66 quarters. This indicates that the real estate market in Taiwan remains in contraction periods for longer than expansion periods. This also shows the asymmetric relationship between contraction periods and expansion periods within business cycles. This finding is in line with the results released by the Taiwan Real Estate Research Center. The MSIAH (2)-VAR(8) model reports good performance regarding the identification of the turning points of real estate business cycles in Taiwan, generally speaking.

We suggest that follow-up studies be performed to analyze the important economic factors that affect the real estate market. Secondly, the MS model used in this paper assumes the transference mechanism for the switching probabilities to be constant. It is suggested that further studies analyze the real estate cycles using a time-varying Markov-switching model in which switching probabilities change over time (see Peersman and Smets, 2001), or a duration dependent Markov-switching model in which switching probabilities are dependent on duration (see Pelagatti, 2001). Also, models should be established that can capture deepness and steepness of changes in business cycles.

References

[1] C.R. Birchenhall, H. Jessen, D.R. Osborn and P. Simpson, Predicting U.S. business-cycle regimes, *Journal of Business and Economic Statistics*, **17**(3), (1999), 313-323.

[2] G. Bry and C. Boschan, Cyclical analysis of time series: Selected procedures and computer programs, *National Bureau of Economic Research*, *Technical paper*, New York, (1971).

[3] J. Cai, A Markov model of switching-regime ARCH, *Journal of Business and Economic Statistics*, **12**(3), (1994), 309-316.

[4] M. Camacho and G. Perez-Quiros, This is what the leading indicators lead, *Journal of Applied Econometrics*, **17**(1), (2002), 61-80.

[5] Y.F. Chen, C.Y. Yang and Y.C. Chian, *Operational Research*, Chuan Hua, Taipei, 1998.

[6] F.X. Diebold and G..D. Rudebusch, Forecasting output with the composite leading index: A real-time analysis, *Journal of the American Statistical Association*, **86**(415), (1991), 603-610.

[7] F.X. Diebold and G..D. Rudebusch, Measuring business cycles: A modern perspective, *Review of Economics and Statistics*, **78**(1), (1996), 67-77.

[8] Estrella and F.S. Mishkin, Predicting US recessions: Financial variables as leading indicators, *Review of Economic and Statistics*, **80**(1), (1998), 45-61.

[9] J. Filardo, Business cycle phases and their transitional dynamics, *Journal of Business and Economics Statistics*, **12**(3), (1994), 299-308.

[10] R. Garcia, Asymptotic null distribution of the likelihood ratio test in Markov switching model, *International Economic Review*, **39**(3), (1998), 763-788.

[11] C.W.J. Granger and P. Newbold, Spurious regressions in econometrics, *Journal of Econometrics*, **2**(2), (1974), 111-120.

[12] J.D. Hamilton, A new approach to the economic analysis of non stationary time series and the business cycle, *Econometrica*, **57**(2), (1989), 357-384.

[13] J.D. Hamilton, *Time series analysis*, Princeton University Press, Princeton, New Jersey, 1994.

[14] J.D. Hamilton and G.. Perez-Quiros, What do the leading indicators lead? *Journal of Business*, **69**(1), (1996), 27-49.

[15] J.D. Hamolton and R. Susmel, Autoregressive conditional heteroscedasticity and changes in regime, *Journal of Econometrics*, **64**(1-2), (1994), 307-333.

[16] B.E. Hansen, Erratum: The likelihood ratio test under nonstandard conditions: Testing the markov switching model of GNP, *Journal of Applied Econometrics*, **11**(2), (1992), 195-198.

[17] B.E. Hansen, Inference when a nuisance parameter is not identified under the null hypothesis, *Econometrica*, **64**(2), (1996), 413-430.

[18] C.H. Huang, Phases and characteristics of Taiwan business cycles: A Markov switching analysis, *Taiwan Economic Review*, **27**(2), (1999), 185-213.

[19] C.J. Kim and C.R. Nelson, Business cycle turning points, a new coincident index, and tests of curation dependence based on a dynamic factor model with regime switching, *The Review of Economics and Statistics*, **80**(2), (1998), 188-201.

[20] H.M. Krolzig, *Markov-Switching Vector Autoregressions: Modelling, Statistical Inference and Application to Business Cycle Analysis*, Springer-Verlag, Berlin, 1997.

[21] Krystalogianni, G. Matysiak and S. Tsolacos, Forecasting UK commercial real estate cycle phases with leading indicators: A probit approach, *Applied Economics*, **36**(20), (2004), 2347-2356.

[22] K. Lahiri and J.G.. Wang, Predicting cyclical turning points with leading index in a markov switching model, *Journal of Forecasting*, **13**(3), (1994), 245-263.

[23] M.M. McConnell and G. Perez-Quiros, Output fluctuations in the United States: What has changed since the early 1980's? *American Economic Review*, **90**(5), (2000), 1464-1476.

[24] G. Peersman and F. Smets, Are the effects of monetary policy in the euro area greater in recessions than in booms!, *European Central Bank Working Paper Series*, **52**, (2001), 1-30.

[25] M. Pelagatti, Gibbs sampling for a duration dependent Markov switching model with an application to the U.S. business cycle, *Working Paper*, QD2001/2, Department of Statistics, Università degli Studi Milano-Bicocca, (2001).

[26] S.H. Rao, H.W. Lin and M.Y. Li, Phasing of MS model to analyze cycles of Taiwanese cycles, *Thesis on Economics*, **29**(3), (2001), 297-319.

[27] D.E. Sichel, Business cycle asymmetry: A deeper look, *Economic Inquiry*, **31**(2), (1993), 224-236.

[28] J. H. Smith, *Essays on Persistence in the Stock Market and the Business Cycle, Master thesis*, Queens and University of Technology, Australia, 2000.

[29] J. H. Stock and M.W. Watson, *New indexes of coincident and leading economic indicators*, Cambridge, MIT Press, 351-394, 1989.

[30] J. H. Stock and M.W. Watson, *A probability model of the coincident economic indicators*, Cambridge University Press, 63-89, 1991.