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The Amendment and Empirical Test of Arbitrage Pricing Models

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Abstract

The classical APT model is of the form $r_j - E(r_j) = \beta_j (I - EI) + \varepsilon_j$, where $r_j - E(r_j)$ is the earning deviation (called basic variance-profit) of the security j, I is a common factor. This paper considers the impact on the securities return caused by the skewness and kurtosis of the stock returns distributions, and poses a re-modified the arbitrage pricing model as follows

$$r_j = E(r_j) + \beta_j (I - EI) + \theta_j (I - EI)^2 + \lambda_j (I - EI)^3 + \delta_j (I - EI)^4 + \varepsilon_j$$

Based on the regression analysis method, and the fitting degree, one can arrive at this re-modified model has a more reasonable explanation level for securities pricing.

JEL classification numbers: D46, E17, G11, G17

Keywords: Arbitrage Pricing Models, Skewness, Kurtosis, Empirical Analysis

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1 Introduction

It is known that "Portfolio Selection Theory" by Markowitz [13] has become the beginning of modern portfolio theory. Following this celebrated work, Sharpe [21], Lintner [12], and Mossin [15], respectively, proposed the remarkable capital asset pricing model. Later, Roll [17] recognized that the real market would never be inspected, the capital asset pricing model can not be tested, and questioned subsequently. Ross [19] first proposed arbitrage pricing theory (APT), which greatly simplifies the CAPM assumptions. Recently, we also try to study the related problems and obtain a little corresponding conclusions (see Wu et al [25] studied recently no-arbitrage properties of frictional markets; Yang et al [26] studied the portfolio with transaction costs).

On the empirical test, R. Roll and S. Ross [18] proposed "Empirical study on the arbitrage pricing", and tested and verified firstly the arbitrage pricing theory in the view of empirical validation. Chen, Roll and Ross [2] through empirical research trying to find out important factors affecting stock prices, they assume that there are some specific factors that can explain the covariance between securities, and select a large number of sample data to estimate β and factor prices, the empirical analysis shows that there are four specific factors can explain most of the unexpected change in the covariance between securities:

(1) The term of the yield difference between government bonds;

(2) The different between the credit rating of the bond yield difference;

(3) The inflation rate;

(4) The industrial market growth.

Later, Chen, Roll and Ross [3] take first to filter some impact factors of return on assets, and then to construct a linear model by using these factors as the dependent variable, and use the model to arrive at the APT empirical analysis, they obtain that there are seven kind factors in return on assets having the greater impacts. These seven factors are as follows: inflation, industrial production, interest rate term structure, stock market index, real consumption, risk compensation, crude oil prices.

A. Craig MacKinlay [5] also constructed a model of an empirical to find that the empirical results of CAPM model is deviating from the security market because of lacking the risk factors considered in CAPM, and to be tested positive. Similarly, the multi-factor CAPM model does not explain this problem.

For the multi-factor arbitrage pricing theory and empirical studies, Fama and French [8] take the U.S. stock market during the period 1962-1989 number samples to study the stock returns and market size, market β value, cash flow, price ratio, financial leverage, earnings price ratio, book value ratio, historical sales growth, return on such factors as the historical long-term relationship. Through the study they found that the market β value, financial leverage and earnings price ratio on the explanatory power of stock returns are relatively small, and the book value ratio, market size may explain the majority of stock returns.

Fama and French [9] also did the empirical analysis based on the U.S. stock market during the period 1963-1993 and posed the famous three-factor model. They believed that the stock return premiums can be explained by a market risk premium, size factor premium, and B/M factors of these three factors. Haugen [10] did the similar empirical analysis by using the samples from the 3000 U.S. stock market, and used the six factors selected to test the predictive power of APT. Haugen's time-series test results show that the APT model does have some explanatory power, and probably does help us to understand what factors can affect the rational pricing of risk depending on the covariance of the market rate of return on assets, and the difference between the expected rate of return for assets not reflecting the value of the predictive power.

Antje Berndt and Lulian Obreja [1] proved that the system risk in yield security risks is not the only source, and pointed out the common factor (CMF) including the term of time, credit rating, etc., and gave the regression analysis for these factors based on the ICAPM and KTV pricing model and concluded that in the European stock market the systematic risk could explain 21% return on assets, a common factor 63%. This implied that it is also failed to find the remaining 16% to be given a reasonable explained.

Chui and Wei [4] studied first Asian stock market by using asset pricing theory, they confirmed the Fama-French Three Factor Model in Asian stock market applicability by virtue of the empirical analysis. Through the research of market returns and the market β factor, B/M and the size of the relationship among stocks in Hong Kong, Taiwan, Korea and Malaysia stock markets, they found that the average stock market returns and the correlation coefficient β is small, but the B/M and the correlation among company sizes are large.

ME Drew, T. Naughton, M. Veeraraghavan [6] conducted a study on the Chinese stock market based on the Shanghai A-share market as a sample, and first confirmed the Fama-French three factor model in the Chinese stock market applicability, and found that the B/M and company sizes effected in the Shanghai A-share market are not established, but the scale of the market portfolio and β factors combined can generate a positive excess return.

From these partial statements, we can see that, for the study of arbitrage pricing theory, many researches almost pay their attention to the majority of the common factors. However, the reality show: basis risk factors, kurtosis, skewness and other factors may also impact on the capital gains rate, but to our knowledge, for this aspect of the research is rare. Based on this consideration, this paper intends to "public factors of basis risk" and other factors, the impact of securities gains, the establishment of a hypothetical model with income securities, and the empirical analysis shows that this research is of practical significance. This new model for assessment of capital gains can get a more reasonable explanation.

The organization of this paper is as follows. Section 2 introduces some necessary notations and terminologies. Section 3 gives the modified APT model, and proposes some empirical analysis. The conclusions posed in this paper can be regarded as a natural generalization with respect to APT.

2 Preliminary Notes

APT Model Based on Common Basis Risk Factors

The classical APT model, can be divided into single-factor APT model and the multi-factor APT model.

(1) Single-Factor Models

The so-called single-factor model is the economic environment may affect the security of all factors that benefit as a combined effect of the macro factors (stock market index can be replaced), and assuming the asset return generating process is only affected by this common factor, As a result, the yield of the securities alone generated model can be used to express the following linear equation [21]:

$$r_j = \gamma_j + \beta_j I + \varepsilon_j \tag{2.1}$$

where r_j is the return rate of security j, γ_j is the expected return rate if security j, β_j is the sensitive index for security j with respect to factor I. ε_j is the residual term with $E(\varepsilon_j) = 0$. It is obvious that there is

$$E(r_j) = \gamma_j + \beta_j I \tag{2.2}$$

This shows that there holds

$$r_j = E(r_j) + \beta_j (I - E(I)) + \varepsilon_j \tag{2.3}$$

Formula (2.3) is exactly the basic idea of APT model, it shows clearly the asset formation process in view of APT.

(2) Multi-Factor Models

Assume that there are k-factors $(I_1, I_2, \cdots, I_k)$, and the j-th security was affected with a corresponding sensitivity by $(\beta_{j1}, \cdots, \beta_{jk})$, then the corresponding mutil-factor APT can be obtained with a similar argument as follows [3]:

$$r_j = \lambda_j + \beta_{j1} I_1 + \beta_{j2} I_2 + \cdots + \beta_{jk} I_k + \varepsilon_j \tag{2.4}$$

3 Arbitrage Pricing Theory With Skewness and Kurtosis

3.1 Arbitrage Pricing Models Based on Skewness and Kurtosis

Based on the Markowitz [13] mean-variance criteria, Tobin [24] further derived

the two-fund separation theorem, Sharpe et al [21,22,23] analyzed the economic significance of non-systematic risk, given based on the mean-variance criteria under which the market equilibrium empirical theory that the capital asset pricing model(CAPM). For mean-variance criteria and practical significance in the economic limitations, thus the development of the corresponding portfolio selection theory and methods. According to the Arrow's utility function characteristics, the marginal utility is positive, decreasing and non increasing absolute risk aversion, namely, non-meet, risk avoidance and risk assets for non-bads, and the absolute risk aversion utility function is increasing, the non-bads means that $u''' > 0$ and the investors admit with a slope of preference $\frac{\partial E[u(\tilde{r}_p)]}{\partial m_p^3}>0$ (m_p^3 is of the third moment of $\tilde{r}_p$), Kraus and Litzenberger [11]

arrived at a three-fund separation theorem and the corresponding asset pricing model. However, Markowitz and Levy and other calculations show [14], in the mean-variance effects of uncertainty similar to the occasion, the third-order moments of the approximate degree of improvement is minimal, while the fourth moment while filling the approximation greatly improved. However, the description of changes in securities gains, due to the incompleteness of market information generated by the positive feedback will result in severe liquidity in the market plummeted (Peters [16]), resulting in extreme value distribution of income appears The so-called income effect of fat tails, and describes the income distribution of the fat tail effect (eg, stable distribution) may be only the first moment. Therefore, Samuelson [20], Fama [7] studied such a portfolio based on stable distribution problems. Taking into account the analytical solutions portfolio selection problem the difficulty of solving the actual economic effects and model, Markowitz portfolio model still has the approximate solution to the problem are clear economic implications. In other words, the perspective from the econometric model is a meaningful reconstruction of APT, is also a need to ascertain the will.

As we know, in the classic theory of financial investment, investors generally focus on asset return means and variances, and thus the rate of return on assets, skewness (Skewness) (ie Basis skewness) and kurtosis (ie kurtosis Basis) are not given adequate attention. However, in the portfolio selection in this focus only mean - variance of the model assumption is not entirely consistent and practical! In fact, as noted above, asset returns are usually not strictly symmetrical distribution, and risk averse investors tend to have a preference for positive skewness. Kurtosis as the number of distribution is another important feature, often together determine the skewness and the return distribution is not close to the normal distribution. Thus, the distribution of stock returns, especially in the distribution of skewness can affect the trend of stock returns. Therefore, factors skewness and kurtosis will also affect the rate of return on securities, where we base the difference between common factors skewness, kurtosis Basis Stock Returns also take into account the formation process was proposed based on common factors Basis of skewness and kurtosis of the arbitrage pricing model:

$$r_j = \alpha_j + \beta_j (I - EI) + \theta_j (I - EI)^2 + \lambda_j (I - EI)^3 + \delta_j (I - EI)^4 + \varepsilon_j \quad (3.1.1)$$

Taking the expected value to (3.1.1), we get the following

$$E(r_j) = \alpha_j + \theta_j \sigma_I^2 + \lambda_j \sigma_I^3 + \delta_j \sigma_I^4 \quad (3.1.2)$$

where r_j is the yield of j-th security, I refers to the revenue-generating value of the common factors, α_j mines the impact-value except from the common factor variance and kurtosis; the coefficient factors $\beta_j, \theta_j, \lambda_j, \delta_j$ measure the sensitivity with r_j corresponding to the expected variance $I - EI$, basis risk $(I - EI)^2$, basis skewness $(I - EI)^3$, basis kurtosis $(I - EI)^4$, respectively, and ε_j is the company-specific risk.

This model is more comprehensive and integrated basis of common factors that may affect the poor, which in theory guarantees the rationality of the model. The empirical analysis shows that this model is valid and can be regarded as a natural generalization of the classical APT model.

3.2 The Model Coefficients

We rewrite (3.1.1) in time t as following

$$r_{jt} = \alpha_j + \beta_j (I_t - EI_t) + \theta_j (I_t - EI_t)^2 + \lambda_j (I_t - EI_t)^3 + \delta_j (I_t - EI_t)^4 + \varepsilon_{jt} \quad (3.2.1)$$

For convenience, we make the following substitution:

$$y_t = r_{jt}; a_0 = \alpha_j, a_1 = \beta_j, a_2 = \theta_j, a_3 = \lambda_j, a_4 = \delta_j; \varepsilon_t = \varepsilon_{jt} \sim N(0, \sigma^2)$$

$$x_{t1} = I_t - EI_t, x_{t2} = (I_t - EI_t)^2, x_{t3} = (I_t - EI_t)^3, x_{t4} = (I_t - EI_t)^4$$

Then, the equation (3.2.1) can be transformed into

$$\begin{cases} y_t = a_0 + a_1 x_{t1} + a_2 x_{t2} + a_3 x_{t3} + a_4 x_{t4} + \varepsilon_t, t = 1, 2, \cdots, n, \cdots \\ \varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n, \cdots, i.i.d., \varepsilon_i \sim N(0, \sigma^2) \end{cases} \quad (3.2.2)$$

Considering the following function

$$D(a_0, a_1, a_2, a_3, a_4) = \sum_{t=1}^{n} (y_t - a_0 - a_1 x_{t1} - a_2 x_{t2} - a_3 x_{t3} - a_4 x_{t4})^2 \quad (3.2.3)$$

Let $\hat{a} = (\hat{a}_0, \cdots, \hat{a}_4)$ be a solution to (3.2.3) such that there holds

$$D(\hat{a}_0, \hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4) = \min_{a_0, \cdots, a_4} \sum_{t=1}^{n} (y_t - a_0 - a_1 x_{t1} - a_2 x_{t2} - a_3 x_{t3} - a_4 x_{t4})^2 \quad (3.2.4)$$

It is not hard to derive that the coefficients can be posed by the classical least-square estimation. We omit them here.

3.3 The Empirical Analysis

3.3.1 The Empirical Tests

This section tests mainly the predictive power of the remodified model (3.2.1) constructed by virtue of the skewness and kurtosis of the common factors. For convenience, we rewrite down the formula (3.2.1) again as follows：

$$r_{jt} = \alpha_j + \beta_j (I_t - EI_t) + \theta_j (I_t - EI_t)^2 + \lambda_j (I_t - EI_t)^3 + \delta_j (I_t - EI_t)^4 + \varepsilon_{jt} \quad (3.3.1)$$

We choose firstly 30 stocks in May 2005 to February 2010 the historical prices from China-Stock-Market to give the coefficient of the regression for model (3.3.1).

The selected standard is of using industry classification principle and from Shanghai A shares with: HNGJ, XDDC, YNCT, ZJDC, MGGF, BGGF, WGGF, GJZQ, HXYH, MSYH, XNZQ, ZGYH, ZXZQ, HZYY, HLSW, HRSJ, JLAD, SHYY, TRT, XHC, YNBY, ZLYY, NFHK, SGJT, SHJC, TJG, ZGHK, ZGYY, ZHFZ, ZHHY.

Denote by p_{jt} the price of j-th security at time t, $r_{jt} = \ln(p_{j,t+1} / p_{j,t})$ the rate of return of j-th security. For convenience, we state only eight stocks in the regression coefficient.

We can use MATLAB program to analyze this model and to estimate the coefficient in equation (3.3.1). Here we omit the special computations. Table 3.1 below is the partial regression coefficients.

Here we list only the estimation equations of HNGJ, ZGYH as follows:

HNGJ:

$$E(r_t) = -0.4486 - 1.4774(I_t - EI_t) + 218.9382(I_t - EI_t)^2 - 164.2580(I_t - EI_t)^3 - 1871.4023(I_t - EI_t)^4$$

ZGYH:

$$E(r_t) = 0.4788 - 2.0691(I_t - EI_t) + 18.4652(I_t - EI_t)^2 + 183.3574(I_t - EI_t)^3 + 122.9043(I_t - EI_t)^4$$

The second step is to give the predicting price in term of the modified model (3.3.1). We take the 30 securities in this regression model the prices of securities in 2010 to predict, and the predicted results with the original model and the true price were analyzed.

Choose the same 30-stocks with 186 time points, we use the remodified model

(3.3.1) to predict the price, we find the model (3.3.1) has 89.5% of the time point of the forecast price is closer than the real price. Here is only listed HNGJ, ZGGH, BGGF, ZGYH from May 17, 2010 pushed the price of the 15 point forecast results were compared with the real price as follows:

Comparing the data in Table 3.2-1 and Table 3.2-2, we find the predicting prices with Model (3.3.1) in the tables below the values predicted are close to the basis prices.

As an example, we also list the predictive yielding curve (see Figure 3.1) of BGGF here by using the dates from May 17, 2010 186 time points to push the price curve below(including BGGF's detailed comparison curves (Figure 3.2) with time periods from 5 to carry out).

4 Conclusions

From these showing results in graphs and tables (see Table 3.2-1, Table 3.2-2 and Figure 3.1, Figure 3.2), we can give some conclusions as follows:

1) The empirical results with remodified (3.3.1) imply that the remodified model (3.3.1) indeed yields the security prices being more realistic predictions. On the other hand, in view of the stability, this remodified model (3.3.1) can fully explain the security price.

2) The remodified model considers the basis skewness, kurtosis of the return on a security possible impact based on the basis of "common factors", this tells roughly us that the remodified model can be regarded as a reasonable model in view of theory. In fact, since the asset returns are not strictly symmetrical distribution, and the public factors skewness, kurtosis as an important feature of the distribution rate of return will also impact naturally on the capital gains rate, thus we take, in this paper, the skewness, kurtosis into account and introduce them into remodified model (3.3.1), this will in some extent reduce the model error for predicting the price of securities, and the price predicted will be more realistic

than that before. These conclusions have been derived step by step with the tables and graphs above.

5 Labels of figures and tables

Table 3.1 Model regression coefficient table

	Model-(3.3.1)				
Securities	α_j	β_j	θ_j	λ_j	δ_j
HNGJ	-0.4486	-1.4774	218.9382	-164.2580	-1871.4023
BGGF	0.2076	12.7844	111.3092	-166.9820	-546.8190
WGGF	-0.4564	17.0571	263.8388	-341.1484	-1865.3981
MSYH	0.5702	3.1277	3.16490	52.4667	157.9574
ZGYH	0.4788	-2.0691	18.4652	183.3574	122.9043
NFHK	-0.6200	3.2066	325.6343	69.2560	-1823.1731
ZGGH	-0.0544	-3.0480	325.3103	-134.0727	-2002.5237
ZHHY	0.1724	-5.1347	179.8298	29.9744	-1011.5920

Table 3.2-1Predicted prices and the real price comparison

Date	HNGJ		ZGGH	
	Real Price	Model(3.3.1) price	Real Price	Model (3.3.1)price
20100329	7.3700	7.2727	12.6400	12.4175
20100330	7.3500	7.4033	12.5400	12.6477
20100331	7.3200	7.3832	12.4700	12.5473
20100402	7.3400	7.3531	12.1800	12.4778
20100406	7.3100	7.3730	12.3900	12.1866
20100407	7.2800	7.3430	12.4200	12.3973
20100408	7.1900	7.3127	13.1500	12.4267
20100409	7.2100	7.2221	13.5400	13.1564
20100413	7.2500	7.2424	13.8300	13.5474
20100506	6.5100	7.2783	12.9000	13.8240
20100507	6.3300	6.5389	12.2900	12.9055
20100512	6.2000	6.3585	10.7200	12.2967
20100513	6.2900	6.2277	11.2200	10.7253
20100514	6.3700	6.3181	11.0000	11.2255
20100517	6.1500	6.3982	10.3100	11.0047

Table 3.2-2 Predicted prices and the real price comparison

Date	BGGF		ZGYH	
	Real Price	Model(3.3.1) price	Real Price	Model (3.3.1) price
20100329	7.9800	7.7808	4.3600	4.2100
20100330	8.0100	7.9604	4.3700	4.3394
20100331	7.8800	7.9919	4.2900	4.3493
20100402	8.0500	7.8591	4.2900	4.2699
20100406	8.0900	8.0333	4.2900	4.2695
20100407	7.9500	8.0717	4.2600	4.2696
20100408	7.7500	7.9335	4.2300	4.2397
20100409	7.8000	7.7353	4.2600	4.2097
20100413	7.7100	7.7838	4.2900	4.2397
20100506	6.4400	7.7055	4.0500	4.2682
20100507	6.4300	6.4290	4.0300	4.0304
20100512	6.4900	6.4167	4.0600	4.0108
20100513	6.6400	6.4777	4.1300	4.0405
20100514	6.6100	6.6275	4.0900	4.4401
20100517	6.2300	6.5987	3.9600	4.0702

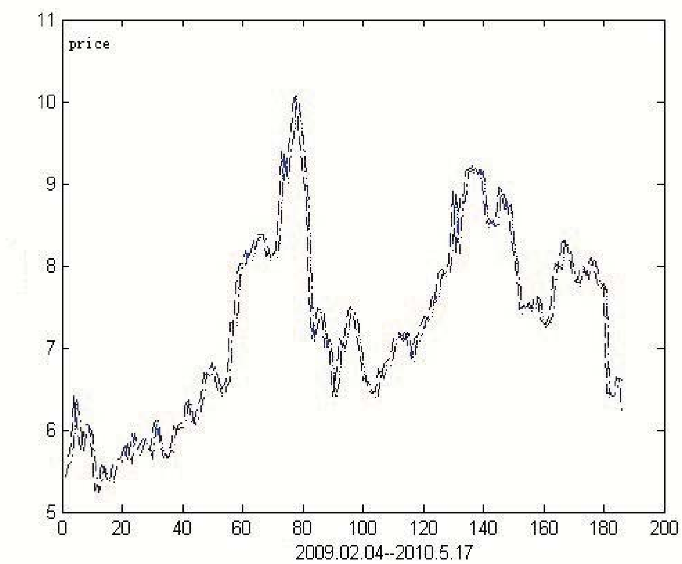

Figure 3.1 The dotted line is the actual price curve, point dashed curve for the price forecasting model of (3.3.1).

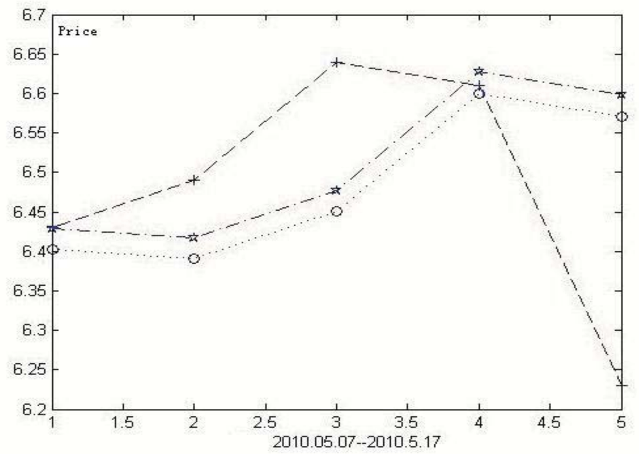

Figure 3.2: BGGF five times the price figure contrasts: The dotted line is the actual price curve, point dashed curve for the price forecasting model of (3.3.1).

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