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An inverse near-field data method for electromagnetic scattering for chiral bodies

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Abstract

We consider a time-harmonic spherical electromagnetic wave that propagates in achiral medium and is incident upon a chiral body. In order to study the inverse scattering problem we formulate the direct transmission electromagnetic problem, we define the corresponding interior transmission problem and study its well-posedness.We use appropriately the Reciprocity Gap Functional and study its properties in order to introduce a new method to identify the unknown chiral scatterer.Such problems arise in medicine, target identification, chemistry and detection of chemical waste deposit.

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1 Introduction

In this work we study a reconstruction method of a chiral body embedded in an achiral background medium using electromagnetic waves. Chirality is a physical phenomenon related to handedness and optical activity and interacts with passing electromagnetic wave by changing its polarization [1]. The micro-structure of chiral materials, either artificial or physical, causes this alteration. For the last past years, chiral materials have been studied in different fields of science such as medicine, chemistry, mathematics and engineering [1],[2],[3]. The fact that scientists approach this research region from different points of view indicates that applications are numerous and discrete. For the mathematical approach and study of electromagnetic scattering due to chiral bodies either in chiral [4], or achiral enviroment [2], there is rich literature.

The issue of detection and determination of geometrical and physical properties of buried objects is the main reason that many reconstruction methods using acoustic, electromagnetic and elastic waves are introduced in the last past years [5],[6],[7].

Nowadays, this kind of inverse scattering problems arises in many fields of science. A few typical examples are target identification, medical imaging and detection of chemical waste deposit.

A typical reconstruction method and maybe the most known one, is Linear Sampling Method which was introduced by Colton and Kirsch. Though this imaging method is not depending on a priori knowledge of geometrical and physical characteristics of the scatterer it requires computation of Greens tensor for the background medium . This is the reason why when it is used for solving problems with buried scatterers [8] the method is ineffective.

In order to overcome these kind of problems Cakoni, Fares and Haddar in [5] introduced a modified version of the classical Linear Sampling Method that was based on the study of an ill-posed integral equation of the first kind including Reciprocity Gap Functional. This new method has a ″more flexible mathematical framework″ than the classical Linear Sampling Method due to the fact that Greens tensor does’t have to be computed [9]. On the other hand the scatterer must be embeded in a bounded, homogeneous region and the tagential components of both the electric and magnetic field have to be known on the boundary of this region.

At this point, it is important to mention that chiral nature of the scatterer affects the structure of the transmission problem and modifies the interior problem that corresponds to achiral scatterer [10]. Consequently, we have to define the corresponding interior transmission problem and study its well-posedness. In general, the inverse scattering problem is related with the interior transmission problem [11] and the existence of transmission eigenvalues. In our case, after defining the interior problem and transmission eigenvalues, by solving the integral equation that contains Reciprocity Gap Functional we get approximating solutions of the interior transmission problem. To state the theorem that reconstructs the scatterer, providing us a boundary characterization, we first need to define the Reciprocity Gap Operator and study its properties. More specifically, in section 2 we formulate the transmission problem. In section 3 we define the interior transmission problem for chiral bodies and study its well-posedness. In section 4 we define the Reciprocity Gap Functional, the Reciprocity Gap Operator and prove required properties for the reconstruction algorithm that we present in section 5.

2 Formulation of the problem

We begin with creating the appropriate mathematical framework in order to study the problem of identifying buried chiral objects. Let D be an isotropic, homogeneous chiral scatterer embedded in an achiral homogeneous background medium Ω. Let ε_1 be the electric permitivity, μ_1 be the magnetic permeability and β_1 the chirality measure. Additionally, ε_0, μ_0 characterize the background medium. The electric permitivity and the magnetic permeability of both of the scatterer and the background medium are real values whereas the chirality measure in general takes complex values.

We consider an auxiliary open surface S where an electric dipole producing the incident electric field is located at $\mathbf{x}_0 \in S$ with polarization $\mathbf{p} \in \mathbb{R}^3$ given by

$$\mathbf{E}_d(\mathbf{x}, \mathbf{x}_0, \mathbf{p}, \kappa) = \frac{i}{\kappa} \nabla \times \nabla \times \mathbf{p} \frac{e^{i\kappa|\mathbf{x}-\mathbf{x}_0|}}{4\pi|\mathbf{x}-\mathbf{x}_0|}.$$

In the interior of the scatterer D the total electric field $\mathbf{E}_1$ satisfies the modified

Helmholtz equation

$$\nabla \times \nabla \times \mathbf{E}_1 - 2\beta_1\gamma_1^2 \nabla \times \mathbf{E}_1 - \gamma_1^2 \mathbf{E}_1 = \mathbf{0} \tag{1}$$

where $\kappa_1^2 = \varepsilon_1\mu_1\omega^2, \quad \gamma_1^2 = \dfrac{\kappa_1^2}{1-\beta_1^2\kappa_1^2}$ and ω is the angular frequency. In the region $\Omega \setminus \overline{D}$ the total electric $\mathbf{E}_0$ field satisfies the Helmholtz equation

$$\nabla \times \nabla \times \mathbf{E}_0 - \kappa_0^2 \mathbf{E}_0 = \mathbf{0} \tag{2}$$

where

$$\kappa_0^2 = \varepsilon_0\mu_0\omega^2.$$

The electric fields $\mathbf{E}_0, \mathbf{E}_1$ satisfy appropriate transmission conditions on the boundary of the scatterer ∂D.

$$\hat{\mathbf{n}} \times \mathbf{E}_0 = \hat{\mathbf{n}} \times \mathbf{E}_1 \quad \text{on} \quad \partial D, \tag{3}$$

$$\hat{\mathbf{n}} \times \nabla \times \mathbf{E}_0 = \mathbf{a}\hat{\mathbf{n}} \times \nabla \times \mathbf{E}_1 - \mathbf{b}\hat{\mathbf{n}} \times \mathbf{E}_1 \quad \text{on} \quad \partial D. \tag{4}$$

where $\mathbf{a} = \dfrac{\varepsilon_1}{\varepsilon_0}\dfrac{\kappa_0^2}{\gamma_1^2}$, $\mathbf{b} = \kappa_0^2\beta_1\dfrac{\varepsilon_1}{\varepsilon_0}$. Finally the scattered electric field satisfy the Silver-Müller radiation condition

$$lim_{\mathbf{r}\to\infty}(\nabla \times \mathbf{E}^s \times \mathbf{x} - i\kappa\mathbf{r}\mathbf{E}^s) = \mathbf{0}, \mathbf{r} = |\mathbf{x}|, \hat{\mathbf{x}} = \frac{\mathbf{x}}{|\mathbf{x}|}. \tag{5}$$

In this work, by using the known measurements of the tagential components of the total electromagnetic wave on the surface of a bounded domain Ω and by the transmission conditions, we get information about the physical and geometrical characteristics of the scatterer. In the following section we state and study the interior transmission problem which is crucial for solving the inverse scattering problem.

3 Interior Transmission Problem

The interior transmission problem and transmission eigenvalues are related to the inverse transmission scattering problem. More precisely, the existence of non-trivial solution of the interior transmission problem implies the existence of an incident wave with no corresponding scattered wave. The values of

κ for which that happens are called transmission eigenvalues. The issue of the interior transmission problem has been studied in detail for the case of electromagnetic waves in inhomogeneous and homogeneous media [10],[12]. For the case of chiral materials in achiral medium we present the following results. The homogeneous interior transmission problem for chiral scatterer is about to find $\mathbf{E} \in H^{inc}(D), \mathbf{E}_1 \in L^2(D)$ such that

$$\nabla \times \mathbf{E} = i\kappa \mathbf{H} \quad \text{in} \quad D, \tag{6}$$

$$\nabla \times \mathbf{H} = -i\kappa \mathbf{E} \quad \text{in} \quad D, \tag{7}$$

$$\nabla \times \mathbf{E}_1 = \frac{\kappa^2 \varepsilon \mu \beta_1}{1 - \kappa^2 \varepsilon \mu \beta_1^2} \mathbf{E}_1 + \frac{i\omega \mu_1}{1 - \kappa^2 \varepsilon \mu \beta_1^2} \mathbf{H}_1 \quad \text{in} \quad D, \tag{8}$$

$$\nabla \times \mathbf{H}_1 = \frac{\kappa^2 \varepsilon \mu \beta_1}{1 - \kappa^2 \varepsilon \mu \beta_1^2} \mathbf{H}_1 - \frac{i\omega \varepsilon_1}{1 - \kappa^2 \varepsilon \mu \beta_1^2} \mathbf{E}_1 \quad \text{in} \quad D \tag{9}$$

and

$$\hat{\mathbf{n}} \times \mathbf{E}_1 = \hat{\mathbf{n}} \times \mathbf{E} \quad \text{on} \quad \partial D, \tag{10}$$

$$\hat{\mathbf{n}} \times \mathbf{H}_1 = \hat{\mathbf{n}} \times \mathbf{H} \quad \text{on} \quad \partial D, \tag{11}$$

where $\kappa_0 = \kappa$ and $\varepsilon = \dfrac{\varepsilon_1}{\varepsilon_0}, \mu = \dfrac{\mu_1}{\mu_0}$ are the corresponding relative electric permitivity and magnetic permeability and

$$H^{inc} = \{\mathbf{W} \in H(curl, D) : \nabla \times \nabla \times \mathbf{W} - \kappa_0^2 \mathbf{W} = \mathbf{0} \text{ in } D\}.$$

Before we continue with the study of the well-posedeness of the interior transmission problem we first have to refer to some basic related concepts such as transmission eigenvalues. The values of κ for which the interior transmission problem has a nontrivial solution are called transmission eigenvalues.Using Bohren's decomposition [1] we rewrite the electromagnetic field

$$\mathbf{E} = \mathbf{Q}_{R0} - i\eta \mathbf{Q}_{L0}, \;\; \mathbf{H} = \frac{1}{i\eta}\mathbf{Q}_{L0} + \mathbf{Q}_{R0},$$

$$\mathbf{E}_1 = \mathbf{Q}_{L1} - i\eta \mathbf{Q}_{R1}, \;\; \mathbf{H}_1 = \frac{1}{i\eta}\mathbf{Q}_{L1} + \mathbf{Q}_{R1}.$$

where $\mathbf{Q}_{R1}, \mathbf{Q}_{L1}$ are the Beltrami fields that correspond to the interior electromagnetic wave and $\mathbf{Q}_{R0}, \mathbf{Q}_{L0}$, the Beltrami fields that correspond to the exterior electromagnetic wave. The latter satisfy the following relationships

$$\nabla \times \mathbf{Q}_{L0} = \kappa_0 \mathbf{Q}_{L0}, \;\; \nabla \times \mathbf{Q}_{R0} = -\kappa_0 \mathbf{Q}_{R0},$$

$$\nabla \times \mathbf{Q}_{L1} = \gamma_{L1}\mathbf{Q}_{L1}, \quad \nabla \times \mathbf{Q}_{R1} = -\gamma_{R1}\mathbf{Q}_{R1},$$

where $\kappa_0 = \kappa$, and $\gamma_{L1} = \dfrac{\kappa_1}{1-\kappa_1\beta_1}$, $\gamma_{R1} = \dfrac{-\kappa_1}{1-\kappa_1\beta_1}$.

Lemma 3.1 *We assume that $Im\gamma_{L1} > 0$ and $Im\gamma_{R1} > 0$.Then the interior transmission problem $(6)-(11)$ admits only the trivial solution.*

Proof. Applying Gauss' theorem for $\mathbf{E}, \overline{\mathbf{H}}$ in D and using Beltrami Fields we get

$$\begin{aligned}
&\int_{\partial D} \hat{\mathbf{n}} \cdot (\mathbf{E} \times \overline{\mathbf{H}}) ds \\
&= \int_D \{\overline{\mathbf{H}} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \overline{\mathbf{H}})\} dx \\
&= \int_D \{(-\frac{1}{i\eta}\overline{\mathbf{Q}}_{L0} + \overline{\mathbf{Q}}_{R0}) \cdot \nabla \times (\mathbf{Q}_{L0} - i\eta\mathbf{Q}_{R0}) \\
&-(\mathbf{Q}_{L0} - i\eta\mathbf{Q}_{R0}) \cdot \nabla \times (-\frac{1}{i\eta}\overline{\mathbf{Q}}_{L0} + \overline{\mathbf{Q}}_{R0})\} dx \\
&= \int_D 4i\kappa Im\mathbf{Q}_{L0} \cdot \overline{\mathbf{Q}}_{R0} dx
\end{aligned}$$

Now,applying Gauss' theorem for $\mathbf{E}_1, \overline{\mathbf{H}}_1$ in D and using Beltrami Fields we get

$$\begin{aligned}
&\int_{\partial D} \hat{\mathbf{n}} \cdot (\mathbf{E}_1 \times \overline{\mathbf{H}}_1) ds \\
&= \int_D \{\overline{\mathbf{H}}_1 \cdot (\nabla \times \mathbf{E}_1) - \mathbf{E}_1 \cdot (\nabla \times \overline{\mathbf{H}}_1)\} dx \\
&= \int_D \{(-\frac{1}{i\eta}\overline{\mathbf{Q}}_{L1} + \overline{\mathbf{Q}}_{R1}) \cdot \nabla \times (\mathbf{Q}_{L1} - i\eta\mathbf{Q}_{R1}) \\
&-(\mathbf{Q}_{L1} - i\eta\mathbf{Q}_{R1}) \cdot \nabla \times (-\frac{1}{i\eta}\overline{\mathbf{Q}}_{L1} + \overline{\mathbf{Q}}_{R1})\} dx \\
&= \int_D \{-2Im\gamma_{L1}\frac{|\mathbf{Q}_{L1}|^2}{\eta} - 2\eta Im\gamma_{R1}|\mathbf{Q}_{R1}|^2 + 2iIm[\mathbf{Q}_{L1} \cdot \overline{\mathbf{Q}}_{R1}(\gamma_{L1} + \overline{\gamma}_{R1})]\} dx
\end{aligned}$$

By using the transmission condition (3) we get

$$\begin{aligned}
&\int_D \{[2Im\gamma_{L1}\frac{|\mathbf{Q}_{L1}|^2}{\eta} + 2\eta Im\gamma_{R1}|\mathbf{Q}_{R1}|^2]\} dx \\
&+i\int_D \{[-2Im\mathbf{Q}_{L1} \cdot \overline{\mathbf{Q}}_{R1}(\gamma_{L1} + \overline{\gamma}_{R1}) + 4\kappa Im\mathbf{Q}_{L0} \cdot \overline{\mathbf{Q}}_{R0}\} dx = \mathbf{0}
\end{aligned}$$

From the real part of the above equation and by taking into account that $Im\gamma_{L1} > 0$ and $Im\gamma_{R1} > 0$ we get $\mathbf{Q}_{R1} = \mathbf{Q}_{L1} = \mathbf{0}$ which also implies that $\mathbf{E} = \mathbf{H} = \mathbf{0}$.Using unique continuation principle [11] and taking into account the transmission conditions $(10), (11)$ we get $\hat{\mathbf{n}} \times \mathbf{E} = \hat{\mathbf{n}} \times \mathbf{E}_1 = \mathbf{0},\ \hat{\mathbf{n}} \times \mathbf{H} = \hat{\mathbf{n}} \times \mathbf{H}_1 = \mathbf{0}$.Finally, by using Stratton-Chu formula [11] we get

$$\mathbf{E}_1 = \mathbf{E} = \mathbf{0}, \quad \mathbf{H}_1 = \mathbf{H} = \mathbf{0}.$$

4 Reciprocity Gap Functional

First we define the function spaces that we will use later.

$$\begin{aligned}
&H(curl, \Omega) = \{\mathbf{u} \in L^2(D) : \nabla \times \mathbf{u} \in L^2(D)\}, \\
&\mathbb{H}(\Omega) = \{\mathbf{W} \in H(curl, \Omega) : \nabla \times \nabla \times \mathbf{W} - {\kappa_0}^2 \mathbf{W} = \mathbf{0}\}, \\
&L_t^2(S) = \{\mathbf{u} \in S : \hat{\mathbf{n}} \cdot \mathbf{u} = \mathbf{0} \text{ on } \ S\}.
\end{aligned}$$

Let $\mathbf{E}, \mathbf{H}$ be the total electric and magnetic field,respectively.For every $\mathbf{W} \in \mathbb{H}(\Omega)$ we define the Reciprocity Gap Functional by

$$\mathcal{R}(\mathbf{E}, \mathbf{W}) = \int_{\partial\Omega} [(\hat{\mathbf{n}} \times \mathbf{E}) \cdot (\nabla \times \mathbf{W}) - (\hat{\mathbf{n}} \times \mathbf{W}) \cdot (\nabla \times \mathbf{E})] ds \tag{12}$$

In order to study the inverse scattering problem we need to treat the Reciprocity Gap Functional in the sense of an operator.Hence, we define the Reciprocity Gap Operator $R : \mathbb{H}(\Omega) \to L_t^2(S)$ by

$$R(\mathbf{W})(\mathbf{x}_0) = \mathcal{R}(\mathbf{E}(\cdot, \mathbf{x}_0, \mathbf{p}(\mathbf{x}_0)), \mathbf{W})\mathbf{p}(\mathbf{x}_0) \tag{13}$$

for all point sources $\mathbf{x}_0 \in S$ and polarization $\mathbf{p}$ tangent to S at $\mathbf{x}_0$.Subsequently,we study the properties of the Reciprocity Gap Operator.

Lemma 4.1 *We assume that κ is not a transmission eigenvalue for D.The opeator $R : \mathbb{H}(\Omega) \to L_t^2(S)$ defined by* (13) *is injective.*

Proof.
We assume that $R\mathbf{W} = \mathbf{0}$ and we will prove that $\mathbf{W} = \mathbf{0}$.

$$R\mathbf{W} = \mathbf{0} \text{ or } \mathcal{R}(\mathbf{E}_0, \mathbf{W})\mathbf{p}(\mathbf{x}_0) = \mathbf{0} \text{ or } \mathcal{R}(\mathbf{E}_0, \mathbf{W}) = \mathbf{0},$$

$$\int_{\partial\Omega} [(\hat{\mathbf{n}} \times \mathbf{E}_0) \cdot (\nabla \times \mathbf{W}) - (\hat{\mathbf{n}} \times \mathbf{W}) \cdot (\nabla \times \mathbf{E}_0)] ds = \mathbf{0}.$$

Using 2nd Green's theorem for $\mathbf{E}_0, \mathbf{W}$ in $\Omega \setminus \bar{D}$ and the transmission conditions (3),(4) we get

$$\begin{aligned}
&\int_{\partial\Omega} [(\hat{\mathbf{n}} \times \mathbf{E}_0) \cdot (\nabla \times \mathbf{W}) - (\hat{\mathbf{n}} \times \mathbf{W}) \cdot (\nabla \times \mathbf{E}_0)] ds \\
&= \int_{\partial D} [(\hat{\mathbf{n}} \times \mathbf{E}_0) \cdot (\nabla \times \mathbf{W}) - (\hat{\mathbf{n}} \times \mathbf{W}) \cdot (\nabla \times \mathbf{E}_0)] ds \\
&= \int_{\partial D} [(\hat{\mathbf{n}} \times \mathbf{E}_1) \cdot (\nabla \times \mathbf{W}) + \mathbf{W} \cdot (\mathbf{a}\hat{\mathbf{n}} \times \nabla \times \mathbf{E}_1 - \mathbf{b}\hat{\mathbf{n}} \times \mathbf{E}_1)] ds
\end{aligned}$$

Now, let $(\tilde{\mathbf{E}}_0, \tilde{\mathbf{E}}_1)$ be the unique solution to the following transmission problem

$$\begin{aligned}
&\nabla \times \nabla \times \tilde{\mathbf{E}}_1 - 2\beta_1\gamma_1^2 \nabla \times \tilde{\mathbf{E}}_1 - \gamma_1^2 \tilde{\mathbf{E}}_1 = \mathbf{0} \quad \text{in} \quad D, \\
&\nabla \times \nabla \times \tilde{\mathbf{E}}_0 - \kappa_0^2 \tilde{\mathbf{E}}_0 = \mathbf{0} \quad \text{in} \quad \mathbb{R}^3 \setminus \bar{D}, \\
&\hat{\mathbf{n}} \times (\tilde{\mathbf{E}}_0 + W) = \hat{\mathbf{n}} \times \tilde{\mathbf{E}}_1 \quad \text{on} \quad \partial D, \\
&\hat{\mathbf{n}} \times \nabla \times (\tilde{\mathbf{E}}_0 + W) = \frac{\varepsilon_1}{\varepsilon_0} \frac{\kappa_0^2}{\gamma_1^2} \hat{\mathbf{n}} \times \nabla \times \tilde{\mathbf{E}}_1 - \kappa_0^2 \beta \frac{\varepsilon_1}{\varepsilon_0} \hat{\mathbf{n}} \times \tilde{\mathbf{E}}_1 \quad \text{on} \quad \partial D.
\end{aligned}$$

Then,

$$\begin{aligned}
&\int_{\partial D} [-\mathbf{E}_1 \cdot (\hat{\mathbf{n}} \times \nabla \times \mathbf{W}) + (\hat{\mathbf{n}} \times \mathbf{W}) \cdot (-\mathbf{a}(\nabla \times \mathbf{E}_1) + \mathbf{b}\mathbf{E}_1)] ds \\
&= \int_{\partial D} [(\hat{\mathbf{n}} \times \tilde{\mathbf{E}}_0) \cdot (\nabla \times \mathbf{E}_0) - (\hat{\mathbf{n}} \times \mathbf{E}_0) \cdot (\nabla \times \tilde{\mathbf{E}}_0)] ds \\
&+ \int_{\partial D} [(\hat{\mathbf{n}} \times \mathbf{E}_1)(\mathbf{a}\nabla \times \tilde{\mathbf{E}}_1 - \mathbf{b}\tilde{\mathbf{E}}_1) - (\hat{\mathbf{n}} \times \tilde{\mathbf{E}}_1)(\mathbf{a}\nabla \times \mathbf{E}_1 - \mathbf{b}\mathbf{E}_1)] ds
\end{aligned}$$

Using appropriately 2nd Green's and Gauss' theorem for $\tilde{\mathbf{E}}_0, \mathbf{E}_0$ and $\tilde{\mathbf{E}}_1, \mathbf{E}_1$ in D we get

$$\int_{\partial D} [(\hat{\mathbf{n}} \times \tilde{\mathbf{E}}_0) \cdot (\nabla \times \mathbf{E}_0) - (\hat{\mathbf{n}} \times \mathbf{E}_0) \cdot (\nabla \times \tilde{\mathbf{E}}_0)] ds = \mathbf{0}$$

But $\mathbf{E}_0 = \mathbf{E}^s + \mathbb{G}p$ where $\mathbb{G}$ is the background Green's function.

$$\begin{aligned}
\mathbf{0} &= \int_{\partial D} [(\hat{\mathbf{n}} \times \tilde{\mathbf{E}}_0) \cdot (\nabla \times \mathbf{E}_0) - (\hat{\mathbf{n}} \times \mathbf{E}_0) \cdot (\nabla \times \tilde{\mathbf{E}}_0)] ds \\
&= \int_{\partial D} [(\hat{\mathbf{n}} \times \tilde{\mathbf{E}}_0) \cdot (\nabla \times (\mathbf{E}^s + \mathbb{G}p)) - (\hat{\mathbf{n}} \times (\mathbf{E}^s + \mathbb{G}p)) \cdot (\nabla \times \tilde{\mathbf{E}}_0)] ds \\
&= \mathbf{p} \cdot \tilde{\mathbf{E}}(\mathbf{x}_0)
\end{aligned}$$

Due to $\mathbf{p}$ being arbitary polarization on the tangent plane to S at $\mathbf{x}_0$, $\hat{\mathbf{n}} \times \tilde{\mathbf{E}}_0(\mathbf{x}_0) = \mathbf{0}$ for all $\mathbf{x}_0 \in S$ as .

Using uniqueness theorem for perfect conductor and unique continuation principle [11] we have that $\tilde{\mathbf{E}}_0 = \mathbf{0}$ inside and outside D. Therefore, $(\mathbf{E}_0, \mathbf{E}_1)$ is a solution to the interior transmission problem.Finally, from the assumption that κ is not a transmission eigenvalue we conclude to $\mathbf{W} = \mathbf{0}$.

Lemma 4.2 *We assume that κ is not a transmission eigenvalue for D.The opeator $R : \mathbb{H}(\Omega) \to L_t^2(S)$ defined by (13) has dense range.*

Proof.
We assume that $(R\mathbf{W}, \mathbf{c})_{L_t^2(S)} = \mathbf{0}$ for all $\mathbf{W} \in \mathbb{H}(\Omega)$ and we will prove that $\mathbf{c} = \mathbf{0}$.

$$(R\mathbf{W}, \mathbf{c})_{L_t^2(S)} = \int_S (R\mathbf{W}) \cdot \mathbf{c} ds = \int_S \mathcal{R}(\mathbf{E}_0(\cdot, \mathbf{x}_0, \mathbf{p}(\mathbf{x}_0)), \mathbf{W})\mathbf{p}(\mathbf{x}_0) \cdot \mathbf{c}\ ds(\mathbf{x}_0)$$
$$= \int_S \mathcal{R}(\boldsymbol{E}_0(\cdot, \mathbf{x}_0, \alpha(\mathbf{x}_0)), \mathbf{W}) ds(\mathbf{x}_0) = \mathcal{R}(\boldsymbol{E}_0, \mathbf{W}),$$

where
$$\boldsymbol{E}_0(\mathbf{x}) = \int_S \mathbf{E}_0(\mathbf{x}, \mathbf{x}_0, \alpha(\mathbf{x}_0)) ds(\mathbf{x}_0), \quad \alpha = (\mathbf{c} \cdot \mathbf{p})\mathbf{p}.$$

We define $\boldsymbol{E}_1(\mathbf{x}) = \int_S \mathbf{E}_1(\mathbf{x}, \mathbf{x}_0, \alpha(\mathbf{x}_0)) ds(\mathbf{x}_0)$ and one can easily see that $\boldsymbol{E}_0, \boldsymbol{E}_1$ satisfy the transmission problem (1)-(5).Using 2nd Green's and Gauss' theorem for $\boldsymbol{E}_0, \mathbf{W}$ in $\Omega \setminus \bar{D}$ and the transmission conditions $(3)-(4)$ we get

$$\int_{\partial\Omega} [(\hat{\mathbf{n}} \times \boldsymbol{E}_0) \cdot (\nabla \times \mathbf{W}) - (\hat{\mathbf{n}} \times \mathbf{W}) \cdot (\nabla \times \boldsymbol{E}_0)] ds$$
$$= \int_{\partial D} [(\hat{\mathbf{n}} \times \boldsymbol{E}_0) \cdot (\nabla \times \mathbf{W}) - (\hat{\mathbf{n}} \times \mathbf{W}) \cdot (\nabla \times \boldsymbol{E}_0)] ds$$
$$= \int_{\partial D} [(\hat{\mathbf{n}} \times \boldsymbol{E}_1) \cdot (\nabla \times \mathbf{W}) + \mathbf{W} \cdot (\mathbf{a}\hat{\mathbf{n}} \times \nabla \times \boldsymbol{E}_1 - \mathbf{b}\mathbf{n} \times \boldsymbol{E}_1)] ds$$

By using 2nd Green's and Gauss' theorem we conclude to

$$\mathbf{0} = \int_D \mathbf{W} \cdot [\kappa^2(\frac{\varepsilon_1}{\varepsilon_0} - 1)\boldsymbol{E}_1 + (\kappa^2\beta_1\frac{\varepsilon_1}{\varepsilon_0})(\nabla \times \boldsymbol{E}_1)] dx$$
$$+ \int_D \nabla \times \mathbf{W} \cdot [(\kappa^2\frac{\varepsilon_1}{\varepsilon_0})\boldsymbol{E}_1 - (\frac{\varepsilon_1}{\varepsilon_0}\frac{\kappa^2}{\gamma_1^2} - 1)(\nabla \times \boldsymbol{E}_1)] dx.$$

Now let H be the space of the vector spherical wave functions

$$H = span\{\mathbf{M}_n^m, \mathbf{N}_n^m, n = 1, ..., \ m = -n, ..., n\}$$

H is dense in, [11]

$$H^{inc} = \{\mathbf{W} \in H(curl, D) : \nabla \times \nabla \times \mathbf{W} - \kappa_0^2 \mathbf{W} = \mathbf{0} \text{ in } D\}.$$

So,$\boldsymbol{E}_1 = \mathbf{0}$, $\nabla \times \boldsymbol{E}_1 = \mathbf{0}$ in D and also $\hat{\mathbf{n}} \times \boldsymbol{E}_1 = \hat{\mathbf{n}} \times \nabla \times \boldsymbol{E}_1 = \mathbf{0}$ on ∂D which implies that $\hat{\mathbf{n}} \times \boldsymbol{E}_0 = \hat{\mathbf{n}} \times \nabla \times \boldsymbol{E}_0 = \mathbf{0}$ on ∂D.
Furthermore, using unique continuation principle [11] we can see that $\boldsymbol{E}_0 = \mathbf{0}$ inside and outside D bounded by S.From the fact that

$$\boldsymbol{E}_0(\mathbf{x}) = \int_S (\boldsymbol{E}_0^s(\mathbf{x}, \mathbf{x}_0, \alpha(\mathbf{x}_0)) + \mathbb{G}(\mathbf{x}, \mathbf{x}_0)\alpha(\mathbf{x}_0))ds(\mathbf{x}_0), \ \ \alpha = (\mathbf{c} \cdot \mathbf{p})\mathbf{p}$$

we can see that $\hat{\mathbf{n}} \times \boldsymbol{E}_0$ is continuous across S. Also, from the uniqueness theorem for the exterior problem for the perfect conductor [11] $\boldsymbol{E}_0 = \mathbf{0}$ outside S. From the jump relations for the vector potential across S [11] we get

$$\alpha = (\mathbf{c} \cdot \mathbf{p})\mathbf{p} = \mathbf{0},$$

$$\mathbf{c} = \mathbf{0}.$$

5 Reconstruction of the chiral body

At this point we recall the definition of the Herglotz functions.
Let $\upsilon_{\mathbf{g}}$ is a Herglotz wave function defined by

$$\upsilon_{\mathbf{g}}(\mathbf{x}) = \int_\Omega \mathbf{g}(\hat{\mathbf{d}}) e^{ik\mathbf{x}\cdot\hat{\mathbf{d}}} ds(\hat{\mathbf{d}}), \mathbf{g} \in L^2(\Omega) \tag{14}$$

and S^2 is the unit sphere and κ is the transmission eigenvalue with non-trivial corresponding solution of the interior transmission problem. An electromagnetic Herglotz pair

$$\mathbf{E}(\mathbf{r}) = \int_\Omega \mathbf{g}(\hat{\mathbf{d}}) e^{ik\mathbf{x}\cdot\hat{\mathbf{d}}} ds(\hat{\mathbf{d}}), \ \ \mathbf{H}(\mathbf{r}) = \frac{1}{ik} \nabla \times \mathbf{E}(\mathbf{r}) \tag{15}$$

where the square integrable tagential field $\mathbf{g} \in L^2(\Omega)$ is the kernel of the electromagnetic pair $\mathbf{E}, \mathbf{H}$.Our aim is to investigate the solvability of the equation

$$\mathcal{R}(\mathbf{E}, \mathcal{H}\mathbf{g}) = \mathcal{R}(\mathbf{E}_d, \mathbf{W}) \tag{16}$$

for every $\mathbf{W} \in \mathbb{H}(\Omega)$
where $\mathbb{H}(\Omega) = \{\mathbf{W} \in H(curl, \Omega) : \nabla \times \nabla \times \mathbf{W} - \kappa_0^2 \mathbf{W} = \mathbf{0}\}$.
In particular, we want to get approximating solutions of the equation (16) by the parametric family of Herglotz wave functions. As a norm-depending reconstructing approach, this method provides us a boundary characterization of the chiral body for a set of sampling points $\mathbf{z}$. Finally, the proof follows the steps as in [9].

Theorem 5.1 *Assume that the interior transmission problem is well-posed. Then*

1. *For* $\mathbf{z} \in D$ *and a given* $\epsilon > 0$ *there exists a* $\mathbf{g}_\mathbf{z}^\epsilon \in L_t^2(S^2)$ *such that*

$$||\mathcal{R}(\mathbf{E}, \mathcal{H}\mathbf{g}_\mathbf{z}^\epsilon) - \mathcal{R}(\mathbf{E}, \mathbf{E}_d(\cdot, \mathbf{z}, \mathbf{q}, \kappa))||_{L_t^2(S)} < \epsilon$$

and the corresponding electric Herglotz wave function $\mathcal{H}\mathbf{g}_\mathbf{z}^\epsilon$ *converges to the solution of the interior transmission problem, as* $\epsilon \to 0$.

2. *For a fixed* $\epsilon > 0$, *we have that*

$$lim_{\mathbf{z}\to\partial D}||\mathcal{H}\mathbf{g}_\mathbf{z}^\epsilon||_{H^{inc}} = \infty \ \ and \ \ lim_{\mathbf{z}\to\partial D}||\mathbf{g}_\mathbf{z}^\epsilon||_{L_t^2(S)} = \infty.$$

3. *For* $\mathbf{z} \in \mathbb{R}^3 \setminus \bar{D}$ *and a given* $\epsilon > 0$ *every* $\mathbf{g}_\mathbf{z}^\epsilon \in L_t^2(S^2)$ *that satisfies*

$$||\mathcal{R}(\mathbf{E}, \mathcal{H}\mathbf{g}_\mathbf{z}^\epsilon) - \mathcal{R}(\mathbf{E}, \mathbf{E}_\mathbf{d}(\cdot, \mathbf{z}, \mathbf{q}, \kappa))||_{L_t^2(S)} < \epsilon$$

is such that

$$lim_{\mathbf{z}\to\partial D}||\mathcal{H}\mathbf{g}_\mathbf{z}^\epsilon||_{H^{inc}} = \infty \ \ and \ \ lim_{\mathbf{z}\to\partial D}||\mathbf{g}_\mathbf{z}^\epsilon||_{L_t^2(S)} = \infty.$$

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