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Analysis of the Preference Shift of Customer Brand Selection among Multiple Genres of Jewelry/Accessory and Its Matrix Structure

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Abstract

It is often observed that consumers select upper class brand when they buy next time. Suppose that former buying data and current buying data are gathered. Also suppose that upper brand is located upper in the variable array. Then transition matrix becomes upper triangular matrix under the supposition that former buying variables are set input and current buying variables are set output. Takeyasu et al. (2007) analyzed the brand selection and its matrix structure before. In that paper, products of one genre are analyzed. In this paper, brand selection among multiple genre of Jewelry/Accessory purchasing case and its matrix structure are analyzed.

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For example, there is a case that customer selects bracelet or earrings besides selecting upper brand of necklace she already has. There may be also the case that customer selects lower brand to seek suitable price when she already has higher ranked brand. Then the transition matrix contains items in lower triangular part. Utilizing purchase history record of jewelry / accessory on-line shopping (Necklace/Pendant, Pierced earrings, Ring, Bracelet/Bangle) over three years, above matrix structure is investigated and confirmed. Analyzing such structure provides useful applications. Unless planner for products does not notice its brand position whether it is upper or lower than another products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach enables to make effective marketing plan and/or establishing new brand.

Keywords: brand selection; matrix structure; brand position; jewelry; accessory

1 Introduction

It is often observed that consumers select upper class brand when they buy next time after they are bored to use current brand. Suppose that former buying data and current buying data are gathered. Also suppose that upper brand is located upper in the variable array. Then the transition matrix becomes upper triangular matrix under the supposition that former buying variables are set input and current buying variables are set output. The analysis of the brand selection in the same brand group is analyzed by Takeyasu et al.[6].

In this paper, we expand this scheme to products of multiple genres and examine them by utilizing the case of jewelry/accessory purchasing. For example, we consider the case of necklace. If she is accustomed to use necklace, she would buy higher priced necklace. On the other hand, she may buy bracelet or earring for her total coordination in fashion. Hearing from the retailer, both can be seen in selecting upper class brand and selecting another genre product. There may be also the case that customer selects lower brand to seek suitable price when she already has higher ranked brand. Then the transition matrix contains items in lower triangular part. Utilizing purchase history record of jewelry / accessory on-line shopping (Necklace/Pendant, Pierced earrings, Ring, Bracelet/Bangle) over three years, above matrix structure is investigated and confirmed.

Therefore, this analysis is very meaningful for the practical use, which occurs actually. If transition matrix is identified, we can make various analysis using it and s-step forecasting can be executed. Unless planners for products notice its brand position whether it is upper or lower than other products, matrix structure makes it possible to identify those by calculating consumers' activities for brand selection. Thus, this proposed approach makes it effective to execute marketing plan and/or establish new brand.

Quantitative analysis concerning brand selection has been executed by Yamanaka[5], Takahashi et al.[4]. Yamanaka[5] examined purchasing process by

Markov Transition Probability with the input of advertising expense. Takahashi et al.[4] made analysis by the Brand Selection Probability model using logistics distribution.

Takeyasu et al.[6] analyzed the preference shift of customer brand selection for a single brand group. In this paper, we try to expand this scheme to products of multiple genres, and the analysis for the jewelry/accessory purchasing case is executed. Actually, this scheme can often be seen. Such research is quite a new one.

Hereinafter, matrix structure for a single brand group is clarified for the selection of brand in section 2. Expansion to multiple brand selection is executed and analyzed in section 3. s-step forecasting is stated in section 4. Numerical calculation is executed in section 5. Remarks are stated in section 6.

2 Brand selection and its matrix structure

2.1 Upper shift of Brand selection

Now, suppose that x is the most upper class brand, y is the second upper class brand, and z is the lowest class brand.

Consumer's behavior of selecting brand might be $z \to y$, $y \to x$, $z \to x$ etc. $x \to z$ might be few.

Suppose that x is current buying variable, and x_b is previous buying variable.

Shift to x is executed from x_b, y_b, or z_b.

Therefore, x is stated in the following equation. a_{ij} represents transition probability from j-th to i-th brand.

$$x = a_{11}x_b + a_{12}y_b + a_{13}z_b$$

Similarly,

$$y = a_{22}y_b + a_{23}z_b$$

And

$$z = a_{33}z_b$$

These are re-written as follows.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} \tag{1}$$

Set

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}, \qquad \mathbf{X_b} = \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

then, $\mathbf{X}$ is represented as follows.

$$\mathbf{X} = \mathbf{A}\mathbf{X_b} \tag{2}$$

Here,

$\mathbf{X} \in \mathbf{R}^3, \mathbf{A} \in \mathbf{R}^{3\times 3}, \mathbf{X_b} \in \mathbf{R}^3$

$\mathbf{A}$ is an upper triangular matrix.

To examine this, generating following data, which are all consisted by the data in which transition is made from lower brand to upper brand,

$$\mathbf{X^i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \cdots \qquad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{3}$$

$$\mathbf{X_b^i} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \cdots \qquad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

$$i = 1 \quad , \qquad 2 \qquad \cdots \qquad N$$

parameter can be estimated using least square method.

Suppose

$$\mathbf{X}^i = \mathbf{A}\mathbf{X}_{\mathbf{b}}^i + \boldsymbol{\varepsilon}^i \tag{5}$$

where

$$\varepsilon^i = \begin{pmatrix} \varepsilon_1^i \\ \varepsilon_2^i \\ \varepsilon_3^i \end{pmatrix} \qquad i = 1,2,\cdots,N$$

and minimize following J

$$J = \sum_{i=1}^{N} \boldsymbol{\varepsilon}^{iT} \boldsymbol{\varepsilon}^i \to Min \tag{6}$$

$\hat{\mathbf{A}}$ which is an estimated value of $\mathbf{A}$ is obtained as follows.

$$\hat{\mathbf{A}} = \left(\sum_{i=1}^{N} \mathbf{X}^i \mathbf{X}_{\mathbf{b}}^{iT} \right) \left(\sum_{i=1}^{N} \mathbf{X}_{\mathbf{b}}^i \mathbf{X}_{\mathbf{b}}^{iT} \right)^{-1} \tag{7}$$

In the data group which are all consisted by the data in which transition is made from lower brand to upper brand, estimated value $\hat{\mathbf{A}}$ should be upper triangular matrix.

If following data which shift to lower brand are added only a few in equation (3) and (4),

$$\mathbf{X}^i = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \mathbf{X}_{\mathbf{b}}^i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\hat{\mathbf{A}}$ would contain minute items in the lower part of triangle.

2.2 Sorting brand ranking by re-arranging row

In a general data, variables may not be in order as x, y, z. In that case, large and small value lie scattered in $\hat{\mathbf{A}}$. But re-arranging this, we can set in order by shifting row. The large value parts are gathered in upper triangular matrix, and the small value parts are gathered in lower triangular matrix.

$\hat{\mathbf{A}}$

$\hat{\mathbf{A}}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \\ \varepsilon & \varepsilon & \bigcirc \end{pmatrix} \xleftarrow{\text{Shifting row}} \begin{pmatrix} z \\ x \\ y \end{pmatrix} \begin{pmatrix} \varepsilon & \varepsilon & \bigcirc \\ \bigcirc & \bigcirc & \bigcirc \\ \varepsilon & \bigcirc & \bigcirc \end{pmatrix} \tag{8}$$

2.3 Matrix structure under the case skipping intermediate class brand is skipped

It is often observed that some consumers select the most upper class brand from the most lower class brand and skip selecting the intermediate class brand.

We suppose v, w, x, y, z brands (suppose they are laid from upper position to lower position as $v > w > x > y > z$).

In the above case, selection shifts would be:

$$v \leftarrow z$$

$$v \leftarrow y$$

Suppose they do not shift to y, x, w from z, to x, w from y, and to w from x, then Matrix structure would be as follows.

$$\begin{pmatrix} v \\ w \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & a_{22} & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 \\ 0 & 0 & 0 & 0 & a_{55} \end{pmatrix} \begin{pmatrix} v_b \\ w_b \\ x_b \\ y_b \\ z_b \end{pmatrix} \tag{9}$$

We confirm this by numerical example in section 5.

3 Expansion of the model to multiple genre products

Expanding Eq.(2) to multiple genre products, we obtain following equations. First of all, we state the generalized model of Eq.(2).

$$\mathbf{X} = \mathbf{A}\mathbf{X_b} \tag{10}$$

Where

$$\mathbf{X} = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^p \end{pmatrix}, \quad \mathbf{X_b} = \begin{pmatrix} x_b^1 \\ x_b^2 \\ \vdots \\ x_b^p \end{pmatrix} \tag{11}$$

Here

$$\mathbf{X} \in \mathbf{R}^p, \mathbf{A} \in \mathbf{R}^{p \times p}, \mathbf{X_b} \in \mathbf{R}^p. \tag{12}$$

If the brand selection is executed towards upper class, then $\mathbf{A}$ becomes as follows.

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ 0 & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & a_{pp} \end{pmatrix} \tag{13}$$

Expanding above equations to products of 3 genres, we obtain following equations.

$$\begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \\ \mathbf{Z} \end{pmatrix} = \begin{pmatrix} \mathbf{A}^{11} & \mathbf{A}^{12} & \mathbf{A}^{13} \\ \mathbf{A}^{21} & \mathbf{A}^{22} & \mathbf{A}^{23} \\ \mathbf{A}^{31} & \mathbf{A}^{32} & \mathbf{A}^{33} \end{pmatrix} \begin{pmatrix} \mathbf{X_b} \\ \mathbf{Y_b} \\ \mathbf{Z_b} \end{pmatrix} \tag{14}$$

Where

$$\mathbf{X} = \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^p \end{pmatrix}, \quad \mathbf{X_b} = \begin{pmatrix} x_b^1 \\ x_b^2 \\ \vdots \\ x_b^p \end{pmatrix}, \quad \mathbf{Y} = \begin{pmatrix} y^1 \\ y^2 \\ \vdots \\ y^q \end{pmatrix}, \quad \mathbf{Y_b} = \begin{pmatrix} y_b^1 \\ y_b^2 \\ \vdots \\ y_b^q \end{pmatrix}, \quad \mathbf{Z} = \begin{pmatrix} z^1 \\ z^2 \\ \vdots \\ z^r \end{pmatrix}, \quad \mathbf{Z_b} = \begin{pmatrix} z_b^1 \\ z_b^2 \\ \vdots \\ z_b^r \end{pmatrix} \tag{15}$$

$$\mathbf{A}^{11}=\begin{pmatrix} a_{11}^{11} & a_{12}^{11} & \cdots & a_{1p}^{11} \\ a_{21}^{11} & a_{22}^{11} & \cdots & a_{2p}^{11} \\ \vdots & \vdots & & \vdots \\ a_{p1}^{11} & a_{p2}^{11} & \cdots & a_{pp}^{11} \end{pmatrix}, \qquad \mathbf{A}^{12}=\begin{pmatrix} a_{11}^{12} & a_{12}^{12} & \cdots & a_{1q}^{12} \\ a_{21}^{12} & a_{22}^{12} & \cdots & a_{2q}^{12} \\ \vdots & \vdots & & \vdots \\ a_{p1}^{12} & a_{p2}^{12} & \cdots & a_{pq}^{12} \end{pmatrix},$$

$$\mathbf{A}^{13}=\begin{pmatrix} a_{11}^{13} & a_{12}^{13} & \cdots & a_{1r}^{13} \\ a_{21}^{13} & a_{22}^{13} & \cdots & a_{2r}^{13} \\ \vdots & \vdots & & \vdots \\ a_{p1}^{13} & a_{p2}^{13} & \cdots & a_{pr}^{13} \end{pmatrix},$$

$$\mathbf{A}^{21}=\begin{pmatrix} a_{11}^{21} & a_{12}^{21} & \cdots & a_{1p}^{21} \\ a_{21}^{21} & a_{22}^{21} & \cdots & a_{2p}^{21} \\ \vdots & \vdots & & \vdots \\ a_{q1}^{21} & a_{q2}^{21} & \cdots & a_{qp}^{21} \end{pmatrix}, \qquad \mathbf{A}^{22}=\begin{pmatrix} a_{11}^{22} & a_{12}^{22} & \cdots & a_{1q}^{22} \\ a_{21}^{22} & a_{22}^{22} & \cdots & a_{2q}^{22} \\ \vdots & \vdots & & \vdots \\ a_{q1}^{22} & a_{q2}^{22} & \cdots & a_{qq}^{22} \end{pmatrix},$$

$$\mathbf{A}^{23}=\begin{pmatrix} a_{11}^{23} & a_{12}^{23} & \cdots & a_{1r}^{23} \\ a_{21}^{23} & a_{22}^{23} & \cdots & a_{2r}^{23} \\ \vdots & \vdots & & \vdots \\ a_{q1}^{23} & a_{q2}^{23} & \cdots & a_{qr}^{23} \end{pmatrix}$$

$$\mathbf{A}^{31}=\begin{pmatrix} a_{11}^{31} & a_{12}^{31} & \cdots & a_{1p}^{31} \\ a_{21}^{31} & a_{22}^{31} & \cdots & a_{2p}^{31} \\ \vdots & \vdots & & \vdots \\ a_{r1}^{31} & a_{r2}^{31} & \cdots & a_{rp}^{31} \end{pmatrix}, \qquad \mathbf{A}^{32}=\begin{pmatrix} a_{11}^{32} & a_{12}^{32} & \cdots & a_{1q}^{32} \\ a_{21}^{32} & a_{22}^{32} & \cdots & a_{2q}^{32} \\ \vdots & \vdots & & \vdots \\ a_{r1}^{32} & a_{r2}^{32} & \cdots & a_{rq}^{32} \end{pmatrix},$$

$$\mathbf{A}^{33}=\begin{pmatrix} a_{11}^{33} & a_{12}^{33} & \cdots & a_{1r}^{33} \\ a_{21}^{33} & a_{22}^{33} & \cdots & a_{2r}^{33} \\ \vdots & \vdots & & \vdots \\ a_{r1}^{33} & a_{r2}^{33} & \cdots & a_{rr}^{33} \end{pmatrix}$$

$\mathrm{X}\in \mathrm{R}^{p}$, $\mathrm{X_b}\in \mathrm{R}^{p}$, $\mathrm{Y}\in \mathrm{R}^{q}$, $\mathrm{Y_b}\in \mathrm{R}^{q}$, $\mathrm{Z}\in \mathrm{R}^{r}$, $\mathrm{Z_b}\in \mathrm{R}^{r}$, $\mathrm{A}^{11}\in \mathrm{R}^{p\times p}$, $\mathrm{A}^{12}\in \mathrm{R}^{p\times q}$, $\mathrm{A}^{13}\in \mathrm{R}^{p\times r}$, $\mathrm{A}^{21}\in \mathrm{R}^{q\times p}$, $\mathrm{A}^{22}\in \mathrm{R}^{q\times q}$, $\mathrm{A}^{23}\in \mathrm{R}^{q\times r}$, $\mathrm{A}^{31}\in \mathrm{R}^{r\times p}$, $\mathrm{A}^{32}\in \mathrm{R}^{r\times q}$, $\mathrm{A}^{33}\in \mathrm{R}^{r\times r}$

Re-writing Eq.(14) as :

$$\mathbf{W} = \mathbf{A}\mathbf{W}_{\mathbf{b}} \tag{17}$$

then, transition matrix $\mathbf{A}$ is derived as follows in the same way with Eq.(7).

$$\hat{\mathbf{A}} = \left(\sum_{i=1}^{N} \mathbf{W}^{i} \mathbf{W}_{\mathbf{b}}^{iT} \right) \left(\sum_{i=1}^{N} \mathbf{W}_{\mathbf{b}}^{i} \mathbf{W}_{\mathbf{b}}^{iT} \right)^{-1} \tag{18}$$

Here,

$$\mathbf{W} = \begin{pmatrix} \mathbf{X} \\ \mathbf{Y} \\ \mathbf{Z} \end{pmatrix}, \quad \mathbf{W}_{\mathbf{b}} = \begin{pmatrix} \mathbf{X}_{\mathbf{b}} \\ \mathbf{Y}_{\mathbf{b}} \\ \mathbf{Z}_{\mathbf{b}} \end{pmatrix} \tag{19}$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}^{11} & \mathbf{A}^{12} & \mathbf{A}^{13} \\ \mathbf{A}^{21} & \mathbf{A}^{22} & \mathbf{A}^{23} \\ \mathbf{A}^{31} & \mathbf{A}^{32} & \mathbf{A}^{33} \end{pmatrix} \tag{20}$$

$$\mathbf{W}^{i} = \mathbf{A}\mathbf{W}_{\mathbf{b}}^{i} + \boldsymbol{\varepsilon}^{i} \qquad i = 1,2,\cdots,N \tag{21}$$

$$\boldsymbol{\varepsilon}^{i} = \begin{pmatrix} \varepsilon_{1}^{i} \\ \vdots \\ \varepsilon_{p}^{i} \\ \varepsilon_{p+1}^{i} \\ \vdots \\ \varepsilon_{p+q}^{i} \\ \varepsilon_{p+q+1}^{i} \\ \vdots \\ \varepsilon_{p+q+r}^{i} \end{pmatrix} \qquad i = 1,2,\cdots,N \tag{22}$$

If the brand selection is executed towards upper class brand in the same genre, transition matrix, for example $\mathbf{A}_{11}, \mathbf{A}_{22}, \mathbf{A}_{33}$, become upper triangular matrix as seen in 2. Suppose $\mathbf{X}$ as necklace, $\mathbf{Y}$ as pierced earring and $\mathbf{Z}$ as ring. If we only see $\mathbf{Z}$, we can examine whether there is an upper brand shift in $\mathbf{A}_{33}$. But there is a case that brand selection is executed towards other genre products. There occurs brand selection shift from a certain brand level of $\mathbf{Z}$ to a certain brand level of $\mathbf{X}$ or $\mathbf{Y}$. For example, suppose there are five levels in each $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ and their levels include from bottom to top brand level. In that case, if there is a brand selection shift from the middle brand level in $\mathbf{Z}$ to another genre product, we can obtain interesting result by examining how the brand selection shift is executed toward the same level or upper or lower level of another genre product. If we can see the trend of brand selection shift, we can foresee the brand selection shift towards another genre brand. Retailer can utilize the result of this to make effective marketing plan. We confirm this by the numerical example in 5.

Next, we examine the case in brand groups. Matrices are composed by Block Matrix.

3.1 Brand shift group — In the case of two groups

Suppose brand selection shifts from Necklace class to Ring. Selection of jewelry/accessory is executed in a group and brand shift is considered to be done from group to group. Suppose brand groups at time n are as follows.

$\mathbf{X}$ consists of p varieties of goods, and $\mathbf{Y}$ consists of q varieties of goods.

$$\mathbf{X_n} = \begin{pmatrix} x_1^n \\ x_2^n \\ \vdots \\ x_p^n \end{pmatrix}, \quad \mathbf{Y_n} = \begin{pmatrix} y_1^n \\ y_2^n \\ \vdots \\ y_q^n \end{pmatrix} \quad \begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}, & \mathbf{A_{12}} \\ \mathbf{0}, & \mathbf{A_{22}} \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-1}} \\ \mathbf{Y_{n-1}} \end{pmatrix} \tag{23}$$

Here, $\mathbf{X_n} \in \mathbf{R}^p \ (n = 1,2,\cdots)$, $\mathbf{Y_n} \in \mathbf{R}^q \ (n = 1,2,\cdots)$, $\mathbf{A_{11}} \in \mathbf{R}^{p \times p}$, $\mathbf{A_{12}} \in \mathbf{R}^{p \times q}$, $\mathbf{A_{22}} \in \mathbf{R}^{q \times q}$. Make one more step of shift, then we obtain the following equation.

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^2, & \mathbf{A_{11}}\mathbf{A_{12}} + \mathbf{A_{12}}\mathbf{A_{22}} \\ \mathbf{0}, & \mathbf{A_{22}}^2 \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-2}} \\ \mathbf{Y_{n-2}} \end{pmatrix} \tag{24}$$

Make one more step of shift again, then we obtain the following equation.

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^3, & \mathbf{A_{11}}^2\mathbf{A_{12}} + \mathbf{A_{11}}\mathbf{A_{12}}\mathbf{A_{22}} + \mathbf{A_{12}}\mathbf{A_{22}}^2 \\ \mathbf{0}, & \mathbf{A_{22}}^3 \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-3}} \\ \mathbf{Y_{n-3}} \end{pmatrix} \tag{25}$$

Similarly,

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^4, & \mathbf{A_{11}}^3\mathbf{A_{12}} + \mathbf{A_{11}}^2\mathbf{A_{12}}\mathbf{A_{22}} + \mathbf{A_{11}}\mathbf{A_{12}}\mathbf{A_{22}}^2 + \mathbf{A_{12}}\mathbf{A_{22}}^3 \\ \mathbf{0}, & \mathbf{A_{22}}^4 \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-4}} \\ \mathbf{Y_{n-4}} \end{pmatrix} \tag{26}$$

$$\begin{pmatrix} \mathrm{X_n} \\ \mathrm{Y_n} \end{pmatrix} = \begin{pmatrix} \mathrm{A_{11}}^5, & \mathrm{A_{11}}^4\mathrm{A_{12}} + \mathrm{A_{11}}^3\mathrm{A_{12}}\mathrm{A_{22}} + \mathrm{A_{11}}^2\mathrm{A_{12}}\mathrm{A_{22}}^2 + \mathrm{A_{11}}\mathrm{A_{12}}\mathrm{A_{22}}^3 + \mathrm{A_{12}}\mathrm{A_{22}}^4 \\ 0, & \mathrm{A_{22}}^5 \end{pmatrix} \begin{pmatrix} \mathrm{X_{n-5}} \\ \mathrm{Y_{n-5}} \end{pmatrix} \tag{27}$$

Finally, we get generalized equation for s-step shift as follows.

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^s, & \mathbf{A_{11}}^{s-1}\mathbf{A_{12}} + \sum_{k=2}^{s-1} \mathbf{A_{11}}^{s-k}\mathbf{A_{12}}\mathbf{A_{22}}^{k-1} + \mathbf{A_{12}}\mathbf{A_{22}}^{s-1} \\ \mathbf{0}, & \mathbf{A_{22}}^s \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-s}} \\ \mathbf{Y_{n-s}} \end{pmatrix} \tag{28}$$

If we replace $n - s \to n, n \to n + s$ in equation (28), we can make s-step forecast.

3.2 Brand shift group — In the case of three groups

Suppose brand selection is executed in the same group or to the upper group, and also suppose that brand position is $x > y > z$ (x is upper position). Then brand selection transition matrix would be expressed as follows.

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \\ \mathbf{Z_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{11}, & \mathbf{A}_{12}, & \mathbf{A}_{13} \\ \mathbf{0}, & \mathbf{A}_{22}, & \mathbf{A}_{23} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33} \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-1}} \\ \mathbf{Y_{n-1}} \\ \mathbf{Z_{n-1}} \end{pmatrix} \tag{29}$$

Where $\mathbf{X_n} = \begin{pmatrix} x_1^n \\ x_2^n \\ \vdots \\ x_p^n \end{pmatrix}$, $\mathbf{Y_n} = \begin{pmatrix} y_1^n \\ y_2^n \\ \vdots \\ y_q^n \end{pmatrix}$, $\mathbf{Z_n} = \begin{pmatrix} z_1^n \\ z_2^n \\ \vdots \\ z_r^n \end{pmatrix}$

Here,

$\mathbf{X_n} \in \mathbf{R}^p \quad (n = 1,2,\cdots)$, $\mathbf{Y_n} \in \mathbf{R}^q \quad (n = 1,2,\cdots)$, $\mathbf{Z_n} \in \mathbf{R}^r \quad (n = 1,2,\cdots)$,

$\mathbf{A}_{11} \in R^{p\times p}$, $\mathbf{A}_{12} \in R^{p\times q}$, $\mathbf{A}_{13} \in R^{p\times r}$, $\mathbf{A}_{22} \in R^{q\times q}$, $\mathbf{A}_{23} \in R^{q\times r}$, $\mathbf{A}_{33} \in R^{r\times r}$

These are re-stated as

$$\boldsymbol{V}_\mathbf{n} = \mathbf{A}\boldsymbol{V}_\mathbf{n-1} \tag{30}$$

where,

$$\boldsymbol{V}_\mathbf{n} = \begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \\ \mathbf{Z_n} \end{pmatrix}, \qquad \mathbf{A} = \begin{pmatrix} \mathbf{A}_{11}, & \mathbf{A}_{12}, & \mathbf{A}_{13} \\ \mathbf{0}, & \mathbf{A}_{22}, & \mathbf{A}_{23} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33} \end{pmatrix}, \qquad \boldsymbol{V}_\mathbf{n-1} = \begin{pmatrix} \mathbf{X_{n-1}} \\ \mathbf{Y_{n-1}} \\ \mathbf{Z_{n-1}} \end{pmatrix}$$

Hereinafter, we shift steps as is done in the previous section.

In the general description, we state them as follows.

$$\boldsymbol{V}_{\mathbf{n}} = \mathbf{A}^{(\mathbf{s})}\boldsymbol{V}_{\mathbf{n-s}} \tag{31}$$

Here,

$$\mathbf{A}^{(\mathbf{s})} = \begin{pmatrix} \mathbf{A}_{11}{}^{(\mathbf{s})}, & \mathbf{A}_{12}{}^{(\mathbf{s})}, & \mathbf{A}_{13}{}^{(\mathbf{s})} \\ \mathbf{0}, & \mathbf{A}_{22}{}^{(\mathbf{s})}, & \mathbf{A}_{23}{}^{(\mathbf{s})} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}{}^{(\mathbf{s})} \end{pmatrix}, \qquad V_{\mathbf{n-s}} = \begin{pmatrix} \mathbf{X}_{\mathbf{n-s}} \\ \mathbf{Y}_{\mathbf{n-s}} \\ \mathbf{Z}_{\mathbf{n-s}} \end{pmatrix}$$

Generalizing them to m groups, they are expressed as follows.

$$\begin{pmatrix} \mathbf{X}_{\mathbf{n}}^{(1)} \\ \mathbf{X}_{\mathbf{n}}^{(2)} \\ \vdots \\ \mathbf{X}_{\mathbf{n}}^{(m)} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{\mathbf{11}} & \mathbf{A}_{\mathbf{12}} & \cdots & \mathbf{A}_{\mathbf{1m}} \\ \mathbf{A}_{\mathbf{21}} & \mathbf{A}_{\mathbf{22}} & \cdots & \mathbf{A}_{\mathbf{2m}} \\ \vdots & \vdots & & \vdots \\ \mathbf{A}_{\mathbf{m1}} & \mathbf{A}_{\mathbf{m2}} & \cdots & \mathbf{A}_{\mathbf{mm}} \end{pmatrix} \begin{pmatrix} \mathbf{X}_{\mathbf{n-1}}^{(1)} \\ \mathbf{X}_{\mathbf{n-1}}^{(2)} \\ \vdots \\ \mathbf{X}_{\mathbf{n-1}}^{(m)} \end{pmatrix} \tag{32}$$

$$\mathbf{X}_{\mathbf{n}}^{(1)} \in R^{k_1}, \quad \mathbf{X}_{\mathbf{n}}^{(2)} \in R^{k_2}, \ \cdots, \quad \mathbf{X}_{\mathbf{n}}^{(m)} \in R^{k_m}, \ \mathbf{A}_{\mathbf{ij}} \in R^{k_i \times k_j}, \ i, j = 1, \cdots, m$$

4 s-Step forecasting

Now, we see Eq.(14) in time series. Set $\mathbf{X}, \mathbf{Y}, \mathbf{Z}$ at time n as :

$$\mathbf{X}_{\mathbf{n}} = \begin{pmatrix} x_1^n \\ x_2^n \\ \vdots \\ x_p^n \end{pmatrix}, \quad \mathbf{Y}_{\mathbf{n}} = \begin{pmatrix} y_1^n \\ y_2^n \\ \vdots \\ y_q^n \end{pmatrix}, \quad \mathbf{Z}_{\mathbf{n}} = \begin{pmatrix} z_1^n \\ z_2^n \\ \vdots \\ z_r^n \end{pmatrix} \tag{33}$$

then, Eq.(14) can be re-stated as :

$$\begin{pmatrix} \mathbf{X}_{\mathbf{n}} \\ \mathbf{Y}_{\mathbf{n}} \\ \mathbf{Z}_{\mathbf{n}} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{A}_{13} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{A}_{23} \\ \mathbf{A}_{31} & \mathbf{A}_{32} & \mathbf{A}_{33} \end{pmatrix} \begin{pmatrix} \mathbf{X}_{\mathbf{n-1}} \\ \mathbf{Y}_{\mathbf{n-1}} \\ \mathbf{Z}_{\mathbf{n-1}} \end{pmatrix} \tag{34}$$

where suffix is written in the lower part of right hand side because there arises a multiplier in the equation of forecasting.

s-step forecasting is executed by the following equation.

$$\begin{pmatrix} \mathbf{X_{n+s}} \\ \mathbf{Y_{n+s}} \\ \mathbf{Z_{n+s}} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{A}_{13} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{A}_{23} \\ \mathbf{A}_{31} & \mathbf{A}_{32} & \mathbf{A}_{33} \end{pmatrix}^{s} \begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \\ \mathbf{Z_n} \end{pmatrix} \tag{35}$$

5 Numerical example

First of all, the framework of jewelry/accessory purchasing via on-line shopping is as follows.

- On-line shop: Ciao! / Happy gift

  Host site: http://www.happy-gift.jp/

  Branch site: http://www.rakuten.co.jp/ciao/

  http://store.shopping.yahoo.co.jp/b-ciao/index.html

  Managed by Cherish Co.Ltd.

- Customers: all over Japan (Every Prefecture)
- Data gathering period: October 2006 – May 2009
- Order number: 2438 (limited to the order number which has repeated order)
- Main residents of customers

  Tokyo 12.9%

  Kanagawa 8.8%

Chiba 6.9%

Osaka 5.9%

Saitama 5.8%

Aichi 5.0%

The share of Tokyo capital area consists of 34.4%.

- Sales goods:

Necklace / Pendant

Pierced earrings

Ring

Bracelet / Bangle

Brooch

Necktie Pin

Miscellaneous （Package/Ribbon etc.）

- Classification of goods by price

Rank	Price(Yen)	Rank	Price(Yen)
Necklace / Pendant		Ring	
N6	40001～	R6	40001～
N5	～40000	R5	～40000
N4	～30000	R4	～28000
N3	～20000	R3	～22000
N2	～15000	R2	～15000
N1	～10000	R1	～10000

Pierced earrings		Bracelet / Bungle	
P6	24001～	B6	40001～
P5	～24000	B5	～40000
P4	～16000	B4	～28000
P3	～10000	B3	～22000
P2	～6000	B2	～15000
P1	～2000	B1	～10000

We consider the case of four variable blocks $\boldsymbol{N}, \boldsymbol{P}, \boldsymbol{R}$ and $\boldsymbol{B}$.

The variable $\boldsymbol{N}, \boldsymbol{P}, \boldsymbol{R}$ and $\boldsymbol{B}$ stands for Necklace/Pendant, Pierced earring, Ring and Bracelet/Bungle respectively.

Here, each of them consists of 6 varieties of goods.

$$\boldsymbol{N}_n = \begin{pmatrix} \boldsymbol{N}_1^n \\ \boldsymbol{N}_2^n \\ \vdots \\ \boldsymbol{N}_6^n \end{pmatrix}, \quad \boldsymbol{P}_n = \begin{pmatrix} \boldsymbol{P}_1^n \\ \boldsymbol{P}_2^n \\ \vdots \\ \boldsymbol{P}_6^n \end{pmatrix}, \quad \boldsymbol{R}_n = \begin{pmatrix} \boldsymbol{R}_1^n \\ \boldsymbol{R}_2^n \\ \vdots \\ \boldsymbol{R}_6^n \end{pmatrix}, \quad \boldsymbol{B}_n = \begin{pmatrix} \boldsymbol{B}_1^n \\ \boldsymbol{B}_2^n \\ \vdots \\ \boldsymbol{B}_6^n \end{pmatrix}$$

$$\begin{pmatrix} \boldsymbol{N}_n \\ \boldsymbol{P}_n \\ \boldsymbol{R}_n \\ \boldsymbol{B}_n \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{A}_{13} & \mathbf{A}_{14} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{A}_{23} & \mathbf{A}_{24} \\ \mathbf{A}_{31} & \mathbf{A}_{32} & \mathbf{A}_{33} & \mathbf{A}_{34} \\ \mathbf{A}_{41} & \mathbf{A}_{42} & \mathbf{A}_{43} & \mathbf{A}_{44} \end{pmatrix} \begin{pmatrix} \boldsymbol{N}_{n-1} \\ \boldsymbol{P}_{n-1} \\ \boldsymbol{R}_{n-1} \\ \boldsymbol{B}_{n-1} \end{pmatrix}$$

$$N_n \in R^6 (n = 1,2,\cdots), P_n \in R^6 (n = 1,2,\cdots),$$
$$R_n \in R^6 (n = 1,2,\cdots), B_n \in R^6 (n = 1,2,\cdots),$$
$$A_{ij} \in R^{6\times 6} (i = 1,\cdots,6)(j = 1,\cdots 6)$$

Set

$$\boldsymbol{N} = \{\boldsymbol{N}_1, \boldsymbol{N}_2, \cdots\}$$
$$\boldsymbol{P} = \{\boldsymbol{P}_1, \boldsymbol{P}_2, \cdots\}$$
$$\boldsymbol{R} = \{\boldsymbol{R}_1, \boldsymbol{R}_2, \cdots\}$$
$$\boldsymbol{B} = \{\boldsymbol{B}_1, \boldsymbol{B}_2, \cdots\}$$

Now, we investigate all cases.

Total numbers of shifts among each block are as follows.

$\mathbf{A}_{11}(\boldsymbol{N} \to \boldsymbol{N})$: 615
$\mathbf{A}_{12}(\boldsymbol{P} \to \boldsymbol{N})$: 71
$\mathbf{A}_{13}(\boldsymbol{R} \to \boldsymbol{N})$: 79
$\mathbf{A}_{14}(\boldsymbol{B} \to \boldsymbol{N})$: 37
$\mathbf{A}_{21}(\boldsymbol{N} \to \boldsymbol{P})$: 84
$\mathbf{A}_{22}(\boldsymbol{P} \to \boldsymbol{P})$: 117
$\mathbf{A}_{23}(\boldsymbol{R} \to \boldsymbol{P})$: 16
$\mathbf{A}_{24}(\boldsymbol{B} \to \boldsymbol{P})$: 4
$\mathbf{A}_{31}(\boldsymbol{N} \to \boldsymbol{R})$: 111
$\mathbf{A}_{32}(\boldsymbol{P} \to \boldsymbol{R})$: 15
$\mathbf{A}_{33}(\boldsymbol{R} \to \boldsymbol{R})$: 161
$\mathbf{A}_{34}(\boldsymbol{B} \to \boldsymbol{R})$: 10
$\mathbf{A}_{41}(\boldsymbol{N} \to \boldsymbol{B})$: 38
$\mathbf{A}_{42}(\boldsymbol{P} \to \boldsymbol{B})$: 5
$\mathbf{A}_{43}(\boldsymbol{R} \to \boldsymbol{B})$: 12
$\mathbf{A}_{44}(\boldsymbol{B} \to \boldsymbol{B})$: 22

The shift to N3 to P4 in $\boldsymbol{N}_{n-1}, \boldsymbol{P}_n$, for example, is expressed as follows when one event arises.

$$P_n = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad N_{n-1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Substituting these to Equation (7), we obtain following equations.

$$\hat{\mathbf{A}} = \begin{pmatrix}
9 & 1 & 3 & 5 & 1 & 1 & 0 & 2 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 10 & 4 & 4 & 6 & 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 2 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\
2 & 5 & 18 & 11 & 16 & 9 & 0 & 3 & 0 & 2 & 2 & 0 & 0 & 0 & 2 & 1 & 4 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
5 & 7 & 10 & 36 & 26 & 16 & 0 & 3 & 1 & 1 & 2 & 0 & 0 & 0 & 1 & 5 & 8 & 2 & 0 & 0 & 0 & 0 & 2 & 3 \\
1 & 7 & 18 & 25 & 150 & 42 & 0 & 4 & 2 & 7 & 14 & 1 & 0 & 2 & 0 & 1 & 13 & 5 & 0 & 0 & 0 & 0 & 7 & 2 \\
0 & 2 & 3 & 8 & 33 & 118 & 0 & 1 & 2 & 6 & 13 & 1 & 0 & 1 & 1 & 2 & 15 & 10 & 2 & 0 & 0 & 1 & 6 & 9 \\
0 & 0 & 0 & 0 & 1 & 1 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 1 & 1 & 2 & 5 & 1 & 1 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 3 & 3 & 3 & 3 & 1 & 2 & 3 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 2 & 7 & 4 & 0 & 0 & 3 & 14 & 1 & 2 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 3 & 7 & 12 & 12 & 1 & 1 & 1 & 0 & 38 & 4 & 0 & 0 & 1 & 0 & 6 & 1 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 3 & 4 & 1 & 0 & 0 & 1 & 4 & 29 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 2 & 0 & 1 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 2 & 6 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 3 & 1 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 3 & 3 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 3 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 2 & 1 & 9 & 32 & 15 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 2 & 1 & 1 & 64 & 4 & 0 & 0 & 0 & 0 & 5 & 1 \\
0 & 0 & 0 & 3 & 6 & 15 & 1 & 0 & 2 & 0 & 1 & 1 & 2 & 2 & 2 & 1 & 18 & 41 & 0 & 0 & 0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 5 & 8 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 2 & 1 & 0 & 0 & 0 & 6 & 1 \\
0 & 1 & 2 & 1 & 4 & 7 & 0 & 1 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 3 & 1 & 0 & 0 & 0 & 0 & 1 & 12
\end{pmatrix}$$

$$
\times\begin{pmatrix}
22&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&40&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&74&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&124&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&321&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&267&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&9&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&21&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&19&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&38&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&81&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&40&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&7&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&19&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&14&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&16&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&143&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&68&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&3&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&3&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&32&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&33
\end{pmatrix}^{-1}
$$

$$
= \begin{pmatrix}
\frac{1}{22} & \frac{1}{40} & \frac{3}{74} & \frac{5}{124} & \frac{1}{321} & \frac{1}{267} & 0 & \frac{2}{21} & \frac{1}{19} & \frac{1}{38} & 0 & 0 & \frac{1}{7} & 0 & 0 & 0 & \frac{1}{143} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{4} & \frac{2}{37} & \frac{1}{31} & \frac{2}{107} & \frac{1}{89} & \frac{1}{9} & \frac{1}{21} & 0 & 0 & 0 & 0 & 0 & \frac{1}{19} & 0 & \frac{1}{16} & \frac{2}{143} & 0 & 0 & 1 & 1 & 0 & \frac{1}{32} & 0 \\
\frac{1}{11} & \frac{1}{8} & \frac{9}{37} & \frac{11}{124} & \frac{16}{321} & \frac{3}{89} & 0 & \frac{1}{7} & 0 & \frac{1}{19} & \frac{2}{81} & 0 & 0 & 0 & \frac{1}{7} & \frac{1}{16} & \frac{4}{143} & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{32} & 0 \\
\frac{5}{22} & \frac{7}{40} & \frac{5}{37} & \frac{9}{31} & \frac{26}{321} & \frac{16}{267} & 0 & \frac{1}{7} & \frac{1}{19} & \frac{1}{38} & \frac{2}{81} & 0 & 0 & 0 & \frac{1}{14} & \frac{5}{16} & \frac{8}{143} & \frac{1}{34} & 0 & 0 & 0 & 0 & \frac{1}{16} & \frac{1}{11} \\
\frac{1}{22} & \frac{7}{40} & \frac{9}{37} & \frac{25}{124} & \frac{50}{107} & \frac{14}{89} & 0 & \frac{4}{21} & \frac{2}{19} & \frac{7}{38} & \frac{14}{81} & \frac{1}{40} & 0 & \frac{2}{19} & 0 & \frac{1}{16} & \frac{1}{11} & \frac{5}{68} & 0 & 0 & 0 & 0 & \frac{7}{32} & \frac{2}{33} \\
0 & \frac{1}{20} & \frac{3}{74} & \frac{2}{31} & \frac{11}{107} & \frac{118}{267} & 0 & \frac{1}{21} & \frac{2}{19} & \frac{3}{19} & \frac{13}{81} & \frac{1}{40} & 0 & \frac{1}{19} & \frac{1}{14} & \frac{1}{8} & \frac{15}{143} & \frac{5}{34} & \frac{2}{3} & 0 & 0 & \frac{1}{3} & \frac{3}{16} & \frac{3}{11} \\
0 & 0 & 0 & 0 & \frac{1}{321} & \frac{1}{267} & \frac{2}{9} & 0 & 0 & 0 & \frac{2}{81} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{11} & \frac{1}{40} & \frac{1}{74} & \frac{1}{124} & \frac{2}{321} & \frac{5}{267} & \frac{1}{9} & \frac{1}{21} & \frac{2}{19} & 0 & \frac{1}{81} & 0 & 0 & \frac{1}{19} & 0 & 0 & \frac{1}{143} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{3}{74} & \frac{3}{124} & \frac{1}{107} & \frac{1}{89} & \frac{1}{9} & \frac{2}{21} & \frac{3}{19} & \frac{1}{19} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{22} & \frac{1}{40} & \frac{1}{74} & \frac{1}{62} & \frac{7}{321} & \frac{4}{267} & 0 & 0 & \frac{3}{19} & \frac{7}{19} & \frac{1}{81} & \frac{1}{20} & 0 & 0 & \frac{1}{14} & \frac{1}{16} & \frac{1}{143} & \frac{1}{68} & 0 & 0 & 0 & 0 & \frac{1}{32} & 0 \\
0 & \frac{1}{40} & \frac{3}{74} & \frac{7}{124} & \frac{4}{107} & \frac{4}{89} & \frac{1}{9} & \frac{1}{21} & \frac{1}{19} & 0 & \frac{38}{81} & \frac{1}{10} & 0 & 0 & \frac{1}{14} & 0 & \frac{6}{143} & \frac{1}{68} & 0 & 0 & 0 & 0 & 0 & \frac{2}{33} \\
0 & 0 & 0 & 0 & \frac{1}{107} & \frac{4}{267} & \frac{1}{9} & 0 & 0 & \frac{1}{38} & \frac{4}{81} & \frac{29}{40} & 0 & 0 & \frac{1}{14} & 0 & 0 & \frac{1}{68} & 0 & 0 & 0 & 0 & 0 & \frac{1}{33} \\
0 & \frac{1}{40} & \frac{1}{74} & \frac{1}{124} & 0 & \frac{2}{267} & 0 & 0 & \frac{1}{19} & 0 & 0 & 0 & \frac{1}{7} & \frac{1}{19} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{22} & 0 & \frac{1}{37} & 0 & \frac{1}{321} & \frac{2}{267} & 0 & \frac{1}{21} & 0 & 0 & 0 & 0 & \frac{2}{7} & \frac{6}{19} & \frac{1}{14} & 0 & \frac{1}{143} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{40} & \frac{3}{74} & \frac{1}{124} & \frac{2}{321} & \frac{2}{267} & 0 & 0 & 0 & \frac{1}{38} & 0 & 0 & 0 & 0 & \frac{3}{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{62} & \frac{1}{107} & \frac{1}{89} & 0 & 0 & 0 & 0 & \frac{2}{81} & 0 & \frac{1}{7} & \frac{3}{19} & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{32} & 0 \\
0 & \frac{1}{20} & \frac{1}{74} & \frac{9}{124} & \frac{32}{321} & \frac{5}{89} & \frac{1}{9} & \frac{1}{21} & \frac{1}{19} & \frac{1}{38} & 0 & \frac{1}{40} & 0 & \frac{2}{19} & \frac{1}{14} & \frac{1}{16} & \frac{64}{143} & \frac{1}{17} & 0 & 0 & 0 & 0 & \frac{5}{32} & \frac{1}{33} \\
0 & 0 & 0 & \frac{3}{124} & \frac{2}{107} & \frac{5}{89} & \frac{1}{9} & 0 & \frac{2}{19} & 0 & \frac{1}{81} & \frac{1}{40} & \frac{2}{7} & \frac{2}{19} & \frac{1}{7} & \frac{1}{16} & \frac{18}{143} & \frac{41}{68} & 0 & 0 & 0 & 0 & \frac{1}{32} & \frac{2}{33} \\
0 & 0 & 0 & 0 & \frac{2}{321} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{22} & 0 & 0 & 0 & \frac{1}{321} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{143} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{74} & 0 & \frac{2}{321} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & \frac{5}{124} & \frac{8}{321} & \frac{1}{89} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{143} & \frac{1}{34} & \frac{1}{3} & 0 & 0 & 0 & \frac{3}{16} & \frac{1}{33} \\
0 & \frac{1}{40} & \frac{1}{37} & \frac{1}{124} & \frac{4}{321} & \frac{7}{267} & 0 & \frac{1}{21} & 0 & \frac{1}{19} & \frac{1}{81} & \frac{1}{40} & 0 & 0 & 0 & 0 & \frac{3}{143} & \frac{1}{68} & 0 & 0 & 0 & 0 & \frac{1}{32} & \frac{4}{11}
\end{pmatrix}
$$

We examine all cases by setting case number.

Case 1 $\mathbf{A}_{11}(\boldsymbol{N} \rightarrow \boldsymbol{N})$

Total number of upper shift from N1 : 71
Total number of lower shift from N1 : -
Total number of upper shift from N2 : 49
Total number of lower shift from N2 : 33
Total number of upper shift from N3 : 20
Total number of lower shift from N3 : 33
Total number of upper shift from N4 : 7
Total number of lower shift from N4 : 31
Total number of upper shift from N5 : 1
Total number of lower shift from N5 : 21
Total number of upper shift from N6 : -
Total number of lower shift from N6 : 8

We can observe that upper shift from N1 and N2 is dominant and on the contrary, lower shift from N3, N4, N5 and N6 is dominant. This implies that customers buy rather cheap goods at first for the trial and after confirming the quality, they make upper shift in selecting brands. After reaching higher brands, they buy cheaper goods and that leads to a lower shift in brand selection.

Hearing form customers, we can also find that she buy necklace for herself and confirm the quality. After that, she makes gift by selecting upper brand. Sometimes she buys lower brand goods for herself after making gift. That scene can often be seen and the result shows its sequence well. When the shop owner introduces new brand goods, he/she has to determine the price. If it is not reasonable, customers do not select their brand position as the shop owner assumes. These are confirmed by the brand shift transition, which forces the shop owner to re-consider the price and brand position of the new brand goods.

Case 2 $\mathbf{A}_{12}(\boldsymbol{P} \to \boldsymbol{N})$

Total number of upper shift from P1 : 1
Total number of lower shift from P1 : -
Total number of upper shift from P2 : 4
Total number of lower shift from P2 : 13
Total number of upper shift from P3 : 3
Total number of lower shift from P3 : 13
Total number of upper shift from P4 : 1
Total number of lower shift from P4 : 5
Total number of upper shift from P5 : 2
Total number of lower shift from P5 : 11
Total number of upper shift from P6 : -
Total number of lower shift from P6 : 1

We can observe that upper shift from P1 can be seen and on the contrary, lower shift from P2, P3, P4, P5 and P6 is dominant.

Case 3 $\mathbf{A}_{13}(\boldsymbol{R} \to \boldsymbol{N})$

Total number of upper shift from R1 : 7
Total number of lower shift from R1 : -
Total number of upper shift from R2 : 15
Total number of lower shift from R2 : 15
Total number of upper shift from R3 : 2
Total number of lower shift from R3 : 3
Total number of upper shift from R4 : 0
Total number of lower shift from R4 : 2
Total number of upper shift from R5 : 0
Total number of lower shift from R5 : 3
Total number of upper shift from R6 : -
Total number of lower shift from R6 : 0

We can observe that upper shift from R1 is dominant and on the contrary, lower shifts from R3, R4 and R5 is dominant. This characteristics is rather common in each case, although subtle difference may occur.

Case 4 $\mathbf{A}_{14}(\boldsymbol{B} \to \boldsymbol{N})$
Total number of upper shift from B1 : 5
Total number of lower shift from B1 : -
Total number of upper shift from B2 : 4
Total number of lower shift from B2 : 6
Total number of upper shift from B3 : 1
Total number of lower shift from B3 : 1
Total number of upper shift from B4 : 1
Total number of lower shift from B4 : 0
Total number of upper shift from B5 : 0
Total number of lower shift from B5 : 0
Total number of upper shift from B6 : -
Total number of lower shift from B6 : 2

We can observe the similar characteristics as seen before, although there is subtle difference.

Case 5 $\mathbf{A}_{21}(\boldsymbol{N} \to \boldsymbol{P})$
Total number of upper shift from N1 : 25
Total number of lower shift from N1 : -
Total number of upper shift from N2 : 13
Total number of lower shift from N2 : 3
Total number of upper shift from N3 : 4
Total number of lower shift from N3 : 7
Total number of upper shift from N4 : 1
Total number of lower shift from N4 : 4

Total number of upper shift from N5 : 0
Total number of lower shift from N5 : 2
Total number of upper shift from N6 : -
Total number of lower shift from N6 : 3

We can observe clearly that the upper shift from N1 and N2 is dominant and on the contrary, the lower shift from N3, N4, N5 and N6 is dominant.

Case 6 $\mathbf{A}_{22}(\boldsymbol{P} \to \boldsymbol{P})$
Total number of upper shift from P1 : 6
Total number of lower shift from P1 : -
Total number of upper shift from P2 : 4
Total number of lower shift from P2 : 4
Total number of upper shift from P3 : 2
Total number of lower shift from P3 : 1
Total number of upper shift from P4 : 2
Total number of lower shift from P4 : 4
Total number of upper shift from P5 : 0
Total number of lower shift from P5 : 3
Total number of upper shift from P6 : -
Total number of lower shift from P6 : 4

We can observe clearly that the upper shift from P1 and P3 is dominant and on the contrary, the lower shift from P4, P5 and P6 is dominant.

Case 7 $\mathbf{A}_{23}(\boldsymbol{R} \to \boldsymbol{P})$
Total number of upper shift from R1 : 2
Total number of lower shift from R1 : -
Total number of upper shift from R2 : 2
Total number of lower shift from R2 : 0
Total number of upper shift from R3 : 0

Total number of lower shift from R3 : 0
Total number of upper shift from R4 : 0
Total number of lower shift from R4 : 3
Total number of upper shift from R5 : 0
Total number of lower shift from R5 : 0
Total number of upper shift from R6 : -
Total number of lower shift from R6 : 0

We can observe the upper shift from R1 and R2.

Case 8 $\mathbf{A}_{24}(\boldsymbol{B} \to \boldsymbol{P})$
Total number of upper shift from B1 : 2
Total number of lower shift from B1 : -
Total number of upper shift from B2 : 1
Total number of lower shift from B2 : 0
Total number of upper shift from B3 : 0
Total number of lower shift from B3 : 0
Total number of upper shift from B4 : 0
Total number of lower shift from B4 : 0
Total number of upper shift from B5 : 0
Total number of lower shift from B5 : 0
Total number of upper shift from B6 : -
Total number of lower shift from B6 : 0

We can observe the upper shift from B1 and B2.

Case 9 $\mathbf{A}_{31}(\boldsymbol{N} \to \boldsymbol{R})$
Total number of upper shift from N1 : 24
Total number of lower shift from N1 : -
Total number of upper shift from N2 : 6
Total number of lower shift from N2 : 6
Total number of upper shift from N3 : 2

Total number of lower shift from N3 : 12
Total number of upper shift from N4 : 3
Total number of lower shift from N4 : 1
Total number of upper shift from N5 : 1
Total number of lower shift from N5 : 3
Total number of upper shift from N6 : -
Total number of lower shift from N6 : 1

We can observe that the upper shift from N1 is dominant and on the contrary, the lower shift from N3, N5 and N6 is dominant.

Case 10 $\mathbf{A}_{32}(\boldsymbol{P} \rightarrow \boldsymbol{R})$
Total number of upper shift from P1 : 1
Total number of lower shift from P1 : -
Total number of upper shift from P2 : 2
Total number of lower shift from P2 : 1
Total number of upper shift from P3 : 1
Total number of lower shift from P3 : 1
Total number of upper shift from P4 : 1
Total number of lower shift from P4 : 3
Total number of upper shift from P5 : 0
Total number of lower shift from P5 : 1
Total number of upper shift from P6 : -
Total number of lower shift from P6 : 2

We can observe the upper shift from P1 and P2 and the lower shift from P4, P5 and P6.

Case 11 $\mathbf{A}_{33}(\boldsymbol{R} \rightarrow \boldsymbol{R})$
Total number of upper shift from R1 : 4
Total number of lower shift from R1 : -

Total number of upper shift from R2 : 1
Total number of lower shift from R2 : 18
Total number of upper shift from R3 : 0
Total number of lower shift from R3 : 2
Total number of upper shift from R4 : 1
Total number of lower shift from R4 : 3
Total number of upper shift from R5 : 1
Total number of lower shift from R5 : 7
Total number of upper shift from R6 : -
Total number of lower shift from R6 : 5

We can observe clearly that the upper shift from R1 is dominant and on the contrary, the lower shift from R2, R3, R4, R5 and R6 is dominant.

Case 12 $\mathbf{A}_{34}(\boldsymbol{B} \rightarrow \boldsymbol{R})$

Total number of upper shift from B1 : 1
Total number of lower shift from B1 : -
Total number of upper shift from B2 : 1
Total number of lower shift from B2 : 1
Total number of upper shift from B3 : 0
Total number of lower shift from B3 : 0
Total number of upper shift from B4 : 0
Total number of lower shift from B4 : 0
Total number of upper shift from B5 : 0
Total number of lower shift from B5 : 0
Total number of upper shift from B6 : -
Total number of lower shift from B6 : 0

We can observe the upper shift from B1.

Case 13 $\mathbf{A}_{41}(\boldsymbol{N} \rightarrow \boldsymbol{B})$

Total number of upper shift from N1 : 3
Total number of lower shift from N1 : -
Total number of upper shift from N2 : 5
Total number of lower shift from N2 : 4
Total number of upper shift from N3 : 0
Total number of lower shift from N3 : 6
Total number of upper shift from N4 : 0
Total number of lower shift from N4 : 3
Total number of upper shift from N5 : 0
Total number of lower shift from N5 : 1
Total number of upper shift from N6 : -
Total number of lower shift from N6 : 1

We can observe that the upper shift from N1 and N2 is dominant and on the contrary, the lower shift from N3, N4, N5 and N6 is dominant.

Case 14 $\mathbf{A}_{42}(\boldsymbol{P} \rightarrow \boldsymbol{B})$

Total number of upper shift from P1 : 0
Total number of lower shift from P1 : -
Total number of upper shift from P2 : 0
Total number of lower shift from P2 : 1
Total number of upper shift from P3 : 0
Total number of lower shift from P3 : 2
Total number of upper shift from P4 : 0
Total number of lower shift from P4 : 0
Total number of upper shift from P5 : 0
Total number of lower shift from P5 : 1
Total number of upper shift from P6 : -
Total number of lower shift from P6 : 0

We can observe the lower shift from P2, P3 and P5.

Case 15 $\mathbf{A}_{43}(\boldsymbol{R} \to \boldsymbol{B})$

Total number of upper shift from R1 : 2
Total number of lower shift from R1 : -
Total number of upper shift from R2 : 1
Total number of lower shift from R2 : 3
Total number of upper shift from R3 : 0
Total number of lower shift from R3 : 0
Total number of upper shift from R4 : 0
Total number of lower shift from R4 : 0
Total number of upper shift from R5 : 0
Total number of lower shift from R5 : 0
Total number of upper shift from R6 : -
Total number of lower shift from R6 : 0

We can observe the upper shift from R1 and the lower shift from R2.

Case 16 $\mathbf{A}_{44}(\boldsymbol{B} \to \boldsymbol{B})$

Total number of upper shift from B1 : 1
Total number of lower shift from B1 : -
Total number of upper shift from B2 : 0
Total number of lower shift from B2 : 1
Total number of upper shift from B3 : 0
Total number of lower shift from B3 : 0
Total number of upper shift from B4 : 0
Total number of lower shift from B4 : 0
Total number of upper shift from B5 : 0
Total number of lower shift from B5 : 0
Total number of upper shift from B6 : -
Total number of lower shift from B6 : 1

We can observe the upper shift from B1 and the lower shift from B2 and B6.

6 Remarks

The retailer and customers including authors discussed about jewelry/accessory on-line shopping based upon their own experiences. Some of them imply the reason of the lower shift from the higher ranked brand. Some of them suggest the future works to be investigated. Nearly 70% customers are men and they mainly purchase jewelry/accessory for gift. Therefore analysis for men becomes dominant in the following discussion.

<In the case male buys>

a. He usually has budget range in making present to his lovers. Therefore there often happens buying the same range level of brand.
b. He is apt to spend much money at the first present. But from the second, it would decrease. –There are Japanese saying that he does not give much food to the fish he caught. When the lover is changed, this scheme would be repeated.
c. Budget may change according to the characteristics of the event. If it is a Xmas or a birthday present, the budget is high and less for those of the white day or other events.
d. If he buys presents with other goods (for example, necklace and flower etc.), jewelry/accessory price would decrease even if the total budget increases.
e. Male is not so severe to the price whether it is qualified or not for its quality.
f. If he buys goods at the real shop, a saleswoman would recommend the goods and he would be apt to buy much higher one than those at the virtual shop.

g. In the case of the couple of separated ages, male is apt to show off, therefore brand selection of upper shift is often the case.

h. After married, brand selection of upper shift is rare for his wife in making present.

i. While young, he makes upper shift after having relationship with partner for about one year. After that, it tends to make lower shift.

j. Male is often simple-minded. Therefore, if she is glad at the present, he repeats the item, not considering another choice.

<In the case female buys>

k. She makes confirmation of the goods about how it would be at the first purchasing. If she is satisfied, then she makes repeated purchasing. If the goods is well, she also buys them for herself.

l. The price of goods may not change for female whether she buys at the real shop or virtual shop.

7 Conclusion

Consumers often buy higher grade brand products as they are accustomed or bored to use current brand products they have. There may be also the case that customer selects lower brand to seek suitable price when she already has higher ranked brand.

In this paper, matrix structure was clarified when brand selection was executed toward higher grade brand and/or lower grade brand. Expanding brand selection from single brand group to multiple genre brand group, we could make much more exquisite and multi-dimensional analysis. Utilizing purchase history record of jewelry/accessory on-line shopping (Necklace/Pendant, Pierced earrings, Ring, Bracelet/Bungle) over three years, above matrix structure was investigated and confirmed. This new method should be examined in various cases.

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