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Mathematical Study of Various Types of Cyber-Attacks and Protection

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Abstract

The paper documents a holistic mathematical modeling theory to provide a rigorous description of cyber-attacks and cyber-security. After determining valuations and vulnerabilities of parts of a node constituent, we recall the definitions of cyber-effect and cyber-interaction. Based on these concepts, we give the mathematical definitions of cyber navigation and infected node and we explain what is meant by dangerous cyber navigation and protection of cyber nodes from unplanned attacks. Our discussion proceeds to a rigorous description of passive and active cyber-attacks, as well as the relevant protections.

Keywords: Mathematical modeling (models of systems), internet topics, measure theory, complex spaces, valuation of a part of node constituent, vulnerability of a part of node constituent, node supervision, cyber-effect, cyber-interaction, germ of cyber-attack, cyber defense, proactive cyber protection.

1. Introduction

In many modern scientific studies, quantifying assumptions, data and variables can contribute to the accurate description of the phenomena through appropriate

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mathematical models. So, in many disciplines, the analysts resort to a mathematical foundation of the concepts, in order to create a solid base for the theoretical formulation and solving all relevant problems. In this direction, there have been numerous significant contributions on the mathematical modeling of several branches of Theoretical Engineering disciplines, such as Theoretical Computer Science, Network Security, Electronics, and Artificial Intelligence etc. Especially, in the case of cyber-security, we may mention several remarkable contributions (**[1, 6-8** and **10-15]**). Indicative of the great interest shown for the mathematization of cyber-security is the regular organization of international conferences and workshops of major interest. However, although these presentations are innovative and promising, it seems that they lack a holistic view of the cyber environment. Moreover, there is no predictability vision of cyber-attacks, nor any opportunity to have given a strict definition of defensive protection so that we can look for an optimal design and organization of cyber defense. As a consequence, thereof, one cannot build a solid foundation for a complete theory containing assumptions, definitions, theorems and conclusions. But, this prevents the researcher to understand deeper behaviours, and imposeslimiting ourselves solely to practical techniques (**[9, 16]**).

The aim of the present paper is to document a holistic mathematical theory to provide a rigorous description of cyber-attacks and cyber-security. To this end, Section 2 recalls in brief the mathematical definition of cyberspace given in **[3]**. Next, in Section 3, we first remind the concepts of valuations and vulnerabilities of the parts of a node constituent, and then, based on these two concepts, we give the definitions of node supervision, cyber-effect and, cyber-interaction. With this background, in Section 4, we provide a mathematical definition of cyber navigation and, after giving the rigorous meaning of infected nodes, we determine what is meant by dangerous cyber navigation and protection of cyber nodes from unplanned attacks. Next, in Section 5, we describe a rigorous outline of passive and active cyber-attacks as well as an identification of the relevant proactive defense (**[2]**) from such attacks. Concrete examples to special types of germs of cyber-attacks are given in **[**4 and 5**]**. Based on the above background, in Section 6 we proceed with a mathematical description of several representative cyber-attacks. The paper concludes with the introduction of some guiding general ideas and thoughts for future research.

2. Mathematical definition of cyberspace

A multilayered weighted (finite or infinite) graph $\boldsymbol{\mathcal{X}}$ with $\boldsymbol{N}$ interconnected layers is said to be an $\boldsymbol{N}-$ **cyber-archetype germ**. An $\boldsymbol{e}$ **−manifestation** gives a geographical qualifier at each node of $\boldsymbol{\mathcal{X}}$. It is an embedding of $\boldsymbol{\mathcal{X}}$ into a Cartesian product of $\boldsymbol{N}$ complex projective spaces $\mathbb{C}\mathbf{P}^{\boldsymbol{n_k}} \equiv \mathbf{P}(\mathbb{C}^{\boldsymbol{n_k}+\mathbf{1}})$, such that all nodes of $\boldsymbol{\mathcal{X}}$ in the $\boldsymbol{k}$ −layer, called $\boldsymbol{e}$ **−node manifestations**, are illustrated at weighted points of the set $\mathbb{C}\mathbf{P}^{\boldsymbol{n_k}}$ and all directed edges (flows) of $\boldsymbol{\mathcal{X}}$ in the $\boldsymbol{k}$ −layer, called $\boldsymbol{e}$ **−edge manifestations**, are given by simple weighted edges, i.e. by weighted homeomorphic images of the closed interval$[\mathbf{0},\mathbf{1}]$ on $\mathbb{C}\mathbf{P}^{\boldsymbol{n_k}}$, so that, for any $\boldsymbol{k}=\mathbf{1},\mathbf{2},\ldots,\boldsymbol{N}$,

- the end points of each e −edge manifestation on $\mathbb{C}\mathbf{P}^{\boldsymbol{n_k}}$ must be images of end points of a corresponding original directed edge of $\boldsymbol{\mathcal{X}}$ in the $\boldsymbol{k}$ −layer
- there should not be any e −edge manifestation on $\mathbb{C}\mathbf{P}^{\boldsymbol{n_k}}$ derived from directed e −edge of $\boldsymbol{\mathcal{X}}$ in the $\boldsymbol{k}$ −layer into which belong points of e −edge manifestations that are defined by other nodes of $\boldsymbol{\mathcal{X}}$ in the same layer.

The set $\boldsymbol{\mathcal{S}_e}=\boldsymbol{\mathcal{S}_e}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\ldots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})$ of e −manifestations of $\boldsymbol{N}$ −cyber archetype germs is the $\boldsymbol{e}-$ **superclass** in $\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\ldots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}}$**.** An $\boldsymbol{e}$**—graph category** $\boldsymbol{\mathcal{E}_{\mathcal{C}}}=\boldsymbol{\mathcal{E}_{\mathcal{C}}}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\ldots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})$ is a category consisting of the class $\boldsymbol{ob}(\boldsymbol{\mathcal{E}_{\mathcal{C}}})$, whose elements, called $\boldsymbol{e}$**—objects**,are the pairs $\boldsymbol{\mathcal{X}}=(\boldsymbol{V},\boldsymbol{E})\in\boldsymbol{\mathcal{S}_e}$, endowed with a class $\boldsymbol{hom}(\boldsymbol{\mathcal{E}_{\mathcal{C}}})$ of $\boldsymbol{e}$**—morphisms** on $\boldsymbol{ob}(\boldsymbol{\mathcal{E}_{\mathcal{C}}})$ and an associative binary operation $\circ$with identity.

Generalizing, one may consider additionally the following other four basic $\boldsymbol{e}$**—categories**: The $\boldsymbol{e}$ **−set category**$\boldsymbol{e_{Set}}=\boldsymbol{e_{Set}}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\ldots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})$ where the objects are subsets of $\boldsymbol{\mathcal{E}_{\mathcal{C}}}$, the $\boldsymbol{e}$ **−homomorphism category** $\boldsymbol{e_{Hom}}=\boldsymbol{e_{Hom}}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\ldots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})$where the objects are sets of homomorphisms between subsets of $\boldsymbol{e_{Set}}$, the $\boldsymbol{e}$ **−group category** $\boldsymbol{e_{Grp}}=\boldsymbol{e_{Gpr}}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\ldots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})$ where the objects are the groups of $\boldsymbol{\mathcal{E}_{\mathcal{C}}}$ and the $\boldsymbol{e}$ **−topological category** $\boldsymbol{e_{Top}}=\boldsymbol{e_{Top}}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\ldots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})$where the objects are topological subcategories of $\boldsymbol{\mathcal{E}_{\mathcal{C}}}$. For reasons of homogenization of symbolism, we will adopt the following common notation $\boldsymbol{\mathcal{W}_e}=\{\boldsymbol{\mathcal{E}_{\mathcal{C}}},\boldsymbol{e_{Set}},\boldsymbol{e_{Hom}},\boldsymbol{e_{Grp}},\boldsymbol{e_{Top}}\}$. The objects of each e −category $\boldsymbol{W_e}=\boldsymbol{W_e}(\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}}\times\ldots\times\mathbb{C}\mathbf{P}^{\boldsymbol{n_N}})\in\boldsymbol{\mathcal{W}_e}$ will be called $\boldsymbol{e}$ **−manifestations**. An easy **algebraic** structure in the (infinite) set of all these e—manifestations $(\boldsymbol{V},\boldsymbol{E})$ and simultaneously, a compatible **topological** structure to allow for a detailed analytic study of $\boldsymbol{\mathcal{S}_e}$ is given in **[3]**. Further, **[3]** investigates the possibility of allocating suitable vector weights to all the objects and morphisms of any e −category $\boldsymbol{W_e} \in \boldsymbol{\mathcal{W}_e} = \{\boldsymbol{\mathcal{E}_{\mathcal{C}}}, \boldsymbol{e_{Set}}, \boldsymbol{e_{Grp}}, \boldsymbol{e_{Top}}\}$. Towards this end, we consider two types of vector weights that can be attached to any object and/or morphism of such an e −category: the maximum weight and the square weight. Any such weight will be a point in the positive quadrant of the plane. Taking this into account, any$e$ −category $\boldsymbol{W_e} \in \boldsymbol{\mathcal{W}_e} = \{\boldsymbol{\mathcal{E}_{\mathcal{C}}}, \boldsymbol{e_{Set}}, \boldsymbol{e_{Hom}}, \boldsymbol{e_{Gpr}}, \boldsymbol{e_{Top}}\}$can be viewed as an **infinite** e −*graph*$(\mathbb{V},\mathbb{E})$ with *vector weights*, in such a way that the e −nodes in $\mathbb{V}$ are the e −objects $\boldsymbol{X} \in \boldsymbol{ob}(\boldsymbol{W_e})$, while the$e$ −edges in $\mathbb{E}$ are the e −morphisms $\boldsymbol{h} \in \boldsymbol{hom}(\boldsymbol{W_e})$. For such an e −graph $\mathfrak{G}_{\boldsymbol{W_e}}$ corresponding to an e −category $\boldsymbol{W_e} \in \boldsymbol{\mathcal{W}_e}$, the vector weight of the e −node associated to the e −manifestation $\boldsymbol{\mathcal{X}} = (\boldsymbol{V},\boldsymbol{E}) \in \mathbb{V} \equiv \boldsymbol{ob}(\boldsymbol{W_e})$ is equal to a weightof$\boldsymbol{\mathcal{X}}$. Bearing all this in mind, in **[3]**, we introduce a suitable intrinsic metric $\boldsymbol{d_{W_e}}$ in the set $\boldsymbol{ob}(\boldsymbol{W_e})$ of objects of an e −category $\boldsymbol{W_e}$. The most significant benefits coming from such a consideration can be derived from the definitions of *cyber-evolution* and *cyber-domain*. To do this, we first defined the concept of e—*dynamics*, as a mapping of the form $\mathcal{cy}: [0,1] \to \left(\boldsymbol{ob}(\boldsymbol{W_e}), \boldsymbol{d_{W_e}}\right)$; its image is an e—*arrangement*. Each point $\mathcal{cy}(\boldsymbol{t}) \in \mathcal{cy}([0,1])$ is an (instantaneous) local e −node manifestation with an interrelated$e$ −edge manifestation. An e −arrangement together with all of its (instantaneous) e—morphisms is an e −*regularization*. The elements of the completion $\overline{\boldsymbol{ob}(\boldsymbol{W_e})}$ of $\boldsymbol{ob}(\boldsymbol{W_e})$ in $\overline{\mathbb{C}\mathbf{P}^{\boldsymbol{n_1}} \times \ldots \times \mathbb{C}\mathbf{P}^{\boldsymbol{n_N}}}$ are the *cyber-elements*, while the topological space $\left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d_{W_e}}\right)$ is a *cyber-domain*. With this notation, a continuous e −dynamics $\mathcal{cy}: [\mathbf{0},\mathbf{1}] \to \left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d_{W_e}}\right)$is said to be a *cyber-evolutionary path* or simply *cyber-evolution* in the cyber-domain $\left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d_{W_e}}\right)$. Its image is said to be a *cyber-arrangement*. A cyber-arrangement together with all of its (instantaneous) cyber-morphisms is called a *cyberspace*.

In view of the above concepts, **[3]** investigates conditions under which an$e$ −regularization may be susceptible of aprojective e −limit. It is important to know if a e −*sub*-regularization is projective e −system. Subsequently, we defined and discussed the concept of the *length* in a cyber-domain. For the intrinsic cyber-metric $\boldsymbol{d_{W_e}}$, the distance between two cyber-elements is the length of the "shortest cyber-track" between these cyber-elements. The term shortest cyber-track is defined and is crucial for understanding the concept of *cyber-geodesic*. Although every shortest cyber track on a cyber-length space is a cyber-geodesic, the reverse argument is not valid. In fact, *some cyber-geodesics may fail to be shortest cyber-tracks on large scales*. However, since each cyber-domain $\left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d}_{\boldsymbol{W_e}}\right)$ is a compact, complete metric space, and since for any pair of cyber-elements in $\overline{\boldsymbol{ob}(\boldsymbol{W_e})}$ there is a cyber-evolutionary path of finite length joining them, one can easily ascertain the following converse result: any *pair of two cyber-elements in each cyber-domain* $\left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d}_{\boldsymbol{W_e}}\right)$ *has a shortest cyber track joining them*. Finally, **[3]** gives a discussion about the *speed* (: *cyber-speed*) of a cyber-evolution and the *convergence* of a sequence of cyber-evolutions.

3. Mathematical description of cyber-attacks

At any moment $\boldsymbol{t}$, a **node** $\boldsymbol{V}$ in the cyber-domain $\left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d}_{\boldsymbol{W_e}}\right)$ is composed of cyber constituents consisting in devices $\boldsymbol{D}_{\boldsymbol{j}}^{(\boldsymbol{V})}$ (:sensors, regulators of information flow, etc) and resources $\boldsymbol{R}_{\boldsymbol{k}}^{(\boldsymbol{V})}$ (:services, data, messages etc), the number of which depend potentially from the three geographical coordinates $\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}$ and the time $\boldsymbol{t}$. The order of any used quote of devices $\boldsymbol{D}_{\mathbf{1}}^{(\boldsymbol{V})}$, $\boldsymbol{D}_{\mathbf{2}}^{(\boldsymbol{V})}$,… and resources $\boldsymbol{R}_{\mathbf{1}}^{(\boldsymbol{V})}$, $\boldsymbol{R}_{\mathbf{2}}^{(\boldsymbol{V})}$,… is assumed to be given, pre-assigned and well defined. We will assume uninterruptedly that:

- the potential number of all ***possible*** devices and resources of $\boldsymbol{V}$ is equal to $\boldsymbol{\mathcal{M}_V} \gg \mathbf{0}$ and $\boldsymbol{\mathcal{L}_V} \gg \mathbf{0}$, respectively, and
- the number of $\boldsymbol{V}$'s ***available*** devices and resources is only $\boldsymbol{m_V} = \boldsymbol{m_V}$ and $\boldsymbol{\ell_V} = \boldsymbol{\ell_V}(\boldsymbol{t})$ respectively, with $\boldsymbol{m_V} < \boldsymbol{\mathcal{M}_V}$ and $\boldsymbol{\ell_V} < \boldsymbol{\mathcal{L}_V}$.

3.1 Valuations and vulnerabilities of parts of a node constituent

Let $\boldsymbol{U}$,$\boldsymbol{V}$ be two nodes in the cyber-domain $\left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d}_{\boldsymbol{W_e}}\right)$ and let $\boldsymbol{\mathcal{K}}^{(\boldsymbol{V})}$ be an available constituent in $\boldsymbol{V}$:

$$\boldsymbol{\mathcal{K}} = \begin{cases} \boldsymbol{D}, if\ the\ constituent\ is\ a\ device, \\ \boldsymbol{R}, if\ the\ constituent\ is\ a\ resource\ element. \end{cases}$$

Obviously, $\mathcal{K}^{(V)}$ may also be viewed as a nonempty collection of a number of elements. It is easy to see that one can make as much finite σ −algebras as partitions on $\mathcal{K}^{(V)}$.

Definition3.1 For every partition $\mathcal{P}$ of $\mathcal{K}^{(V)}$, let us consider a corresponding σ −algebra $\mathfrak{U}_{\mathcal{P}}$ of subsets of $\mathcal{K}^{(V)}$ as well as a monotonic measure μ defined on $\mathfrak{U}_{\mathcal{P}}$. Let also $Cr_1, Cr_2, \ldots, Cr_{\mathfrak{N}}$ be $\mathfrak{N} = \mathfrak{N}(\mathcal{K}^{(V)}, \mathcal{P})$ objective quantifiable criteria for the assessment of the points of $\mathcal{K}^{(V)}$. Denoting by $Cr_j(p) \in \mathbb{R}$ the value of Cr_j on $p \in \mathcal{K}^{(V)}$ at a point $(x_1, x_2, x_3, t) \in \mathbb{R}^3 \times [0, 1]$, suppose:

1) the functions $Cr_j(p)$ are measurable with respect to μ and
2) a **valuation weight** $u_j(p)$ is attributed by (the user(s) of) U to the Criterion Cr_j on $p \in \mathcal{K}^{(V)}$ at $(x_1, x_2, x_3, t) \in \mathbb{R}^4$.

If $E \in \mathfrak{U}_{\mathcal{P}}$ is a part of $\mathcal{K}^{(V)}$and $\mathfrak{n} \leq \mathfrak{N}$, then a **relative valuation of E from the viewpoint (of user(s)) of node U**with respect to the $\mathfrak{n}$ criteria $Cr_1, Cr_2, \ldots, Cr_{\mathfrak{n}}$ at the spatiotemporal point $(x_1, x_2, x_3, t) \in \mathbb{R}^4$ is any vector

$$A^{(U \rightsquigarrow V)}(E) = \left(a_1^{(U \rightsquigarrow V)}(E), a_2^{(U \rightsquigarrow V)}(E), \ldots, a_{\mathfrak{n}}^{(U \rightsquigarrow V)}(E)\right)^T \in \mathbb{R}^{\mathfrak{n}}$$

where each definite integral

$$a_j^{(U \rightsquigarrow V)}(E) := \int_E Cr_j(p) u_j(p) d\mu(p).$$

is the **component valuation of E from the viewpoint (of user(s)) of the node U into the constituent $\mathcal{K}^{(V)}$**at (x_1, x_2, x_3, t). The number $\mathfrak{n}$ is the **dimension of the valuation**. ■

There is a special category of valuations of particular interest, determined in regards to the low degree of "security" of the constituents of the node. The low degree of security is described completely by the concept of vulnerability.

Definition 3.2 For every partition $\mathcal{P}$ of $\mathcal{K}^{(V)}$, let us consider a corresponding σ −algebra$\mathfrak{U}_{\mathcal{P}}$ of subsets of $\mathcal{K}^{(V)}$as well as a monotonic measure λ defined on $\mathfrak{U}_{\mathcal{P}}$. Let also $SCr_1, SCr_2, \ldots, SCr_{\mathfrak{M}}$ be $\mathfrak{M} = \mathfrak{M}(\mathcal{K}^{(V)}, \mathcal{P})$ objective quantifiable criteria for the security assessment of the points of $\mathcal{K}^{(V)}$. Denoting by $SCr_j(p) \in \mathbb{R}$the value of $SeCr_j$ on $p \in \mathcal{K}^{(V)}$ at a spatiotemporal point $(x_1, x_2, x_3, t) \in \mathbb{R}^3 \times [0, 1]$, suppose:

1) the functions $SCr_j(p)$ are measurable with respect to λ and

2) a **vulnerability weight** $\boldsymbol{u_j(p)}$ is attributed by (the (user(s) of) node $\boldsymbol{U}$ to the security criterion $\boldsymbol{SCr_j}$ on $\boldsymbol{p} \in \boldsymbol{\mathcal{K}^{(V)}}$ at $(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{t}) \in \mathbb{R}^4$.

If $\boldsymbol{E} \in \boldsymbol{\mathfrak{U}_{\mathcal{P}}}$ is a part of $\boldsymbol{\mathcal{K}^{(V)}}$and $\boldsymbol{\mathfrak{m}} \leq \boldsymbol{\mathfrak{M}}$, then a **relative vulnerability of $\boldsymbol{E}$ from the viewpoint (of the user(s)) of node $\boldsymbol{U}$**with respect to the $\boldsymbol{\mathfrak{m}}$ security criteria $\boldsymbol{SCr_1}, \boldsymbol{SCr_2}, \dots, \boldsymbol{SCr_{\mathfrak{m}}}at(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{t}) \in \mathbb{R}^4$ is any vector

$$\boldsymbol{B}^{(U\rightsquigarrow V)}(\boldsymbol{E}) = \left(\boldsymbol{b}_{\boldsymbol{1}}^{(U\rightsquigarrow V)}(\boldsymbol{E}), \boldsymbol{b}_{\boldsymbol{2}}^{(U\rightsquigarrow V)}(\boldsymbol{E}), \dots, \boldsymbol{b}_{\boldsymbol{\mathfrak{m}}}^{(U\rightsquigarrow V)}(\boldsymbol{E})\right)^{\boldsymbol{T}} \in \mathbb{R}^{m}$$

where each definite integral

$$\boldsymbol{b}_{\boldsymbol{j}}^{(U\rightsquigarrow V)}(\boldsymbol{E}) = \int_{\boldsymbol{E}} \boldsymbol{SCr_j(p) u_j(p) d\lambda(p)}.$$

is the **component vulnerability of $\boldsymbol{E}$ from the viewpoint (of the (user(s)) of the node $\boldsymbol{U}$ into the constituent $\boldsymbol{\mathcal{K}^{(V)}}$** at $(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{t})$. The number $\boldsymbol{\mathfrak{m}}$ is the **dimension of the vulnerability**. ■

In what follows, a part $\boldsymbol{E}$ of a ***possible*** device $\boldsymbol{D_k^{(V)}}$ or/and resource $\boldsymbol{R_\xi^{(V)}}$ of $\boldsymbol{V}$ that is evaluated from the viewpoint (of the user(s)) of node $\boldsymbol{U}$ may be denoted by $\boldsymbol{fr}\left(\boldsymbol{D_\kappa^{(V)}}\right)$ or/and $\boldsymbol{fr}\left(\boldsymbol{R_\xi^{(V)}}\right)$, respectively $(\boldsymbol{\kappa} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{\mathcal{M}_V}, \quad \boldsymbol{\xi} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{\mathcal{L}_V})$. However, to denote both $\boldsymbol{A}^{(U\rightsquigarrow V)}\left(\boldsymbol{fr}\left(\boldsymbol{D_\kappa^{(V)}}\right)\right)$ and $\boldsymbol{A}^{(U\rightsquigarrow V)}\left(\boldsymbol{fr}\left(\boldsymbol{R_\xi^{(V)}}\right)\right)$ we will prefer to use the common notation $\boldsymbol{A_v^{(U\rightsquigarrow V)}}$**:**

$$\boldsymbol{A}_{\boldsymbol{v}}^{(U\rightsquigarrow V)} = \left(\boldsymbol{a}_{\boldsymbol{1},\boldsymbol{v}}^{(U\rightsquigarrow V)}, \dots, \boldsymbol{a}_{\boldsymbol{\mathfrak{n}},\boldsymbol{v}}^{(U\rightsquigarrow V)}\right)^{\boldsymbol{T}} =$$

$$\begin{cases} \boldsymbol{A_U}\left(\boldsymbol{fr}\left(\boldsymbol{D_v^{(V)}}\right)\right), \boldsymbol{if}\ \boldsymbol{v} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{\mathcal{M}_V} \\ \boldsymbol{A_U}\left(\boldsymbol{fr}\left(\boldsymbol{R_{v-\mathcal{M}_V}^{(V)}}\right)\right)\ \boldsymbol{if}\ \boldsymbol{v} = \boldsymbol{\mathcal{M}_V} + \boldsymbol{1}, \boldsymbol{\mathcal{M}_V} + \boldsymbol{2}, \dots, \boldsymbol{\mathcal{M}_V} + \boldsymbol{\mathcal{L}_V}. \end{cases}$$

Similarly, to denote both $\boldsymbol{B}^{(U\rightsquigarrow V)}\left(\boldsymbol{fr}\left(\boldsymbol{D_\kappa^{(V)}}\right)\right)$, $\boldsymbol{\kappa} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{\mathcal{M}_V}$ and $\boldsymbol{B}^{(U\rightsquigarrow V)}\left(\boldsymbol{fr}\left(\boldsymbol{R_\xi^{(V)}}\right)\right)$, $\boldsymbol{\xi} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{\mathcal{L}_V}$, we will prefer to adopt the notation

$$\boldsymbol{B}_{\boldsymbol{v}}^{(U\rightsquigarrow V)} = \left(\boldsymbol{b}_{\boldsymbol{1},\boldsymbol{v}}^{(U\rightsquigarrow V)}, \dots, \boldsymbol{b}_{\boldsymbol{\mathfrak{m}},\boldsymbol{v}}^{(U\rightsquigarrow V)}\right)^{\boldsymbol{T}} =$$

$$\begin{cases} \boldsymbol{B_U}\left(\boldsymbol{fr}\left(\boldsymbol{D_v^{(V)}}\right)\right), \boldsymbol{if}\ \boldsymbol{v} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{\mathcal{M}_V} \\ \boldsymbol{B_U}\left(\boldsymbol{fr}\left(\boldsymbol{R_{v-\mathcal{M}_V}^{(V)}}\right)\right)\ \boldsymbol{if}\ \boldsymbol{v} = \boldsymbol{\mathcal{M}_V} + \boldsymbol{1}, \boldsymbol{\mathcal{M}_V} + \boldsymbol{2}, \dots, \boldsymbol{\mathcal{M}_V} + \boldsymbol{\mathcal{L}_V}. \end{cases}$$

3.2 Cyber-effects and cyber-interactions

We are now in position to proceed towards a description of homomorphisms between cyber nodes. Let $\boldsymbol{U}$,$\boldsymbol{V}$ be two nodes in the cyber-domain $\left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d_{W_e}}\right)$. Without loss of generality, we may suppose the numbers $\boldsymbol{\mathcal{M}_V} + \boldsymbol{\mathcal{L}_V}$ and $\boldsymbol{\mathcal{M}_U} + \boldsymbol{\mathcal{L}_U}$ are both enough large, so that $\boldsymbol{k} := \boldsymbol{\mathcal{M}_V} + \boldsymbol{\mathcal{L}_V} = \boldsymbol{\mathcal{M}_U} + \boldsymbol{\mathcal{L}_U}$.

We consider the following sets.

1) $\mathfrak{C}^{(\boldsymbol{fraction})}(\boldsymbol{V}) =$

$$\left\{\left(\boldsymbol{fr}\left(\boldsymbol{D}_{\boldsymbol{1}}^{(\boldsymbol{V})}\right), \dots, \boldsymbol{fr}\left(\boldsymbol{D}_{\boldsymbol{\mathcal{M}_V}}^{(\boldsymbol{V})}\right), \boldsymbol{fr}\left(\boldsymbol{R}_{\boldsymbol{1}}^{(\boldsymbol{V})}\right), \dots, \boldsymbol{fr}\left(\boldsymbol{R}_{\boldsymbol{\mathcal{L}_V}}^{(\boldsymbol{V})}\right)\right): \boldsymbol{fr}\left(\boldsymbol{D}_{\boldsymbol{k}}^{(\boldsymbol{V})}\right), \boldsymbol{fr}\left(\boldsymbol{R}_{\boldsymbol{\xi}}^{(\boldsymbol{V})}\right) \in \mathfrak{U}_{\boldsymbol{\mathcal{P}}}, \boldsymbol{\kappa} \leq \boldsymbol{\mathcal{M}_V}, \boldsymbol{\xi} \leq \boldsymbol{\mathcal{L}_V}\right\}:$$

the set of ordered columns of possible parts of constituents of $\boldsymbol{V}$;

2) $\boldsymbol{\mathcal{A}_U}\mathfrak{C}^{(\boldsymbol{fraction})}(\boldsymbol{V}) =$

$$\left\{\left(\boldsymbol{A}_{\boldsymbol{1}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}, \dots, \boldsymbol{A}_{\boldsymbol{k}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right): \boldsymbol{A}_{\boldsymbol{v}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})} \in \mathbb{R}^{\mathfrak{n}}, \boldsymbol{v} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{k}\right\} \equiv \mathbb{R}^{\mathfrak{n} \times \boldsymbol{k}}:$$

the set of ordered columns of relative *valuations* of parts of possible constituents of $\boldsymbol{V}$, from the viewpoint of $\boldsymbol{U}$, over the space time$\mathbb{R}^{\boldsymbol{3}} \times [\boldsymbol{0}, \boldsymbol{1}]$;

3) $\boldsymbol{\mathcal{B}_U}\mathfrak{C}^{(\boldsymbol{fraction})}(\boldsymbol{V}) =$

$$\left\{\left(\boldsymbol{B}_{\boldsymbol{1}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}, \dots, \boldsymbol{B}_{\boldsymbol{k}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right): \boldsymbol{B}_{\boldsymbol{v}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})} \in \mathbb{R}^{\mathfrak{n}}, \boldsymbol{v} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{k}\right\} \equiv \mathbb{R}^{\mathfrak{n} \times \boldsymbol{k}}:$$

the set of all ordered columns of relative *vulnerabilities* of parts of possible constituents in $\boldsymbol{V}$, from the viewpoint of $\boldsymbol{U}$, over $\mathbb{R}^{\boldsymbol{3}} \times [\boldsymbol{0}, \boldsymbol{1}]$.

Definition 3.3 The triplet

$$\boldsymbol{\mathcal{P}} = \boldsymbol{\mathcal{P}}(\boldsymbol{V}) = \left(\mathfrak{C}^{(\boldsymbol{fraction})}(\boldsymbol{V}), \boldsymbol{\mathcal{A}_U}\mathfrak{C}^{(\boldsymbol{fraction})}(\boldsymbol{V}), \boldsymbol{\mathcal{B}_U}\mathfrak{C}^{(\boldsymbol{fraction})}(\boldsymbol{V})\right)$$

is called the **cyber-range of $\boldsymbol{V}$ from the viewpoint of (the users of)$\boldsymbol{U}$**.Its elements $\boldsymbol{p}$ are the (**threefold**) **cyber situations**. Especially, if $\boldsymbol{U} = \boldsymbol{V}$, the cyber-field $\boldsymbol{\mathcal{P}} = \boldsymbol{\mathcal{P}}(\boldsymbol{V})$ is the **cyber-purview** of $\boldsymbol{V}$ and is denoted $\boldsymbol{\mathcal{P}}^{(\boldsymbol{self})} = \boldsymbol{\mathcal{P}}^{(\boldsymbol{self})}(\boldsymbol{V})$. Its elements are represented by$\hat{\boldsymbol{p}}$.■

Given an ordered set

$$\boldsymbol{FR}^{(\boldsymbol{V})} := \left(\boldsymbol{fr}\left(\boldsymbol{D}_{\boldsymbol{1}}^{(\boldsymbol{V})}\right), \dots, \boldsymbol{fr}\left(\boldsymbol{D}_{\boldsymbol{\mathcal{M}_V}}^{(\boldsymbol{V})}\right), \boldsymbol{fr}\left(\boldsymbol{R}_{\boldsymbol{1}}^{(\boldsymbol{V})}\right), \dots, \boldsymbol{fr}\left(\boldsymbol{R}_{\boldsymbol{\mathcal{L}_V}}^{(\boldsymbol{V})}\right)\right)$$

of ordered columns of parts of constituents of $\boldsymbol{V}$, a cyber situation $\boldsymbol{p}$ on $\boldsymbol{V}$can be viewed as an ordered pair of matrices

$$\boldsymbol{p} = \left(\mathbb{A}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}, \mathbb{B}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right) = \left(\left(\boldsymbol{a_{i,j}}\right), \left(\boldsymbol{b_{i,j}}\right)\right) \in \mathbb{R}^{\mathfrak{n} \times \boldsymbol{k}} \times \mathbb{R}^{\mathfrak{m} \times \boldsymbol{k}}$$

where:

$$\mathbb{A}^{(U\rightsquigarrow V)} = \left(\boldsymbol{A}_{\mathbf{1}}^{(U\rightsquigarrow V)}, \dots, \boldsymbol{A}_{\boldsymbol{k}}^{(U\rightsquigarrow V)}\right) = \left(\boldsymbol{a}_{i,j}\right) = \begin{pmatrix} \boldsymbol{a}_{1,1}^{(U\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{1,\boldsymbol{k}}^{(U\rightsquigarrow V)} \\ \vdots & \vdots & \vdots \\ \boldsymbol{a}_{\mathfrak{n},\mathbf{1}}^{(U\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{\mathfrak{n},\boldsymbol{k}}^{(U\rightsquigarrow V)} \end{pmatrix} \text{ and}$$

$$\mathbb{B}^{(U\rightsquigarrow V)} = \left(\boldsymbol{B}_{\mathbf{1}}^{(U\rightsquigarrow V)}, \dots, \boldsymbol{B}_{\boldsymbol{k}}^{(U\rightsquigarrow V)}\right) = \left(\boldsymbol{b}_{i,j}\right) = \begin{pmatrix} \boldsymbol{b}_{1,1}^{(U\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{1,\boldsymbol{k}}^{(U\rightsquigarrow V)} \\ \vdots & \vdots & \vdots \\ \boldsymbol{b}_{\mathfrak{m},\mathbf{1}}^{(U\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{\mathfrak{m},\boldsymbol{k}}^{(U\rightsquigarrow V)} \end{pmatrix}.$$

In particular, any purview $\widehat{\boldsymbol{p}}$ on $\boldsymbol{V}$, can simply be viewed as an ordered pair

$$\widehat{\boldsymbol{p}} = \left(\widehat{\mathbb{A}}^{(V\rightsquigarrow V)}, \widehat{\mathbb{B}}^{(V\rightsquigarrow V)}\right) = \left(\left(\widehat{\boldsymbol{a}}_{i,j}\right), \left(\widehat{\boldsymbol{b}}_{i,j}\right)\right) \in \mathbb{R}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{R}^{\mathfrak{m}\times\boldsymbol{k}}$$

with

$$\widehat{\mathbb{A}}^{(V\rightsquigarrow V)} = \left(\widehat{\boldsymbol{a}}_{i,j}\right) = \begin{pmatrix} \boldsymbol{a}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{1,\boldsymbol{k}}^{(V\rightsquigarrow V)} \\ \vdots & \vdots & \vdots \\ \boldsymbol{a}_{\mathfrak{n},\mathbf{1}}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{\mathfrak{n},\boldsymbol{k}}^{(V\rightsquigarrow V)} \end{pmatrix} \text{ and}$$

$$\widehat{\mathbb{B}}^{(V\rightsquigarrow V)} = \left(\widehat{\boldsymbol{b}}_{i,j}\right) = \begin{pmatrix} \boldsymbol{b}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{1,\boldsymbol{k}}^{(V\rightsquigarrow V)} \\ \vdots & \vdots & \vdots \\ \boldsymbol{b}_{\mathfrak{m},\mathbf{1}}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{\mathfrak{m},\boldsymbol{k}}^{(V\rightsquigarrow V)} \end{pmatrix}.$$

To simplify our approach, in what follows we will assume that ***the location*** $(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}) \in \mathbb{R}^{\mathbf{3}}$ ***of V remains constantly fixed***.

Definition 3.4 The **supervision vector of $\boldsymbol{V}$ in the node system** $(\boldsymbol{V}, \boldsymbol{U})$ at a given time moment $\boldsymbol{t} \in [\mathbf{0}, \mathbf{1}]$ is defined to be the pair

$$(\mathbb{z}, \mathbb{w})(\boldsymbol{t}) = \left(\mathbb{A}_{\boldsymbol{U}\to\boldsymbol{V}} + \boldsymbol{i}\widehat{\mathbb{A}}_{\boldsymbol{V}\to\boldsymbol{V}}, \mathbb{B}_{\boldsymbol{U}\to\boldsymbol{V}} + \boldsymbol{i}\widehat{\mathbb{B}}_{\boldsymbol{V}\to\boldsymbol{V}}\right)(\boldsymbol{t}) \in \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}$$

with $\boldsymbol{i} := \sqrt{-\mathbf{1}} \in \mathbb{C}$. Especially, the complex matrices $\mathbb{z}$ and $\mathbb{w}$ are called **supervisory perceptions of $\boldsymbol{V}$ in the node system** $(\boldsymbol{V}, \boldsymbol{U})$ at moment $\boldsymbol{t}$. The mapping defined by

$$\boldsymbol{\gamma_V}: [\mathbf{0}, \mathbf{1}] \to \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}: \boldsymbol{t} \mapsto \boldsymbol{\gamma_V}(\boldsymbol{t}) = (\mathbb{z}, \mathbb{w})(\boldsymbol{t})$$

is the **supervisory perception curve of $\boldsymbol{V}$ in the node system**$(\boldsymbol{V}, \boldsymbol{U})$ **during the whole of time interval** $[\mathbf{0}, \mathbf{1}]$. The **supervisory perception domain of $\boldsymbol{V}$ in the node system**$(\boldsymbol{V}, \boldsymbol{U})$is the range $\boldsymbol{\gamma_V}([\mathbf{0}, \mathbf{1}])$ of $\boldsymbol{\gamma_V}$, denoted by $\boldsymbol{\gamma_V^*}$.■

Theoretically, each point in the space $\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}$ may be viewed as a supervision vector of $\boldsymbol{V}$ in the system of nodes $\boldsymbol{V}$ and $\boldsymbol{U}$. Since in many cases, it suffices (or is preferable) to use only specific supervisions from the viewpoint of $\boldsymbol{U}$ or $\boldsymbol{V}$:

$(\mathbb{A}_{U\to V}, \mathbb{B}_{U\to V})(\boldsymbol{t})$ or $(\widehat{\mathbb{A}}_{V\to V}, \widehat{\mathbb{B}}_{V\to V})(\boldsymbol{t})$ or $(\mathbb{A}_{U\to V}, \boldsymbol{i}\widehat{\mathbb{B}}_{V\to V})(\boldsymbol{t})$ or $(\boldsymbol{i}\widehat{\mathbb{A}}_{V\to V}, \mathbb{B}_{U\to V})(\boldsymbol{t})$ it is natural and imperative to consider two main vector fields $\boldsymbol{X1}$ and $\boldsymbol{X2}$ defined on $\boldsymbol{\gamma}_V^*$, as follows.

– The vector field $\boldsymbol{X1}$which assigns to each point

$$(\mathbb{z}, \mathbb{w})(\boldsymbol{t}) = \left(\mathbb{A}_{U\to V} + \boldsymbol{i}\widehat{\mathbb{A}}_{V\to V}, \mathbb{B}_{U\to V} + \boldsymbol{i}\widehat{\mathbb{B}}_{V\to V}\right)(\boldsymbol{t})$$

of $\boldsymbol{\gamma}_V^*$ the vector

$$(\boldsymbol{Re}\mathbb{z}, \boldsymbol{Re}\mathbb{w})(\boldsymbol{t}) \equiv (\mathbb{A}_{U\to V} + \boldsymbol{i0}, \mathbb{B}_{U\to V} + \boldsymbol{i0})(\boldsymbol{t}) \in \mathbb{R}^{\mathfrak{n}\times\mathcal{k}} \times \mathbb{R}^{\mathfrak{m}\times\mathcal{k}},$$

i.e., the vector of the relative *valuations* and *vulnerabilities* of $\boldsymbol{FR}^{(V)}$at $\boldsymbol{t}$, considered from the viewpoint of $\boldsymbol{U}$**;**in particular, we may also define the vector fields $\boldsymbol{Y1}$ and $\boldsymbol{Z1}$ assigning to each point $(\mathbb{z}, \mathbb{w})(\boldsymbol{t}) = \left(\mathbb{A}_{U\to V} + \boldsymbol{i}\widehat{\mathbb{A}}_{V\to V}, \mathbb{B}_{U\to V} + \boldsymbol{i}\widehat{\mathbb{B}}_{V\to V}\right)(\boldsymbol{t})$ of $\boldsymbol{\gamma}_V^*$ the relative *valuations* and relative *vulnerabilities* of $\boldsymbol{FR}^{(V)}$ at $\boldsymbol{t}$, considered from the viewpoint of $\boldsymbol{U}$:

$$\boldsymbol{Re}\mathbb{z}(\boldsymbol{t}) \equiv \mathbb{A}_{U\to V}(\boldsymbol{t}) \in \mathbb{R}^{\mathfrak{n}\times\mathcal{k}} \text{ and } \boldsymbol{Re}\mathbb{w}(\boldsymbol{t}) \equiv \mathbb{B}_{U\to V}(\boldsymbol{t}) \in \mathbb{R}^{\mathfrak{m}\times\mathcal{k}}.$$

– The vector field $\boldsymbol{X2}$ which assigns to each point

$$(\mathbb{z}, \mathbb{w})(\boldsymbol{t}) = \left(\mathbb{A}_{U\to V} + \boldsymbol{i}\widehat{\mathbb{A}}_{V\to V}, \mathbb{B}_{U\to V} + \boldsymbol{i}\widehat{\mathbb{B}}_{V\to V}\right)(\boldsymbol{t})$$

of $\boldsymbol{\gamma}_V^*$ the vector

$$(\boldsymbol{Im}\mathbb{z}, \boldsymbol{Im}\mathbb{w})(\boldsymbol{t}) \equiv \left(\boldsymbol{0} + \boldsymbol{i}\widehat{\mathbb{A}}_{V\to V}, \boldsymbol{0} + \boldsymbol{i}\widehat{\mathbb{B}}_{V\to V}\right)(\boldsymbol{t}) \in \mathbb{R}^{\mathfrak{n}\times\mathcal{k}} \times \mathbb{R}^{\mathfrak{m}\times\mathcal{k}},$$

i.e., the vector of the *valuations* and *vulnerabilities* of $\boldsymbol{FR}^{(V)}$ at $\boldsymbol{t}$, considered from the viewpoint of $\boldsymbol{V}$ itself**;** in subsequently, we may define the vector fields $\boldsymbol{Y2}$ and $\boldsymbol{Z2}$ assigning to each point $(\mathbb{z}, \mathbb{w})(\boldsymbol{t}) = \left(\mathbb{A}_{U\to V} + \boldsymbol{i}\widehat{\mathbb{A}}_{V\to V}, \mathbb{B}_{U\to V} + \boldsymbol{i}\widehat{\mathbb{B}}_{V\to V}\right)(\boldsymbol{t})$ of $\boldsymbol{\gamma}_V^*$ the vectors of *valuations* and *vulnerabilities* of $\boldsymbol{FR}^{(V)}$ at $\boldsymbol{t}$, considered from the viewpoint of $\boldsymbol{V}$ itself:

$$\boldsymbol{Im}\mathbb{z}(\boldsymbol{t}) \equiv \widehat{\mathbb{A}}_{V\to V}(\boldsymbol{t}) \in \mathbb{R}^{\mathfrak{n}\times\mathcal{k}} \text{ and } \boldsymbol{Im}\mathbb{w}(\boldsymbol{t}) \equiv \widehat{\mathbb{B}}_{V\to V}(\boldsymbol{t}) \in \mathbb{R}^{\mathfrak{m}\times\mathcal{k}}.$$

Of course, we may consider combinatorial vector fields, for instance the vector field $\boldsymbol{X3}$ which assigns to each point

$$(\mathbb{z}, \mathbb{w})(\boldsymbol{t}) = \left(\mathbb{A}_{U\to V} + \boldsymbol{i}\widehat{\mathbb{A}}_{V\to V}, \mathbb{B}_{U\to V} + \boldsymbol{i}\widehat{\mathbb{B}}_{V\to V}\right)(\boldsymbol{t})$$

of $\boldsymbol{\gamma}_V^*$ the vector

$$(\boldsymbol{Re}\mathbb{z}, \boldsymbol{Im}\mathbb{w})(\boldsymbol{t}) \equiv \left(\mathbb{A}_{U\to V} + \boldsymbol{i0}, \boldsymbol{0} + \boldsymbol{i}\widehat{\mathbb{B}}_{V\to V}\right)(\boldsymbol{t}) \in \mathbb{R}^{\mathfrak{n}\times\mathcal{k}} \times \mathbb{R}^{\mathfrak{m}\times\mathcal{k}},$$

i.e., the vector containing relative *valuations* of $\boldsymbol{FR}^{(\boldsymbol{V})}$ at $\boldsymbol{t}$ considered from the viewpoint of $\boldsymbol{U}$ and *vulnerabilities* of $\boldsymbol{FR}^{(\boldsymbol{V})}$at $\boldsymbol{t}$ considered from the viewpoint of $\boldsymbol{V}$ itself, or the vector field $\boldsymbol{X4}$ which assigns to each point

$$(\mathbb{z}, \mathbb{w})(\boldsymbol{t}) = \left(\mathbb{A}_{\boldsymbol{U}\to\boldsymbol{V}} + \boldsymbol{i}\widehat{\mathbb{A}}_{\boldsymbol{V}\to\boldsymbol{V}}, \mathbb{B}_{\boldsymbol{U}\to\boldsymbol{V}} + \boldsymbol{i}\widehat{\mathbb{B}}_{\boldsymbol{V}\to\boldsymbol{V}}\right)(\boldsymbol{t})$$

of $\boldsymbol{\gamma}_{\boldsymbol{V}}^{*}$ the vector

$$(\boldsymbol{Im}\mathbb{z}, \boldsymbol{Re}\mathbb{w})(\boldsymbol{t}) \equiv \left(\boldsymbol{0} + \boldsymbol{i}\widehat{\mathbb{A}}_{\boldsymbol{V}\to\boldsymbol{V}}, \mathbb{B}_{\boldsymbol{U}\to\boldsymbol{V}} + \boldsymbol{i0}\right)(\boldsymbol{t}).$$

i.e., the vector containing *valuations* of $\boldsymbol{FR}^{(\boldsymbol{V})}$ at $\boldsymbol{t}$ considered from the viewpoint of $\boldsymbol{V}$ itself and relative *vulnerabilities* of $\boldsymbol{FR}^{(\boldsymbol{V})}$ at $\boldsymbol{t}$ considered from the viewpoint of $\boldsymbol{U}$ itself.

With these definitions and notations, we can go further. The concept of supervisory perception curve is a concept that provides a clear overall relative evaluation of a node along time and particularly contain the changes of the quantitative overview on the node. In this sense, the supervisory perception curve could be considered as a concept that provides for the appearance of an action which could lead to changes. However, such a concept may not contain neither describe any action that could lead to changes. For this purpose, it should be noted that the concept of such an action is clearly local. Having regard to the above, it is therefore necessary to seek for a momentary consideration of the above defined supervisory concepts and, particularly, to proceed to a local study of the relevant curves. To this end, we fix a time moment $\boldsymbol{t_0} \in]\boldsymbol{0}, \boldsymbol{1}[$. A **supervision element** $(\boldsymbol{\gamma_V}, \boldsymbol{\mathcal{N}})$at $\boldsymbol{t_0}$ consists of a supervisory perception curve of $\boldsymbol{V}$(in the system of nodes $\boldsymbol{V}$and $\boldsymbol{U}$) defined on an open neighborhood $\boldsymbol{\mathcal{N}} =]\boldsymbol{t_0} - \boldsymbol{\varepsilon}, \boldsymbol{t_0} + \boldsymbol{\varepsilon}[$ of $\boldsymbol{t_0}$. Two supervision elements $(\boldsymbol{\gamma_V}, \boldsymbol{\mathcal{N}})$ and $(\boldsymbol{\delta_V}, \boldsymbol{\mathcal{M}})$ at $\boldsymbol{t_0}$ are equivalent (at $\boldsymbol{t_0}$) if there is an open neighborhood $\boldsymbol{\mathcal{J}} \subset \boldsymbol{\mathcal{N}} \cap \boldsymbol{\mathcal{M}}$of $\boldsymbol{t_0}$, such that $\boldsymbol{\gamma_V}|_{\boldsymbol{\mathcal{J}}} = \boldsymbol{\delta_V}|_{\boldsymbol{\mathcal{J}}}$. The set of equivalence classes of supervision elements at $\boldsymbol{t_0}$ is called **the set of germs of supervisory perceptions** of $\boldsymbol{V}$ (in the system of nodes $\boldsymbol{V}$ and $\boldsymbol{U}$) at a given time moment $\boldsymbol{t_0}$ and is denoted by $\mathbb{G}_{\boldsymbol{t_0}}^{(\boldsymbol{V})}$:

$$\mathbb{G}_{\boldsymbol{t_0}}^{(\boldsymbol{V})} := \{\boldsymbol{\gamma_V}(\boldsymbol{t}) = (\mathbb{z}, \mathbb{w})(\boldsymbol{t}) \equiv \left(\mathbb{A}_{\boldsymbol{U}\to\boldsymbol{V}} + \boldsymbol{i}\widehat{\mathbb{A}}_{\boldsymbol{V}\to\boldsymbol{V}}, \mathbb{B}_{\boldsymbol{U}\to\boldsymbol{V}} + \boldsymbol{i}\widehat{\mathbb{B}}_{\boldsymbol{V}\to\boldsymbol{V}}\right)(\boldsymbol{t}): \\ \boldsymbol{t} \in]\boldsymbol{t_0} - \boldsymbol{\varepsilon}, \boldsymbol{t_0} + \boldsymbol{\varepsilon}[, \boldsymbol{\varepsilon} > \boldsymbol{0}\}.$$

The equivalence class of $(\boldsymbol{\gamma_V}, \boldsymbol{\mathcal{N}})$ at $\boldsymbol{t_0}$ is denoted by $\boldsymbol{\Gamma}_{\boldsymbol{V}}^{\boldsymbol{t_0}}$, and $(\boldsymbol{\gamma_V}, \boldsymbol{\mathcal{N}})$ is called a **representative** of the germ $\boldsymbol{\Gamma}_{\boldsymbol{V}}^{\boldsymbol{t_0}}$. A supervision element $(\boldsymbol{\gamma_V}, \boldsymbol{\mathcal{N}})$ defines germs $\boldsymbol{\Gamma}_{\boldsymbol{V}}^{\boldsymbol{t_0}}$ of supervisory perceptions for each $\boldsymbol{t_0} \in \boldsymbol{\mathcal{N}}$. Since $\mathbb{G}_{\boldsymbol{t_0}}^{(\boldsymbol{V})} \cap \mathbb{G}_{\boldsymbol{s_0}}^{(\boldsymbol{V})} = \emptyset$ for $\boldsymbol{t_0} \neq \boldsymbol{s_0}$, we also have $\boldsymbol{\Gamma}_{\boldsymbol{V}}^{\boldsymbol{t_0}} \neq \boldsymbol{\Gamma}_{\boldsymbol{V}}^{\boldsymbol{s_0}}$.

Definition 3.5 A **cyber-effect** (or **momentary homomorphism**) of $\boldsymbol{U}$ on $\boldsymbol{V}$ in the cyber-domain $\left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d_{W_e}}\right)$ is a collection of mappings $\boldsymbol{g_t}$ from the set $\mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{U})}$ of germs of supervisory perceptions of $\boldsymbol{U}$ at time $\boldsymbol{t} \in]\boldsymbol{\sigma}, \boldsymbol{\tau}[\subset\subset]\boldsymbol{0}, \boldsymbol{1}[$ into the set $\mathbb{G}_{\boldsymbol{t+\Delta t}}^{(\boldsymbol{V})}$ of germs of supervisory perceptions of $\boldsymbol{V}$ into a cyber field $\boldsymbol{\mathcal{P}}$ of $\boldsymbol{V}$ at another time $\boldsymbol{t'} := \boldsymbol{t} + \Delta \boldsymbol{t} \in]\boldsymbol{\sigma}, \boldsymbol{\tau}[$:

$$\left(\boldsymbol{g_t} : \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{U})} \to \mathbb{G}_{\boldsymbol{t+\Delta t}}^{(\boldsymbol{V})} : \boldsymbol{\delta_U}(\boldsymbol{t}) \longmapsto \boldsymbol{\gamma_V}(\boldsymbol{t'})\right)_{\boldsymbol{t} \in]\boldsymbol{\sigma}, \boldsymbol{\tau}[} (\boldsymbol{t'} := \boldsymbol{t} + \Delta \boldsymbol{t}).$$

Each mapping $\boldsymbol{g_t}$ is said to be a **momentary homomorphism** between $\boldsymbol{U}$, $\boldsymbol{V} \in \left(\overline{\boldsymbol{ob}(\boldsymbol{W_e})}, \boldsymbol{d_{W_e}}\right)$. Notice that the case $\Delta \boldsymbol{t} = \boldsymbol{0}$ is not excluded. ■

Although the concept of cyber-effect at a time moment $\boldsymbol{t}$ seems to be rather sufficient, we care to describe the **interaction** that has one cyber-node on each other, as well as the mutual effects resulting at a later time. In this case, the putative mutuality is influenced directly by the users' **subjectivity** of the cyber nodes. So, frequently, instead of the concept of a momentary cyber-effect, we are forced to consider mappings describing mutual influences.

Definition 3.6 A **cyber-activity** of $\boldsymbol{U}$ on $\boldsymbol{V}$ over the time interval $]\boldsymbol{\sigma}, \boldsymbol{\tau}[\subset\subset]\boldsymbol{0}, \boldsymbol{1}[$ is a collection of correspondences from the product $\mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{U})} \times \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{V})}$ into the set $\mathbb{G}_{\boldsymbol{t+\Delta t}}^{(\boldsymbol{U})} \times \mathbb{G}_{\boldsymbol{t+\Delta t}}^{(\boldsymbol{V})}$:

$$\left(\boldsymbol{\mathcal{G}_t} : \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{V})} \times \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{U})} \to \mathbb{G}_{\boldsymbol{t+\Delta t}}^{(\boldsymbol{V})} \times \mathbb{G}_{\boldsymbol{t+\Delta t}}^{(\boldsymbol{U})} : \left(\boldsymbol{\gamma_V}(\boldsymbol{t}), \boldsymbol{\delta_U}(\boldsymbol{t})\right) \longmapsto \left(\boldsymbol{\gamma_V}(\boldsymbol{t'}), \boldsymbol{\delta_U}(\boldsymbol{t'})\right)\right)_{\boldsymbol{t} \in]\boldsymbol{\sigma}, \boldsymbol{\tau}[}$$
$$(\boldsymbol{t'} := \boldsymbol{t} + \Delta \boldsymbol{t} \in]\boldsymbol{\sigma}, \boldsymbol{\tau}[).$$

Notice that the case $\Delta \boldsymbol{t} = \boldsymbol{0}$ is not excluded. A **cyber-interplay** of the ordered cyber pair $(\boldsymbol{V}, \boldsymbol{U})$ over the time interval $]\boldsymbol{\sigma}, \boldsymbol{\tau}[\subset\subset [\boldsymbol{0}, \infty[$ is an open shift curve

$$\boldsymbol{\mathcal{G}}:]\boldsymbol{\sigma}, \boldsymbol{\tau}[\to \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{V})} \times \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{U})} \times \mathbb{G}_{\boldsymbol{t+\Delta t}}^{(\boldsymbol{V})} \times \mathbb{G}_{\boldsymbol{t+\Delta t}}^{(\boldsymbol{U})}:$$
$$\boldsymbol{t} \mapsto \boldsymbol{\mathcal{G}}(\boldsymbol{t}) := \left(\boldsymbol{\gamma_V}(\boldsymbol{t}), \boldsymbol{\delta_U}(\boldsymbol{t}), \boldsymbol{\gamma_V}(\boldsymbol{t} + \Delta \boldsymbol{t}), \boldsymbol{\delta_U}(\boldsymbol{t} + \Delta \boldsymbol{t})\right)$$
$$(\boldsymbol{t} + \Delta \boldsymbol{t} \in]\boldsymbol{\sigma}, \boldsymbol{\tau}[).$$

If the cyber-interplay $\boldsymbol{\mathcal{G}}$ is composition of several separate interplays, we say that $\boldsymbol{\mathcal{G}}$ is **sequential**; otherwise is called **elementary**. ■

In that regard to the concept of cyber-activity, we have the concept of cyber-interaction.

Definition 3.7 A **cyber-interaction** between $\boldsymbol{U}$ and $\boldsymbol{V}$ at a given time moment $\boldsymbol{t_0} \in \,]\boldsymbol{\sigma}, \boldsymbol{\tau}[$ is a tetrad:

$$\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(\boldsymbol{U},\boldsymbol{V})}(\boldsymbol{t_0}) = \big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big) \in \left(\mathbb{C}^{\mathfrak{n}\times\mathfrak{k}} \times \mathbb{C}^{\mathfrak{m}\times\mathfrak{k}}\right)^{\boldsymbol{4}}$$

for which there is an associated cyber-activity of $\boldsymbol{U}$ on $\boldsymbol{V}$:

$$\Big(\boldsymbol{\mathcal{G}_t} = \boldsymbol{\mathcal{G}_t^{(\mathcal{Z})}} : \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{V})} \times \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{U})} \to \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{V})} \times \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{U})} :$$

$$\big(\boldsymbol{\gamma_V}(\boldsymbol{t}), \boldsymbol{\delta_U}(\boldsymbol{t})\big) \longmapsto \big(\boldsymbol{\gamma_V}(\boldsymbol{t}'), \boldsymbol{\delta_U}(\boldsymbol{t}')\big)\Big)_{\boldsymbol{t}\in]\boldsymbol{\sigma},\boldsymbol{\tau}[}$$

$$(\boldsymbol{t}' := \boldsymbol{t} + \Delta\boldsymbol{t} \in \,]\boldsymbol{\sigma}, \boldsymbol{\tau}[),$$

such that:

$$(\mathbb{z}_1, \mathbb{w}_1) = \boldsymbol{\gamma_V}(\boldsymbol{t_0}) = \big(\mathbb{A}_{\boldsymbol{U}\to\boldsymbol{V}} + \boldsymbol{i}\widehat{\mathbb{A}}_{\boldsymbol{V}\to\boldsymbol{V}}, \mathbb{B}_{\boldsymbol{U}\to\boldsymbol{V}} + \boldsymbol{i}\widehat{\mathbb{B}}_{\boldsymbol{V}\to\boldsymbol{V}}\big) \in \mathbb{C}^{\mathfrak{n}\times\mathfrak{k}} \times \mathbb{C}^{\mathfrak{m}\times\mathfrak{k}},$$

$$(\mathbb{z}_2, \mathbb{w}_2) = \boldsymbol{\delta_U}(\boldsymbol{t_0}) = \big(\mathbb{A}_{\boldsymbol{V}\to\boldsymbol{U}} + \boldsymbol{i}\widehat{\mathbb{A}}_{\boldsymbol{U}\to\boldsymbol{U}}, \mathbb{B}_{\boldsymbol{V}\to\boldsymbol{U}} + \boldsymbol{i}\widehat{\mathbb{B}}_{\boldsymbol{U}\to\boldsymbol{U}}\big) \in \mathbb{C}^{\mathfrak{n}\times\mathfrak{k}} \times \mathbb{C}^{\mathfrak{m}\times\mathfrak{k}},$$

$$(\mathbb{z}'_1, \mathbb{w}'_1) = \boldsymbol{\gamma_V}(\boldsymbol{t}'_0) = \big(\mathbb{A}'_{\boldsymbol{U}\to\boldsymbol{V}} + \boldsymbol{i}\widehat{\mathbb{A}}'_{\boldsymbol{V}\to\boldsymbol{V}}, \mathbb{B}'_{\boldsymbol{U}\to\boldsymbol{V}} + \boldsymbol{i}\widehat{\mathbb{B}}'_{\boldsymbol{V}\to\boldsymbol{V}}\big) \in \mathbb{C}^{\mathfrak{n}\times\mathfrak{k}} \times \mathbb{C}^{\mathfrak{m}\times\mathfrak{k}},$$

$$(\mathbb{z}'_2, \mathbb{w}'_2) = \boldsymbol{\delta_U}(\boldsymbol{t}'_0) = \big(\mathbb{A}'_{\boldsymbol{V}\to\boldsymbol{U}} + \boldsymbol{i}\widehat{\mathbb{A}}'_{\boldsymbol{U}\to\boldsymbol{U}}, \mathbb{B}'_{\boldsymbol{V}\to\boldsymbol{U}} + \boldsymbol{i}\widehat{\mathbb{B}}'_{\boldsymbol{U}\to\boldsymbol{U}}\big) \in \mathbb{C}^{\mathfrak{n}\times\mathfrak{k}} \times \mathbb{C}^{\mathfrak{m}\times\mathfrak{k}}. \blacksquare$$

Obviously, keeping a fixed supervisory perception $\boldsymbol{\gamma_V}(\boldsymbol{t_0})$ in the archetype germ $\mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{V})}$and a fixed supervisory perception $\boldsymbol{\gamma_U}(\boldsymbol{t} + \Delta\boldsymbol{t})$ in the component image germ $\mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{U})}$, the corresponding cyber-interaction becomes a cyber-effect. And, as we see below, proper management of cyber-effects is enough to study cyber navigations. However, in most cases, as in the case of cyber-attacks, it is necessary to consider cyber-interactions. So, because cyber-effects are a partial case of cyber-interactions, we will give a slight priority in the most general context of cyber-interactions.

It is easily verified that the general form of a cyber-interaction is as follows.

$$\boldsymbol{\mathcal{Z}} = \big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big)$$

$$= \big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big)(\boldsymbol{t_0})$$

$$= \left(\left(\underbrace{\begin{pmatrix} a_{1,1}^{(W\rightsquigarrow V)} + i\ \hat{a}_{1,1}^{(V\rightsquigarrow V)} & \cdots\cdots\cdots & a_{1,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{a}_{1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & & \cdots \\ a_{m_V,1}^{(W\rightsquigarrow V)} + i\ \hat{a}_{m_V,1}^{(V\rightsquigarrow V)} & & a_{m_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{a}_{m_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & & \cdots \\ a_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V,1}^{(V\rightsquigarrow V)} & \cdots\cdots\cdots & a_{\mathcal{M}_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ a_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & & a_{\mathcal{M}_V+1,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & & \cdots \\ a_{\mathcal{M}_V+\mathcal{L}_V,1}^{(W\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V+\mathcal{L}_V,1}^{(V\rightsquigarrow V)} & \cdots\cdots\cdots & a_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}}^{(W\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \end{pmatrix}, \begin{pmatrix} b_{1,1}^{(W\rightsquigarrow V)} + i\ \hat{b}_{1,1}^{(V\rightsquigarrow V)} & \cdots\cdots\cdots & b_{1,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{b}_{1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & & \cdots \\ b_{m_V,1}^{(W\rightsquigarrow V)} + i\ \hat{b}_{m_V,1}^{(V\rightsquigarrow V)} & & b_{m_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{b}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & & \cdots \\ b_{\mathcal{M}_V,1}^{(W\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V,1}^{(V\rightsquigarrow V)} & \cdots\cdots\cdots & b_{\mathcal{M}_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ b_{\mathcal{M}_V+1,1}^{(W\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & & b_{\mathcal{M}_V+1,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & & \cdots \\ b_{\mathcal{M}_V+\mathcal{L}_V,1}^{(W\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V+\mathcal{L}_V,1}^{(V\rightsquigarrow V)} & \cdots\cdots\cdots & b_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}}^{(W\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \end{pmatrix}}_{(\mathbb{z}_1,\mathbb{w}_1)=\boldsymbol{\gamma_V}(\boldsymbol{t_0})=\left(\mathbb{A}_{U\to V}+\boldsymbol{i}\widehat{\mathbb{A}}_{V\to V},\mathbb{B}_{U\to V}+\boldsymbol{i}\widehat{\mathbb{B}}_{V\to V}\right)\in\mathbb{C}^{\mathfrak{n}\times\mathfrak{k}}\times\mathbb{C}^{\mathfrak{m}\times\mathfrak{k}}}\right),\right.$$

$$\left(\underbrace{\begin{pmatrix} a_{1,1}^{(V\rightsquigarrow W)}+i\ \hat{a}_{1,1}^{(W\rightsquigarrow W)} & \dots\dots\dots & a_{1,\mathfrak{n}}^{(V\rightsquigarrow W)}+i\ \hat{a}_{1,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & & \cdots \\ a_{m_W,1}^{(V\rightsquigarrow W)}+i\ \hat{a}_{m_W,1}^{(W\rightsquigarrow W)} & & a_{m_W,\mathfrak{n}}^{(V\rightsquigarrow W)}+i\ \hat{a}_{m_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & & \cdots \\ a_{\mathcal{M}_W,1}^{(V\rightsquigarrow W)}+i\ \hat{a}_{\mathcal{M}_W,1}^{(W\rightsquigarrow W)} & \dots\dots\dots & a_{\mathcal{M}_W,\mathfrak{n}}^{(V\rightsquigarrow W)}+i\ \hat{a}_{\mathcal{M}_W,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ a_{\mathcal{M}_W+1,1}^{(V\rightsquigarrow W)}+i\ \hat{a}_{\mathcal{M}_W+1,1}^{(W\rightsquigarrow W)} & & a_{\mathcal{M}_W+1,\mathfrak{n}}^{(V\rightsquigarrow W)}+i\ \hat{a}_{\mathcal{M}_W+1,\mathfrak{n}}^{(W\rightsquigarrow W)} \\ \cdots & & \cdots \\ a_{\mathcal{M}_W+\mathcal{L}_W,1}^{(V\rightsquigarrow W)}+i\ \hat{a}_{\mathcal{M}_W+\mathcal{L}_W,1}^{(W\rightsquigarrow W)} & \dots\dots\dots & a_{\mathcal{M}_W+\mathcal{L}_W,1}^{(V\rightsquigarrow W)}+i\ \hat{a}_{\mathcal{M}_W+\mathcal{L}_W,1}^{(W\rightsquigarrow W)} \end{pmatrix},\begin{pmatrix} b_{1,1}^{(V\rightsquigarrow W)}+i\ \hat{b}_{1,1}^{(W\rightsquigarrow W)} & \dots\dots\dots & b_{1,\mathfrak{m}}^{(V\rightsquigarrow W)}+i\ \hat{b}_{1,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & & \cdots \\ b_{m_W,1}^{(V\rightsquigarrow W)}+i\ \hat{b}_{m_W,1}^{(W\rightsquigarrow W)} & & b_{m_W,\mathfrak{m}}^{(V\rightsquigarrow W)}+i\ \hat{b}_{m_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & & \cdots \\ b_{\mathcal{M}_W,1}^{(V\rightsquigarrow W)}+i\ \hat{b}_{\mathcal{M}_W,1}^{(W\rightsquigarrow W)} & \dots\dots\dots & b_{\mathcal{M}_W,\mathfrak{m}}^{(V\rightsquigarrow W)}+i\ \hat{b}_{\mathcal{M}_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ b_{\mathcal{M}_W+1,1}^{(V\rightsquigarrow W)}+i\ \hat{b}_{\mathcal{M}_W+1,1}^{(W\rightsquigarrow W)} & & b_{\mathcal{M}_W+1,\mathfrak{m}}^{(V\rightsquigarrow W)}+i\ \hat{b}_{\mathcal{M}_W+1,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \cdots & & \cdots \\ b_{\mathcal{M}_W+\mathcal{L}_W,1}^{(V\rightsquigarrow W)}+i\ \hat{b}_{\mathcal{M}_W+\mathcal{L}_W,1}^{(W\rightsquigarrow W)} & \dots\dots\dots & b_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{m}}^{(V\rightsquigarrow W)}+i\ \hat{b}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{m}}^{(W\rightsquigarrow W)} \end{pmatrix}}_{(\mathbb{z}_2,\mathbb{w}_2)=\boldsymbol{\delta}_U(t_0)=(\mathbb{A}_{V\to U}+i\hat{\mathbb{A}}_{U\to U},\mathbb{B}_{V\to U}+i\hat{\mathbb{B}}_{U\to U})\in\mathbb{C}^{\mathfrak{n}\times\mathscr{k}}\times\mathbb{C}^{\mathfrak{m}\times\mathscr{k}}}\right)$$

$$\left(\underbrace{\begin{pmatrix} a'^{(W\rightsquigarrow V)}_{1,1}+i\ \widehat{a'}^{(V\rightsquigarrow V)}_{1,1} & \dots\dots\dots & a'^{(W\rightsquigarrow V)}_{1,\mathfrak{n}}+i\ \widehat{a'}^{(V\rightsquigarrow V)}_{1,\mathfrak{n}} \\ \cdots & & \cdots \\ a'^{(W\rightsquigarrow V)}_{m_V,1}+i\ \widehat{a'}^{(V\rightsquigarrow V)}_{m_V,1} & & a'^{(W\rightsquigarrow V)}_{m_V,\mathfrak{n}}+i\ \widehat{a'}^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\ \cdots & & \cdots \\ a'^{(W\rightsquigarrow V)}_{\mathcal{M}_V,1}+i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V,1} & \dots\dots\dots & a'^{(W\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{n}}+i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{n}} \\ a'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & a'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}}+i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\ \cdots & & \cdots \\ a'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1}+i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1} & \dots\dots\dots & a'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}}+i\ \widehat{a'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{n}} \end{pmatrix},\begin{pmatrix} b'^{(W\rightsquigarrow V)}_{1,1}+i\ \widehat{b'}^{(V\rightsquigarrow V)}_{1,1} & \dots\dots\dots & b'^{(W\rightsquigarrow V)}_{1,\mathfrak{m}}+i\ \widehat{b'}^{(V\rightsquigarrow V)}_{1,\mathfrak{m}} \\ \cdots & & \cdots \\ b'^{(W\rightsquigarrow V)}_{m_V,1}+i\ \widehat{b'}^{(V\rightsquigarrow V)}_{m_V,1} & & b'^{(W\rightsquigarrow V)}_{m_V,\mathfrak{m}}+i\ \widehat{b'}^{(V\rightsquigarrow V)}_{m_V,\mathfrak{m}} \\ \cdots & & \cdots \\ b'^{(W\rightsquigarrow V)}_{\mathcal{M}_V,1}+i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V,1} & \dots\dots\dots & b'^{(W\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{m}}+i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V,\mathfrak{m}} \\ b'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & b'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}}+i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\ \cdots & & \cdots \\ b'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1}+i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,1} & \dots\dots\dots & b'^{(W\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}}+i\ \widehat{b'}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\mathcal{L}_V,\mathfrak{m}} \end{pmatrix}}_{(\mathbb{z}'_1,\mathbb{w}'_1)=\boldsymbol{\gamma}_V(t'_0)=(\mathbb{A}'_{U\to V}+i\hat{\mathbb{A}}'_{V\to V},\mathbb{B}'_{U\to V}+i\hat{\mathbb{B}}'_{V\to V})\in\mathbb{C}^{\mathfrak{n}\times\mathscr{k}}\times\mathbb{C}^{\mathfrak{m}\times\mathscr{k}}}\right).$$

$$\left(\underbrace{\begin{pmatrix} a'^{(V\rightsquigarrow W)}_{1,1}+i\ \widehat{a'}^{(W\rightsquigarrow W)}_{1,1} & \dots\dots\dots & a'^{(V\rightsquigarrow W)}_{1,\mathfrak{n}}+i\ \widehat{a'}^{(W\rightsquigarrow W)}_{1,\mathfrak{n}} \\ \cdots & & \cdots \\ a'^{(V\rightsquigarrow W)}_{m_W,1}+i\ \widehat{a'}^{(W\rightsquigarrow W)}_{m_W,1} & & a'^{(V\rightsquigarrow W)}_{m_W,\mathfrak{n}}+i\ \widehat{a'}^{(W\rightsquigarrow W)}_{m_W+1,\mathfrak{n}} \\ \cdots & & \cdots \\ a'^{(V\rightsquigarrow W)}_{\mathcal{M}_W,1}+i\ \widehat{a'}^{(W\rightsquigarrow W)}_{\mathcal{M}_W,1} & \dots\dots\dots & a'^{(V\rightsquigarrow W)}_{\mathcal{M}_W,\mathfrak{n}}+i\ \widehat{a'}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} \\ a'^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,1}+i\ \widehat{a'}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & & a'^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}}+i\ \widehat{a'}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{n}} \\ \cdots & & \cdots \\ a'^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,1}+i\ \widehat{a'}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,1} & \dots\dots\dots & a'^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{n}}+i\ \widehat{a'}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{n}} \end{pmatrix},\begin{pmatrix} b'^{(V\rightsquigarrow W)}_{1,1}+i\ \widehat{b'}^{(W\rightsquigarrow W)}_{1,1} & \dots\dots\dots & b'^{(V\rightsquigarrow W)}_{1,\mathfrak{m}}+i\ \widehat{b'}^{(W\rightsquigarrow W)}_{1,\mathfrak{m}} \\ \cdots & & \cdots \\ b'^{(V\rightsquigarrow W)}_{m_W,1}+i\ \widehat{b'}^{(W\rightsquigarrow W)}_{m_W,1} & \dots\dots\dots & b'^{(V\rightsquigarrow W)}_{m_W,\mathfrak{m}}+i\ \widehat{b'}^{(W\rightsquigarrow W)}_{m_W,\mathfrak{m}} \\ \cdots & & \\ b'^{(V\rightsquigarrow W)}_{\mathcal{M}_W,1}+i\ \widehat{b'}^{(W\rightsquigarrow W)}_{\mathcal{M}_W,1} & & b'^{(V\rightsquigarrow W)}_{\mathcal{M}_W,\mathfrak{m}}+i\ \widehat{b'}^{(W\rightsquigarrow W)}_{\mathcal{M}_W,\mathfrak{m}} \\ b'^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,1}+i\ \widehat{b'}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,1} & \dots\dots\dots & b'^{(V\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}}+i\ \widehat{b'}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+1,\mathfrak{m}} \\ \cdots & & \cdots \\ b'^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,1}+i\ \widehat{b'}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,1} & & b'^{(V\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{m}}+i\ \widehat{b'}^{(W\rightsquigarrow W)}_{\mathcal{M}_W+\mathcal{L}_W,\mathfrak{m}} \end{pmatrix}}_{(\mathbb{z}'_2,\mathbb{w}'_2)=\boldsymbol{\delta}_U(t'_0)=(\mathbb{A}'_{V\to U}+i\hat{\mathbb{A}}'_{U\to U},\mathbb{B}'_{V\to U}+i\hat{\mathbb{B}}'_{U\to U})\in\mathbb{C}^{\mathfrak{n}\times\mathscr{k}}\times\mathbb{C}^{\mathfrak{m}\times\mathscr{k}}}\right)\Bigg)$$

Suppose now:

$$\boldsymbol{\mathcal{Z}}=\boldsymbol{\mathcal{Z}}_{(U,V)}(\boldsymbol{t_0})=$$

$$\big((\mathbb{z}_1,\mathbb{w}_1),(\mathbb{z}_2,\mathbb{w}_2),(\mathbb{z}'_1,\mathbb{w}'_1),(\mathbb{z}'_2,\mathbb{w}'_2)\big)\in\mathbb{C}^{\mathfrak{n}\times\mathscr{k}}\times\mathbb{C}^{\mathfrak{m}\times\mathscr{k}}\times\mathbb{C}^{\mathfrak{n}\times\mathscr{k}}\times\mathbb{C}^{\mathfrak{m}\times\mathscr{k}}$$

is a cyber interaction between $\boldsymbol{U}$, $\boldsymbol{V}$ at a fixed time moment $\boldsymbol{t_0}\in\,]\boldsymbol{\sigma},\boldsymbol{\tau}[\,\subset\subset\,]\boldsymbol{0},\boldsymbol{1}[$, with corresponding cyber- interplay

$$\boldsymbol{\mathcal{G}}:\,]\boldsymbol{\sigma},\boldsymbol{\tau}[\,\to\mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{V})}\times\mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{U})}\times\mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{V})}\times\mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{U})}:$$

$$\boldsymbol{t}\mapsto\boldsymbol{\mathcal{G}}(\boldsymbol{t}):=\big(\boldsymbol{\gamma_V}(\boldsymbol{t}),\boldsymbol{\delta_U}(\boldsymbol{t}),\boldsymbol{\gamma_V}(\boldsymbol{t}+\Delta\boldsymbol{t}),\boldsymbol{\delta_U}(\boldsymbol{t}+\Delta\boldsymbol{t})\big)$$

$$(\boldsymbol{t}+\Delta\boldsymbol{t}\in\,]\boldsymbol{\sigma},\boldsymbol{\tau}[)$$

and cyber-activity

$$\Big(\boldsymbol{\mathcal{G}_t}=\boldsymbol{\mathcal{G}}_{\boldsymbol{t}}^{(\boldsymbol{\mathcal{Z}})}:\mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{V})}\times\mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{U})}\to\mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{V})}\times\mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{U})}:$$

$$\big(\boldsymbol{\gamma_V}(\boldsymbol{t}),\boldsymbol{\delta_U}(\boldsymbol{t})\big)\longmapsto\big(\boldsymbol{\gamma_V}(\boldsymbol{t}'),\boldsymbol{\delta_U}(\boldsymbol{t}')\big)\Big)_{\boldsymbol{t}\in]\boldsymbol{\sigma},\boldsymbol{\tau}[}$$

$$(\boldsymbol{t}':=\boldsymbol{t}+\Delta\boldsymbol{t}\in\,]\boldsymbol{\sigma},\boldsymbol{\tau}[).$$

Definition 3.8 A **forced cyber-reflection** of $\boldsymbol{\mathcal{Z}}$ at a time moment $\boldsymbol{t_0}\in\,]\boldsymbol{\sigma},\boldsymbol{\tau}[$ is another cyber-interaction

$$\boldsymbol{\mathcal{Z}}' = \boldsymbol{\mathcal{Z}}'_{(\boldsymbol{U},\boldsymbol{V})}(\boldsymbol{t_0}) = \left((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2), (\mathbb{z}''_1, \mathbb{w}''_1), (\mathbb{z}''_2, \mathbb{w}''_2)\right)$$
$$= \left((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2), (\mathbb{z}''_1, \mathbb{w}''_1), (\mathbb{z}''_2, \mathbb{w}''_2)\right)(\boldsymbol{t'_0}) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{4}}$$

between $\boldsymbol{U}$ and $\boldsymbol{V}$ at a next time moment $\boldsymbol{t'_0} = \boldsymbol{t_0} + \boldsymbol{\Delta t_0} \in]\boldsymbol{\sigma}, \boldsymbol{\tau}[$, with corresponding **forced cyber- interplay**

$$\boldsymbol{\mathcal{G}}' :]\boldsymbol{\sigma}, \boldsymbol{\tau}[\to \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{V})} \times \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{U})} \times \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{V})} \times \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{U})} :$$
$$\boldsymbol{t} \mapsto \boldsymbol{\mathcal{G}}'(\boldsymbol{t}) := \left(\boldsymbol{\gamma_V}(\boldsymbol{t}'), \boldsymbol{\delta_U}(\boldsymbol{t}'), \boldsymbol{\gamma_V}(\boldsymbol{t}''), \boldsymbol{\delta_U}(\boldsymbol{t}'')\right)$$
$$(\boldsymbol{t}'' := \boldsymbol{t}' + \Delta\boldsymbol{t}' \in]\boldsymbol{\sigma}, \boldsymbol{\tau}[)$$

and associated **forced cyber-activity**:

$$\left(\boldsymbol{\mathcal{G}}'_{\boldsymbol{t}'} = \boldsymbol{\mathcal{G}}'^{(\boldsymbol{\mathcal{Z}})}_{\boldsymbol{t}'} : \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{V})} \times \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{U})} \to \mathbb{G}_{\boldsymbol{t}'+\Delta\boldsymbol{t}'}^{(\boldsymbol{V})} \times \mathbb{G}_{\boldsymbol{t}'+\Delta\boldsymbol{t}'}^{(\boldsymbol{U})} : \right.$$
$$\left. \left(\boldsymbol{\gamma_V}(\boldsymbol{t}'), \boldsymbol{\delta_U}(\boldsymbol{t}')\right) \longmapsto \left(\boldsymbol{\gamma_V}(\boldsymbol{t}''), \boldsymbol{\delta_U}(\boldsymbol{t}'')\right)\right)_{\boldsymbol{t}\in]\boldsymbol{\sigma},\boldsymbol{\tau}[}$$
$$(\boldsymbol{t}'' := \boldsymbol{t}' + \Delta\boldsymbol{t}' \in]\boldsymbol{\sigma}, \boldsymbol{\tau}[),$$

that satisfies the following property :*there is an open neighborhood* $]\boldsymbol{t_0} - \boldsymbol{\varepsilon}, \boldsymbol{t_0} + \boldsymbol{\varepsilon}[\subset]\boldsymbol{\sigma}, \boldsymbol{\tau}[$ *of* $\boldsymbol{t_0}$*, into which presence of cyber-reflection* of $\boldsymbol{\mathcal{Z}}$ *forces application of* $\boldsymbol{\mathcal{Z}}'$*,in the sense that the activity* $\boldsymbol{\mathcal{G}_t}$ *is obliged to push forward its composition with activity* $\boldsymbol{\mathcal{G}}'_{\boldsymbol{t}'}$ *in such a way that occurrence of* $\boldsymbol{\mathcal{G}}$ *guarantees the appearance of the composition* $\boldsymbol{\mathcal{G}}'_{\boldsymbol{t}'} \circ \boldsymbol{\mathcal{G}_t}$. In such a case, the cyber-activity $\boldsymbol{\mathcal{G}_t}$ together with its forced cyber-activity $\boldsymbol{\mathcal{G}}'_{\boldsymbol{t}'}$ is a **reflexive cyber-activity** between $\boldsymbol{U}$ and $\boldsymbol{V}$ during the period $]\boldsymbol{t_0} - \boldsymbol{\varepsilon}, \boldsymbol{t_0} + \boldsymbol{\varepsilon}[$. Their composition

$$\boldsymbol{\mathcal{G}}' \circ \boldsymbol{\mathcal{G}} : \boldsymbol{t} \longmapsto \boldsymbol{g}'_{\boldsymbol{t}'} \circ \boldsymbol{g}_{\boldsymbol{t}}$$

is a **self-inflicted cyber-activity** between $\boldsymbol{U}$ and $\boldsymbol{V}$ during the period$]\boldsymbol{t_0} - \boldsymbol{\varepsilon}, \boldsymbol{t_0} + \boldsymbol{\varepsilon}[$. In particular, the interaction $\boldsymbol{\mathcal{Z}}' = \boldsymbol{\mathcal{Z}}'_{(\boldsymbol{U},\boldsymbol{V})}(\boldsymbol{t'_0})$ is called **forced cyber-reflection** of $\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(\boldsymbol{U},\boldsymbol{V})}(\boldsymbol{t_0})$ at $\boldsymbol{t_0}$. A mapping

$$\boldsymbol{\Phi} : \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}} \to \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{2}}$$

which maps the cyber-interaction $\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(\boldsymbol{U},\boldsymbol{V})}(\boldsymbol{t_0})$ to its forced cyber-reflection $\boldsymbol{\mathcal{Z}}' = \boldsymbol{\mathcal{Z}}'_{(\boldsymbol{U},\boldsymbol{V})}(\boldsymbol{t'_0})$ is called **reflexive cyber-interaction mapping** at $\boldsymbol{t_0}$. ■

It is frequent that, under a self-inflicted cyber-activity

$$\left(\boldsymbol{\mathcal{G}}'_{\boldsymbol{t}'} \circ \boldsymbol{\mathcal{G}_t} : \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{V})} \times \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{U})} \to \mathbb{G}_{\boldsymbol{t}'+\Delta\boldsymbol{t}'}^{(\boldsymbol{V})} \times \mathbb{G}_{\boldsymbol{t}'+\Delta\boldsymbol{t}'}^{(\boldsymbol{U})} : \right.$$
$$\left. \left(\boldsymbol{\gamma_V}(\boldsymbol{t}), \boldsymbol{\delta_U}(\boldsymbol{t})\right) \longmapsto \left(\left(\boldsymbol{\gamma_V}(\boldsymbol{t}' + \Delta\boldsymbol{t}'), \boldsymbol{\delta_U}(\boldsymbol{t}' + \Delta\boldsymbol{t}')\right)\right)\right)_{\boldsymbol{t}\in]\boldsymbol{\sigma},\boldsymbol{\tau}[}$$
$$(\boldsymbol{t}' := \boldsymbol{t} + \Delta\boldsymbol{t} \in]\boldsymbol{\sigma}, \boldsymbol{\tau}[),$$

between $\boldsymbol{U}$ and $\boldsymbol{V}$ during the period $]\boldsymbol{t_0} - \boldsymbol{\varepsilon}, \boldsymbol{t_0} + \boldsymbol{\varepsilon}[\subset]\boldsymbol{\sigma}, \boldsymbol{\tau}[$, some valuations and vulnerabilities of the initial node $\boldsymbol{U}$ change at the moment $\boldsymbol{t_0}$.

For emphasis, this "new" node is called **variant node** of $\boldsymbol{U}$ and is denoted by $\boldsymbol{U'}$, or sometimes, without any risk of confusion, again by $\boldsymbol{U}$. In such a case, the forced cyber-reflection $\mathcal{Z}' = \mathcal{Z}'_{(U,V)}(\boldsymbol{t'_0})$ is a **cyber parallax** of the cyber-interaction $\mathcal{Z} = \mathcal{Z}_{(U,V)}(\boldsymbol{t_0})$ at $\boldsymbol{t_0}$ and the forced cyber-activity $\mathcal{g}'_{t'}$ is a **parallactic cyber-activity** which gives rise to a **parallactic cyber-interaction** at $\boldsymbol{t_0}$.

Definition 3.9 Let $\boldsymbol{E} \in \mathfrak{U}_{\mathcal{P}}$ be a part of $\mathcal{K}^{(U)}$where:

$$\mathcal{K} = \begin{cases} \boldsymbol{D}, if \ \ the \ \ constituent \ \ is \ \ a \ \ device, \\ \boldsymbol{R}, if \ \ the \ \ constituent \ \ is \ \ a \ \ resource \ \ element. \end{cases}$$

i. A **node shield** containing $\boldsymbol{E}$ in the node $\boldsymbol{U}$at $\boldsymbol{t}$ is an intermediate fixed node $\overline{\overline{\boldsymbol{U}}} = \overline{\overline{\boldsymbol{U}}}_{\boldsymbol{t}}$ which, at this time, is interposed in each cyber parallax $\mathcal{Z}'$ that aims at $\boldsymbol{E}$ in the node $\boldsymbol{U}$, so that the self-inflicted parallactic cyber-activity $\mathcal{G}' \circ \mathcal{G}$ between $\boldsymbol{U}$ and $\boldsymbol{V}$at moment time $\boldsymbol{t}$ends up in the intermediate node $\overline{\overline{\boldsymbol{U}}}$, and never can reach part $\boldsymbol{E}$ of the initial target $\boldsymbol{U}$.

ii. A **node filter** in part $\boldsymbol{E}$ of the constituent $\mathcal{K}^{(U)}$in $\boldsymbol{U}$ at $\boldsymbol{t}$ is an intermediate fixed node $\overline{\boldsymbol{U}}^{(E)}$ which, at this time moment, is interposed in each parallactic cyber-activity $\mathcal{G}'$ that aims at part $\boldsymbol{E}$ of node $\boldsymbol{U}$, so that the filter $\overline{\boldsymbol{U}}^{(E)}$ allows the self-inflicted parallactic cyber-activity $\mathcal{G}' \circ \mathcal{G}$ at $\boldsymbol{t}$ to reach only constituent parts of the initial target $\boldsymbol{U}$ that differ from $\boldsymbol{E}$. ■

4 Description of Cyber Navigations and Protection from Unplanned Attacks

4.1 Cyber navigations

Cyber navigation refers to the process of navigating a network of information resources in cyberspace, which is organized as hypertext or hypermedia. The mathematical modeling of cyber-navigation and its risks, as well as protection against such risks will be the main theme of this session. To this direction, let us begin with the following definition.

Definition 4.1 Suppose $t = t_0 < t_1 < \cdots < t_k = t'$ is a partition of the interval $[t, t'] \subset]0, 1[$.

i. The corresponding **cyber walk** with start node $V_{(x_1,x_2,x_3,t_0)}$ in the source $ob(cy(t_0))$ and final node $V_{(x_1,x_2,x_3,t_k)}$ in the ending $ob(cy(t_k))$ is an ordered node quote

$$V_0 V_1 \dots V_k = \underbrace{V_{(x_1,x_2,x_3,t_0)}}_{\in ob(cy(t_0))} \underbrace{V_{(x_1,x_2,x_3,t_1)}}_{\in ob(F_1[cy(t_0)])} \cdots \underbrace{V_{(x_1,x_2,x_3,t_k)}}_{\in ob([F_k \circ \dots \circ F_1][cy(t_0)])},$$

defined by given mappings

$$F_i: \underbrace{\{cy: \mathbb{I} \to ([ob(W_e)], d_{W_e})\}}_{T} \to \underbrace{\{cy: \mathbb{I} \to ([ob(W_e)], d_{W_e})\}}_{T}, \; i = 1, 2, \dots, k$$

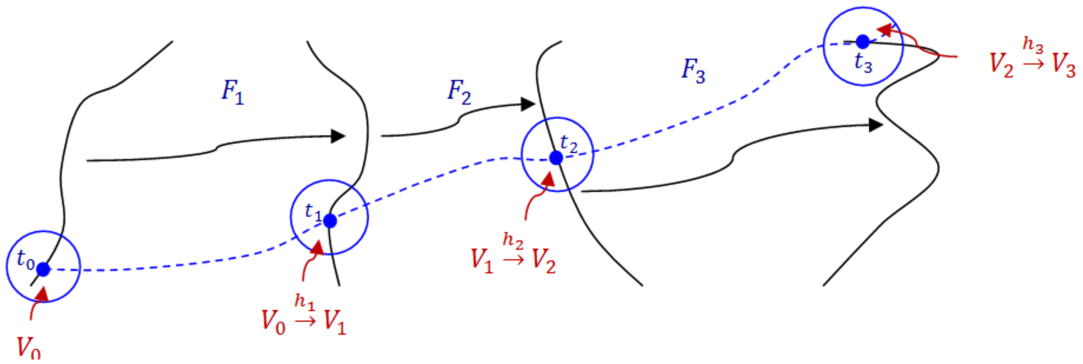
with the following three properties

1) $cy(t_\nu) = [F_\nu \circ \dots \circ F_1][cy(t_0)], \nu = 1, 2, \dots, k$

2) $V_0, V_1 \in ob(F_1[cy(t_0)]), V_1, V_2 \in ob([F_2 \circ F_1][cy(t_0)]), \dots$

$\dots, V_{k-1}, V_k \in ob([F_k \circ \dots \circ F_1][cy(t_0)])$

3) $h_1 = [V_0, V_1] \in hom(F_1[cy(t_0)]), \dots, h_k = [V_{k-1}, V_k] \in hom([F_k \circ \dots \circ F_1][cy(t_0)])$.



ii. A **cyber navigation** of the cyber node $U = U_{(x_1,x_2,x_3,t)} \in \bigcap_{\alpha=1}^{k} ob(cy(t_\alpha))$ (over a cyber walk from the node V_0 up to the node V_k) is a finite sequence of reflexive cyber-effects

$$\aleph = \Big(g_0 \equiv g_{t_0}: \mathbb{G}_t^{(U)} \to \mathbb{G}_{t+\Delta t}^{(V_0)} \forall t \in [t_0, t_1[,$$

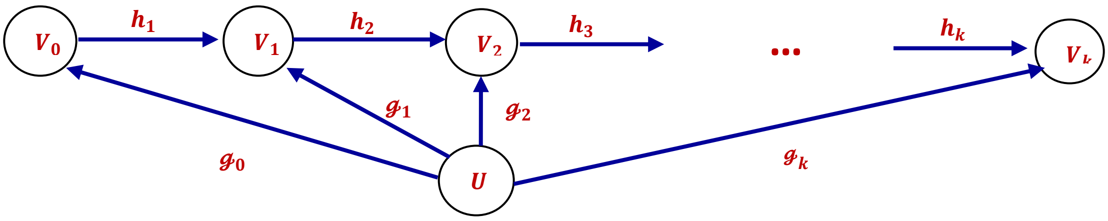
$$g_1 \equiv g_{t_1}: \mathbb{G}_t^{(U)} \to \mathbb{G}_{t+\Delta t}^{(V_1)} \forall t \in [t_1, t_2[,$$

$$\dots$$

$$g_{k-1} \equiv g_{t_{k-1}}: \mathbb{G}_t^{(U)} \to \mathbb{G}_{t+\Delta t}^{(V_{k-1})} \forall t \in [t_{k-1}, t_k[,$$

$$g_k \equiv g_{t_k}: \mathbb{G}_{t_k}^{(U)} \to \mathbb{G}_{t_k}^{(V_k)} \Big)$$

such that the ordered node quote $V_0V_1 \dots V_k$ is a cyber walk and the diagrams below commute



in the sense that $g_1 = h_1 \circ g_0, g_2 = h_2 \circ g_1, \dots, g_k = h_k \circ g_{k-1}$ It is clear that

$$g_k = h_k \circ h_{k-1} \circ \dots \circ h_2 \circ h_1 \circ g_0 = h \circ g_0 \text{ where } h := h_k \circ \dots \circ h_1. \blacksquare$$

4.2 Inadequacy of Cyber Nodes

Suppose $t = t_0 < t_1 < \cdots < t_k = t'$ is a partition of the interval $[t, t'] \subset]0, 1[$. Let

$$V_0V_1 \dots V_k = \underbrace{V_{(x_1,x_2,x_3)(t_0)}}_{\in ob(cy(t_0))} \underbrace{V_{(x_1,x_2,x_3)(t_1)}}_{\in ob(F_1[cy(t_0)])} \cdots \underbrace{V_{(x_1,x_2,x_3)(t_n)}}_{\in ob([F_k \circ \dots \circ F_1][cy(t_0)])}$$

be a corresponding walk with starting node $V_0 = V_{(x_1,x_2,x_3)(t_0)}$ in the source $ob(cy(t_0))$ and defined by the mappings

$$F_i : \underbrace{\{cy: \mathbb{I} \to ([ob(W_e)], d_{W_e})\}}_{T} \to \underbrace{\{cy: \mathbb{I} \to ([ob(W_e)], d_{W_e})\}}_{T}, \; i = 1, 2, \dots, k.$$

Let also a cyber navigation $\aleph = (g_0, g_1, \dots g_{k-1}, g_k)$ of a cyber node $U = U_{(x_1,x_2,x_3,t)} \in \bigcap_{\alpha=1}^{k} ob(cy(t_\alpha))$ over a cyber walk from the node V_0 up to the node V_k.

Definition 4.2 To each part $E = fr(\mathcal{K}^{(U)})$in the$\sigma-$algebra $\mathfrak{U}_{\mathcal{P}}$ of subsets of available (or not) constituents in the node U:

$$\mathcal{K} = \begin{cases} \boldsymbol{dev}, \textit{if the constituent is a device,} \\ \boldsymbol{res}, \textit{if the constituent is a resource element} \end{cases}$$

the users of a cyber-node Z (possibly identical to U) associate an **efficiency threshold vector**

$$\mathcal{T}(E) = (\mathcal{T}_1(E), \dots, \mathcal{T}_n(E)) \in [0, +\infty[^n.$$

i. The cyber node U is said to be **partially inadequate** in its part E over the cyber walk $V_0V_1 \dots V_k$ from the viewpoint of the user(s) of Z, if there is a variant node $U' = U_{t_\lambda}$ and a valuation

$$A^{(Z\rightsquigarrow U')}(E)=\left(a_1^{(Z\rightsquigarrow U')}(E),a_2^{(Z\rightsquigarrow U')}(E),\dots,a_{\mathfrak{n}}^{(Z\rightsquigarrow U)}(E)\right)^T$$

of $\mathcal{K}^{(U)}$ in $U'$from the viewpoint of the user(s) of Z, with some coordinates **less** than the corresponding coordinates of the efficiency threshold vector:

$$a_{i_j}^{(Z\rightsquigarrow U')}(E)<\mathcal{T}_{i_j}(E),\ 1\le j\le \mathfrak{n}.$$

The number

$$\varrho:=max_{1\le j\le \mathfrak{n}}\left(\mathcal{T}_{i_j}(E)-a_{i_j}^{(Z\rightsquigarrow U')}(E)\right)$$

is called the **degree of partial inadequacy** of part E in the cyber node U over the cyber walk $V_0V_1\dots V_k$ from the viewpoint of the user(s) of Z. In the particular case where $a_j^{(Z\rightsquigarrow U')}(E)<\mathcal{T}_j(E)$ whenever $j=1,2,\dots,\mathfrak{n}$, we say that U is **completely inadequate** in its part E over the cyber walk $V_0V_1\dots V_k$from the viewpoint of the user(s) of Z.

ii. The cyber node U is said to be **totally inadequate** in its part E over the cyber walk $V_0V_1\dots V_k$ from the viewpoint of the user(s) of Z, if there is a variant node $U'=U_{t_\lambda}$ and a valuation

$$A^{(Z\rightsquigarrow U')}(E)=\left(a_1^{(Z\rightsquigarrow U')}(E),a_2^{(Z\rightsquigarrow U')}(E),\dots,a_{\mathfrak{n}}^{(Z\rightsquigarrow U')}(E)\right)^T$$

of $\mathcal{K}^{(U)}$ in U' from the viewpoint of the user(s) of Z, with (Euclidean or not) norm **less** than the (corresponding Euclidean or not) norm of the efficiency threshold vector:

$$\left\|A^{(Z\rightsquigarrow U)}(E)\right\|<\|\mathcal{T}(E)\|.$$

The number

$$\varrho^{(\infty)}:=\|B(E)\|-\left\|A^{(Z\rightsquigarrow U')}(E)\right\|$$

is the **degree of total inadequacy** of part E in the cyber node U over the cyber walk $V_0V_1\dots V_k$ from the viewpoint of the user(s) of Z. In the contrary case, where U is **not partially inadequate** and **not totally inadequate** in its part Eover the cyber walk $V_0V_1\dots V_k$ from the viewpoint of the user(s) of Z, the node U is said to be **adequate** in its part $E=fr\left(\mathcal{A}^{(U)}\right)$over the cyber walk $V_0V_1\dots V_k$ from the viewpoint of the user(s) of Z. ■

4.3 Infected Cyber Nodes

Suppose $\boldsymbol{t = t_0 < t_1 < \cdots < t_k = t'}$ is a partition of the interval $[\boldsymbol{t}, \boldsymbol{t'}] \subset]\boldsymbol{0}, \boldsymbol{1}[$. Let

$$\boldsymbol{V_0 V_1 \dots V_k} = \underbrace{\boldsymbol{V_{(x_1,x_2,x_3)(t_0)}}}_{\in \boldsymbol{ob}(\boldsymbol{cy}(\boldsymbol{t_0}))} \underbrace{\boldsymbol{V_{(x_1,x_2,x_3)(t_1)}}}_{\in \boldsymbol{ob}(\boldsymbol{F_1}[\boldsymbol{cy}(\boldsymbol{t_0})])} \cdots \underbrace{\boldsymbol{V_{(x_1,x_2,x_3)(t_n)}}}_{\in \boldsymbol{ob}([\boldsymbol{F_k} \circ \dots \circ \boldsymbol{F_1}][\boldsymbol{cy}(\boldsymbol{t_0})])}$$

be corresponding walk with starting node $\boldsymbol{V_0} = \boldsymbol{V_{(x_1,x_2,x_3)(t_0)}}$ in the source $\boldsymbol{ob}(\boldsymbol{cy}(\boldsymbol{t_0}))$ and defined by the mappings

$$\boldsymbol{F_i}: \underbrace{\{\boldsymbol{cy}: \mathbb{I} \to ([\boldsymbol{ob}(\boldsymbol{W_e})], \boldsymbol{d_{W_e}})\}}_{\boldsymbol{T}} \to \underbrace{\{\boldsymbol{cy}: \mathbb{I} \to ([\boldsymbol{ob}(\boldsymbol{W_e})], \boldsymbol{d_{W_e}})\}}_{\boldsymbol{T}}, \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{k}.$$

Let also a cyber navigation $\aleph = (\boldsymbol{g_0}, \ \boldsymbol{g_1}, \ \dots \ \boldsymbol{g_{k-1}}, \ \boldsymbol{g_k})$of a cyber node $\boldsymbol{U} = \boldsymbol{U_{(x_1,x_2,x_3,t)}} \in \bigcap_{\boldsymbol{\alpha=1}}^{\boldsymbol{k}} \boldsymbol{ob}(\boldsymbol{cy}(\boldsymbol{t_\alpha}))$ over a cyber walk from the node $\boldsymbol{V_0}$ up to the node $\boldsymbol{V_k}$.

To each part $\boldsymbol{E} = \boldsymbol{fr}(\boldsymbol{\mathcal{K}^{(U)}})$ in the $\boldsymbol{\sigma}$ $-$algebra$\boldsymbol{\mathfrak{U}_{\mathcal{P}}}$ of subsets of available (or not) constituents of the node $\boldsymbol{U}$:

$$\boldsymbol{\mathcal{K}} = \begin{cases} \boldsymbol{dev}, \mathit{if\ the\ constituent\ is\ a\ device}, \\ \boldsymbol{res}, \mathit{if\ the\ constituent\ is\ a\ resource\ element} \end{cases}$$

the user(s) of a cyber-node $\boldsymbol{Z}$ (possibly identical to $\boldsymbol{U}$) associate a **health tolerance vector**

$$\boldsymbol{\mathfrak{T}}(\boldsymbol{E}) = (\boldsymbol{\mathfrak{T}_1}(\boldsymbol{E}), \dots, \boldsymbol{\mathfrak{T}_{\mathfrak{m}}}(\boldsymbol{E})) \in [\boldsymbol{0}, +\infty[^{\boldsymbol{\mathfrak{m}}}.$$

Definition 4.3. i The cyber node $\boldsymbol{U}$ is said to be **partially infected** in its part $\boldsymbol{E} = \boldsymbol{fr}(\boldsymbol{\mathcal{A}^{(U)}})$over the cyber walk $\boldsymbol{V_0 V_1 \dots V_k}$from the viewpoint of the user(s) of $\boldsymbol{Z}$, if there is a variant node $\boldsymbol{U'} = \boldsymbol{U_{t_\lambda}}$ and a vulnerability

$$\boldsymbol{B^{(Z \rightsquigarrow U')}}(\boldsymbol{E}) = \left(\boldsymbol{b_1^{(Z \rightsquigarrow U')}}(\boldsymbol{E}), \boldsymbol{b_2^{(Z \rightsquigarrow U')}}(\boldsymbol{E}), \dots, \boldsymbol{b_{\mathfrak{m}}^{(Z \rightsquigarrow U')}}(\boldsymbol{E})\right)^{\boldsymbol{T}}$$

of $\boldsymbol{\mathcal{K}^{(U)}}$ in $\boldsymbol{U'}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, with some coordinates **greater** than the corresponding coordinates of the **health tolerance vector**:

$$\boldsymbol{b_{i_j}^{(Z \rightsquigarrow U')}}(\boldsymbol{E}) > \boldsymbol{\mathfrak{T}_{i_j}}(\boldsymbol{E}), \boldsymbol{1} \leq \boldsymbol{j} \leq \boldsymbol{\mathfrak{m}}.$$

The number

$$\boldsymbol{\delta} := \boldsymbol{min_{1 \leq j \leq \mathfrak{m}}}\left(\boldsymbol{\mathfrak{T}_{i_j}}(\boldsymbol{E}) - \boldsymbol{b_{i_j}^{(Z \rightsquigarrow U')}}(\boldsymbol{E})\right)$$

is the **degree of partial infection** of part $\boldsymbol{E}$ in the cyber node $\boldsymbol{U}$ over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$. In the particular case where $\boldsymbol{b}_{\boldsymbol{j}}^{(\boldsymbol{Z} \rightsquigarrow \boldsymbol{U}')}(\boldsymbol{E}) > \mathfrak{T}_{\boldsymbol{j}}(\boldsymbol{E})$ whenever $\boldsymbol{j} = \boldsymbol{1}, \boldsymbol{2}, \dots, \mathfrak{m}$, we say that $\boldsymbol{U}$ is **completely infected** in its part $\boldsymbol{E}$over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$from the viewpoint of the user(s) of $\boldsymbol{Z}$.

ii The cyber node $\boldsymbol{U}$ is said to be **totally infected** (or **totally compromised**) in its part $\boldsymbol{E} = \boldsymbol{fr}\left(\boldsymbol{\mathcal{A}}^{(\boldsymbol{U})}\right)$over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, if there is a variant node $\boldsymbol{U}' = \boldsymbol{U}_{\boldsymbol{t}_\lambda}$ and a valuation

$$\boldsymbol{B}^{(\boldsymbol{Z} \rightsquigarrow \boldsymbol{U}')}(\boldsymbol{E}) = \left(\boldsymbol{b}_{\boldsymbol{1}}^{(\boldsymbol{Z} \rightsquigarrow \boldsymbol{U}')}(\boldsymbol{E}), \boldsymbol{b}_{\boldsymbol{2}}^{(\boldsymbol{Z} \rightsquigarrow \boldsymbol{U}')}(\boldsymbol{E}), \dots, \boldsymbol{b}_{\mathfrak{m}}^{(\boldsymbol{Z} \rightsquigarrow \boldsymbol{U})}(\boldsymbol{E})\right)^{\boldsymbol{T}}$$

of $\boldsymbol{\mathcal{K}}^{(\boldsymbol{U})}$ in $\boldsymbol{U}'$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, with (Euclidean or not) norm **greater** than the (corresponding Euclidean or not) norm of the health tolerance vector:

$$\left\|\boldsymbol{B}^{(\boldsymbol{Z} \rightsquigarrow \boldsymbol{U})}(\boldsymbol{E})\right\| > \|\mathfrak{T}(\boldsymbol{E})\|.$$

The number:

$$\boldsymbol{\delta}^{(\infty)} := \left\|\boldsymbol{B}^{(\boldsymbol{Z} \rightsquigarrow \boldsymbol{U}')}(\boldsymbol{E})\right\| - \|\mathfrak{T}(\boldsymbol{E})\|$$

is the **degree of the total infection** of part $\boldsymbol{E}$ in the cyber node $\boldsymbol{U}$ over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$. In the contrary case, where $\boldsymbol{U}$ is **not partially infected** and **not totally infected** in its part $\boldsymbol{E}$ over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, the node $\boldsymbol{U}$ is said to be **healthy** in its part $\boldsymbol{E}$ over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$.■

4.4 Dangerous Navigations

Let again $\boldsymbol{E} = \boldsymbol{fr}\left(\boldsymbol{\mathcal{K}}^{(\boldsymbol{U})}\right)$be a set in the $\boldsymbol{\sigma}$ −algebra $\mathfrak{U}_{\boldsymbol{\mathcal{P}}}$ of subsets of available (or not) constituents of the cyber node $\boldsymbol{U}$:

$$\boldsymbol{\mathcal{K}} = \begin{cases} \boldsymbol{dev}, \textit{if the constituent is a device}, \\ \boldsymbol{res}, \textit{if the constituent is a resource element} \end{cases}$$

Suppose the user(s) of a cyber-node $\boldsymbol{Z}$ (possibly identical to $\boldsymbol{U}$) associate an efficiency threshold vector

$$\boldsymbol{\mathcal{T}}(\boldsymbol{E}) = \left(\boldsymbol{\mathcal{T}}_{\boldsymbol{1}}(\boldsymbol{E}), \dots, \boldsymbol{\mathcal{T}}_{\mathfrak{n}}(\boldsymbol{E})\right) \in [\boldsymbol{0}, +\infty[^{\mathfrak{n}},$$

as well as a health tolerance vector

$$\mathfrak{T}(\boldsymbol{E}) = \left(\mathfrak{T}_{\boldsymbol{1}}(\boldsymbol{E}), \dots, \mathfrak{T}_{\mathfrak{m}}(\boldsymbol{E})\right) \in [\boldsymbol{0}, +\infty[^{\mathfrak{m}}.$$

Definition 4.4 The navigation $\aleph = (\mathcal{g}_0, \mathcal{g}_1, \dots \mathcal{g}_{k-1}, \mathcal{g}_k)$ of an adequate and healthy cyber node $\boldsymbol{U} = \boldsymbol{U}_{(x_1,x_2,x_3,t)} \in \bigcap_{\alpha=1}^{k} \boldsymbol{ob}\big(\boldsymbol{cy}(\boldsymbol{t_\alpha})\big)$ (over a cyber node homomorphism from a node $\boldsymbol{V_0}$ up to an infected node $\boldsymbol{V_k}$) is said to be a **dangerous navigation** or an **unplanned attack with degree of danger** $\boldsymbol{d} := \boldsymbol{max}\{\boldsymbol{\varrho}, \boldsymbol{\varrho}^{(\infty)}\} + \boldsymbol{max}\{\boldsymbol{\delta}, \boldsymbol{\delta}^{(\infty)}\}$ in its part $\boldsymbol{E}$ over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, if the node $\boldsymbol{U}$ becomes

- inadequate in its part $\boldsymbol{E} = \boldsymbol{fr}\big(\boldsymbol{\mathcal{A}}^{(\boldsymbol{U})}\big)$over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, with degree of partial inadequacy equal to $\boldsymbol{\varrho}$ and degree of total inadequacy equal to $\boldsymbol{\varrho}^{(\infty)}$ and
- infected in its part $\boldsymbol{E} = \boldsymbol{fr}\big(\boldsymbol{\mathcal{A}}^{(\boldsymbol{U})}\big)$ over the cyber walk $\boldsymbol{V_0 V_1} \dots \boldsymbol{V_k}$ from the viewpoint of the user(s) of $\boldsymbol{Z}$, with degree of partial infection equal to $\boldsymbol{\delta}$ and degree of total infection equal to $\boldsymbol{\delta}^{(\infty)}$.■

4.5 Protection of cyber nodes from unplanned attacks

Let again $\boldsymbol{E} = \boldsymbol{fr}\big(\boldsymbol{\mathcal{K}}^{(\boldsymbol{U})}\big)$ in the $\boldsymbol{\sigma}$ −algebra $\mathfrak{U}_{\boldsymbol{\mathcal{P}}}$ of subsets of an available or not constituent $\boldsymbol{\mathcal{K}}^{(\boldsymbol{U})}$ in node $\boldsymbol{U}$:

$$\boldsymbol{\mathcal{K}} = \begin{cases} \boldsymbol{dev}, if\ the\ constituent\ is\ a\ device, \\ \boldsymbol{res}, if\ the\ constituent\ is\ a\ resource\ element \end{cases}$$

Suppose the user(s) of a cyber-node $\boldsymbol{Z}$ (possibly identical to $\boldsymbol{U}$) associate an efficiency threshold vector $\boldsymbol{\mathcal{T}}(\boldsymbol{E}) \in [\boldsymbol{0}, +\infty[^{\mathbf{n}}$, as well as a **health tolerance vector** $\mathfrak{T}(\boldsymbol{E}) \in [\boldsymbol{0}, +\infty[^{\mathbf{m}}$.

Definition 4.5. i At a given time, the constituent part $\boldsymbol{E}$ of node $\boldsymbol{U}$ is said to be **protected from unplanned attacks, with degree of protection** $\boldsymbol{p} \in]\boldsymbol{0}, \boldsymbol{1}]$, if, at this time, there is a nodal fixed filter system $\overline{\boldsymbol{U}}^{(\boldsymbol{E})}$in part $\boldsymbol{E}$ that allows every **self-inflicted parallactic cyber-effect** $\mathcal{g}'_j \circ \mathcal{g}_j$ in any cyber-navigation of degree of danger $\boldsymbol{d} \leq -\boldsymbol{logp}$ to reach only constituent parts of the initial target $\boldsymbol{U}$ that are different from part $\boldsymbol{E}$ of $\boldsymbol{\mathcal{K}}^{(\boldsymbol{U})}$.

ii At a given time, the node $\boldsymbol{U}$ is said to be **completely protected from unplanned attacks of danger** degree $\boldsymbol{d}$, if, at this time, any part of every constituent of $\boldsymbol{U}$ is protected from unplanned attacks with degree of protection $\boldsymbol{p} \leq \boldsymbol{e}^{-\boldsymbol{d}}$. The node $\boldsymbol{U}$ is said to be **completely protected from unplanned attacks** at a given time, if, at this time, any constituent part of $\boldsymbol{U}$ is protected from unplanned attacks with degree of protection $\boldsymbol{p} = \mathbf{1}$. ■

5 Description of Various Types of Cyber Attacks and Protection

5.1 Passive cyber-attacks

A passive attack is a network attack in which a system is monitored and sometimes scanned for open inbound ports and vulnerabilities. The purpose is solely to gain information about the target and no data is changed on the target. So, a passive attack contrasts with an active attack, in which an intruder attempts to alter data on the target system or data en route for the target system.

Let $\boldsymbol{U}, \boldsymbol{V} \in \boldsymbol{ob}\big(\boldsymbol{cy}(\boldsymbol{t})\big)$, whenever $\boldsymbol{t}$ is in an arbitrary subset $\mathbb{I} =]\boldsymbol{\sigma}, \boldsymbol{\tau}[\subset\subset [\mathbf{0}, \mathbf{1}]$. Let also

$$\boldsymbol{\delta_U}: [\mathbf{0}, \mathbf{1}] \to \mathbb{C}^{\mathfrak{n}\times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times \boldsymbol{k}}: \boldsymbol{t} \mapsto \boldsymbol{\delta_W}(\boldsymbol{t}) = (\mathbb{z}_1, \mathbb{w}_1)(\boldsymbol{t}) \text{ and}$$

$$\boldsymbol{\gamma_V}: [\mathbf{0}, \mathbf{1}] \to \mathbb{C}^{\mathfrak{n}\times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times \boldsymbol{k}}: \boldsymbol{t} \mapsto \boldsymbol{\gamma_V}(\boldsymbol{t}) = (\mathbb{z}_2, \mathbb{w}_2)(\boldsymbol{t})$$

be two supervisory perception curves of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the node system $(\boldsymbol{U}, \boldsymbol{V})$.

A family of interactions

$$\boldsymbol{\mathcal{F}} = \Big\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(Y,X)}(\boldsymbol{t}) = \big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}_1', \mathbb{w}_1'), (\mathbb{z}_2', \mathbb{w}_2')\big)(\boldsymbol{t}) \in \big(\mathbb{C}^{\mathfrak{n}\times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times \boldsymbol{k}}\big)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\Big\},$$

$\boldsymbol{X}, \boldsymbol{Y} \in \{\boldsymbol{U}, \boldsymbol{V}\}$, with associated family of cyber-interplays

$$\boldsymbol{\mathcal{D}_{\mathcal{F}}} = \Big\{\boldsymbol{\mathcal{G}} = \boldsymbol{\mathcal{G}}^{(\mathcal{Z})}: \mathbb{I} \to \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{X})} \times \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{Y})} \times \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{X})} \times \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{Y})}:$$
$$\boldsymbol{t} \mapsto \boldsymbol{\mathcal{G}}(\boldsymbol{t}) = \Big(\boldsymbol{\delta}_{\boldsymbol{Y}}^{(\mathcal{Z})}(\boldsymbol{t}), \boldsymbol{\gamma}_{\boldsymbol{X}}^{(\mathcal{Z})}(\boldsymbol{t}), \boldsymbol{\delta}_{\boldsymbol{Y}}^{(\mathcal{Z})}(\boldsymbol{t} + \Delta\boldsymbol{t}), \boldsymbol{\gamma}_{\boldsymbol{X}}^{(\mathcal{Z})}(\boldsymbol{t} + \Delta\boldsymbol{t})\Big): \boldsymbol{t} + \Delta\boldsymbol{t} \in \mathbb{I}, \boldsymbol{\mathcal{Z}} \in \boldsymbol{\mathcal{F}}\Big\}$$

of the ordered cyber pair $(\boldsymbol{Y}, \boldsymbol{X})$ over the time $\boldsymbol{t} \in \mathbb{I}$**,** is called **coherent interactive family** in $\mathbb{I}$, if there is a homotopy

$$\boldsymbol{H}: \mathbb{I} \times [\mathbf{0}, \mathbf{1}] \to \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{X})} \times \mathbb{G}_{\boldsymbol{t}}^{(\boldsymbol{Y})} \times \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{X})} \times \mathbb{G}_{\boldsymbol{t}+\Delta\boldsymbol{t}}^{(\boldsymbol{Y})}$$

such that, for each cyber-interplay $\boldsymbol{\mathcal{G}} = \boldsymbol{\mathcal{G}}^{(\mathcal{Z})} \in \boldsymbol{\mathcal{D}_{\mathcal{F}}}$ there is a $\boldsymbol{p} \in [\mathbf{0}, \mathbf{1}]$ satisfying $\boldsymbol{H}(\boldsymbol{t}, \boldsymbol{p}) = \boldsymbol{\mathcal{G}}(\boldsymbol{t})$ at any moment time $\boldsymbol{t} \in \mathbb{I}$ on which the cyber-interplay $\boldsymbol{\mathcal{G}} = \boldsymbol{\mathcal{G}}^{(\mathcal{Z})}$ implements the interaction $\boldsymbol{\mathcal{Z}}$.

Definition 5.1 A family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \left\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(\boldsymbol{Y},\boldsymbol{X})}(\boldsymbol{t}) = \left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\right)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{\hbar}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{\hbar}}\right)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\right\},$$

lying in (a partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U}\to\boldsymbol{V}}$ of) the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ is a **germ of (partial) passive attack from $\boldsymbol{U}$ against the $(\boldsymbol{\kappa}_1, \dots, \boldsymbol{\kappa}_\lambda) -$ resource parts** $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa}_1}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa}_2}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa}_\lambda}\right)$ of $\boldsymbol{V}$ during an entire time interval $\mathbb{I}(=]\boldsymbol{\sigma}, \boldsymbol{\tau}[\subset\subset [\mathbf{0}, \mathbf{1}])$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, there is an integer $\boldsymbol{\nu} = \boldsymbol{\nu}(\boldsymbol{t}) > \mathbf{0}$ such that the pair $\left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\right) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{\hbar}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{\hbar}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\right) =$$

$$\left(\left(\left(\begin{array}{ccc} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \\ \boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\mathbf{1},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\mathbf{1},\mathbf{1}} & \dots\dots\dots & \boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\mathbf{1},\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\mathbf{1},\mathfrak{n}} \\ \vdots & & \vdots \\ \boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,\mathbf{1}} & & \boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,\mathfrak{n}} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{array}\right),\right.\right.$$

$$\left.\left.\left(\begin{array}{ccc} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \\ \boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\mathbf{1},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\mathbf{1},\mathbf{1}} & \dots\dots\dots & \boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\mathbf{1},\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\mathbf{1},\mathfrak{m}} \\ \vdots & & \vdots \\ \boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,\mathbf{1}} & & \boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_V+\boldsymbol{\ell}_V,\mathfrak{m}} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{array}\right)\right)\right),$$

$$\left(\left(\begin{array}{ccc} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \\ \boldsymbol{a}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}}_U+\mathbf{1},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}}_U+\mathbf{1},\mathbf{1}} & \dots\dots\dots & \boldsymbol{a}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}}_U+\mathbf{1},\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}}_U+\mathbf{1},\mathfrak{n}} \\ \vdots & & \vdots \\ \boldsymbol{a}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}}_U+\boldsymbol{\ell}_U,\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}}_U+\boldsymbol{\ell}_U,\mathbf{1}} & & \boldsymbol{a}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}}_U+\boldsymbol{\ell}_U,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}}_U+\boldsymbol{\ell}_U,\mathfrak{n}} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{array}\right),\right.\right.$$

$$\left.\left.\left.\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \\ \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & \cdots\cdots\cdots & \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} \\ \cdots & & \cdots \\ \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\boldsymbol{\ell}_U,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\boldsymbol{\ell}_U,1} & & \boldsymbol{b}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\boldsymbol{\ell}_U,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\boldsymbol{\ell}_U,\mathfrak{m}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right)\right)$$

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$, via the associated family of cyber-activities

$$\boldsymbol{\mathcal{D}}_{\boldsymbol{\mathcal{F}}} = \Big(\boldsymbol{\mathscr{g}}_{\boldsymbol{t}} = \boldsymbol{\mathscr{g}}_{\boldsymbol{t}}^{(\mathcal{Z})} : \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}} \to \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}} :$$

$$\big(\boldsymbol{\delta}_{\boldsymbol{U}}(\boldsymbol{t}), \boldsymbol{\gamma}_{\boldsymbol{V}}(\boldsymbol{t})\big) \longmapsto \big(\boldsymbol{\delta}'_{\boldsymbol{U}}(\boldsymbol{t}'), \boldsymbol{\gamma}'_{\boldsymbol{V}}(\boldsymbol{t}')\big)\Big)_{\boldsymbol{t}\in\mathbb{I}}$$

over the time $\boldsymbol{t} \in \mathbb{I}$, at $\big((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\big) \in \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form

$$\big((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\big) =$$

$$\left(\left(\left(\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \\ \boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots\cdots\cdots & \boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\ \cdots & & \cdots \\ \boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\boldsymbol{\ell}_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\boldsymbol{\ell}_V,1} & & \boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\boldsymbol{\ell}_V,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\boldsymbol{\ell}_V,\mathfrak{n}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.\right.$$

$$\left.\left.\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \\ \boldsymbol{b}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots\cdots\cdots & \boldsymbol{b}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\ \cdots & & \cdots \\ \boldsymbol{b}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\boldsymbol{\ell}_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\boldsymbol{\ell}_V,1} & & \boldsymbol{b}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\boldsymbol{\ell}_V,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\boldsymbol{\ell}_V,\mathfrak{m}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right),$$

$$\left(\left(\begin{matrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \\ \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & \cdots\cdots\cdots & \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\ \vdots & & \vdots \\ \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\ \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} + i\ \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} & & \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_{\backslash U}+1,\mathfrak{n}} \\ \vdots & \cdots\cdots\cdots & \vdots \\ \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,1} + i\ \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,1} & & \boldsymbol{a'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,\mathfrak{n}} \\ \mathbf{0} & & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right),\right.$$

$$\left.\left(\begin{matrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \\ \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & \cdots\cdots\cdots & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} \\ \vdots & & \vdots \\ \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} \\ \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} + i\ \widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{m}} \\ \vdots & \cdots\cdots\cdots & \vdots \\ \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,1} + i\ \widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\nu,\mathfrak{m}} \\ \mathbf{0} & & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{matrix}\right)\right)\Bigg). \blacksquare$$

It is quite easy to prove/verify the next two results.

Proposition 5.2 In a passive attack $\boldsymbol{\mathcal{F}}$ from $\boldsymbol{U}$ against $\boldsymbol{V}$, the number of resource parts in $\boldsymbol{U}$ at a moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$ has increased by at least $\boldsymbol{\lambda}$ new resource parts, say $\boldsymbol{fr}\left(\boldsymbol{res}^{(U)}_{\mathcal{M}_U+\ell_U+1}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(U)}_{\mathcal{M}_U+\ell_U+2}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}^{(U)}_{\mathcal{M}_U+\ell_U+\lambda}\right)$, derived from the resource parts $\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_1}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_2}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_\lambda}\right)$ that existed in the node $\boldsymbol{V}$ the previous moment $\boldsymbol{t}$, in such a way that the following elementary properties hold.

i. If the relative valuations of $\boldsymbol{fr}\left(\boldsymbol{res}^{(U)}_{\mathcal{M}_V+\ell_V+1}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(U)}_{\mathcal{M}_V+\ell_V+2}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}^{(U)}_{\mathcal{M}_V+\ell_V+\lambda}\right)$ from the viewpoint of the (user(s) of) node $\boldsymbol{U}$ at the previous moment $\boldsymbol{t}$ are $\left(\boldsymbol{a}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\mu_1,1}, \dots, \boldsymbol{a}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\mu_1,\mathfrak{n}}\right)$,…,$\left(\boldsymbol{a}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\mu_\lambda,1}, \dots, \boldsymbol{a}^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\mu_\lambda,\mathfrak{n}}\right)$ respectively, with $\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\lambda} \in \{\mathbf{1}, \mathbf{2}, \dots, \boldsymbol{\ell_V}\}$, then the resulting valuation vectors $\left(\widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1}, \dots, \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{n}}\right)$,…,$\left(\widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\lambda,1}, \dots, \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\lambda,\mathfrak{n}}\right)$ of the **new**

resource parts $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U+1}}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U+2}}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U+\lambda}}\right)$ in $\boldsymbol{U}$, as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at a next moment $\boldsymbol{t'}=\boldsymbol{t}+\Delta\boldsymbol{t}$ are equal to $\left(\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\mu_1,1}},\dots,\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\mu_1,\mathfrak{n}}}\right)$,…,$\left(\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\mu_\lambda,1}},\dots,\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\mu_\lambda,\mathfrak{n}}}\right)$**:**

$$\left(\widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U+\alpha,1}},\dots,\widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U+\alpha,\mathfrak{n}}}\right)=\left(\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\mu_\alpha,1}},\dots,\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\mu_\alpha,\mathfrak{n}}}\right),\ \forall\boldsymbol{\alpha}\in\{\boldsymbol{1},\boldsymbol{2},\dots,\boldsymbol{\lambda}\}.$$

ii. All resulting valuations and vulnerabilities of **new** resource parts $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U+1}}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U+\lambda}}\right)$ in $\boldsymbol{U}$ from the viewpoint of the user(s) of $\boldsymbol{V}$ remain equal to $\boldsymbol{0}$**:**

$$\forall\boldsymbol{j}\in\{\boldsymbol{1},\boldsymbol{2},\dots,\mathfrak{n}\}\ \boldsymbol{and}\ \forall\boldsymbol{\alpha}\in\{\boldsymbol{1},\boldsymbol{2},\dots,\boldsymbol{\lambda}\}\Rightarrow\boldsymbol{a}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U+\alpha,j}}=\boldsymbol{0},$$

$$\forall\boldsymbol{k}\in\{\boldsymbol{1},\boldsymbol{2},\dots,\mathfrak{m}\}\ \boldsymbol{and}\ \forall\boldsymbol{\alpha}\in\{\boldsymbol{1},\boldsymbol{2},\dots,\boldsymbol{\lambda}\}\Rightarrow\boldsymbol{b}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U+\alpha,k}}=\boldsymbol{0}.$$

iii. There is at least one resulting valuation $\boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\lambda_\alpha,j}}$ of a part $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa_\alpha}}\right)$ in $\boldsymbol{V}$ from the viewpoint of the user(s) of $\boldsymbol{U}$ which decreases:

$$\exists\boldsymbol{j}\in\{\boldsymbol{1},\boldsymbol{2},\dots,\mathfrak{n}\}\ \boldsymbol{and}\ \exists\boldsymbol{\lambda_\alpha}\in\{\boldsymbol{\mathcal{M}_V}+\boldsymbol{1},\dots,\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V}\}:\boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\lambda_\alpha,j}}<\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\lambda_\alpha,j}};$$

similarly, there is at least one vulnerability $\boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\rho_\alpha,k}}$ of part $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa_\alpha}}\right)$ in $\boldsymbol{V}$ from the viewpoint of the user(s) of $\boldsymbol{U}$ which increases

$$\exists\boldsymbol{k}\in\{\boldsymbol{1},\boldsymbol{2},\dots,\mathfrak{m}\}\ \boldsymbol{and}\ \exists\boldsymbol{\rho_\alpha}\in\{\boldsymbol{\mathcal{M}_V}+\boldsymbol{1},\dots,\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V}\}:$$
$$\boldsymbol{b}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\rho_\alpha,k}}>\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\rho_\alpha,k}}.$$

iv. The valuations and vulnerabilities of each part $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa_\alpha}}\right)$ in $\boldsymbol{V}$ from the viewpoint of the user(s) of $\boldsymbol{V}$ remain unchanged:

$$\forall\boldsymbol{j}\in\{\boldsymbol{1},\boldsymbol{2},\dots,\mathfrak{n}\}\ \boldsymbol{and}\ \forall\boldsymbol{\lambda_\alpha}\in\{\boldsymbol{\mathcal{M}_V}+\boldsymbol{1},\dots,\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V}\}\Rightarrow$$
$$\widehat{\boldsymbol{a}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\lambda_\alpha,j}}=\widehat{\boldsymbol{a}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\lambda_\alpha,j}},$$

$$\forall\boldsymbol{k}\in\{\boldsymbol{1},\boldsymbol{2},\dots,\mathfrak{m}\}\ \boldsymbol{and}\ \forall\boldsymbol{\mu_\alpha}\in\{\boldsymbol{\mathcal{M}_V}+\boldsymbol{1},\dots,\boldsymbol{\mathcal{M}_V}+\boldsymbol{\ell_V}\}\Rightarrow$$
$$\widehat{\boldsymbol{b}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V+\mu_\alpha,k}}=\widehat{\boldsymbol{b}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V+\mu_\alpha,k}}.\ \blacksquare$$

Proposition 5.3 In a passive attack $\boldsymbol{\mathcal{F}}$ from $\boldsymbol{U}$ against $\boldsymbol{V}$, the number of resource parts in $\boldsymbol{U}$ at a moment $\boldsymbol{t'}=\boldsymbol{t}+\Delta\boldsymbol{t}$ has increased by at least $\boldsymbol{\lambda}$ new resource parts, say $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U+1}}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U+2}}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\ell_U+\lambda}}\right)$, derived from the resource parts $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa_1}}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa_2}}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\kappa_\lambda}}\right)$ that existed in the node $\boldsymbol{V}$ the previous moment $\boldsymbol{t}$, in such a way that the following elementary properties hold.

i. The (Euclidean) norm $\left\|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\nu=1}^{\boldsymbol{\ell}_{U}+\boldsymbol{\lambda}} \left|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}}_{U}+\boldsymbol{\nu},\boldsymbol{j}}\right|^{\boldsymbol{2}}\right)^{1/2}$ of the resulting overall valuation in the variant node $\boldsymbol{U}'$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is **greater** than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{a}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\nu=1}^{\boldsymbol{\ell}_{U}} \left|\widehat{\boldsymbol{a}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}}_{U}+\boldsymbol{\nu},\boldsymbol{j}}\right|^{2}\right)^{\boldsymbol{1/2}} \text{ and}$$

$$\left\|\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\nu=1}^{\boldsymbol{\ell}_{V}} \left|\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_{V}+\boldsymbol{\nu},\boldsymbol{j}}\right|^{2}\right)^{\boldsymbol{1/2}}$$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the users of $\boldsymbol{U}$at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| > max\left\{\left\|\widehat{\boldsymbol{a}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|, \left\|\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|\right\}.$$

ii. The norm $\left\|\boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\nu=1}^{\boldsymbol{\ell}_{V}} \left|\boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_{V}+\boldsymbol{\nu},\boldsymbol{j}}\right|^{2}\right)^{1/2}$ of the resulting overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{W}$ at the next moment $\boldsymbol{t}'$ is **less** than the norm $\left\|\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\nu=1}^{\boldsymbol{\ell}_{V}} \left|\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_{V}+\boldsymbol{\nu},\boldsymbol{j}}\right|^{2}\right)^{\boldsymbol{1/2}}$ of the initial overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the users of $\boldsymbol{U}$at the preceding moment $\boldsymbol{t}$:

$$\left\|\boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\| < \left\|\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|.$$

iii. The norm $\left\|\widehat{\boldsymbol{b}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\boldsymbol{\ell}_{U}+\boldsymbol{\nu}} \left|\widehat{\boldsymbol{b}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}}_{U}+\boldsymbol{\lambda},\boldsymbol{j}}\right|^{\boldsymbol{2}}\right)^{1/2}$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is **less or equal** than the norms

$$\left\|\widehat{\boldsymbol{b}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\nu=1}^{\boldsymbol{\ell}_{W}} \left|\widehat{\boldsymbol{b}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}_{\boldsymbol{\mathcal{M}}_{U}+\boldsymbol{\nu},\boldsymbol{j}}\right|^{2}\right)^{\boldsymbol{1/2}} \text{ and}$$

$$\left\|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\nu=1}^{\boldsymbol{\ell}_{V}} \left|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}}_{V}+\boldsymbol{\nu},\boldsymbol{j}}\right|^{2}\right)^{\boldsymbol{1/2}}$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the users of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{b}}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| \leq \boldsymbol{min}\left\{\left\|\widehat{\boldsymbol{b}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|, \left\|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|\right\}.$$

iv. The norm $\left\|\boldsymbol{b}'^{(U \rightsquigarrow V)}\right\| := \left(\sum_{j=1}^{m} \sum_{\nu=1}^{\ell_V} \left|\boldsymbol{b}'^{(U \rightsquigarrow V)}_{\mathcal{M}_V+\nu,j}\right|^{2}\right)^{1/2}$ of the resulting overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the users of $\boldsymbol{U}$at the next moment $\boldsymbol{t}'$ is **greater** than the norm $\left\|\boldsymbol{b}^{(U \rightsquigarrow V)}\right\| := \left(\sum_{j=1}^{m} \sum_{\nu=1}^{\ell_V} \left|\boldsymbol{b}^{(U \rightsquigarrow V)}_{\mathcal{M}_U+\nu,j}\right|^{2}\right)^{1/2}$ of the initial overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\boldsymbol{b}'^{(U \rightsquigarrow V)}\right\| > \left\|\boldsymbol{b}^{(U \rightsquigarrow V)}\right\|.\blacksquare$$

The **degree** $\boldsymbol{d} = \boldsymbol{d}_{\boldsymbol{\kappa_1},\dots,\boldsymbol{\kappa_\nu}}$ **of the passive attack** $\boldsymbol{f}$ **against the resource parts** $\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_1}\right), \boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_2}\right),\dots,\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_\lambda}\right)$ of node $\boldsymbol{V}$ from the offensive node $\boldsymbol{U}$ at time moment $\boldsymbol{t} \in \mathbb{I}$ is the maximum of the two quotients

$$\boldsymbol{d_1} := \left\|\widehat{\boldsymbol{a}}'^{(U \rightsquigarrow U)}\right\| / \left\|\boldsymbol{a}'^{(U \rightsquigarrow V)}\right\| \text{ and } \boldsymbol{d_2} := \left(\left\|\widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)}\right\| / \left\|\boldsymbol{b}'^{(U \rightsquigarrow V)}\right\|\right)^{-1}.$$

Thus

$$\boldsymbol{d} = \boldsymbol{d}_{\boldsymbol{\kappa_1},\dots,\boldsymbol{\kappa_\lambda}} := \boldsymbol{max}\{\boldsymbol{d_1}, \boldsymbol{d_2}\}.$$

If the degree $\boldsymbol{d}$ surpasses a given threshold $\boldsymbol{S}^{(W,V)}_{\kappa_1,\dots,\kappa_\lambda} \in [\boldsymbol{0}, \infty[$, called the **passive attack threshold in the resource parts** $\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_1}\right), \boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_2}\right),\dots,\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_\lambda}\right)$ of $\boldsymbol{V}$ at time moment$\boldsymbol{t} \in \mathbb{I}$,we say that the passive attack $\boldsymbol{f}$ is **dangerous with degree of danger** $\boldsymbol{d}$ in theresource parts $\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_1}\right), \boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_2}\right),\dots,\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_\lambda}\right)$ of $\boldsymbol{V}$.

5.2 Protected cyber nodes from passive attacks

Definition 5.4.

i. The node $\boldsymbol{V}$ is said to be **protected from passive attacks, with degree of protection** $\boldsymbol{p} \in]\boldsymbol{0}, \boldsymbol{1}]$ over the resource parts $\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_1}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_2}\right),\dots,\boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_\nu}\right)$ of $\boldsymbol{V}$ over a time period $\mathbb{I}$, if, during this time period, there is a nodal fixed filter system $\overline{\boldsymbol{V}}^{(\kappa_1,\dots,\kappa_\nu)}$in the union $\boldsymbol{E} = \boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_1}\right) \cup \boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_2}\right) \cup \dots \cup \boldsymbol{fr}\left(\boldsymbol{res}^{(V)}_{\kappa_\nu}\right)$that allow every parallactic cyber passive attack against the resource parts (from any offensive node $\boldsymbol{U}$)with degree of danger $\boldsymbol{d} \leq -\boldsymbol{logp}$ to reach only resource parts $\boldsymbol{K}$ of the initial target $\boldsymbol{V}$ that are disjoint from $\boldsymbol{E}$.

ii. During the time period $\mathbb{I}$, the node $\boldsymbol{V}$ is said to be **completely protected from passive attacks of danger degree** $\boldsymbol{d}$, if, at this time period, any resource part in $\boldsymbol{V}$ is protected from passive attacks against $\boldsymbol{V}$, with degree of protection $\boldsymbol{p} \leq \boldsymbol{e}^{-\boldsymbol{d}}$. The node $\boldsymbol{V}$ is said to be **completely protected from passive attacks at a given time period**, if, during this time period, any resource part of $\boldsymbol{V}$ is protected from active attacks against $\boldsymbol{V}$ with degree of protection $\boldsymbol{p} = \mathbf{1}$.■

5.3 Active cyber-attacks

An attack is active if it is an attack with data transmission to all parties thereby acting as a liaison enabling severe compromise. The purpose is to alter system resources or affect their operation. So, in an active attack, an intruder attempts to alter data on the target system or data "en route" for the target system.

Let $\boldsymbol{U}, \boldsymbol{V} \in \boldsymbol{ob}(\boldsymbol{cy}(\boldsymbol{t}))$, whenever $\boldsymbol{t}$ is in an arbitrary interval $\mathbb{I} =]\boldsymbol{\sigma}, \boldsymbol{\tau}[\subset\subset [\mathbf{0}, \mathbf{1}]$. Let also

$$\boldsymbol{\delta}_{\boldsymbol{U}}: [\mathbf{0}, \mathbf{1}] \to \mathbb{C}^{\mathfrak{n} \times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m} \times \boldsymbol{k}}: \boldsymbol{t} \mapsto \boldsymbol{\delta}_{\boldsymbol{U}}(\boldsymbol{t}) = (\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}})(\boldsymbol{t}) \text{ and}$$

$$\boldsymbol{\gamma}_{\boldsymbol{V}}: [\mathbf{0}, \mathbf{1}] \to \mathbb{C}^{\mathfrak{n} \times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m} \times \boldsymbol{k}}: \boldsymbol{t} \mapsto \boldsymbol{\gamma}_{\boldsymbol{V}}(\boldsymbol{t}) = (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})(\boldsymbol{t})$$

be two supervisory perception curves of $\boldsymbol{V}$ and $\boldsymbol{U}$ in the node system $(\boldsymbol{V}, \boldsymbol{U})$.

Definition 5.5 A family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \left\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(\boldsymbol{Y},\boldsymbol{X})}(\boldsymbol{t}) = \left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}}), (\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\right)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathfrak{n} \times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m} \times \boldsymbol{k}}\right)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\right\},$$

lying in (the partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U} \to \boldsymbol{V}}$ to) the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ during the entire time set $\mathbb{I}$, is a **germ of (partial) active attack** against the $(\boldsymbol{\mu}_{\mathbf{1}}, \dots, \boldsymbol{\mu}_{\boldsymbol{\nu}})$ −device parts $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_{\mathbf{1}}}^{(\boldsymbol{V})}\right)$, $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_{\mathbf{2}}}^{(\boldsymbol{V})}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu}_{\boldsymbol{\nu}}}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ and the $(\boldsymbol{\kappa}_{\mathbf{1}}, \dots, \boldsymbol{\kappa}_{\boldsymbol{\lambda}})$ − resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_{\mathbf{1}}}^{(\boldsymbol{V})}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_{\mathbf{2}}}^{(\boldsymbol{V})}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa}_{\boldsymbol{\lambda}}}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$, during the time interval $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, there is an integer $\boldsymbol{N} = \boldsymbol{N}(\boldsymbol{t}) > \mathbf{0}$ such that the pair $\left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\right) \in \left(\mathbb{C}^{\mathfrak{n} \times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m} \times \boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\big) =$$

$$\left(\left(\begin{pmatrix} \boldsymbol{a}_{1,1}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{1,\mathfrak{n}}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{m_V,1}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{m_V,1}^{(V\rightsquigarrow V)} & & \boldsymbol{a}_{m_V,\mathfrak{n}}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{m_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{a}_{\mathcal{M}_V+1,1}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{\mathcal{M}_V+1,\mathfrak{n}}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{\mathcal{M}_V+\ell_V,1}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.$$

$$\left.\begin{pmatrix} \boldsymbol{b}_{1,1}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{1,\mathfrak{m}}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}_{m_V,1}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{m_V,\mathfrak{m}}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{b}_{\mathcal{M}_V+1,1}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{\mathcal{M}_V+1,\mathfrak{m}}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & & \cdots \\ \boldsymbol{b}_{\mathcal{M}_V+\ell_V,1}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(U\rightsquigarrow V)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right),$$

$$\left(\begin{pmatrix} \boldsymbol{a}_{1,1}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{a}}_{1,1}^{(U\rightsquigarrow U)} & \cdots & \boldsymbol{a}_{1,\mathfrak{n}}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{a}}_{1,\mathfrak{n}}^{(U\rightsquigarrow U)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{m_U,1}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{a}}_{m_U,1}^{(U\rightsquigarrow U)} & \cdots & \boldsymbol{a}_{m_U,\mathfrak{n}}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{a}}_{m_U,\mathfrak{n}}^{(U\rightsquigarrow U)} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{a}_{\mathcal{M}_U+1,1}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_U+1,1}^{(U\rightsquigarrow U)} & \cdots & \boldsymbol{a}_{\mathcal{M}_U+1,\mathfrak{n}}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_U+1,\mathfrak{n}}^{(U\rightsquigarrow U)} \\ \cdots & & \cdots \\ \boldsymbol{a}_{\mathcal{M}_U+\ell_U,1}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_U+\ell_U,1}^{(U\rightsquigarrow U)} & \cdots & \boldsymbol{a}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{a}}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}^{(U\rightsquigarrow U)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.$$

$$\left.\left.\begin{pmatrix} \boldsymbol{b}_{1,1}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{b}}_{1,1}^{(U\rightsquigarrow U)} & \cdots & \boldsymbol{b}_{1,\mathfrak{m}}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{b}}_{1,\mathfrak{m}}^{(U\rightsquigarrow U)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}_{m_U,1}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{b}}_{m_U,1}^{(U\rightsquigarrow U)} & \cdots & \boldsymbol{b}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow W)} + i\ \widehat{\boldsymbol{b}}_{m_V,\mathfrak{m}}^{(W\rightsquigarrow W)} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{b}_{\mathcal{M}_U+1,1}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_U+1,1}^{(U\rightsquigarrow U)} & \cdots & \boldsymbol{b}_{\mathcal{M}_U+1,\mathfrak{m}}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_U+1,\mathfrak{m}}^{(U\rightsquigarrow U)} \\ \cdots & & \cdots \\ \boldsymbol{b}_{\mathcal{M}_U+\ell_U,1}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_U+\ell_U,1}^{(U\rightsquigarrow U)} & \cdots & \boldsymbol{b}_{\mathcal{M}_U+\ell_U,\mathfrak{m}}^{(V\rightsquigarrow U)} + i\ \widehat{\boldsymbol{b}}_{\mathcal{M}_U+\ell_U,\mathfrak{m}}^{(U\rightsquigarrow U)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right)$$

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta \boldsymbol{t}$, via the associated family of cyber-activities

$$\boldsymbol{\mathcal{D}_{\mathcal{F}}} = \Big(\boldsymbol{g_t} = \boldsymbol{g}_{\boldsymbol{t}}^{(\mathcal{Z})} : \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}} \to \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}} :$$

$$\big(\boldsymbol{\delta_U}(\boldsymbol{t}), \boldsymbol{\gamma_V}(\boldsymbol{t})\big) \mapsto \big(\boldsymbol{\delta}'_{\boldsymbol{U}}(\boldsymbol{t}'), \boldsymbol{\gamma}'_{\boldsymbol{V}}(\boldsymbol{t}')\big)\Big)_{\boldsymbol{t}\in\mathbb{I}}$$

over the time $\boldsymbol{t} \in \mathbb{I}$, at $\big((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\big) \in \mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form

$\big((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\big) =$

$$\left(\left(\left(\begin{array}{ccc}
\boldsymbol{a}'^{(U\rightsquigarrow V)}_{1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{1,1} & \cdots & \boldsymbol{a}'^{(U\rightsquigarrow V)}_{1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\boldsymbol{a}'^{(U\rightsquigarrow V)}_{m_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{m_V,1} & & \boldsymbol{a}'^{(U\rightsquigarrow V)}_{m_V,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
\boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\
\cdots & & \cdots \\
\boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & \boldsymbol{a}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{array}\right),\right.\right.$$

$$\left.\left.\left(\begin{array}{ccc}
\boldsymbol{b}'^{(U\rightsquigarrow V)}_{1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{1,1} & \cdots & \boldsymbol{b}'^{(U\rightsquigarrow V)}_{1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
\boldsymbol{b}'^{(U\rightsquigarrow V)}_{m_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{m_V,1} & \cdots & \boldsymbol{b}'^{(U\rightsquigarrow V)}_{m_V,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{m_V,\mathfrak{m}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
\boldsymbol{b}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \cdots & \boldsymbol{b}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\
\cdots & & \cdots \\
\boldsymbol{b}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \cdots & \boldsymbol{b}'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\
\mathbf{0} & \cdots & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{array}\right)\right)\right),$$

$$\left(\left(\begin{array}{ccc}
\boldsymbol{a}'^{(V\rightsquigarrow U)}_{1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{1,1} & \cdots & \boldsymbol{a}'^{(V\rightsquigarrow U)}_{1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\boldsymbol{a}'^{(V\rightsquigarrow U)}_{m_U,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{m_U,1} & \cdots & \boldsymbol{a}'^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
\boldsymbol{a}'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & \cdots & \boldsymbol{a}'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\
\cdots & & \cdots \\
\boldsymbol{a}'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{a}'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\
\boldsymbol{a}'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} & \cdots & \boldsymbol{a}'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
\boldsymbol{a}'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,1} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,1} & \cdots & \boldsymbol{a}'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,\mathfrak{n}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{array}\right),\right.$$

$$\begin{pmatrix} b'^{(V \rightsquigarrow U)}_{1,1} + i\ \hat{b}'^{(U \rightsquigarrow U)}_{1,1} & \cdots & b'^{(V \rightsquigarrow U)}_{1,\mathfrak{m}} + i\ \hat{b}'^{(U \rightsquigarrow U)}_{1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ b'^{(V \rightsquigarrow U)}_{m_U,1} + i\ \hat{b}'^{(U \rightsquigarrow U)}_{m_U,1} & \cdots & b'^{(V \rightsquigarrow U)}_{m_U,\mathfrak{m}} + i\ \hat{b}'^{(U \rightsquigarrow U)}_{m_U,\mathfrak{m}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ b'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \hat{b}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+1,1} & \cdots & b'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} + i\ \hat{b}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} \\ \cdots & & \cdots \\ b'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \hat{b}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & b'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} + i\ \hat{b}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} \\ b'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} + i\ \hat{b}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} & \cdots & b'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{m}} + i\ \hat{b}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,\mathfrak{m}} \\ \cdots & \cdots & \cdots \\ b'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,1} + i\ \hat{b}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,1} & \cdots & b'^{(V \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,\mathfrak{m}} + i\ \hat{b}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+N,\mathfrak{m}} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix} \Bigg)\Bigg). \blacksquare$$

It is easy to prove and/or verify the next two results.

Proposition 5.6 In an active attack $\mathcal{F}$ from U against the $(\mu_1, \dots, \mu_\nu)$ −device parts $fr\left(dev^{(V)}_{\mu_1}\right), \dots, fr\left(dev^{(V)}_{\mu_\nu}\right)$ of V and the $(\kappa_1, \dots, \kappa_\lambda)-$ resource parts $fr\left(res^{(V)}_{\kappa_1}\right), \dots, fr\left(res^{(V)}_{\kappa_\lambda}\right)$ of V, the following elementary properties hold.

i. All **new** resource valuations of the offensive node U are derived from the set of all initial resource valuations of V, i.e., for any $j \in \{\mathcal{M}_U + \ell_U + 1, \dots, \mathcal{M}_U + \ell_U + N\}$ and any $k \in \{1, 2, \dots, \mathfrak{n}\}$, the new valuations

$$a'^{(V \rightsquigarrow U)}_{j,k} + i\hat{a}'^{(U \rightsquigarrow U)}_{j,k}$$

are obtained as functions of the initial valuations

$$a^{(U \rightsquigarrow V)}_{p,l} + i\hat{a}^{(V \rightsquigarrow V)}_{p,l},\ p \in \{1, 2, \dots, m_V, \mathcal{M}_V + 1, \dots, \mathcal{M}_V + \ell_V\},\ l \in \{1, 2, \dots, \mathfrak{n}\}.$$

ii. Similarly, all **new** resource vulnerabilities of the offensive node U are derived from the set of all initial resource vulnerabilities of V, i.e., for any $j \in \{\mathcal{M}_U + \ell_U + 1, \dots, \mathcal{M}_U + \ell_U + N\}$ and any $k \in \{1, 2, \dots, \mathfrak{n}\}$, the new vulnerabilities

$$b'^{(V \rightsquigarrow U)}_{j,k} + i\hat{b}'^{(U \rightsquigarrow U)}_{j,k}$$

are obtained as functions of the initial vulnerabilities

$$b^{(U \rightsquigarrow V)}_{p,l} + i\hat{b}^{(V \rightsquigarrow V)}_{p,l},\ p \in \{1, 2, \dots, m_V, \mathcal{M}_V + 1, \dots, \mathcal{M}_V + \ell_V\},\ k \in \{1, 2, \dots, \mathfrak{m}\}.$$

iii. Finally, from the viewpoint of the (user(s) of) node V, all valuations of U remain unchanged, i.e., if $j \in \{1, 2, \dots, m_U, \mathcal{M}_U + 1, \dots, \mathcal{M}_U + \ell_U\}$, then $a^{(V \rightsquigarrow U)}_{j,k} = a'^{(V \rightsquigarrow U)}_{j,k}$ for any $k \in \{1, 2, \dots, \mathfrak{n}\}$ and $b^{(V \rightsquigarrow U)}_{j,k} = b'^{(V \rightsquigarrow U)}_{j,k}$ for any $k \in \{1, 2, \dots, \mathfrak{m}\}$. $\blacksquare$

Proposition 5.7 In an active attack $\boldsymbol{\mathcal{F}}$ from $\boldsymbol{U}$ against the $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu})-$device parts $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu_1}}^{(\boldsymbol{V})}\right), \dots, \boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu_\nu}}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ and the $(\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda})-$ resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_1}}^{(\boldsymbol{V})}\right), \dots, \boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_\lambda}}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$, the following elementary properties hold.

i. The (Euclidean) norm $\left\|\boldsymbol{a}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\boldsymbol{\ell_V}} \left|\boldsymbol{a}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\lambda,j}}\right|^{\boldsymbol{2}}\right)^{1/2}$ of the resulting overall valuation in node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t'}$ is *less* than the (Euclidean) norm $\left\|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \sum_{\lambda=1}^{\boldsymbol{\ell_V}} \left|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\lambda,j}}\right|^{2}\right)^{\boldsymbol{1/2}}$ of the initial overall valuation in $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\boldsymbol{a}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\| < \left\|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\|.$$

ii. The (Euclidean) norm $\left\|\boldsymbol{b}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\boldsymbol{\ell_V}} \left|\boldsymbol{b}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\lambda,j}}\right|^{\boldsymbol{2}}\right)^{1/2}$ of the resulting overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t'}$ is greater than the (Euclidean) norm $\left\|\boldsymbol{b}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{m}} \sum_{\lambda=1}^{\boldsymbol{\ell_V}} \left|\boldsymbol{b}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\lambda,j}}\right|^{2}\right)^{\boldsymbol{1/2}}$ of the initial overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\boldsymbol{b}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\| > \left\|\boldsymbol{b}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\|.$$

iii. The (Euclidean) norm

$$\left\|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \left\{\sum_{\lambda=1}^{\boldsymbol{m_U}} \left|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}_{\boldsymbol{\lambda,j}}\right|^{\boldsymbol{2}} + \sum_{\lambda=1}^{\boldsymbol{\ell_U+N}} \left|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\lambda,j}}\right|^{\boldsymbol{2}}\right\}\right)^{1/2}$$

of the resulting overall valuation in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t'}$ is greater than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{a}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \left\{\sum_{\lambda=1}^{\boldsymbol{m_U}} \left|\widehat{\boldsymbol{a}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}_{\boldsymbol{\lambda,j}}\right|^{\boldsymbol{2}} + \sum_{\lambda=1}^{\boldsymbol{\ell_W}} \left|\widehat{\boldsymbol{a}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}_{\boldsymbol{\mathcal{M}_U+\lambda,j}}\right|^{\boldsymbol{2}}\right\}\right)^{\boldsymbol{1/2}} \text{ and}$$

$$\left\|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\| := \left(\sum_{j=1}^{\mathfrak{n}} \left\{\sum_{\lambda=1}^{\boldsymbol{m_V}} \left|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\lambda,j}}\right|^{\boldsymbol{2}} + \sum_{\lambda=1}^{\boldsymbol{\ell_V}} \left|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\lambda,j}}\right|^{2}\right\}\right)^{\boldsymbol{1/2}}$$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{a}}'^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\right\| > \boldsymbol{max}\left\{\left\|\widehat{\boldsymbol{a}}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{U})}\right\|, \left\|\boldsymbol{a}^{(\boldsymbol{U} \rightsquigarrow \boldsymbol{V})}\right\|\right\}.$$

iv. The (Euclidean) norm $\left\|\widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)}\right\| := \left(\sum_{j=1}^{m} \sum_{\lambda=1}^{\ell_U+N} \left|\widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)}_{\mathcal{M}_U+\lambda,j}\right|^{2}\right)^{1/2}$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t'}$ is *less or equal* than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{b}}^{(U \rightsquigarrow U)}\right\| := \left(\sum_{j=1}^{m} \sum_{\lambda=1}^{\ell_U} \left|\widehat{\boldsymbol{b}}^{(U \rightsquigarrow U)}_{\mathcal{M}_U+\lambda,j}\right|^{2}\right)^{1/2} \text{ and}$$

$$\left\|\boldsymbol{b}^{(U \rightsquigarrow V)}\right\| := \left(\sum_{j=1}^{m} \sum_{\lambda=1}^{\ell_V} \left|\boldsymbol{b}^{(U \rightsquigarrow V)}_{\mathcal{M}_U+\lambda,j}\right|^{2}\right)^{1/2}$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)}\right\| \leq \boldsymbol{min}\left\{\left\|\widehat{\boldsymbol{b}}^{(U \rightsquigarrow U)}\right\|, \left\|\boldsymbol{b}^{(U \rightsquigarrow V)}\right\|\right\}. \blacksquare$$

The **degree** $\boldsymbol{d} = \boldsymbol{d}_{\{\mu_1,\dots,\mu_\nu\}\cup\{\kappa_1,\dots,\kappa_\lambda\}}$ **of the active attack** $\boldsymbol{f}$ against the $(\boldsymbol{\mu_1},\dots,\boldsymbol{\mu_\nu})$ −device parts $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_1}^{(V)}\right)$, $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_2}^{(V)}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_\nu}^{(V)}\right)$ of $\boldsymbol{V}$ and the $(\boldsymbol{\kappa_1},\dots,\boldsymbol{\kappa_\lambda})$ − resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_1}^{(V)}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_2}^{(V)}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_\lambda}^{(V)}\right)$ of $\boldsymbol{V}$ from the offensive node $\boldsymbol{U}$ at time moment $\boldsymbol{t} \in \mathbb{I}$ is defined to be the maximum of the two quotients

$$\boldsymbol{d_1} := \left\|\widehat{\boldsymbol{a}}'^{(U \rightsquigarrow U)}\right\| / \left\|\boldsymbol{a}'^{(U \rightsquigarrow U)}\right\| \text{ and } \boldsymbol{d_2} := \left(\left\|\widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)}\right\| / \left\|\boldsymbol{b}'^{(U \rightsquigarrow V)}\right\|\right)^{-1}.$$

Thus, $\boldsymbol{d} = \boldsymbol{d}_{\{\mu_1,\dots,\mu_\nu\}\cup\{\kappa_1,\dots,\kappa_\lambda\}} := \boldsymbol{max}\{\boldsymbol{d_1}, \boldsymbol{d_2}\}$. If the degree $\boldsymbol{d}$ surpasses a given threshold $\boldsymbol{\mathcal{T}}^{(U,V)}_{\{\mu_1,\dots,\mu_\nu\}\cup\{\kappa_1,\dots,\kappa_\lambda\}} \in [\boldsymbol{0}, \infty[$, called **threshold of active attack** from $\boldsymbol{U}$ against the $(\boldsymbol{\mu_1},\dots,\boldsymbol{\mu_\nu})$ −device parts $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_1}^{(V)}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_\nu}^{(V)}\right)$ of $\boldsymbol{V}$ and the$(\boldsymbol{\kappa_1},\dots,\boldsymbol{\kappa_\lambda})$ − resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_1}^{(V)}\right)$,,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_\lambda}^{(V)}\right)$ of $\boldsymbol{V}$at time moment $\boldsymbol{t} \in \mathbb{I}$, we say that **the passive attack** $\boldsymbol{f}$ **is dangerous with degree of danger** $\boldsymbol{d}$ **in the** $(\boldsymbol{\mu_1},\dots,\boldsymbol{\mu_\nu})$ **−device parts** $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_1}^{(V)}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_\nu}^{(V)}\right)$ **of** $\boldsymbol{V}$ **and the** $(\boldsymbol{\kappa_1},\dots,\boldsymbol{\kappa_\lambda})$ **− resource parts** $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_1}^{(V)}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_2}^{(V)}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\kappa_\lambda}^{(V)}\right)$ **of** $\boldsymbol{V}$.■

Remark 5.8 It is easy to verify that the following conditions 1 to 4 can be considered as stronger forms of the corresponding conditions in Proposition 5.7.

i. 1st Condition: From the point of view of users of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$, every attacked device part, as well as any attacked resource part, acquire new valuation measures that are smaller than the original corresponding valuations in node $\boldsymbol{V}$, with (at least) one such a valuation measure very reduced, i.e., for any $\boldsymbol{j} \in \{\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu}\} \cup \{\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda}\}$, it holds

$$\sum_{k=1}^{\mathfrak{n}} \left| a_{j,k}^{(X \rightsquigarrow V)} + i\hat{a}_{j,k}^{(X \rightsquigarrow V)} \right|^2 > \sum_{k=1}^{\mathfrak{n}} \left| a'^{(X \rightsquigarrow V)}_{j,k} + i\widehat{a'}^{(X \rightsquigarrow V)}_{j,k} \right|^2$$

with at least one index $\boldsymbol{k} \in \{\boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{\mathfrak{n}}\}$ being such that

$$\left| a_{j,k}^{(X \rightsquigarrow V)} + i\hat{a}_{j,k}^{(X \rightsquigarrow V)} \right| \gg \left| a'^{(X \rightsquigarrow V)}_{j,k} + i\widehat{a'}^{(X \rightsquigarrow V)}_{j,k} \right|$$

whenever $\boldsymbol{X} = \boldsymbol{V}, \boldsymbol{U}$.

ii. <u>2nd Condition:</u> Similarly, from the point of view of users of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$, every attacked device part, as well as any attacked resource part, acquire new vulnerability measures that are smaller than the original corresponding vulnerabilities in node $\boldsymbol{V}$, with (at least) one such a vulnerability measure very reduced, i.e., for any $\boldsymbol{j} \in \{\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu}\} \cup \{\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda}\}$, it holds

$$\sum_{k=1}^{\mathfrak{m}} \left| b_{j,k}^{(X \rightsquigarrow V)} + i\hat{b}_{j,k}^{(X \rightsquigarrow V)} \right|^2 \leq \sum_{k=1}^{\mathfrak{m}} \left| b_{j,k}^{(X \rightsquigarrow V)} + i\hat{b}_{j,k}^{(X \rightsquigarrow V)} \right|^2$$

with at least one index $\boldsymbol{k} \in \{\boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{\mathfrak{m}}\}$ being such that

$$\left| b_{j,k}^{(X \rightsquigarrow V)} + i\hat{b}_{j,k}^{(X \rightsquigarrow V)} \right| < \left| b'^{(X \rightsquigarrow V)}_{j,k} + i\widehat{b'}^{(X \rightsquigarrow V)}_{j,k} \right|$$

whenever $\boldsymbol{X} = \boldsymbol{V}, \boldsymbol{U}$.

iii. <u>3rd Condition:</u> From the viewpoint of the (user(s) of) node $\boldsymbol{U}$, in the offensive node $\boldsymbol{U}$ there are strongly growing valuations, i.e., there are $\boldsymbol{j} \in \{\boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{m_U}, \boldsymbol{\mathcal{M}_U} + \boldsymbol{1}, \dots, \boldsymbol{\mathcal{M}_U} + \boldsymbol{\ell_U}\}$ and $\boldsymbol{k} \in \{\boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{\mathfrak{n}}\}$, such that

$$\left| \hat{a}_{j,k}^{(U \rightsquigarrow U)} \right| \ll \left| \widehat{a'}^{(U \rightsquigarrow U)}_{j,k} \right|.$$

iv. <u>4th Condition:</u> From the viewpoint of the (user(s) of) node $\boldsymbol{U}$, in the offensive node $\boldsymbol{U}$ there is no growing vulnerability, i.e., for any $\boldsymbol{j} \in \{\boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{m_U}, \boldsymbol{\mathcal{M}_U} + \boldsymbol{1}, \dots, \boldsymbol{\mathcal{M}_U} + \boldsymbol{\ell_U}\}$ and any $\boldsymbol{k} \in \{\boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{\mathfrak{m}}\}$, it holds

$$\left| \hat{b}_{j,k}^{(U \rightsquigarrow U)} \right| \geq \left| \widehat{b'}^{(U \rightsquigarrow U)}_{j,k} \right|. \blacksquare$$

5.4 Protected cyber nodes from active attacks

Finally, let's see how we could define the concept of protection from active cyber-attacks.

Definition 5.9.

i. The node $\boldsymbol{V}$ is said to be **protected from active attacks, with degree of protection** $\boldsymbol{p} \in]\boldsymbol{0}, \boldsymbol{1}]$ over the $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu})$ −device parts

$\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu_1}}^{(\boldsymbol{V})}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu_\nu}}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ and the $(\boldsymbol{\kappa_1},\dots,\boldsymbol{\kappa_\lambda})-$ resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_1}}^{(\boldsymbol{V})}\right)$, …,$\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_\lambda}}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ over a time period $\mathbb{I}$, if, during this time period, there is a nodal fixed filter system $\overline{\boldsymbol{V}}^{\{\boldsymbol{\mu_1},\dots,\boldsymbol{\mu_\nu}\}\cup\{\boldsymbol{\kappa_1},\dots,\boldsymbol{\kappa_\lambda}\}}$ in the union $\boldsymbol{E}=\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu_1}}^{(\boldsymbol{V})}\right)\cup\dots\cup\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu_\nu}}^{(\boldsymbol{V})}\right)\cup\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_1}}^{(\boldsymbol{V})}\right)\cup\dots\cup\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_\nu}}^{(\boldsymbol{V})}\right)$ that allow every parallactic cyber active attack against the $(\boldsymbol{\mu_1},\dots,\boldsymbol{\mu_\nu})-$device parts $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu_1}}^{(\boldsymbol{V})}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{dev}_{\boldsymbol{\mu_\nu}}^{(\boldsymbol{V})}\right)$ of $\boldsymbol{V}$ and the $(\boldsymbol{\kappa_1},\dots,\boldsymbol{\kappa_\lambda})-$ resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_1}}^{(\boldsymbol{V})}\right)$,…,$\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_\lambda}}^{(\boldsymbol{V})}\right)$ of node $\boldsymbol{V}$ (from any offensive node $\boldsymbol{U}$) with degree of danger $\boldsymbol{d}\leq-\boldsymbol{logp}$ to reach only resource parts $\boldsymbol{K}$ of the initial target $\boldsymbol{V}$ that are disjoint from $\boldsymbol{E}$.

ii. During the time period $\mathbb{I}$, the node $\boldsymbol{V}$ is said to be **completely protected from active attacks of danger degree$\boldsymbol{d}$**, if, at this time period, any resource part in $\boldsymbol{V}$ is protected from active attacks against $\boldsymbol{V}$, with degree of protection $\boldsymbol{p}\leq\boldsymbol{e}^{-\boldsymbol{d}}$. The node $\boldsymbol{V}$ is said to be **completely protected from active attacks at a given time period**, if, during this time period, any resource part of $\boldsymbol{V}$ is protected from active attacks against $\boldsymbol{V}$ with degree of protection $\boldsymbol{p}=\boldsymbol{1}$. ■

6 Mathematical Description of Representative Cyber Attacks

So, having consistently examined the more general cases of a passive and active attacks, we will try to focus on some indicative, yet quite important, cases, namely the cyber espionage attack, the access attack, the reconnaissance attack, the denial of service attack, and the distributed denial of service attack.

In order to go further and get the full description of these indicative cyber-attacks, it would be wise to mathematically orient and define some further concepts. The sophistication of development of any cyber-attack is a critical issue and can be described as follows.

6.1 Sophistication of Cyber Attacks

The term “sophisticated” is often used inconsistently or incorrectly by the cyber community. Seldom will the victim of a cyber-attack disclose that they have been targeted without characterising either the attack or assailant as “sophisticated”. But the label is often applied inconsistently, either inadvertently or deliberately. The term, even though it is highly important and critical, loses its value when overused, and should instead be employed to differentiate exceptional attacks or attackers from the norm.

Victims of cyber-attacks are not necessarily best placed to identify how exceptional their compromise is compared with other incidents. There may also be reasons for the victim to exaggerate the complexity of the attack, or the perpetrator’s ability. In doing so they imply the breach was unavoidable, absolving them of responsibility in the eyes of potentially litigious customers or shareholders. Wrongly characterizing an attack, however, is not without consequence. If simple, preventable attacks are labeled as sophisticated and inevitable, rather than a product of rectifiable vulnerabilities or security lapses, then those vulnerabilities may be allowed to fester.

It’s obvious that the most sophisticated cyber-attacks have not yet been detected. While sophisticated attacks are often effective, attacks need not be sophisticated to be effective. In that direction, and in order to establish a concrete behavior against sophisticated cyber-attacks, we will try to define the term “sophistication” of a cyber-attack in accordance to the whole concept of this dissertation. We earnestly believe that prescriptive definitions are problematic because there will inevitably be exceptions and the criteria will have to be dynamic enough to reflect the unrelenting pace of cyber capability development and proliferation.

The “sophistication” of a cyber-attack concept is a puzzle of definitions that form the big picture. To enter the structural operational status of such a **“sophisticated” attack puzzle**, suppose *the derivatives*

$$\boldsymbol{\varphi}^{(U \rightsquigarrow V)}(\boldsymbol{t}) := \frac{\partial\{\boldsymbol{a}^{(U \rightsquigarrow V)}\}}{\partial \boldsymbol{t}}(\boldsymbol{t}) =$$

$$\frac{\partial\left\{\left(\boldsymbol{a}_1^{(U \rightsquigarrow V)}, \ldots \boldsymbol{a}_{m_V}^{(U \rightsquigarrow V)}, \boldsymbol{a}_{m_V+1}^{(U \rightsquigarrow V)}, \ldots, \boldsymbol{a}_{\mathcal{M}_V}^{(U \rightsquigarrow V)}, \boldsymbol{a}_{\mathcal{M}_V+1}^{(U \rightsquigarrow V)}, \ldots, \boldsymbol{a}_{\mathcal{M}_V+\ell_V+1}^{(U \rightsquigarrow V)}, \boldsymbol{a}_{\mathcal{M}_V+\ell_V+1}^{(U \rightsquigarrow V)}, \ldots, \boldsymbol{a}_{\mathcal{M}_V+\mathcal{L}_V}^{(U \rightsquigarrow V)}\right)^{T}\right\}}{\partial \boldsymbol{t}}(\boldsymbol{t})$$

and

$$\widehat{\boldsymbol{\varphi}}^{(V \rightsquigarrow V)}(\boldsymbol{t}) := \frac{\partial\{\widehat{\boldsymbol{a}}^{(V \rightsquigarrow V)}[x_1, x_2, x_3, t\,]\}}{\partial \boldsymbol{t}}(\boldsymbol{t}) =$$

$$\frac{\partial\left\{\left(\widehat{\boldsymbol{a}}_1^{(V \rightsquigarrow V)}, \ldots \widehat{\boldsymbol{a}}_{m_V}^{(V \rightsquigarrow V)}, \widehat{\boldsymbol{a}}_{m_V+1}^{(V \rightsquigarrow V)}, \ldots, \widehat{\boldsymbol{a}}_{\mathcal{M}_V}^{(V \rightsquigarrow V)}, \widehat{\boldsymbol{a}}_{\mathcal{M}_V+1}^{(V \rightsquigarrow V)}, \ldots, \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\ell_V+1}^{(V \rightsquigarrow V)}, \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\ell_V+1}^{(V \rightsquigarrow V)}, \ldots, \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\mathcal{L}_V}^{(V \rightsquigarrow V)}\right)^{T}\right\}}{\partial \boldsymbol{t}}(\boldsymbol{t})$$

exist in a time interval $\mathbb{I} =]\boldsymbol{\alpha}, \boldsymbol{\beta}[$ *in the sense of distributions*. In such a case, we say that the relative effectiveness states $\boldsymbol{a}^{(U \rightsquigarrow V)} = \boldsymbol{a}^{(U \rightsquigarrow V)}[\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{t}\,] \in \mathbb{R}^{\boldsymbol{k}}$ and $\widehat{\boldsymbol{a}}^{(V \rightsquigarrow V)} = \widehat{\boldsymbol{a}}^{(V \rightsquigarrow V)}[\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{t}\,] \in \mathbb{R}^{\boldsymbol{k}}$ are two **smooth node valuations** and the distributional derivatives $\boldsymbol{\varphi}^{(U \rightsquigarrow V)}(\boldsymbol{t})$ and $\widehat{\boldsymbol{\varphi}}^{(V \rightsquigarrow V)}(\boldsymbol{t})$ are the **rate changes/slopes of the valuations** $\boldsymbol{a}^{(U \rightsquigarrow V)}$ and $\widehat{\boldsymbol{a}}^{(V \rightsquigarrow V)}$ respectively, at a point $(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\,)$ of a part $\boldsymbol{E}$ into the node $\boldsymbol{V}$ from the viewpoint of the (user(s) of) node $\boldsymbol{U}$ and $\boldsymbol{V}$, respectively, over the time interval $\mathbb{I}$. Here, as usually, $\boldsymbol{k} \coloneqq \boldsymbol{\mathcal{M}_V} + \boldsymbol{\mathcal{L}_V}$.

For $\boldsymbol{\Phi} = \boldsymbol{\varphi}, \widehat{\boldsymbol{\varphi}}$ and $\boldsymbol{X}, \boldsymbol{\Upsilon} \in \{\boldsymbol{U}, \boldsymbol{V}\}$, it is obvious that

1. If $\boldsymbol{\Phi}^{(X \rightsquigarrow \Upsilon)}(\boldsymbol{t}) > \boldsymbol{0}$ whenever $\boldsymbol{t} \in \mathbb{I}$**,** then we are situated definitely in the area $[\boldsymbol{\mathcal{A}}_{\boldsymbol{X}}^{+}(\boldsymbol{\Upsilon})](\mathbb{I})$ of correlated growth for the total valuation of the node $\boldsymbol{\Upsilon}$ as evaluated subjectively from the user(s) of $\boldsymbol{X}$ over the time set $\mathbb{I}$ ([5]).
2. If $\boldsymbol{\Phi}^{(X \rightsquigarrow \Upsilon)}(\boldsymbol{t}) < \boldsymbol{0}$ whenever $\boldsymbol{t} \in \mathbb{I}$**,** then we are situated definitely in the area $[\boldsymbol{\mathcal{A}}_{\boldsymbol{X}}^{-}(\boldsymbol{\Upsilon})](\mathbb{I})$ of correlated reduction for the total valuation of the node $\boldsymbol{\Upsilon}$ as evaluated subjectively from the user(s) of $\boldsymbol{X}$ over the time set $\mathbb{I}$ ([5]).
3. If $\boldsymbol{\Phi}^{(X \rightsquigarrow \Upsilon)}(\boldsymbol{t}) = \boldsymbol{0}$ whenever $\boldsymbol{t} \in \mathbb{I}$, there is no correlated growth or reduction for the total valuation of the node $\boldsymbol{\Upsilon}$ as evaluated subjectively from the user(s) of $\boldsymbol{X}$ over the time set $\mathbb{I}$, due to a multitude of potential reasons.

By analogy, suppose *the derivatives*

$$\boldsymbol{\psi}^{(U \rightsquigarrow V)}(\boldsymbol{t}) := \frac{\partial\{\boldsymbol{b}^{(U \rightsquigarrow V)}[\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{t}\,]\}}{\partial \boldsymbol{t}}(\boldsymbol{t}) =$$

$$\frac{\partial\{\left(\boldsymbol{b}_1^{(U \rightsquigarrow V)}, \dots, \boldsymbol{b}_{m_V}^{(U \rightsquigarrow V)}, \boldsymbol{b}_{m_V+1}^{(U \rightsquigarrow V)}, \dots, \boldsymbol{b}_{\mathcal{M}_V}^{(U \rightsquigarrow V)}, \boldsymbol{b}_{\mathcal{M}_V+1}^{(U \rightsquigarrow V)}, \dots, \boldsymbol{b}_{\mathcal{M}_V+\ell_V+1}^{(U \rightsquigarrow V)}, \boldsymbol{b}_{\mathcal{M}_V+\ell_V+1}^{(U \rightsquigarrow V)}, \dots, \boldsymbol{b}_{\mathcal{M}_V+\mathcal{L}_V}^{(U \rightsquigarrow V)}\right)^{T}\}}{\partial \boldsymbol{t}}(\boldsymbol{t})$$

and

$$\widehat{\boldsymbol{\psi}}^{(V \rightsquigarrow V)}(\boldsymbol{t}) := \frac{\partial\{\widehat{\boldsymbol{b}}^{(V \rightsquigarrow V)}[\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{t}\,]\}}{\partial \boldsymbol{t}}(\boldsymbol{t}) =$$

$$\frac{\partial\{\left(\widehat{\boldsymbol{b}}_1^{(V \rightsquigarrow V)}, \dots, \widehat{\boldsymbol{b}}_{m_V}^{(V \rightsquigarrow V)}, \widehat{\boldsymbol{b}}_{m_V+1}^{(V \rightsquigarrow V)}, \dots, \widehat{\boldsymbol{b}}_{\mathcal{M}_V}^{(V \rightsquigarrow V)}, \widehat{\boldsymbol{b}}_{\mathcal{M}_V+1}^{(V \rightsquigarrow V)}, \dots, \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\ell_V+1}^{(V \rightsquigarrow V)}, \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\ell_V+1}^{(V \rightsquigarrow V)}, \dots, \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\mathcal{L}_V}^{(V \rightsquigarrow V)}\right)^{T}\}}{\partial \boldsymbol{t}}(\boldsymbol{t})$$

exist in a time interval $\mathbb{I} =]\boldsymbol{\alpha}, \boldsymbol{\beta}[$ *in the sense of distributions*. In such a case, we say that the relative effectiveness states $\boldsymbol{b}^{(U \rightsquigarrow V)} = \boldsymbol{b}^{(U \rightsquigarrow V)}[\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{t}\,] \in \mathbb{R}^{\boldsymbol{k}}$ and $\widehat{\boldsymbol{b}}^{(V \rightsquigarrow V)} = \widehat{\boldsymbol{b}}^{(V \rightsquigarrow V)}[\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{t}\,] \in \mathbb{R}^{\boldsymbol{k}}$ are two **smooth node vulnerabilities** and the distributional derivatives $\boldsymbol{\psi}^{(U \rightsquigarrow V)}(\boldsymbol{t})$ and $\widehat{\boldsymbol{\psi}}^{(V \rightsquigarrow V)}(\boldsymbol{t})$ are the **rate changes/slopes of the vulnerabilities** $\boldsymbol{b}^{(U \rightsquigarrow V)}$ and $\widehat{\boldsymbol{b}}^{(V \rightsquigarrow V)}$ respectively, at a point $(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}\,)$ of a part $\boldsymbol{E}$

into the node $\boldsymbol{V}$ from the viewpoint of the (user(s) of) node $\boldsymbol{U}$ and $\boldsymbol{V}$, respectively, over the time interval $\mathbb{I}$.

As above, for $\boldsymbol{\Psi} = \boldsymbol{\psi}, \widehat{\boldsymbol{\psi}}$ and $\boldsymbol{X}, \boldsymbol{\Upsilon} \in \{\boldsymbol{U}, \boldsymbol{V}\}$, it is obvious that:

1. If $\boldsymbol{\Psi}^{(\boldsymbol{X} \rightsquigarrow \boldsymbol{\Upsilon})}(\boldsymbol{t}) > \mathbf{0}$ whenever $\boldsymbol{t} \in \mathbb{I}$**,** then we are situated definitely in the area $[\boldsymbol{\mathcal{B}}_{\boldsymbol{X}}^{+}(\boldsymbol{\Upsilon})](\mathbb{I})$ of correlated growth for the total vulnerability of the node $\boldsymbol{\Upsilon}$ as evaluated subjectively from the user(s) of $\boldsymbol{X}$ over the time set $\mathbb{I}$ ([5]).
2. If $\boldsymbol{\Psi}^{(\boldsymbol{X} \rightsquigarrow \boldsymbol{\Upsilon})}(\boldsymbol{t}) < \mathbf{0}$ whenever $\boldsymbol{t} \in \mathbb{I}$**,** then we are situated definitely in the area $[\boldsymbol{\mathcal{B}}_{\boldsymbol{X}}^{-}(\boldsymbol{\Upsilon})](\mathbb{I})$ of correlated reduction for the total vulnerability of the node $\boldsymbol{\Upsilon}$ as evaluated subjectively from the user(s) of $\boldsymbol{X}$ over the time set $\mathbb{I}$ ([5]).
3. If $\boldsymbol{\Psi}^{(\boldsymbol{X} \rightsquigarrow \boldsymbol{\Upsilon})}(\boldsymbol{t}) = \mathbf{0}$ whenever $\boldsymbol{t} \in \mathbb{I}$, there is no correlated growth or reduction of the total vulnerability for node $\boldsymbol{\Upsilon}$ as evaluated subjectively from the user(s) of $\boldsymbol{X}$ over the time set $\mathbb{I}$, due to a multitude of potential reasons.

Remark 6.1 Having defined the rate change of valuations and vulnerabilities we can proceed to orientation of sophistication in cyber-attacks, definition which will support our further posture in this paper. So, if we have one or combination of the following states that declare a slow infection (constituents' degradation) we assume that there should be a **suspicion of sophistication**

$$\widehat{\boldsymbol{\varphi}}^{(\boldsymbol{V} \rightsquigarrow \boldsymbol{V})} \cong \mathbf{0}^{-} \text{ and } \widehat{\boldsymbol{\psi}}^{(\boldsymbol{V} \rightsquigarrow \boldsymbol{V})} \cong \mathbf{0}^{+}. \ \blacksquare$$

6.2 Man in the Middle Vs Wiretapping Cyber Attacks

It would be very helpful and constructive, for the sake of the smooth development of this dissertation, to mathematically define on parallel the aforementioned attacks. **Man in the Middle attack**, where the attacker secretly relays and possibly alters the communication between two parties who believe they are directly communicating with each other, belongs to active cyber-attacks, and on the other hand, **wiretapping attack** which is a passive attack that consists in the monitoring of cyber activity, often by covert means.

In the **Man in the Middle (MiTM) attack of a node** $\boldsymbol{Z}$ in the cyber-interaction between nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ we have the "active" intersection of node $\boldsymbol{Z}$. Actually in this "active" intersection (MitM) attack, instead of this "normal" interaction we experience an active attack from node $\boldsymbol{Z}$ to either or/and both of other nodes **using some resources of the other interacted node**. In such a case, a family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \Big\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(Y,X)}(\boldsymbol{t}) = \big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big)(\boldsymbol{t}) \in \big(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\big)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\Big\},$$

lying in the partial danger sector $\mathcal{E} = \mathcal{E}_{Z\to V}$ to the node $\boldsymbol{V}$ from the node Z during the entire time set $\mathbb{I}$, is a **germ of (partial) active attack against the** $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu}) -$ **device parts** $\boldsymbol{fr}\Big(\boldsymbol{dev}_{\boldsymbol{\mu_1}}^{(\boldsymbol{V})}\Big)$, $\boldsymbol{fr}\Big(\boldsymbol{dev}_{\boldsymbol{\mu_2}}^{(\boldsymbol{V})}\Big)$,…, $\boldsymbol{fr}\Big(\boldsymbol{dev}_{\boldsymbol{\mu_\nu}}^{(\boldsymbol{V})}\Big)$ **of** $\boldsymbol{V}$ **and the** $(\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda}) -$ **resource parts** $\boldsymbol{fr}\Big(\boldsymbol{res}_{\boldsymbol{\kappa_1}}^{(\boldsymbol{V})}\Big)$, $\boldsymbol{fr}\Big(\boldsymbol{res}_{\boldsymbol{\kappa_2}}^{(\boldsymbol{V})}\Big)$,…, $\boldsymbol{fr}\Big(\boldsymbol{res}_{\boldsymbol{\kappa_\lambda}}^{(\boldsymbol{V})}\Big)$ **of** $\boldsymbol{V}$, during a given time set $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\big) \in \big(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\big)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{Z}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{Z}$ and $\boldsymbol{V}$ has the form

$$\big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\big) =$$

$$\left(\left(\left(\begin{matrix} \boldsymbol{a}_{1,1}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{1,\mathfrak{n}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{m_V,1}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{m_V,\mathfrak{n}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{m_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{a}_{\mathcal{M}_V+1,1}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{\mathcal{M}_V+1,\mathfrak{n}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{\mathcal{M}_V+\ell_V,1}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{a}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{matrix}\right),\right.\right.$$

$$\left.\left(\begin{matrix} \boldsymbol{b}_{1,1}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{1,\mathfrak{m}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}_{m_V,1}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{m_V,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{m_V,\mathfrak{m}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{m_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{b}_{\mathcal{M}_V+1,1}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{\mathcal{M}_V+1,\mathfrak{m}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}_{\mathcal{M}_V+\ell_V,1}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & \boldsymbol{b}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(Z\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{matrix}\right)\right),$$

$$\left(\left(\begin{matrix} \boldsymbol{a}_{1,1}^{(V\rightsquigarrow Z)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{1,1}^{(Z\rightsquigarrow Z)} & \cdots & \boldsymbol{a}_{1,\mathfrak{n}}^{(V\rightsquigarrow Z)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{1,\mathfrak{n}}^{(Z\rightsquigarrow Z)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{m_Z,1}^{(V\rightsquigarrow Z)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{m_Z,1}^{(Z\rightsquigarrow Z)} & \cdots & \boldsymbol{a}_{m_Z,\mathfrak{n}}^{(V\rightsquigarrow Z)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{m_Z,\mathfrak{n}}^{(Z\rightsquigarrow Z)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ \boldsymbol{a}_{\mathcal{M}_Z+1,1}^{(V\rightsquigarrow Z)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_Z+1,1}^{(Z\rightsquigarrow Z)} & \cdots & \boldsymbol{a}_{\mathcal{M}_Z+1,\mathfrak{n}}^{(V\rightsquigarrow Z)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_Z+1,\mathfrak{n}}^{(Z\rightsquigarrow Z)} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{\mathcal{M}_Z+\ell_Z,1}^{(V\rightsquigarrow Z)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_Z+\ell_Z,1}^{(Z\rightsquigarrow Z)} & \cdots & \boldsymbol{a}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}}^{(V\rightsquigarrow Z)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}}^{(Z\rightsquigarrow Z)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{matrix}\right),\right.\right.$$

$$\left.\left.\begin{pmatrix} \boldsymbol{b}_{1,1}^{(V\rightsquigarrow Z)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{1,1}^{(Z\rightsquigarrow Z)} & \dots & \boldsymbol{b}_{1,\mathfrak{m}}^{(V\rightsquigarrow Z)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{1,\mathfrak{m}}^{(Z\rightsquigarrow Z)} \\ \dots & \dots & \dots \\ \boldsymbol{b}_{m_Z,1}^{(V\rightsquigarrow Z)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{m_Z,1}^{(Z\rightsquigarrow Z)} & \dots & \boldsymbol{b}_{m_Z,\mathfrak{m}}^{(V\rightsquigarrow Z)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{m_Z,\mathfrak{m}}^{(Z\rightsquigarrow Z)} \\ \mathbf{0} & & \mathbf{0} \\ \dots & \dots & \dots \\ \mathbf{0} & \dots & \mathbf{0} \\ \boldsymbol{b}_{\mathcal{M}_Z+1,1}^{(V\rightsquigarrow Z)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_Z+1,1}^{(Z\rightsquigarrow Z)} & \dots & \boldsymbol{b}_{\mathcal{M}_Z+1,\mathfrak{m}}^{(V\rightsquigarrow Z)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_Z+1,\mathfrak{m}}^{(Z\rightsquigarrow Z)} \\ \dots & & \dots \\ \boldsymbol{b}_{\mathcal{M}_Z+\ell_Z,1}^{(V\rightsquigarrow Z)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_Z+\ell_Z,1}^{(Z\rightsquigarrow Z)} & \dots & \boldsymbol{b}_{\mathcal{M}_Z+\ell_Z,\mathfrak{m}}^{(V\rightsquigarrow Z)}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_Z+\ell_Z,\mathfrak{m}}^{(Z\rightsquigarrow Z)} \\ \mathbf{0} & \dots & \mathbf{0} \\ \dots & \dots & \dots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right)$$

and is depicted, at a next moment $\boldsymbol{t}'=\boldsymbol{t}+\Delta\boldsymbol{t}$, at a pair $\left((\mathbb{z}_1',\mathbb{w}_1'),(\mathbb{z}_2',\mathbb{w}_2')\right)\in\left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}}\times\mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^2$ of supervisory resource perceptions of $\boldsymbol{Z}$ and $\boldsymbol{V}$ having the form

$\left((\mathbb{z}_1',\mathbb{w}_1'),(\mathbb{z}_2',\mathbb{w}_2')\right)=$

$$\left(\left(\begin{pmatrix} \boldsymbol{a}'^{(Z\rightsquigarrow V)}_{1,1}+\boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{1,1} & \dots & \boldsymbol{a}'^{(Z\rightsquigarrow V)}_{1,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{1,\mathfrak{n}} \\ \dots & \dots & \dots \\ \boldsymbol{a}'^{(Z\rightsquigarrow V)}_{m_V,1}+\boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{m_V,1} & & \boldsymbol{a}'^{(Z\rightsquigarrow V)}_{m_V,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\ \mathbf{0} & \dots & \mathbf{0} \\ \dots & \dots & \dots \\ \mathbf{0} & \dots & \mathbf{0} \\ \boldsymbol{a}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+\boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \dots & \boldsymbol{a}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} \\ \dots & & \dots \\ \boldsymbol{a}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1}+\boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \dots & \boldsymbol{a}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} \\ \mathbf{0} & \dots & \mathbf{0} \\ \dots & \dots & \dots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.$$

$$\left.\left.\begin{pmatrix} \boldsymbol{b}'^{(Z\rightsquigarrow V)}_{1,1}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{1,1} & \dots & \boldsymbol{b}'^{(Z\rightsquigarrow V)}_{1,\mathfrak{m}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{1,\mathfrak{m}} \\ \dots & \dots & \dots \\ \boldsymbol{b}'^{(Z\rightsquigarrow V)}_{m_V,1}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{m_V,1} & \dots & \boldsymbol{b}'^{(Z\rightsquigarrow V)}_{m_V,\mathfrak{m}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{m_V,\mathfrak{m}} \\ \mathbf{0} & & \mathbf{0} \\ \dots & \dots & \dots \\ \mathbf{0} & \dots & \mathbf{0} \\ \boldsymbol{b}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & \dots & \boldsymbol{b}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} \\ \dots & & \dots \\ \boldsymbol{b}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & \dots & \boldsymbol{b}'^{(Z\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}+\boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\ \mathbf{0} & \dots & \mathbf{0} \\ \dots & \dots & \dots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right),$$

$$\left(\begin{pmatrix} \boldsymbol{a}'^{(V\rightsquigarrow Z)}_{1,1}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{1,1} & \dots & \boldsymbol{a}'^{(V\rightsquigarrow Z)}_{1,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{1,\mathfrak{n}} \\ \dots & \dots & \dots \\ \boldsymbol{a}'^{(V\rightsquigarrow Z)}_{m_Z,1}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{m_Z,1} & \dots & \boldsymbol{a}'^{(V\rightsquigarrow Z)}_{m_Z,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{m_Z,\mathfrak{n}} \\ \mathbf{0} & & \mathbf{0} \\ \dots & \dots & \dots \\ \mathbf{0} & \dots & \mathbf{0} \\ \boldsymbol{a}'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,1}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{Z+1,1} & \dots & \boldsymbol{a}'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+1,\mathfrak{n}} \\ \dots & & \dots \\ \boldsymbol{a}'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,1} & & \boldsymbol{a}'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}} \\ \boldsymbol{a}'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1}=\boldsymbol{a}'^{(V\rightsquigarrow Z)}_{\mathcal{M}_W+1,1}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_W+1,1} & \dots & \boldsymbol{a}'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}}=\boldsymbol{a}'^{(V\rightsquigarrow Z)}_{\mathcal{M}_W+1,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_W+1,\mathfrak{n}} \\ \dots & \dots & \dots \\ \boldsymbol{a}'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,1}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,1}=\boldsymbol{a}'^{(V\rightsquigarrow Z)}_{\mathcal{M}_W+\ell_W,1}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_W+\ell_W,1} & \dots & \boldsymbol{a}'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,\mathfrak{n}}=\boldsymbol{a}'^{(V\rightsquigarrow Z)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}}+\boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_W+\ell_W,\mathfrak{n}} \\ \mathbf{0} & & \mathbf{0} \\ \dots & & \dots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right.,$$

$$\left(\begin{array}{ccc}
b'^{(V\rightsquigarrow Z)}_{1,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{1,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{1,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{1,\mathfrak{m}} \\
\cdots & \cdots & \cdots \\
b'^{(V\rightsquigarrow Z)}_{m_V,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{m_V,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{m_V,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{m_V,\mathfrak{m}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & \cdots & \cdots \\
\mathbf{0} & \cdots & \mathbf{0} \\
b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_V+1,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_V+1,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_V+1,\mathfrak{m}} \\
\cdots & & \cdots \\
b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_V+\ell_V,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_V+\ell_V,1} & & b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} \\
b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,1} = b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\,\widehat{b'}^{(Z\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+1,\mathfrak{m}} = b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + i\,\widehat{b'}^{(Z\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\
\cdots & \cdots & \cdots \\
b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,1} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,1} = b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\,\widehat{b'}^{(Z\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & \cdots & b'^{(V\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,\mathfrak{m}} + i\,\widehat{b'}^{(Z\rightsquigarrow Z)}_{\mathcal{M}_Z+\ell_Z+N,\mathfrak{m}} = b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + i\,\widehat{b'}^{(Z\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{array}\right)\Bigg)\Bigg).$$

With exactly the same way, a MitM attack can be conducted against $\boldsymbol{U}$ node without the knowledge of node $\boldsymbol{V}$. Most of the times the sophistication of this attack is low to medium due to active orientation of this attack.

It is obvious that if the nodes have smooth valuations and smooth vulnerabilities, the following states applied during this attack:

$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$
$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) > 0$
$\widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) < 0$	$\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) > 0$
$\widehat{\boldsymbol{\varphi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) < \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(Z\rightsquigarrow V)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(Z\rightsquigarrow V)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(V\rightsquigarrow Z)}(\boldsymbol{t}) > 0$	$\boldsymbol{\psi}^{(V\rightsquigarrow Z)}(\boldsymbol{t}) < 0$
$\widehat{\boldsymbol{\varphi}}^{(Z\rightsquigarrow Z)}(\boldsymbol{t}) > 0$	$\widehat{\boldsymbol{\psi}}^{(Z\rightsquigarrow Z)}(\boldsymbol{t}) < 0$
$\boldsymbol{\varphi}^{(Z\rightsquigarrow U)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(Z\rightsquigarrow U)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(U\rightsquigarrow Z)}(\boldsymbol{t}) > 0$	$\boldsymbol{\psi}^{(U\rightsquigarrow Z)}(\boldsymbol{t}) < 0$

On the other hand, **wiretapping** attack which is, as mentioned, a passive attack that consists in the monitoring of Cyber activity, often by covert means, escalates as follows. A family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \Big\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(Y,X)}(\boldsymbol{t}) = \big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big)(\boldsymbol{t}) \in \big(\mathbb{C}^{\mathfrak{n}\times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times \boldsymbol{k}}\big)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\Big\},$$

lying in (a partial danger sector $\mathcal{E} = \mathcal{E}_{U\to V}$ to) the node $\boldsymbol{V}$ from the node $\boldsymbol{Z}$ during the entire time set $\mathbb{I}$, is a **germ of (partial) passive attack from an intermediate node $\boldsymbol{Z}$ against the $(\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda}) -$ resource parts $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_1}}^{(\boldsymbol{V})}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_2}}^{(\boldsymbol{V})}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}_{\boldsymbol{\kappa_\lambda}}^{(\boldsymbol{V})}\right)$** of $\boldsymbol{V}$, during a given time subset $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\right) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{\hbar}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{\hbar}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$
\left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\right) =
$$

$$
\left(\left(\left(\begin{matrix} \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ a_{\mathcal{M}_V+1,1}^{(Z\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & a_{\mathcal{M}_V+1,\mathfrak{n}}^{(Z\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ a_{\mathcal{M}_V+\ell_V,1}^{(Z\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & a_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(Z\rightsquigarrow V)} + i\ \hat{a}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{matrix}\right), \right.\right.
$$

$$
\left.\left(\begin{matrix} \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ b_{\mathcal{M}_V+1,1}^{(Z\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & \cdots & b_{\mathcal{M}_V+1,\mathfrak{m}}^{(Z\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \cdots & \cdots & \cdots \\ b_{\mathcal{M}_V+\ell_V,1}^{(Z\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & \cdots & b_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(Z\rightsquigarrow V)} + i\ \hat{b}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{matrix}\right)\right),
$$

$$
\left(\left(\begin{matrix} \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ a_{\mathcal{M}_Z+1,1}^{(V\rightsquigarrow Z)} + i\ \hat{a}_{\mathcal{M}_Z+1,1}^{(Z\rightsquigarrow Z)} & \cdots & a_{\mathcal{M}_Z+1,\mathfrak{n}}^{(V\rightsquigarrow Z)} + i\ \hat{a}_{\mathcal{M}_Z+1,\mathfrak{n}}^{(Z\rightsquigarrow Z)} \\ \cdots & \cdots & \cdots \\ a_{\mathcal{M}_Z+\ell_Z,1}^{(V\rightsquigarrow Z)} + i\ \hat{a}_{\mathcal{M}_Z+\ell_Z,1}^{(Z\rightsquigarrow Z)} & \cdots & a_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}}^{(V\rightsquigarrow Z)} + i\ \hat{a}_{\mathcal{M}_Z+\ell_Z,\mathfrak{n}}^{(Z\rightsquigarrow Z)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{matrix}\right), \right.
$$

$$
\left.\left.\left(\begin{matrix} \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \\ b_{\mathcal{M}_Z+1,1}^{(V\rightsquigarrow Z)} + i\ \hat{b}_{\mathcal{M}_Z+1,1}^{(Z\rightsquigarrow Z)} & \cdots & b_{\mathcal{M}_Z+1,\mathfrak{m}}^{(V\rightsquigarrow Z)} + i\ \hat{b}_{\mathcal{M}_Z+1,\mathfrak{m}}^{(Z\rightsquigarrow Z)} \\ \cdots & \cdots & \cdots \\ b_{\mathcal{M}_Z+\ell_Z,1}^{(V\rightsquigarrow Z)} + i\ \hat{b}_{\mathcal{M}_Z+\ell_Z,1}^{(Z\rightsquigarrow Z)} & \cdots & b_{\mathcal{M}_Z+\ell_Z,\mathfrak{m}}^{(V\rightsquigarrow Z)} + i\ \hat{b}_{\mathcal{M}_Z+\ell_Z,\mathfrak{m}}^{(Z\rightsquigarrow Z)} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{matrix}\right)\right)\right)
$$

and is depicted, at a next moment $\boldsymbol{t'} = \boldsymbol{t} + \Delta \boldsymbol{t}$, at a pair $\left((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\right) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{\mathfrak{k}}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{\mathfrak{k}}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{Z}$ and $\boldsymbol{V}$ having the form

$$
\left((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\right) =
$$

$$
\left(\left(\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\boldsymbol{a'}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{\beta}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,1}} & & \boldsymbol{a'}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,\mathfrak{n}}} \\
\cdots & & \cdots \\
\boldsymbol{a'}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,1}} & & \boldsymbol{a'}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,\mathfrak{n}}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.\right.
$$

$$
\left.\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\boldsymbol{b'}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,1}} & & \boldsymbol{b'}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,\mathfrak{m}}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+1,\mathfrak{m}}} \\
\cdots & & \cdots \\
\boldsymbol{b'}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,1}} & & \boldsymbol{b'}^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,\mathfrak{m}}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{\mathcal{M}_V+\ell_V,\mathfrak{m}}} \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right),
$$

$$
\left(\left(\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\boldsymbol{a'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+1,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+1,1}} & & \boldsymbol{a'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+1,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+1,\mathfrak{n}}} \\
\cdots & & \cdots \\
\boldsymbol{a'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z,1}} & & \boldsymbol{a'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z,\mathfrak{n}}} \\
\boldsymbol{a'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+1,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+1,1}} & & \boldsymbol{a'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}}} \\
\cdots & \cdots\cdots\cdots & \cdots \\
\boldsymbol{a'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+\nu,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+\nu,1}} & & \boldsymbol{a'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+\nu,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+\nu,\mathfrak{n}}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix},\right.\right.
$$

$$
\left.\left.\begin{pmatrix}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\
\boldsymbol{b'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+1,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+1,1}} & & \boldsymbol{b'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+1,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+1,\mathfrak{n}}} \\
\cdots & & \cdots \\
\boldsymbol{b'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z,1}} & & \boldsymbol{b'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z,\mathfrak{n}}} \\
\boldsymbol{b'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+1,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+1,1}} & & \boldsymbol{b'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+1,\mathfrak{n}}} \\
\cdots & \cdots\cdots\cdots & \cdots \\
\boldsymbol{b'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+\nu,1}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+\nu,1}} & & \boldsymbol{b'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+\nu,\mathfrak{n}}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(\boldsymbol{Z}\rightsquigarrow\boldsymbol{Z})}_{\boldsymbol{\mathcal{M}_Z+\ell_Z+\nu,\mathfrak{n}}} \\
\mathbf{0} & & \mathbf{0} \\
\cdots & & \cdots \\
\mathbf{0} & & \mathbf{0}
\end{pmatrix}\right)\right).
$$

With exactly the same way, a wiretapping attack can be conducted against $\boldsymbol{U}$ node without the knowledge of node $\boldsymbol{V}$. Most of the times the sophistication of this attack is medium to high due to "passive" orientation of this.

Specifically, during Wiretapping attack the following states applied:

$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t})\boldsymbol{\psi}^{\varphi}, \widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})\ \boldsymbol{\psi}^{c}$
$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) = 0$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) = 0$
$\widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) = 0$	$\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) = 0$
$\boldsymbol{\varphi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) = 0$	$\boldsymbol{\psi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) = 0$
$\widehat{\boldsymbol{\varphi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) = \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) = 0$
$\boldsymbol{\varphi}^{(Z\rightsquigarrow V)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(Z\rightsquigarrow V)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(V\rightsquigarrow Z)}(\boldsymbol{t}) = 0$	$\boldsymbol{\psi}^{(V\rightsquigarrow Z)}(\boldsymbol{t}) = 0$
$\widehat{\boldsymbol{\varphi}}^{(Z\rightsquigarrow Z)}(\boldsymbol{t}) > 0$	$\widehat{\boldsymbol{\psi}}^{(Z\rightsquigarrow Z)}(\boldsymbol{t}) < 0$
$\boldsymbol{\varphi}^{(Z\rightsquigarrow U)}(\boldsymbol{t}) < 0$	$\boldsymbol{\psi}^{(Z\rightsquigarrow U)}(\boldsymbol{t}) > 0$
$\boldsymbol{\varphi}^{(U\rightsquigarrow Z)}(\boldsymbol{t}) = 0$	$\boldsymbol{\psi}^{(U\rightsquigarrow Z)}(\boldsymbol{t}) = 0$

6.3 Access Attack

An **access attack** is actually an attack where intruder gains **access** to a device/system to which he has no right for access. Thus, during this attack the following general form of cyber-effect applies:

$$\boldsymbol{g} = \boldsymbol{g_t}\colon \boldsymbol{\mathcal{Q}}_{\mathbf{5}}^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t}) \rightarrow \boldsymbol{\mathcal{P}}_{\mathbf{11}}^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t}')$$

where $\boldsymbol{\mathcal{Q}}_{\mathbf{5}}^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t})$ and $\boldsymbol{\mathcal{P}}_{\mathbf{11}}^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t}')$ are the combinatorial triplets

$\boldsymbol{\mathcal{Q}}_{\mathbf{5}}^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t}) = \left(\boldsymbol{\mathfrak{D}}^{(\boldsymbol{fraction})}(\boldsymbol{U}), \boldsymbol{\mathcal{S}_V}\boldsymbol{\mathfrak{D}}^{(\boldsymbol{fraction})}(\boldsymbol{U}), \boldsymbol{\mathcal{U}_V}\boldsymbol{\mathfrak{D}}^{(\boldsymbol{fraction})}(\boldsymbol{U})\right)$ and

$\boldsymbol{\mathcal{P}}_{\mathbf{11}}^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t}') = \left(\boldsymbol{\mathfrak{D}}_{\boldsymbol{available}}^{(\boldsymbol{fraction})}(\boldsymbol{V}), \boldsymbol{\mathcal{S}_U}\boldsymbol{\mathfrak{D}}_{\boldsymbol{available}}^{(\boldsymbol{fraction})}(\boldsymbol{V}), \boldsymbol{\mathcal{U}_U}\boldsymbol{\mathfrak{D}}_{\boldsymbol{available}}^{(\boldsymbol{fraction})}(\boldsymbol{V})\right)$,

respectively ([5]).

In such a case, a family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \left\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(\boldsymbol{Y},\boldsymbol{X})}(\boldsymbol{t}) = \left((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\right)(\boldsymbol{t}) \in \left(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\right)^{\mathbf{4}}, \boldsymbol{t} \in \mathbb{I}\right\},$$

lying in (a partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U}\rightarrow\boldsymbol{V}}$ to) the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ during the entire time set $\mathbb{I}$, is a **germ of (partial) access attack against the** $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu})$ **– device parts** $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_1}^{(\boldsymbol{V})}\right)$, $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_2}^{(\boldsymbol{V})}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{dev}_{\mu_\nu}^{(\boldsymbol{V})}\right)$ **of** $\boldsymbol{V}$ during a given time subset $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\right) \in \left(\mathbb{C}^{\mathfrak{n}\times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times \boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\left((\mathbb{z}_{\mathbf{1}}, \mathbb{w}_{\mathbf{1}}), (\mathbb{z}_{\mathbf{2}}, \mathbb{w}_{\mathbf{2}})\right) =$$

$$\left(\left(\begin{pmatrix} \boldsymbol{a}_{\mathbf{1},\mathbf{1}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{a}_{\mathbf{1},\mathfrak{n}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1},\mathfrak{n}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{\boldsymbol{m}_{\boldsymbol{V}},\mathbf{1}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{m}_{\boldsymbol{V}},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & & \boldsymbol{a}_{\boldsymbol{m}_{\boldsymbol{V}},\mathfrak{n}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{m}_{\boldsymbol{V}},\mathfrak{n}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}, \begin{pmatrix} \boldsymbol{b}_{\mathbf{1},\mathbf{1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathbf{1},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{b}_{\mathbf{1},\mathfrak{m}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathbf{1},\mathfrak{m}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}_{\boldsymbol{m}_{\boldsymbol{V}},\mathbf{1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{m}_{\boldsymbol{V}},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} & \cdots & \boldsymbol{b}_{\boldsymbol{m}_{\boldsymbol{V}},\mathfrak{m}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{m}_{\boldsymbol{V}},\mathfrak{m}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}\right), \right.$$

$$\left. \left(\begin{pmatrix} \boldsymbol{a}_{\mathbf{1},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1},\mathbf{1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})} & \cdots & \boldsymbol{a}_{\mathbf{1},\mathfrak{n}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathbf{1},\mathfrak{n}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}_{\boldsymbol{m}_{\boldsymbol{U}},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{m}_{\boldsymbol{U}},\mathbf{1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})} & \cdots & \boldsymbol{a}_{\boldsymbol{m}_{\boldsymbol{U}},\mathfrak{n}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\boldsymbol{m}_{\boldsymbol{U}},\mathfrak{n}}^{(\boldsymbol{W}\rightsquigarrow\boldsymbol{W})} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}, \begin{pmatrix} \boldsymbol{b}_{\mathbf{1},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathbf{1},\mathbf{1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})} & \cdots & \boldsymbol{b}_{\mathbf{1},\mathfrak{m}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathbf{1},\mathfrak{m}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})} \\ \cdots & \cdots & \cdots \\ \boldsymbol{b}_{\boldsymbol{m}_{\boldsymbol{U}},\mathbf{1}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{m}_{\boldsymbol{U}},\mathbf{1}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})} & \cdots & \boldsymbol{b}_{\boldsymbol{m}_{\boldsymbol{U}},\mathfrak{m}}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{U})} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\boldsymbol{m}_{\boldsymbol{U}},\mathfrak{m}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})} \\ \mathbf{0} & & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}\right)\right)$$

and is depicted, at a next moment $\boldsymbol{t}' = \boldsymbol{t} + \Delta\boldsymbol{t}$, at a pair $\left((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\right) \in \left(\mathbb{C}^{\mathfrak{n}\times \boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times \boldsymbol{k}}\right)^{\mathbf{2}}$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form

$$\left((\mathbb{z}'_{\mathbf{1}}, \mathbb{w}'_{\mathbf{1}}), (\mathbb{z}'_{\mathbf{2}}, \mathbb{w}'_{\mathbf{2}})\right) =$$

$$\left(\left(\begin{pmatrix} \boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\mathbf{1},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\mathbf{1},\mathbf{1}} & \cdots & \boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\mathbf{1},\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\mathbf{1},\mathfrak{n}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{m}_{\boldsymbol{V}},\mathbf{1}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{m}_{\boldsymbol{V}},\mathbf{1}} & & \boldsymbol{a}'^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{m}_{\boldsymbol{V}},\mathfrak{n}} + \boldsymbol{i}\ \widehat{\boldsymbol{a}'}^{(\boldsymbol{V}\rightsquigarrow\boldsymbol{V})}_{\boldsymbol{m}_{\boldsymbol{V}},\mathfrak{n}} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix}, \right.\right.$$

$$\begin{pmatrix} \boldsymbol{b'}^{(U\rightsquigarrow V)}_{1,1} + i\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{1,1} & \dots & \boldsymbol{b'}^{(U\rightsquigarrow V)}_{1,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{1,\mathfrak{m}} \\ \dots & \dots & \dots \\ \boldsymbol{b'}^{(U\rightsquigarrow V)}_{m_U,1} + i\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{m_U,1} & \dots & \boldsymbol{b'}^{(U\rightsquigarrow V)}_{m_U,\mathfrak{m}} + i\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{m_U,\mathfrak{m}} \\ \mathbf{0} & \dots & \mathbf{0} \\ \dots & \dots & \dots \\ \mathbf{0} & \dots & \mathbf{0} \end{pmatrix}\Bigg)\Bigg),$$

$$\Bigg(\begin{pmatrix} \boldsymbol{a'}^{(V\rightsquigarrow U)}_{1,1} + i\ \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{1,1} = \boldsymbol{a'}^{(U\rightsquigarrow V)}_{1,1} + i\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{1,1} & \dots & \boldsymbol{a'}^{(V\rightsquigarrow U)}_{1,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} = \boldsymbol{a'}^{(U\rightsquigarrow V)}_{1,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{1,\mathfrak{n}} \\ \dots & \dots & \dots \\ \boldsymbol{a'}^{(V\rightsquigarrow U)}_{m_U,1} + i\ \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{m_U,1} = \boldsymbol{a'}^{(U\rightsquigarrow V)}_{m_V,1} + i\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{m_V,1} & \dots & \boldsymbol{a'}^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}'}^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}} = \boldsymbol{a'}^{(U\rightsquigarrow V)}_{m_V,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\ \boldsymbol{a'}^{(U\rightsquigarrow V)}_{m_V+1,1} + i\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{m_V+1,1} & & \boldsymbol{a'}^{(U\rightsquigarrow V)}_{m_V+1,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{m_V+1,\mathfrak{n}} \\ \dots & \dots & \dots \\ \boldsymbol{a'}^{(U\rightsquigarrow V)}_{m_V+\lambda,1} + i\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{m_V+\lambda,1} & \dots & \boldsymbol{a'}^{(U\rightsquigarrow V)}_{m_V+\lambda,\mathfrak{n}} + i\ \widehat{\boldsymbol{a}'}^{(V\rightsquigarrow V)}_{m_V+\lambda,\mathfrak{n}} \\ & \dots & \\ \mathbf{0} & \dots & \mathbf{0} \\ \dots & \dots & \dots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},$$

$$\begin{pmatrix} \boldsymbol{b'}^{(V\rightsquigarrow U)}_{1,1} + i\ \widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{1,1} = \boldsymbol{b'}^{(U\rightsquigarrow V)}_{1,1} + i\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{1,1} & \dots & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{1,\mathfrak{n}} + i\ \widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{1,\mathfrak{n}} = \boldsymbol{b'}^{(U\rightsquigarrow V)}_{1,\mathfrak{n}} + i\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{1,\mathfrak{n}} \\ \dots & \dots & \dots \\ \boldsymbol{b'}^{(V\rightsquigarrow U)}_{m_U,1} + i\ \widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{m_U,1} = \boldsymbol{b'}^{(U\rightsquigarrow V)}_{m_V,1} + i\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{m_V,1} & \dots & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}} + i\ \widehat{\boldsymbol{b}'}^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}} = \boldsymbol{b'}^{(U\rightsquigarrow V)}_{m_V,\mathfrak{n}} + i\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{m_V,\mathfrak{n}} \\ \boldsymbol{b'}^{(U\rightsquigarrow V)}_{m_V+1,1} + i\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{m_V+1,1} & & \boldsymbol{b'}^{(U\rightsquigarrow V)}_{m_V+1,\mathfrak{n}} + i\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{m_V+1,\mathfrak{n}} \\ \dots & \dots & \dots \\ \boldsymbol{b'}^{(U\rightsquigarrow V)}_{m_V+\lambda,1} + i\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{m_V+\lambda,1} & \dots & \boldsymbol{b'}^{(U\rightsquigarrow V)}_{m_V+\lambda,\mathfrak{n}} + i\ \widehat{\boldsymbol{b}'}^{(V\rightsquigarrow V)}_{m_V+\lambda,\mathfrak{n}} \\ & \dots & \\ \mathbf{0} & \dots & \mathbf{0} \\ \dots & \dots & \dots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\Bigg)\Bigg)\Bigg).$$

Most of the times the sophistication of this attack is medium to high. Specifically, during Access attack the following states applied:

$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(t), \widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(t)$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(t), \widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(t)$
$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(t) < 0$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(t) > 0$
$\widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(t) = 0$	$\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(t) = 0$
$\boldsymbol{\varphi}^{(V\rightsquigarrow U)}(t) = 0$	$\boldsymbol{\psi}^{(V\rightsquigarrow U)}(t) = 0$
$\widehat{\boldsymbol{\varphi}}^{(U\rightsquigarrow U)}(t) > 0$	$\widehat{\boldsymbol{\psi}}^{(U\rightsquigarrow U)}(t) < 0$

Proposition 6.2 It is clear that during an access attack $\boldsymbol{\mathcal{F}}$ from $\boldsymbol{U}$ against the $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu}) -$ device parts $\boldsymbol{fr}\left(\boldsymbol{dev}^{(V)}_{\mu_1}\right)$, $\boldsymbol{fr}\left(\boldsymbol{dev}^{(V)}_{\mu_2}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{dev}^{(V)}_{\mu_\nu}\right)$ of $\boldsymbol{V}$, the following elementary properties hold.

i. The (Euclidean) norm $\| \boldsymbol{a}'^{(U \rightsquigarrow V)} \|$ of the resulting overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is less than the (Euclidean) norm $\|\boldsymbol{a}^{(U \rightsquigarrow V)}\|$ of the initial overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of$\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\| \boldsymbol{a}'^{(U \rightsquigarrow V)} \| < \|\boldsymbol{a}^{(U \rightsquigarrow V)}\|.$$

ii. The (Euclidean) norm $\| \boldsymbol{b}'^{(U \rightsquigarrow V)} \|$ of the resulting overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is greater than the (Euclidean) norm $\|\boldsymbol{b}^{(U \rightsquigarrow V)}\| := \left(\sum_{j=1}^{\boldsymbol{m}} \sum_{\lambda=1}^{\ell_V} \left|\boldsymbol{b}_{\mathcal{M}_U+\lambda,j}^{(U \rightsquigarrow V)}\right|^{2}\right)^{1/2}$ of the initial overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\| \boldsymbol{b}'^{(U \rightsquigarrow V)} \| > \|\boldsymbol{b}^{(U \rightsquigarrow V)}\|.$$

iii. The (Euclidean) norm $\|\widehat{\boldsymbol{a}}'^{(U \rightsquigarrow U)}\|$ of the resulting overall valuation in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is greater than the (Euclidean) norms

$$\|\widehat{\boldsymbol{a}}^{(U \rightsquigarrow U)}\| \text{ and } \|\boldsymbol{a}^{(U \rightsquigarrow V)}\|$$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\|\widehat{\boldsymbol{\beta}}'^{(U \rightsquigarrow U)}\| > \boldsymbol{max}\{\|\widehat{\boldsymbol{\beta}}^{(U \rightsquigarrow U)}\|, \|\boldsymbol{\beta}^{(U \rightsquigarrow V)}\| \}.$$

iv. The (Euclidean) norm $\|\widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)}\|$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is less or equal than the (Euclidean) norms

$$\|\widehat{\boldsymbol{b}}^{(U \rightsquigarrow U)}\| \text{ and } \|\boldsymbol{b}^{(U \rightsquigarrow V)}\|$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\|\widehat{\boldsymbol{b}}'^{(U \rightsquigarrow U)}\| \leq \boldsymbol{min}\{\|\widehat{\boldsymbol{b}}^{(U \rightsquigarrow U)}\|, \|\boldsymbol{b}^{(U \rightsquigarrow V)}\| \}. \blacksquare$$

Remark 6.3 Of course, in the special case where there is a fully successful access attack the following hold:

$$\| \boldsymbol{a}'^{(U \rightsquigarrow V)} \| \approx \boldsymbol{0}, \| \boldsymbol{a}'^{(U \rightsquigarrow U)} \| = \sqrt{\boldsymbol{m}_U}, \| \boldsymbol{b}'^{(U \rightsquigarrow V)} \| = \sqrt{\boldsymbol{m}_U}. \blacksquare$$

An access attack, besides a reflexive homomorphism, can take place **physically** when an attacker $\boldsymbol{U}$, physically gains access of victim node devices $\boldsymbol{V}$.

6.4 Reconnaissance Attack

A **reconnaissance attack** is actually an attack which involves unauthorized detection system mapping and services to steal data. This attack can potentially take place both actively and passively. Specifically, in passive reconnaissance, an intruder monitors systems for vulnerabilities without interaction, through methods like session capture. In active reconnaissance, the intruder engages with the target system through methods like port scans.

Thus, during this attack the following general form of cyber-effect applies:

$$\boldsymbol{g} = \boldsymbol{g_t}: \boldsymbol{\mathcal{Q}_9^{(V)}(U)(t)} \to \boldsymbol{\mathcal{P}_7^{(U)}(V)(t')}$$

where $\boldsymbol{\mathcal{Q}_9^{(V)}(U)(t')}$ and $\boldsymbol{\mathcal{P}_7^{(U)}(V)(t')}$ are the combinatorial triplets

$$\boldsymbol{\mathcal{Q}_9^{(V)}(U)} = \boldsymbol{\mathcal{Q}_9^{(V)}(U)(t')} = (\boldsymbol{\mathfrak{R}_{available}(V)}, \boldsymbol{\mathcal{S}_U \mathfrak{R}_{available}(V)}, \boldsymbol{\mathcal{U}_U \mathfrak{R}_{available}(V)}\ )$$

and

$$\boldsymbol{\mathcal{P}_7^{(U)}(V)(t')} = (\boldsymbol{\mathfrak{C}_{available}(V)}, \boldsymbol{\mathcal{S}_U \mathfrak{C}_{available}(V)}, \boldsymbol{\mathcal{U}_U \mathfrak{C}_{available}(V)}\ )$$

respectively ([5]).

It is obvious that the purpose of this attack is for node $\boldsymbol{U}$ to uncover all constituents' vulnerabilities of node $\boldsymbol{V}$.

A family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \Big\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}_{(Y,X)}(t)} = \big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big)(\boldsymbol{t}) \in \big(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{\hbar}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{\hbar}}\big)^{\boldsymbol{4}}, \boldsymbol{t} \in \mathbb{I}\Big\},$$

lying in (the partial danger sector $\mathcal{E} = \mathcal{E}_{\boldsymbol{U}\to\boldsymbol{V}}$ to) the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ during the entire time set $\mathbb{I}$, is **a germ of reconnaissance attack against the** $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu})-$ **device parts** $\boldsymbol{fr}\left(\boldsymbol{dev_{\mu_1}^{(V)}}\right)$**,** $\boldsymbol{fr}\left(\boldsymbol{dev_{\mu_2}^{(V)}}\right)$**,…,** $\boldsymbol{fr}\left(\boldsymbol{dev_{\mu_\nu}^{(V)}}\right)$ **and the** $(\boldsymbol{\kappa_1}, \dots, \boldsymbol{\kappa_\lambda})-$ **resource parts** $\boldsymbol{fr}\left(\boldsymbol{res_{\kappa_1}^{(V)}}\right)$**,** $\boldsymbol{fr}\left(\boldsymbol{res_{\kappa_2}^{(V)}}\right)$**,…,** $\boldsymbol{fr}\left(\boldsymbol{res_{\kappa_\lambda}^{(V)}}\right)$ **of** $\boldsymbol{V}$ during a given time set $\mathbb{I} \subset\subset [\boldsymbol{0}, \boldsymbol{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\big) \in \big(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{\hbar}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{\hbar}}\big)^{\boldsymbol{2}}$ of supervisory constituents perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\big((\mathbb{z}_1,\mathbb{w}_1),(\mathbb{z}_2,\mathbb{w}_2)\big)=$$

$$\left(\left(\left(\begin{array}{ccc}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
\vdots & & \vdots\\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+i\ \hat{a}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}}+i\ \hat{a}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}}\\
\vdots & & \vdots\\
a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1}+i\ \hat{a}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & a^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}+i\ \hat{a}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}\\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
\vdots & & \vdots\\
\mathbf{0} & & \mathbf{0}
\end{array}\right),\right.\right.$$

$$\left.\left.\left(\begin{array}{ccc}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
\vdots & & \vdots\\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+i\ \hat{b}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}}+i\ \hat{b}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}}\\
\vdots & & \vdots\\
b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1}+i\ \hat{b}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & b^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}+i\ \hat{b}^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}\\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
\vdots & & \vdots\\
\mathbf{0} & & \mathbf{0}
\end{array}\right)\right)\right),$$

$$\left(\left(\begin{array}{ccc}
a^{(V\rightsquigarrow U)}_{1,1}+i\ \hat{a}^{(U\rightsquigarrow U)}_{1,1} & & a^{(V\rightsquigarrow U)}_{1,\mathfrak{n}}+i\ \hat{a}^{(U\rightsquigarrow U)}_{1,\mathfrak{n}}\\
\vdots & \cdots\cdots\cdots & \vdots\\
a^{(V\rightsquigarrow U)}_{m_U,1}+i\ \hat{a}^{(U\rightsquigarrow U)}_{m_U,1} & & a^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}}+i\ \hat{a}^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}}\\
\vdots & & \vdots\\
\mathbf{0} & & \mathbf{0}\\
\vdots & \cdots\cdots\cdots & \vdots\\
a^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1}+i\ \hat{a}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & a^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}}+i\ \hat{a}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}}\\
\vdots & & \vdots\\
a^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1}+i\ \hat{a}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & a^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}+i\ \hat{a}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}\\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
\vdots & & \vdots\\
\mathbf{0} & & \mathbf{0}
\end{array}\right),\right.$$

$$\left.\left(\begin{array}{ccc}
b^{(V\rightsquigarrow U)}_{1,1}+i\ \hat{b}^{(U\rightsquigarrow U)}_{1,1} & & b^{(V\rightsquigarrow U)}_{1,\mathfrak{n}}+i\ \hat{b}^{(U\rightsquigarrow U)}_{1,\mathfrak{n}}\\
\vdots & \cdots\cdots\cdots & \vdots\\
b^{(V\rightsquigarrow U)}_{m_U,1}+i\ \hat{b}^{(U\rightsquigarrow U)}_{m_U,1} & & b^{(V\rightsquigarrow U)}_{m_U,\mathfrak{n}}+i\ \hat{b}^{(U\rightsquigarrow U)}_{m_U,\mathfrak{n}}\\
\vdots & & \vdots\\
\mathbf{0} & & \mathbf{0}\\
\vdots & \cdots\cdots\cdots & \vdots\\
b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1}+i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}}+i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}}\\
\vdots & & \vdots\\
b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1}+i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}+i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}\\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
\vdots & & \vdots\\
\mathbf{0} & & \mathbf{0}
\end{array}\right)\right)\Bigg)\Bigg)$$

and is depicted, at a next moment $\boldsymbol{t}'=\boldsymbol{t}+\Delta\boldsymbol{t}$, at a pair $\big((\mathbb{z}_1',\mathbb{w}_1'),(\mathbb{z}_2',\mathbb{w}_2')\big)\in\left(\mathbb{C}^{\mathfrak{n}\times\mathscr{k}}\times\mathbb{C}^{\mathfrak{m}\times\mathscr{k}}\right)^2$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form

$$\left(\left(\mathbb{z}'_1,\mathbb{w}'_1\right),\left(\mathbb{z}'_2,\mathbb{w}'_2\right)\right)=$$

$$\left(\left(\left(\begin{array}{ccc}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
\cdots & & \cdots\\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+i\ \widehat{a}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}}+i\ \widehat{a}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}}\\
\cdots & & \cdots\\
a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1}+i\ \widehat{a}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}+i\ \widehat{a}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}\\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
\cdots & & \cdots\\
\mathbf{0} & & \mathbf{0}
\end{array}\right),\right.\right.$$

$$\left.\left(\begin{array}{ccc}
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
\cdots & & \cdots\\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1}+i\ \widehat{b}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} & & b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}}+i\ \widehat{b}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}}\\
\cdots & & \cdots\\
b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1}+i\ \widehat{b}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} & & b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}+i\ \widehat{b}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}\\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
\cdots & & \cdots\\
\mathbf{0} & & \mathbf{0}
\end{array}\right)\right),$$

$$\left(\left(\begin{array}{ccc}
a'^{(V\rightsquigarrow U)}_{1,1}+i\ \widehat{a}'^{(U\rightsquigarrow U)}_{1,1} & & a'^{(V\rightsquigarrow U)}_{1,\mathfrak{n}}+i\ \widehat{a}'^{(U\rightsquigarrow U)}_{1,\mathfrak{n}}\\
\cdots & \cdots\cdots\cdots & \cdots\\
a'^{(V\rightsquigarrow U)}_{\mathcal{m}_U,1}+i\ \widehat{a}'^{(U\rightsquigarrow U)}_{\mathcal{m}_U,1} & & a'^{(V\rightsquigarrow U)}_{\mathcal{m}_U,\mathfrak{n}}+i\ \widehat{a}'^{(U\rightsquigarrow U)}_{\mathcal{m}_U,\mathfrak{n}}\\
\cdots & & \cdots\\
\mathbf{0} & & \mathbf{0}\\
\cdots & \cdots\cdots\cdots & \cdots\\
a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1}+i\ \widehat{a}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}}+i\ \widehat{a}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}}\\
\cdots & & \cdots\\
a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1}+i\ \widehat{a}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}+i\ \widehat{a}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}\\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
\cdots & & \cdots\\
\mathbf{0} & & \mathbf{0}
\end{array}\right),\right.$$

$$\left.\left.\left(\begin{array}{ccc}
b'^{(V\rightsquigarrow U)}_{1,1}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{1,1} & & b'^{(V\rightsquigarrow U)}_{1,\mathfrak{n}}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{1,\mathfrak{n}}\\
\cdots & \cdots\cdots\cdots & \cdots\\
b'^{(V\rightsquigarrow U)}_{\mathcal{m}_U,1}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{m}_U,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{m}_U,\mathfrak{n}}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{m}_U,\mathfrak{n}}\\
b'^{(V\rightsquigarrow U)}_{\mathcal{m}_U+1,1}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{m}_U+1,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{m}_U+1,\mathfrak{n}}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{m}_U+1,\mathfrak{n}}\\
\cdots & & \cdots\\
b'^{(V\rightsquigarrow U)}_{\mathcal{m}_U+\ell_V,1}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{m}_U+\ell_V,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{m}_U+\ell_V,n}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{m}_U+\ell_V,n}\\
\mathbf{0} & \cdots\cdots\cdots & \mathbf{0}\\
\cdots & & \cdots\\
\mathbf{0} & & \mathbf{0}\\
b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}}\\
\cdots & & \cdots\\
b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}\\
b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,n}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+1,n}\\
\cdots & \cdots\cdots\cdots & \cdots\\
b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\ell_V1}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\ell_V,1} & & b'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\ell_V,n}+i\ \widehat{b}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U+\ell_V,n}\\
\mathbf{0} & & \mathbf{0}\\
\cdots & & \cdots\\
\mathbf{0} & & \mathbf{0}
\end{array}\right)\right)\right).$$

Most of the times the sophistication of this attack is very low and highly "transparent" to attacked node. Frequently, after this attack a more sophisticated attack is expected. Specifically, during Reconnaissance attack the following states applied:

$\boldsymbol{\varphi^{(U \rightsquigarrow V)}(t), \widehat{\varphi}^{(V \rightsquigarrow V)}(t)}$	$\boldsymbol{\psi^{(U \rightsquigarrow V)}(t), \widehat{\psi}^{(V \rightsquigarrow V)}(t)}$
$\boldsymbol{\varphi^{(U \rightsquigarrow V)}(t) < 0}$	$\boldsymbol{\psi^{(U \rightsquigarrow V)}(t) > 0}$
$\boldsymbol{\widehat{\varphi}^{(V \rightsquigarrow V)}(t) = 0}$	$\boldsymbol{\widehat{\psi}^{(V \rightsquigarrow V)}(t) = 0}$
$\boldsymbol{\varphi^{(V \rightsquigarrow U)}(t) = 0}$	$\boldsymbol{\psi^{(V \rightsquigarrow U)}(t) = 0}$
$\boldsymbol{\widehat{\varphi}^{(U \rightsquigarrow U)}(t) > 0}$	$\boldsymbol{\widehat{\psi}^{(U \rightsquigarrow U)}(t) < 0}$

Proposition 6.4 It is obvious that during a reconnaissance attack $\boldsymbol{\mathcal{F}}$ from $\boldsymbol{U}$ against the $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu}) -$ resource parts $\boldsymbol{fr\left(res^{(V)}_{\mu_1}\right)}$, $\boldsymbol{fr\left(res^{(V)}_{\mu_2}\right)}$,…, $\boldsymbol{fr\left(res^{(V)}_{\mu_\nu}\right)}$ of $\boldsymbol{V}$, the following elementary properties hold:

i. The (Euclidean) norm $\left\| \boldsymbol{a}'^{(U \rightsquigarrow V)} \right\|$ of the resulting overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t'}$ is less than the (Euclidean) norm $\left\|\boldsymbol{a}^{(U \rightsquigarrow V)}\right\|$ of the initial overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\| \boldsymbol{a}'^{(U \rightsquigarrow V)} \right\| < \left\|\boldsymbol{a}^{(U \rightsquigarrow V)}\right\|.$$

ii. The (Euclidean) norm $\left\| \boldsymbol{b}'^{(U \rightsquigarrow V)} \right\|$ of the resulting overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t'}$ is greater than the (Euclidean) norm $\left\|\boldsymbol{b}^{(U \rightsquigarrow V)}\right\| := \left(\sum_{j=1}^{\boldsymbol{m}} \sum_{\lambda=1}^{\boldsymbol{\ell_V}} \left|\boldsymbol{b}^{(U \rightsquigarrow V)}_{\boldsymbol{\mathcal{M}_U+\lambda,j}}\right|^{\boldsymbol{2}}\right)^{\boldsymbol{1/2}}$ of the initial overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\| \boldsymbol{b}'^{(U \rightsquigarrow V)} \right\| > \left\|\boldsymbol{b}^{(U \rightsquigarrow V)}\right\|.$$

iii. The (Euclidean) norm $\left\|\widehat{\boldsymbol{a}}'^{(U \rightsquigarrow U)}\right\|$ of the resulting overall valuation in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t'}$ is greater than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{a}}^{(U \rightsquigarrow U)}\right\| \text{ and } \left\|\boldsymbol{a}^{(U \rightsquigarrow V)}\right\|$$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{a}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| > \boldsymbol{max}\left\{\left\|\widehat{\boldsymbol{a}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|, \left\|\boldsymbol{a}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|\right\}.$$

iv. The (Euclidean) norm $\left\|\widehat{\boldsymbol{b}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t'}$ is less or equal than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{b}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| \text{ and } \left\|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{b}'}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\| \leq \boldsymbol{min}\left\{\left\|\widehat{\boldsymbol{b}}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{U})}\right\|, \left\|\boldsymbol{b}^{(\boldsymbol{U}\rightsquigarrow\boldsymbol{V})}\right\|\right\}. \blacksquare$$

The criticality of this attack is high since most of times it is the omen of a more severe or more sophisticated attack.

6.5 Denial of Service (DoS) attack and Distributed Denial of Service (DDoS) attack

Both attacks intent to deny services and generally resources to authorized users. The attacker makes a computing or memory resource too busy or too full to handle legitimate requests, thus denying legitimate user access to a machine. The difference between a **Denial of Service (DoS) attack** and a **Distributed Denial of Service (DDoS) attack** is the source of attack. In the first attack (DoS) the attack initiated by only one node. On the other hand, in DDoS attack there is the engagement of a multitude of nodes (intentionally or not, e.g. via Botnets).

Thus, during this kind of attack the following general form of cyber-effect applies:

$$\boldsymbol{g} = \boldsymbol{g_t}: \boldsymbol{\mathcal{Q}}_9^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t}) \to \boldsymbol{\mathcal{P}}_9^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t'})$$

where $\boldsymbol{\mathcal{Q}}_9^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t'})$ **and** $\boldsymbol{\mathcal{P}}_9^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t'})$ are the combinatorial triplets

$$\boldsymbol{\mathcal{Q}}_9^{(\boldsymbol{V})}(\boldsymbol{U}) = \boldsymbol{\mathcal{Q}}_9^{(\boldsymbol{V})}(\boldsymbol{U})(\boldsymbol{t'}) = (\boldsymbol{\mathfrak{R}_{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{S}_U}\boldsymbol{\mathfrak{R}_{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{U}_U}\boldsymbol{\mathfrak{R}_{available}}(\boldsymbol{V}))$$

and

$$\boldsymbol{\mathcal{P}}_9^{(\boldsymbol{U})}(\boldsymbol{V})(\boldsymbol{t'}) = (\boldsymbol{\mathfrak{R}_{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{S}_U}\boldsymbol{\mathfrak{R}_{available}}(\boldsymbol{V}), \boldsymbol{\mathcal{U}_U}\boldsymbol{\mathfrak{R}_{available}}(\boldsymbol{V}))$$

respectively ([5]).

It is obvious that the purpose of this attack is for node $\boldsymbol{U}$ to keep all resources/services of node $\boldsymbol{V}$ busy in order to make them unavailable to all users that really need them.

A family of coherent interactions

$$\boldsymbol{\mathcal{F}} = \Big\{\boldsymbol{\mathcal{Z}} = \boldsymbol{\mathcal{Z}}_{(Y,X)}(\boldsymbol{t}) = \big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2), (\mathbb{z}_1', \mathbb{w}_1'), (\mathbb{z}_2', \mathbb{w}_2')\big)(\boldsymbol{t}) \in \big(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\big)^{4}, \boldsymbol{t} \in \mathbb{I}\Big\},$$

lying in the partial danger sector $\mathcal{E} = \mathcal{E}_{U\to V}$ to the node $\boldsymbol{V}$ from the node $\boldsymbol{U}$ during the entire time set $\mathbb{I}$, is a **germ of DoS attack against the** $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu}) -$ $\boldsymbol{fr}\Big(\boldsymbol{dev}_{\mu_2}^{(V)}\Big)$**,…,** $\boldsymbol{fr}\Big(\boldsymbol{dev}_{\mu_\nu}^{(V)}\Big)$ **resource parts** $\boldsymbol{fr}\Big(\boldsymbol{res}_{\kappa_1}^{(V)}\Big)$**,** $\boldsymbol{fr}\Big(\boldsymbol{res}_{\kappa_2}^{(V)}\Big)$**,…,** $\boldsymbol{fr}\Big(\boldsymbol{res}_{\kappa_\lambda}^{(V)}\Big)$ **of** $\boldsymbol{V}$ during a given time set $\mathbb{I} \subset\subset [\mathbf{0}, \mathbf{1}]$, if, whenever $\boldsymbol{t} \in \mathbb{I}$, the pair $\big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\big) \in \big(\mathbb{C}^{\mathfrak{n}\times\boldsymbol{k}} \times \mathbb{C}^{\mathfrak{m}\times\boldsymbol{k}}\big)^{2}$ of supervisory constituents perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ in the system of nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ has the form

$$\big((\mathbb{z}_1, \mathbb{w}_1), (\mathbb{z}_2, \mathbb{w}_2)\big) =$$

$$\left(\left(\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{a}_{\mathcal{M}_V+1,1}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & & \boldsymbol{a}_{\mathcal{M}_V+1,\mathfrak{n}}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+1,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \vdots & & \vdots \\ \boldsymbol{a}_{\mathcal{M}_V+\ell_V,1}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & & \boldsymbol{a}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_V+\ell_V,\mathfrak{n}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.$$

$$\left.\left.\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{b}_{\mathcal{M}_V+1,1}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+1,1}^{(V\rightsquigarrow V)} & & \boldsymbol{b}_{\mathcal{M}_V+1,\mathfrak{m}}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+1,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \vdots & & \vdots \\ \boldsymbol{b}_{\mathcal{M}_V+\ell_V,1}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\ell_V,1}^{(V\rightsquigarrow V)} & & \boldsymbol{b}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(U\rightsquigarrow V)} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}_{\mathcal{M}_V+\ell_V,\mathfrak{m}}^{(V\rightsquigarrow V)} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right),$$

$$\left(\begin{pmatrix} \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \boldsymbol{a}_{\mathcal{M}_U+1,1}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_U+1,1}^{(U\rightsquigarrow U)} & & \boldsymbol{a}_{\mathcal{M}_U+1,\mathfrak{n}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_U+1,\mathfrak{n}}^{(U\rightsquigarrow U)} \\ \vdots & & \vdots \\ \boldsymbol{a}_{\mathcal{M}_U+\ell_U,1}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_U+\ell_U,1}^{(U\rightsquigarrow U)} & & \boldsymbol{a}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}^{(V\rightsquigarrow U)} + \boldsymbol{i}\ \widehat{\boldsymbol{a}}_{\mathcal{M}_U+\ell_U,\mathfrak{n}}^{(U\rightsquigarrow U)} \\ \mathbf{0} & \dots\dots\dots & \mathbf{0} \\ \vdots & & \vdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.$$

$$\left.\left.\left.\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} + i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} \\ \cdots & & \cdots \\ b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & b^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} + i\ \hat{b}^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right)\right)$$

and is depicted, at a next moment $t' = t + \Delta t$, at a pair $\big((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big) \in \left(\mathbb{C}^{\mathfrak{n}\times\mathscr{k}} \times \mathbb{C}^{\mathfrak{m}\times\mathscr{k}}\right)^2$ of supervisory resource perceptions of $\boldsymbol{U}$ and $\boldsymbol{V}$ having the form

$$\big((\mathbb{z}'_1, \mathbb{w}'_1), (\mathbb{z}'_2, \mathbb{w}'_2)\big) =$$

$$\left(\left(\left(\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \widehat{a}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} = \mathbf{0} & & a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} + i\ \widehat{a}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{n}} = \mathbf{0} \\ \cdots & & \cdots \\ a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \widehat{a}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} = \mathbf{0} & & a'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} + i\ \widehat{a}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{n}} = \mathbf{0} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.\right.$$

$$\left.\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,1} + i\ \widehat{b}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,1} = \mathbf{1} & & b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} + i\ \widehat{b}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+1,\mathfrak{m}} = \mathbf{1} \\ \cdots & & \cdots \\ b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} + i\ \widehat{b}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,1} = \mathbf{1} & & b'^{(U\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} + i\ \widehat{b}'^{(V\rightsquigarrow V)}_{\mathcal{M}_V+\ell_V,\mathfrak{m}} = \mathbf{1} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix}\right)\right),$$

$$\left(\left(\begin{pmatrix} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + i\ \widehat{a}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} + i\ \widehat{a}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{n}} \\ \cdots & & \cdots \\ a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + i\ \widehat{a}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & a'^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} + i\ \widehat{a}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{n}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{pmatrix},\right.\right.$$

$$\left(\begin{array}{ccc} \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+1,\mathfrak{m}} \\ \cdots & & \cdots \\ \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,1} & & \boldsymbol{b'}^{(V\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} + \boldsymbol{i}\ \widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}_{\mathcal{M}_U+\ell_U,\mathfrak{m}} \\ \mathbf{0} & \cdots\cdots\cdots & \mathbf{0} \\ \cdots & & \cdots \\ \mathbf{0} & & \mathbf{0} \end{array}\right)\Bigg)\Bigg)\Bigg).$$

During this attack the results depicted in previous matrices are usually temporary and only strictly during the application of the attack. Most of the times the sophistication of this attack is very low and highly “transparent” to attacked node since the lack of resources is more than obvious. Frequently, after or during this attack a more sophisticated attack is expected. Specifically, during DoS and DDoS attacks the following states applied:

$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}), \widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t})$
$\boldsymbol{\varphi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) < \mathbf{0}$	$\boldsymbol{\psi}^{(U\rightsquigarrow V)}(\boldsymbol{t}) > \mathbf{0}$
$\widehat{\boldsymbol{\varphi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) < \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(V\rightsquigarrow V)}(\boldsymbol{t}) > \mathbf{0}$
$\boldsymbol{\varphi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) > \mathbf{0}$	$\boldsymbol{\psi}^{(V\rightsquigarrow U)}(\boldsymbol{t}) < \mathbf{0}$
$\widehat{\boldsymbol{\varphi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) > \mathbf{0}$	$\widehat{\boldsymbol{\psi}}^{(U\rightsquigarrow U)}(\boldsymbol{t}) < \mathbf{0}$

Proposition 6.5 It is obvious that during a DoS and DDoS attack $\boldsymbol{\mathcal{F}}$ from $\boldsymbol{U}$ against the $(\boldsymbol{\mu_1}, \dots, \boldsymbol{\mu_\nu}) -$ resource parts $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\mu_1}}\right)$, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\mu_2}}\right)$,…, $\boldsymbol{fr}\left(\boldsymbol{res}^{(\boldsymbol{V})}_{\boldsymbol{\mu_\nu}}\right)$ of $\boldsymbol{V}$, the following elementary properties hold:

i. The (Euclidean) norm $\parallel \boldsymbol{a}'^{(U\rightsquigarrow V)} \parallel$ of the resulting overall valuation in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t'}$ is temporary $\mathbf{0}$:

$$\parallel \boldsymbol{a}'^{(U\rightsquigarrow V)} \parallel = \mathbf{0}.$$

ii. The (Euclidean) norm $\parallel \boldsymbol{b}'^{(U\rightsquigarrow V)} \parallel$ of the resulting overall vulnerability in the node $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t'}$ is temporary $\mathbf{1}$:

$$\parallel \boldsymbol{b}'^{(U\rightsquigarrow V)} \parallel = \mathbf{1}.$$

iii. The (Euclidean) norm $\left\|\widehat{\boldsymbol{a}}'^{(U\rightsquigarrow U)}\right\|$ of the resulting overall valuation in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is greater than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{a}}^{(U\rightsquigarrow U)}\right\| \text{ and } \left\|\boldsymbol{a}^{(U\rightsquigarrow V)}\right\|$$

of the initial overall valuations in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

$$\left\|\widehat{\boldsymbol{\beta}}'^{(U\rightsquigarrow U)}\right\| > \boldsymbol{max}\left\{\left\|\widehat{\boldsymbol{\beta}}^{(U\rightsquigarrow U)}\right\|, \left\|\boldsymbol{\beta}^{(U\rightsquigarrow V)}\right\|\right\}.$$

iv. The (Euclidean) norm $\left\|\widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}\right\|$ of the resulting overall vulnerability in the variant node $\boldsymbol{U}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the next moment $\boldsymbol{t}'$ is less or equal than the (Euclidean) norms

$$\left\|\widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}\right\| \text{ and } \left\|\boldsymbol{b}^{(U\rightsquigarrow V)}\right\|$$

of the initial overall vulnerabilities in the nodes $\boldsymbol{U}$ and $\boldsymbol{V}$ as evaluated from the viewpoint of the user(s) of $\boldsymbol{U}$ at the preceding moment $\boldsymbol{t}$:

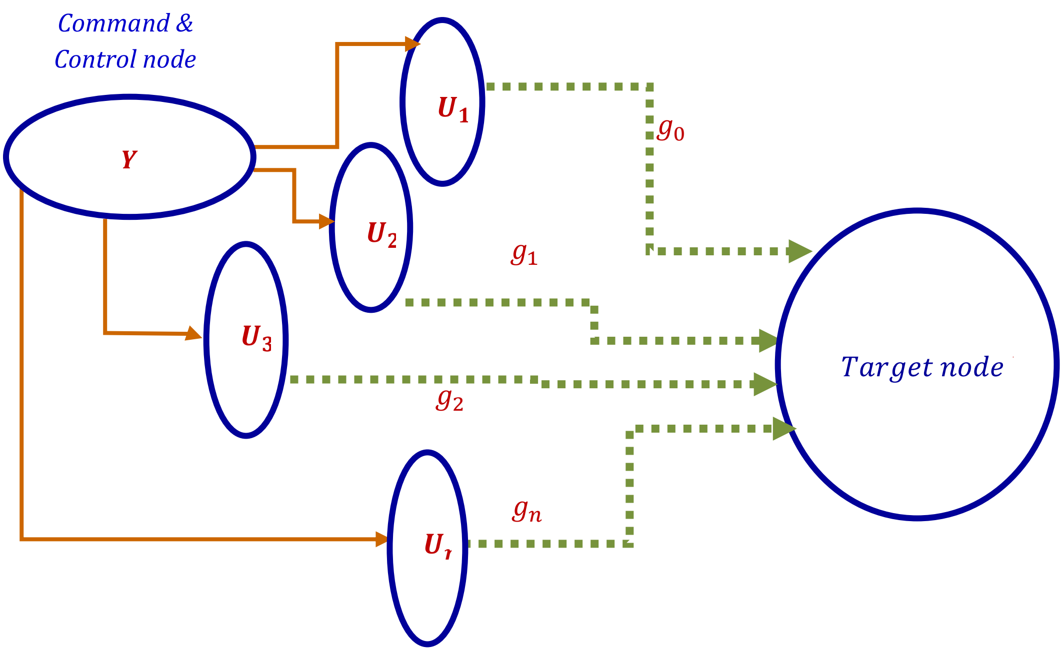
$$\left\|\widehat{\boldsymbol{b}}'^{(U\rightsquigarrow U)}\right\| \leq \boldsymbol{min}\left\{\left\|\widehat{\boldsymbol{b}}^{(U\rightsquigarrow U)}\right\|, \left\|\boldsymbol{b}^{(U\rightsquigarrow V)}\right\|\right\}. \blacksquare$$

The importance of this attack is high since most of the time, especially during DDoS attack, the nodes that participate are already compromised via Access attack that has already discussed.

Accordingly, in DDoS attack, since the attack is being generated by a multitude of already compromised nodes

$$\boldsymbol{U_1}, \boldsymbol{U_2}, \boldsymbol{U_3}, \dots, \boldsymbol{U_n}$$

that compose a botnet, the visualization of this attack can be the following:



In addition and actually in reality, the geographical distribution of $\boldsymbol{U_1}, \boldsymbol{U_2}, \boldsymbol{U_3}, \ldots$ $\ldots, \boldsymbol{U_n}$ is spread evenly. The controller of a botnet (Command and Control node) is able to direct the activities of these compromised computers through e-flows in order to conduct a DDoS attack.

7 General Thoughts

7.1. Internet of Things

Process for devices/entities/services to be connected to cyber-space as $\boldsymbol{ob}\big(\boldsymbol{cy}(\boldsymbol{t_0})\big)$ is gaining high momentum. This whole concept that called **Internet of Things (IoT)** contributes to make our environment looking smart. The IoT is very simple to describe: it's about putting the internet in things/objects which can sense, send/receive signals and capture data. Fridges, smoke alarms, televisions, cars, smartphones and many such products are getting wirelessly connected to the internet and to other devices easily. It is simply an advancement of human vision to stay connected and use technology as a platform to enjoy not only new features and items available to them but also use the same sources to make informed decisions that would better serve their preferences.

We can imagine that everything can be a potential $\boldsymbol{ob}\big(\boldsymbol{cy}(\boldsymbol{t_0})\big)$ or better, in any node $\boldsymbol{V}$ the possible constituents $\boldsymbol{k} \coloneqq \boldsymbol{\mathcal{M}_V} + \boldsymbol{\mathcal{L}_V}$ can be as high as never before IoT era.

It is reasonable that the most critical aspects to the IoT architecture and design are interoperability, compatibility, load balancing, consistency, bandwidth optimization, minimization of information storage and retrieval delay while keeping the cost low, provision for two or more levels of access control and authorization checks, high availability, and multi-protocol support.

7.2. Offensive Defense

A way to proactively defend a node is to develop an offensive strategy. “Returning” a reflexive homomorphism back to the attacker before a parallax homomorphism occurs will be ideal. This can be done before any degradation of target valuation resources.

7.3. Innovative Worms/Viruses/Trojans

Innovative worms/viruses/Trojans (non signature-based malicious software) can be presented by $\mathcal{L}_{V+1}, \mathcal{L}_{V+2}, \ldots, \mathcal{L}_{V+\lambda}$, where λ is the number of malicious software embedded in a V node.

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