Theoretical Mathematics & Applications, vol.3, no.1, 2013, 161-181
ISSN: 1792- 9687 (print), 1792-9709 (online)
Scienpress Ltd, 2013

Isomorphic Properties of Highly Irregular Fuzzy Graph and Its Complement

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Abstract

This paper discusses some properties on isomorphism, weak isomorphism and co-weak isomorphism between highly irregular fuzzy graphs and its complement. Isomorphic properties of $\mu -$ complement, self ($\mu -$) complement and self weak ($\mu -$) complement of highly irregular fuzzy graph are established. Also, isomorphic properties of busy nodes and free nodes in highly irregular fuzzy graph are discussed.

Mathematics Subject Classification: 03E72, 05C72

Keywords: Highly irregular fuzzy graph, complement, $\mu -$complement, isomorphism

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Article Info: *Received* : December 2, 2012. *Revised* : February 12, 2013
Published online : April 15, 2013

1 Introduction

In 1965, Lofti A. Zadeh [14] introduced the notion of a fuzzy subset of a set as a method for representing the phenomena of uncertainty in real life situation. Based on the definitions of fuzzy sets and fuzzy relations, ten years later Azriel Rosenfeld [10] defined fuzzy graph and developed the theory of fuzzy graph in 1975. Yousef Alavi etl., [11] introduced highly irregular fuzzy graph and examined several problems relating to the existence and enumeration of highly irregular graphs. Youesf Alavi etl., [12] extended the concept of highly irregular graphs by introducing k-path irregular graph. Nagoor Gani and Latha [5] defined highly irregular fuzzy graph and discussed some properties on it. The concept of weak isomorphism, co-weak isomorphism and isomorphism between fuzzy graphs was introduced by K.R.Bhutani [2]. Isomorphic properties of highly irregular fuzzy graphs were discussed by Nagoor Gani and Latha [6]. In this paper, some properties on isomorphism of highly irregular fuzzy graph and its complement are established. Also, some isomorphic properties of highly irregular fuzzy graph in connection with $\mu -$ complement of a fuzzy graph introduced by Nagoor Gani and Chandrasekaran [4] are studied. Following definitions are from [1 – 14].

2 Preliminaries

Definition 2.1 A fuzzy subset of a nonempty set S is a mapping σ: S→[0,1]. A fuzzy relation on S is a fuzzy subset of SxS. If μ and ν are fuzzy relations, then $\mu o \nu(u,w) = \text{Sup}\{ \mu(u,v) \Lambda\ \nu(v, w) : v \in S\}$ and
$\mu^k(u,v) = \text{Sup}\{\mu(u ,u_1)\Lambda\nu(u_1 ,u_2)\Lambda\mu(u_2 ,u_3)\ \Lambda\ldots\Lambda\mu(u_{k-1} ,v) : u_1 , u_2 ,\ldots u_{k-1} \in S\}$, where 'Λ' stands for minimum.

Definition 2.2 The underlying crisp graph of a fuzzy graph G = (σ,μ) is denoted by $G^* = (\sigma^*,\mu^*)$, where $\sigma^* = \{ u \in V / \sigma(u) > 0 \}$ and

$\mu^* = \{ (u,v) \in VxV / \mu(u,v) > 0 \}$.

Definition 2.3 Let G = (σ,μ) be a fuzzy graph. The degree of a vertex u is $d(u) = \sum_{u \neq v} \mu(u,v)$.

Definition 2.4 The order of a fuzzy graph G is $O(G) = \sum_{u \in V} \sigma(u)$. The size of a fuzzy graph G is $S(G) = \sum_{u \in V} \mu(u,v)$.

Definition 2.5 A graph G is called regular if every vertex is adjacent only to vertices having the same degree.

Definition 2.6 A graph G is called irregular, if there is a vertex which is adjacent to atleast one vertex with distinct degree.

Definition 2.7 A connected graph G is said to be highly irregular if every vertex of G is adjacent only to vertices with distinct degrees.

Definition 2.8 Let G = (σ,μ) be a connected fuzzy graph. G is said to be a highly irregular fuzzy graph if every vertex of G is adjacent to vertices with distinct degrees.

Definition 2.9 The complement of a fuzzy graph $G: (\sigma, \mu)$ is a fuzzy graph $\overline{G}: (\sigma, \overline{\mu})$, where $\overline{\sigma} = \sigma$ and $\overline{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v)\ \forall\, u, v \in V$.

3 Isomorphic properties of highly irregular fuzzy graph and its complement

Definition 3.1 [6] A homomorphism of highly irregular fuzzy graphs G and G',

$h: G \rightarrow G'$ is a map $h : V \rightarrow V'$such that $h(u) = u'$, , which satisfies

$(i)\ \sigma(u) \leq \sigma'((h(u))\ \forall\, u \in V$ *and* $(ii)\ \mu(u,v) \leq \mu'(h(u),h(v))\ \forall\, u,v \in V.$

Definition 3.2 [6] A weak isomorphism of highly irregular fuzzy graphs $G$and G', $h: G \rightarrow G'$ is a map $h: V \rightarrow V'$ such that $h(u) = u', \forall\, u \in V$, which is a bijective homomorphism that satisfies $\sigma(u) = \sigma'((h(u))\ \forall\, u \in V.$

Definition 3.3 [6] A co- weak isomorphism of highly irregular fuzzy graphs G and G', $h : G \rightarrow G'$ is a map $h : V \rightarrow V'$ such that $h(u) = u', \forall\, u \in V$, which is a bijective homomorphism that satisfies $\mu\ (u\ , v) = \mu'\big(h(u), h(v)\big)\ \forall\, u, v \in V.$

Definition 3.4 [6] An isomorphism of highly irregular fuzzy graphs $h : G \rightarrow G'$ is a map $h : V \rightarrow V'$ which is bijective that satisfies
$(i)\ \sigma(u) = \sigma'((h(u))\ \forall\, u \in V,$ and
$(ii)\ \mu(u,v) = \mu'(h(u),h(v))\ \forall\, u,v \in V.$
It is denoted by $G \cong G'$.

Theorem 3.5 The complement of highly irregular fuzzy graph need not be highly irregular.

Proof: To every vertex, the adjacent vertices with distinct degrees or the non–adjacent vertices with distinct or same degrees may happen to be adjacent vertices with same degrees. This contradicts the definition of highly irregular fuzzy graph.
The following example illustrates that the complement of highly irregular fuzzy graph need not be highly irregular.

Example 3.6

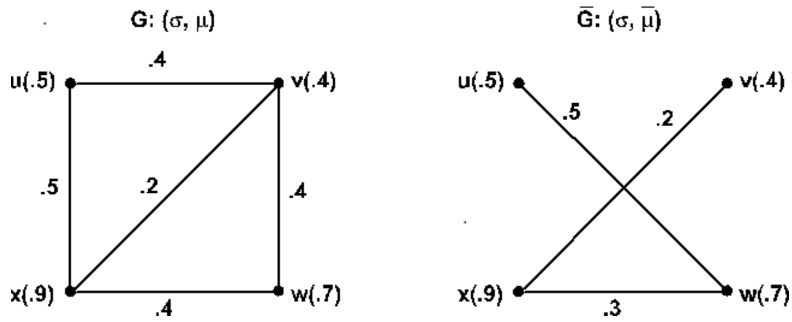


Figure 1

From the Figure 1, we see that $\overline{G}$ is not highly irregular because the degrees of the adjacent vertices of w are the same.

Theorem 3.7 Let $G:(\sigma,\mu)$ and $G':(\sigma',\mu')$ be two highly irregular fuzzy graphs. G and G' are isomorphic if and only if, their complements are isomorphic, but the complements need not be highly irregular.

Proof: Assume $G \cong G'$

$\Rightarrow$ There exists a bijective map $h: V \to V'$ such that $h(u) = u'$ for all $u \in V$, satisfying

$$\sigma(u) = \sigma'(h(u)) \tag{3.1}$$

and $\mu(u,v) = \mu'(h(u),h(v))$

By definition of complement of a fuzzy graph, $\overline{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v)\ \forall\ u,v \in V$

$$\overline{\mu}(u,v) = \sigma'\big(h(u)\big) \wedge \sigma'\big(h(v)\big) - \mu'\big(h(u),h(v)\big)\ \forall\ u,v \in V$$

$$= \overline{\mu'}\big(h(u),h(v)\big)\ \forall\ u,v \in V \tag{3.2}$$

(3.1) and (3.2) $\Rightarrow$ $\overline{G} \cong \overline{G'}$

Conversely, assume that $\overline{G} \cong \overline{G'}$.

(i.e) There is a bijective map $g: V \to V'$ such that $g(u) = u'$ for all u ∈ V satisfying

$\sigma(u) = \sigma'\big(g(u)\big) \forall\, u \in V$ (3.3)

and $\overline{\mu}(u, v) = \overline{\mu'}\big(g(u), g(v)\big) \,\forall\, u, v \in V$

Using the definition of complement of fuzzy graph,

$\sigma(u) \wedge \sigma(v) - \mu(u, v) = \sigma'\big(g(u)\big) \wedge \sigma'\big(g(v)\big) - \mu'(g(u), g(v))$

$\Rightarrow \mu(u, v) = \mu'\big(g(u), g(v)\big) \,\forall\, u, v \in V$ (3.4)

From (3.3) and (3.4), it is proved that $G \cong G'$.

The following example illustrates the above theorem.

Example 3.8 Let $G{:}\,(\sigma, \mu)$ and $G'{:}\,(\sigma', \mu')$ be two highly irregular fuzzy graphs defined by $\sigma(u) = 0.6 = \sigma'(u'), \sigma(v) = 0.8 = \sigma'(v'), \sigma(w) = 1 = \sigma'(w'), \sigma(x) = 1 = \sigma'(x'), \mu(u, v) = 0.6 = \mu'(u', v'), \mu(v, w) = 0.6 = \mu'(v', w'), \mu(x, w) = 0.5 = \mu'(x', w'), \mu(u, x) = 0.6 = \mu'(u', x')$ The complements are defined by $\overline{\mu}(u, w) = 0.6 = \overline{\mu'}(u', w'), \overline{\mu}(v, x) = 0.8 = \overline{\mu'}(v', x'), \overline{\mu}(v, w) = 0.2 = \overline{\mu'}(v', w'), \overline{\mu}(x, w) = 0.5 = \overline{\mu'}(x', w')'$.In this example, $G \cong G'$ and $\overline{G} \cong \overline{G'}$, but the complements are not highly irregular fuzzy graph.

Theorem 3.9 Let G and G' be two highly irregular fuzzy graphs. If G is weak isomorphic with G', then $\overline{G'}$ is weak isomorphic with $\overline{G}$, but the complements need not be highly irregular.

Proof: Assume that G is weak isomorphic with G'. Then there is a bijective map $h{:}\, V \to V'$ such that $h(u) = u' \,\forall\, u \in V$ satisfying ,

$\sigma(u) = \sigma'\big(h(u)\big) \forall u \in V$ (3.5)

and

$\mu(u, v) \leq \mu'\big(h(u), h(v)\big) \forall\, u, v \in V$ (3.6)

Since $h{:}\, V \to V'$ is bijective, $h^{-1}{:}\, V' \to V$ exists for all $u' \in V'$.

$\Rightarrow$ There is an $u' \in V'$ such that

$$h^{-1}(u') = u \tag{3.7}$$

Using (3.7) in (3.5),

$$\sigma\left(h^{-1}(u')\right) = \sigma'(u')\ \forall\, u' \in V' \tag{3.8}$$

Using the definition of complement of a fuzzy graph,

$$\overline{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v)\ \ \forall u,v \in V \tag{3.9}$$

we have

$$\overline{\mu}\left(h^{-1}(u'), h^{-1}(v')\right) = \sigma\left(h^{-1}(u')\right) \wedge \sigma\left(h^{-1}(v')\right) - \mu((h^{-1}(u'), (h^{-1}(v'))$$
$$= \sigma'(u') \wedge \sigma'(v') - \mu(u,v)$$
$$\geq \sigma'(u') \wedge \sigma'(v') - \mu'(u',v')\ \ \forall\ u',v' \in V'$$

(i.e)

$$\overline{\mu'}(u',v') \leq \overline{\mu}\left(h^{-1}(u'), h^{-1}(v')\right)\ \forall\ u',v' \in V' \tag{3.10}$$

From (3.5) and (3.10) it is proved that, $\overline{G'}$ is weak isomorphic with $\overline{G}$.

Example 3.10 Let $G: (\sigma, \mu)$ and $G': (\sigma', \mu'')$ be two highly irregular fuzzy graphs defined by

$\sigma(u) = 0.8. \sigma(v) = 0.9, \sigma(w) = 1, \sigma(x) = 0.9, \mu(u,v) = 0.4, \mu(v,w) = 0.5, \mu(x,w) = 0.4, \mu(u,x) = 0.8, \sigma'(u') = 0.6, \sigma'(v') = 0.9, \sigma'(w') = 1, \sigma'(x') = 0.9, \mu'(u',v') = 0.5, \mu'(v',w') = 0.6, \mu'(x',w') = 0.5, \mu'(u',x') = 0.8.$

The complements of G and G' are defined by $\overline{\mu}(u,v) = 0.4, \overline{\mu}(v,w) = 0.4, \overline{\mu}(u,w) = 0.8, \overline{\mu}(v,x) = 0.9, \overline{\mu}(x,w) = 0.5, \overline{\mu'}(u',v') = 0.3, \overline{\mu'}(v',w') = 0.3, \overline{\mu'}(u',w') = 0.8, \overline{\mu'}(v',x') = 0.9, \overline{\mu'}(x',w') = 0.4.$

From the example, it is seen that G is weak isomorphic with G' and $\overline{G'}$ is weak isomorphic with $\overline{G}$ but $\overline{G}$ and $\overline{G'}$ are not highly irregular.

Theorem 3.11 Let G and G' be two highly irregular fuzzy graphs. If G is a co-weak isomorphic with G', then there exists a homomorphism between $\overline{G}$ and $\overline{G'}$, but the complements need not be highly irregular.

Proof: Assume that G is co-weak isomorphic to G'. Then $h\, V: \to V'$ is a bijective map such that

$h(u) = u'$ satisfying,

$$\sigma(u) \leq \sigma'(h(u)) \; \forall\, u \in V \tag{3.11}$$

and

$$\mu(u,v) = \mu'(h(u), h(v)), \;\; \forall\, u, v \in V. \tag{3.12}$$

By definition of complement of fuzzy graph,

$$\overline{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v) \; \forall\, u, v \in V \tag{3.13}$$

we get,

$$\overline{\mu}(u,v) \leq \sigma'^{(h(u))} \wedge \sigma'^{(h(v))} - \mu'(h(u), h(v))$$

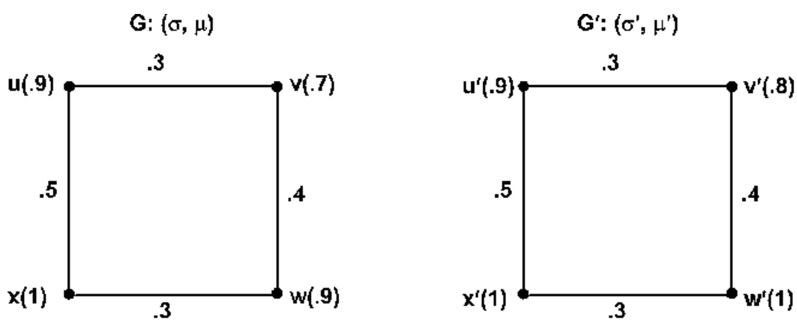
$$= \overline{\mu'}(h(u), h(v)) \; \forall\, u, v \in V \tag{3.14}$$

From (3.11) and (3.14), h is a bijective homomorphism between $\overline{G}$ and $\overline{G'}$.

Proposition 3.12 If there is a co-weak isomorphism between be two highly irregular fuzzy graphs G and G' , then there need not be a co-weak isomorphism between $\overline{G}$ and $\overline{G'}$ and the complements need not be highly irregular.

The following example illustrates the above result.

Example 3.13



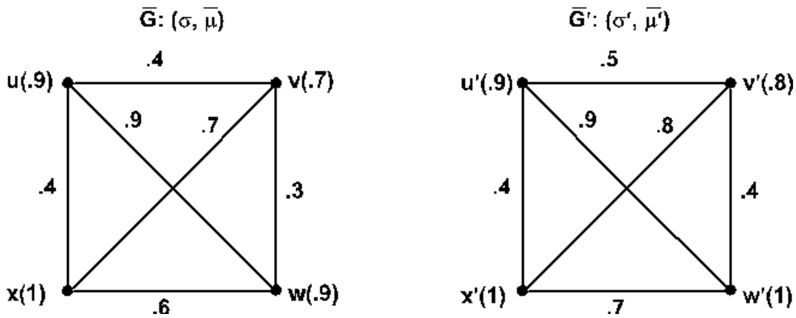


Figure 2

From the Figures 2, it is seen that G is co-weak isomorphic with $\overline{G'}$. But $\overline{G}$ is not co-weak isomorphic with $\overline{G'}$. However there is a homomorphism between $\overline{G}$ and $\overline{G'}$. Also $\overline{G}$ is not highly irregular where $\overline{G'}$ is highly irregular.

Definition 3.14 [7] A fuzzy graph G is said to be a self complementary if $G \cong \overline{G}$.

Definition 3.15 [7] A fuzzy graph G is said to be a self weak complementary if G is weak isomorphic with $\overline{G}$.

Proposition 3.16 A highly irregular fuzzy graph need not be self complementary. The proof follows from Theorem 3.5.

Example 3.17

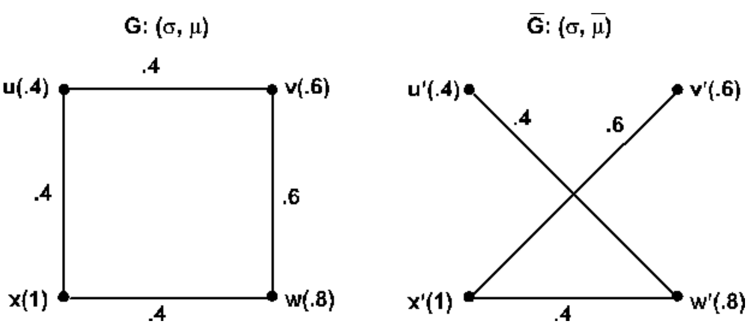


Figure 3

In this example, G is highly irregular but not a self complementary fuzzy graph. The following two examples shows that a highly irregular fuzzy graph cannot necessarily be a self weak complementary fuzzy graph.

Example 3.18

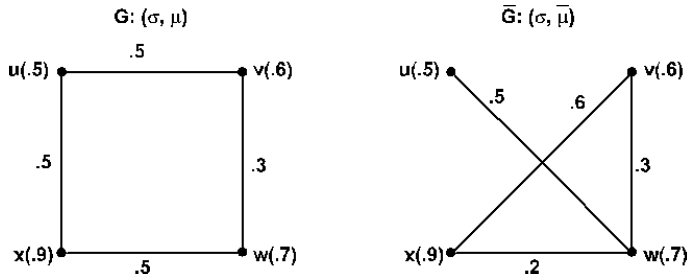


Figure 4

From the above example, we see that G and $\overline{G}$ are highly irregular, but G is not weak isomorphic with $\overline{G}$. Hence G is not a self weak complementary highly irregular fuzzy graph.

Example 3.19

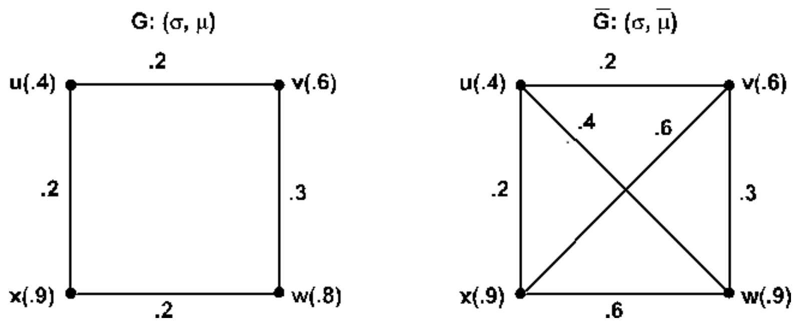


Figure 5

In this example, G and $\overline{G}$ are highly irregular and G is weak isomorphic with $\overline{G}$. Hence G is a self weak complementary highly irregular fuzzy graph.

Theorem 3.20 Let G be a self weak complementary highly irregular fuzzy graph, then $\sum_{u\neq v}\mu(u,v) \leq \frac{1}{2}\sum_{u\neq v}(\sigma(u)\wedge\sigma(v))$.

Proof: Let $G:(\sigma,\mu)$ be a self weak complementary and highly irregular fuzzy graph.

$\Rightarrow G$ is weak isomorphic with G'. Therefore, there exists a bijective map

$h: V \rightarrow V(\overline{V})$ with $h(u) = \overline{u} = u$, satisfying,

$$\sigma(u) = \overline{\sigma}\big(h(u)\big)\ \forall\, u \in V \tag{3.15}$$

and

$$\mu(u,v) \leq \overline{\mu}\big(h(u),h(v)\big)\ \forall\, u,v \in V \tag{3.16}$$

By definition of complement of a fuzzy graph,

$$\overline{\mu}(u,v) = \sigma(u)\wedge\ \sigma(v) - \mu(u,v)\forall u,v\ \in V \tag{3.17}$$

From (3.15) and (3.16),

$$\mu(u,v) \leq\ \overline{\mu}\big(h(u),h(v)\big) = \sigma\big(h(u)\big)\wedge\ \sigma\big(h(v)\big) - \mu\big(h(u),h(v)\big)\forall\, u,v\ \in V$$

$$\Rightarrow \quad \mu(u,v) \leq \sigma(u)\ \wedge\ \sigma(v) - \mu(u,v)$$

$$\Rightarrow \quad \mu(u,v) + \mu(u,v) \leq \sigma(u)\ \wedge\ \sigma(v)$$

$$2\mu(u,v) \leq \sigma(u)\ \wedge\ \sigma(v)$$

$$2\sum_{u\neq v}\mu(u,v) \leq \sum_{u\neq v}(\sigma(u)\ \wedge\ \sigma(v))$$

$$\sum_{u\neq v}\mu(u,v) \leq \frac{1}{2}\sum_{u\neq v}(\sigma(u)\ \wedge\ \sigma(v)).$$

4 Isomorphic properties of $\mu -$ complement of highly irregular fuzzy graph

Definition 4.1 [4] Let $G:(\sigma,\mu)$ be a fuzzy graph. The $\mu -$ complement of G is defined as $G^{\mu}:(\sigma,\mu^{\mu})$ where

$$\mu^{\mu}(u,v) = \begin{cases}(\sigma(u)\wedge\sigma(v) - \mu(u,v), & if\ \mu(u,v) > 0\\ 0 & ,if\ \ \mu(u,v) = 0\end{cases}$$

Theorem 4.2 The μ-complement of a highly irregular fuzzy graph need not be highly irregular.

Proof: The proof is similar to theorem 3.5

The following example illustrates the above theorem.

Example 4.3

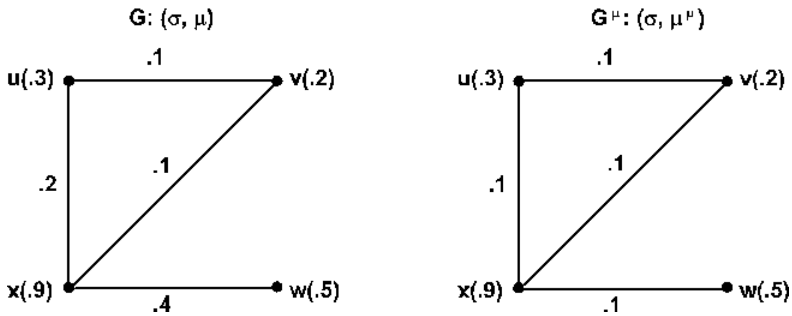


Figure 6

G is a highly irregular fuzzy graph. The degrees of the vertices in G^{μ} contradicts the definition of highly irregular fuzzy graph.

Theorem 4.4 Let G and G' be two highly irregular fuzzy graphs. If G and G' are isomorphic, then μ-complements of G and G' are also isomorphic and vice versa, but the complements need not be highly irregular.

Proof: The proof is similar to Theorem 3.7.

Remark 4.5 Let G and G' be two highly irregular fuzzy graphs. If G is weak isomorphic with G', then neither μ-complement of G is weak isomorphic with $\mu-$complement of G' nor μ-complement of G' is weak isomorphic with $\mu-$complement of G.

The following example illustrates the above result.

Example 4.6

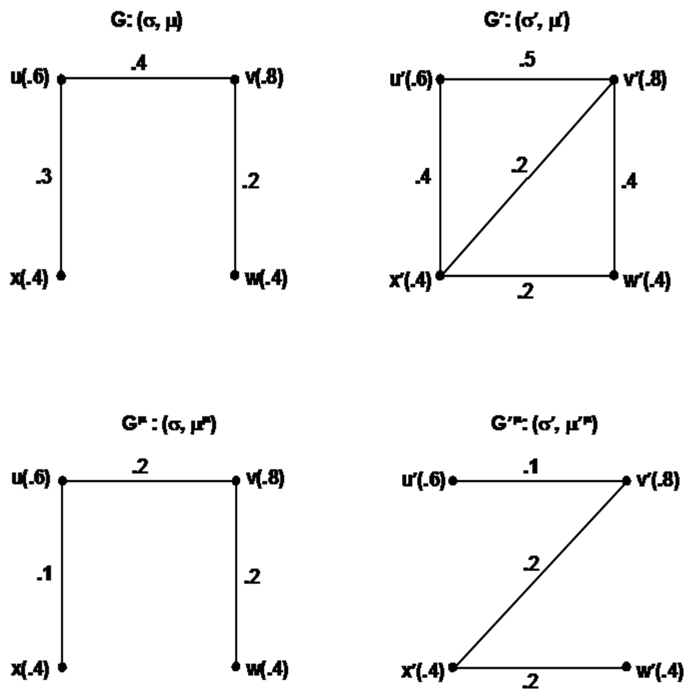


Figure 7

It is easy to observe that G and $G^{'}$ are highly irregular fuzzy graphs and G is weak isomorphic with $G^{'}$, but μ-complement of G is not weak isomorphic with $\mu -$ complement of G' and μ-complement of G' is not weak isomorphic with $\mu -$ complement of G.

Theorem 4.7 If there is a co-weak isomorphism between G and G', then μ-complement of G and G' need not be co-weak isomorphic , but there can be a homomorphism between μ-complement of G and G' .

Proof: The proof is similar to Theorem 3.11

The following example illustrates that there exists a homomorphism between μ-complement of G and G'.

Example 4.8

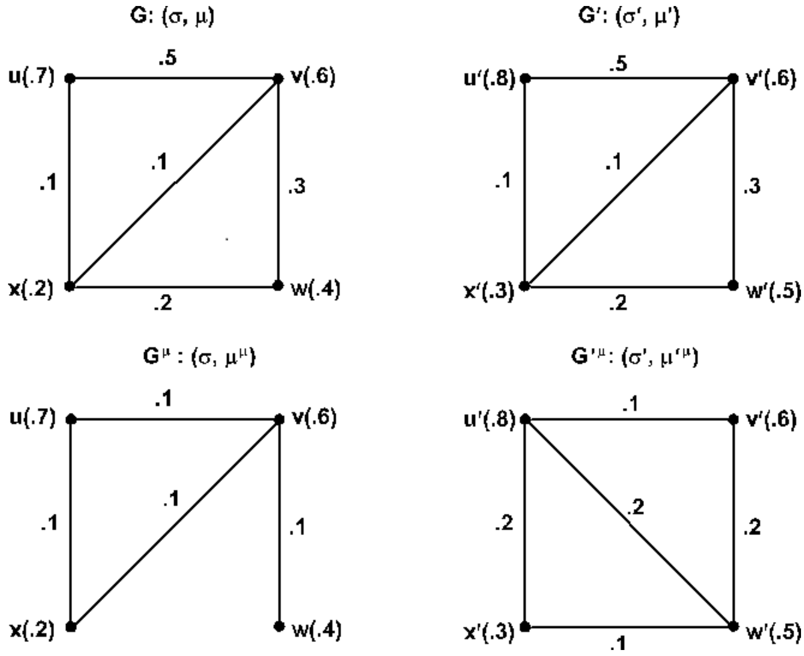


Figure 8

From the figure it is clear that μ-complement of G is not co-weak isomorphic with G', but there is a homomorphism between μ-complement of G and G'.

Definition 4.9 [8] A fuzzy graph $G: (\sigma, \mu)$ is said to be a self $\mu -$ complementary fuzzy graph if $G \cong G^{\mu}$.

Example 4.10

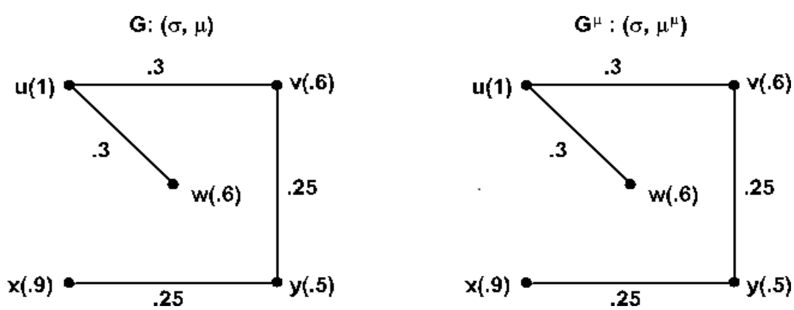


Figure 9

Clearly G and G^{μ} are highly irregular fuzzy graphs and G is isomorphic with G^{μ}. Therefore G is self $\mu -$ complementary fuzzy graph.

Theorem 4.11 Let G be a highly irregular and self $\mu -$ complementary fuzzy graph, then $\sum_{u\neq v} \mu(u,v) = \frac{1}{2}\sum_{u\neq v}[\sigma(u) \wedge \sigma(v)]$.

Proof: Let $G:(\sigma,\mu)$ be a self $\mu -$ complementary highly irregular fuzzy graph.

$\Rightarrow$ $G \cong G^{\mu}$. $\Rightarrow$ There exists a bijective map $h : V \to V$ such that

$$\sigma(u) = \sigma^{\mu}\big(h(u)\big) = \sigma\big(h(u)\big)\ \forall\ u \in V \qquad (4.1)$$

and

$$\mu(u,v) = \mu^{\mu}\big(h(u),h(v)\big)\ \forall\ u,v\ \in V \qquad (4.2)$$

If $(u,v) \in \mu^*$, then $\mu(u,v) > 0$. By the definition of $\mu -$ complement of a fuzzy graph

$$\mu^{\mu}(u,v) = \begin{cases}(\sigma(u) \wedge \sigma(v) - \mu(u,v), & if\ \mu(u,v) > 0\\ 0 & ,if\ \ \mu(u,v) = 0\end{cases}$$

Therefore, $\mu^{\mu}\big(h(u),h(v)\big) = \sigma^{\mu}\big(h(u)\big) \wedge \sigma^{\mu}\big(h(u)\big) - \mu(h(u),h(v))$

$\mu(u,v) = \sigma(h(u) \wedge \sigma\big(h(v)\big) - \mu(h(u),h(v))$ (from (4.1) to (4.2))

$$\mu(u,v) + \mu\big(h(u),h(v)\big) = \sigma(u) \wedge \sigma(v)$$

$2\mu(u,v) = \sigma(u) \wedge \sigma(v)$. Taking summation,

$2\sum_{u\neq v}\mu(u,v) = \sum_{u\neq v}\sigma(u) \wedge \sigma(v) \Rightarrow \sum_{u\neq v}\mu(u,v) = \frac{1}{2}\sum_{u\neq v}[\sigma(u) \wedge \sigma(v)]$.

The following example illustrates the above theorem.

Example 4.12 From Example 4.10 we have

$$\sum_{u\neq v}\mu(u,v) = S(G) = .3 + .3 + .25 + .25 = 1.1$$

$$\frac{1}{2}\sum_{u\neq v}\sigma(u) \wedge \sigma(v) = \frac{1}{2}[.6 + .5 + .5 + .6] = \frac{2.2}{2} = 1.1$$

Thus,

$\sum_{u\neq v}\mu(u,v) = \frac{1}{2}\sum_{u\neq v}[\sigma(u) \wedge \sigma(v)]$.

The following example illustarates that, in a highly irregular fuzzy graph, even if

$$\sum_{u \neq v} \mu(u, v) = \frac{1}{2} \sum_{u \neq v} \sigma(u) \wedge \sigma(v),$$

G need not be self $\mu -$ complementary.

Example 4.13

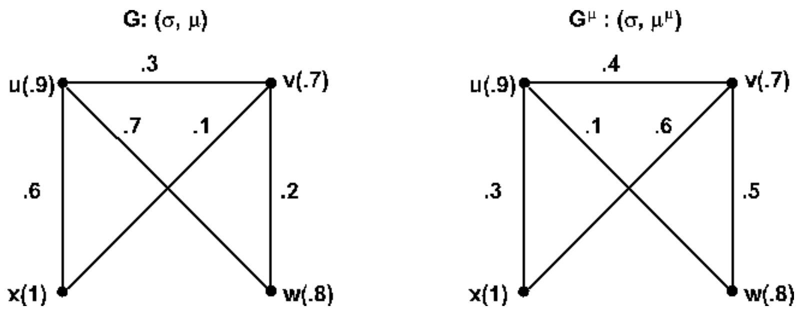


Figure 10

Here G and G^{μ} are highly irregular fuzzy graphs, but G is not self $\mu -$ complement.

Definition 4.14 [8] A fuzzy graph $G: (\sigma, \mu)$ is said to be a self weak $\mu -$ complementary fuzzy graph if G is weak isomorphic with G^{μ}.

Example 4.15

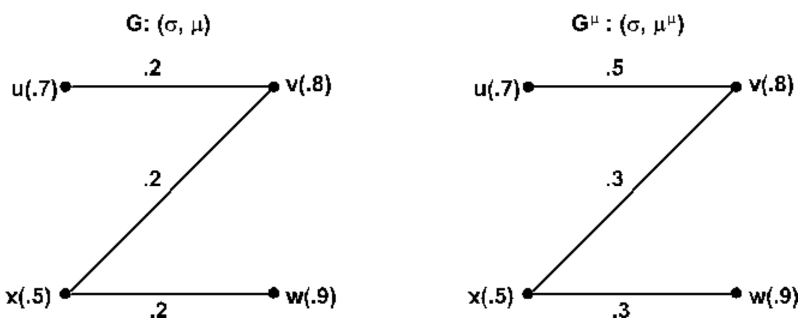


Figure 11

Here G and G^{μ} are highly irregular fuzzy graphs and also G is weak isomorphic with G^{μ}. Thus, G is a self weak $\mu -$ complementary fuzzy graph.

Theorem 4.16 Let G be highly irregular and self weak $\mu -$ complementary fuzzy graph, then $\sum_{u \neq v} \mu(u,v) \leq \frac{1}{2} \sum_{u \neq v} \sigma(u) \wedge \sigma(v)$.

Proof: Using the definition of self weak $\mu -$ complementary fuzzy graph, the proof is similar to Theorem 4.11.

5 Isomorphic properties of busy nodes and free nodes in highly irregular fuzzy graph

Definition 5.1[4] A node "u" in a fuzzy graph is said to be a busy node if $\sigma(u) \leq d(u)$. Otherwise it is called as a free node.

Theorem 5.2 If $G: (\sigma, \mu)$ and $G': (\sigma', \mu')$ are two isomorphic highly irregular fuzzy graphs, then the busy nodes and free nodes are preserved under isomorphism.

Proof: Let $G: (\sigma, \mu)$ and $G': (\sigma', \mu')$ be two highly irregular fuzzy graphs and let $h : G \to G'$ be an isomorphism between the highly irregular fuzzy graphs G and G' with the underlying sets V and V' respectively.

Then $\sigma(u) = \sigma'(h(u))$ for all $u \in V$ and $\mu(u,v) = \mu'(h(u), h(v))$ for all $u, v \in V$.

By theorem 3.22 [6], the bijective mapping h preserves the degree of the vertex u.

(i.e) $d(u) = d(h(u))$.

If u is a busy node in G, then $\sigma(u) \leq d(u) \Rightarrow \sigma'(h(u)) \leq d(h(u))$.

Thus $h(u)$ is a busy node in G'.

If v is a free node in G, then $\sigma(v) > d(v) \Rightarrow \sigma'(h(v)) > d(h(v))$.

Hence $h(v)$ is a free node in G'.

Theorem 5.3 Let $G:(\sigma,\mu)$ and $G':(\sigma',\mu')$ be two highly irregular fuzzy graphs and let G be co-weak isomorphic with G'. Then the image of a free node in G is also a free node in G'.

Proof: Let u be a free node in $G \Rightarrow \sigma(u) > d(u)$ (5.1)

Let $h: G \to G'$ be a co- weak isomorphism between G and G'.

Then $\sigma(u) \leq \sigma'(h(u)) \;\forall\, u \in V'$ (5.2)

and $\mu(u,v) = \mu'(h(u),h(v)) \;\forall\, u,v \in V$ (5.3)

From (5.1) and (5.2), $\sigma'(h(u)) \geq \sigma(u) > d(u)$

Hence $\sigma'(h(u)) > d(u) = \sum_{u\neq v}\mu(u,v) = \sum_{u\neq v}\mu'(h(u),h(v)) = d(h(u))$

Therefore, $\sigma'(h(u)) > d(h(u)) \Rightarrow h(u)$ is a free node in G'.

Remark 5.4 Let $G:(\sigma,\mu)$ and $G':(\sigma',\mu')$ be two highly irregular fuzzy graphs. If G is co-weak isomorphic with G', then the image of a busy node in G need not be a busy node in G'.

The following example illustrates the above result.

Example 5.5

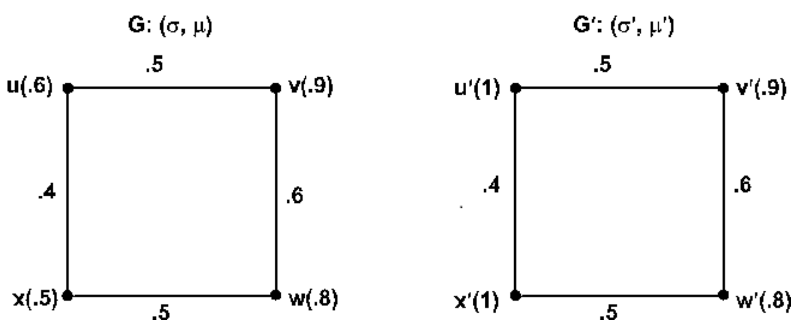


Figure 12

G is co-weak isomorphic with G'. The busy nodes u & x in G are not busy in G'.

Theorem 5.6 Let $G:(\sigma,\mu)$ and $G':(\sigma',\mu')$ be two highly irregular fuzzy graphs and let G be weak isomorphic with G'. Then the image of a busy node in G is a busy node in G'.

Proof: Let $h : G \rightarrow G'$ be a weak isomorphism between G and G'.

Then $\sigma(u) = \sigma'(h(u)) \; \forall \, u \in V$ (5.4)

and

$\mu(u,v) \leq \mu'(h(u),h(v)) \; \forall \, u,v \in V$ (5.5)

Let $u \in V$ be a busy node in G. (i.e) $\sigma(u) \leq d(u)$ (5.6)

From (5.4) and (5.6)

$$\sigma'(h(u)) = \sigma(u) \leq d(u) = \sum_{u \neq v} \mu(u,v) \leq \sum_{u \neq v} \mu'(h(u),h(v)) = d(h(u))$$

Hence $\sigma'(h(u)) \leq d(h(u)) \Rightarrow h(u)$ is a busy node in G'.

Remark 5.7 Under weak isomorphism the image of a free node in G need not be a free node in G'.

This is illustrated in the following example.

Example 5.8

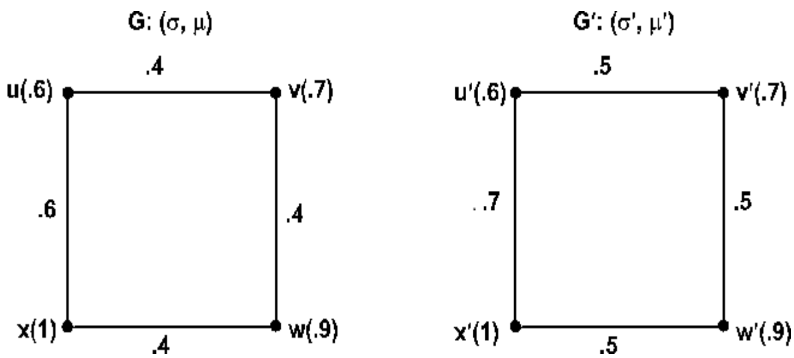


Figure 13

In this example, though w is a free node in G, w' is a busy node in G', because $\sigma'(w') \leq d(w')$.

6 Conclusion

It is proved that $\overline{G'}$ is weak isomorphic with $\overline{G}$, when G is weak isomorphic with G'. Also, existence of homomorphism between $\overline{G}$ and $\overline{G'}$ is established, when G is co-weak isomorphic with G'. Similarly, it is proved that μ-complement of G and G' need not be co-weak isomorphic , however there is a homomorphism between μ-complement of G and G' .Some results on self complementary and self weak complementary are discussed. Finally, some properties of isomorphism with respect to busy nodes and free nodes in highly irregular fuzzy graphs are established.

References

[1] P.Bhattacharya, Some Remarks on fuzzy graphs, *Pattern Recognition Letter*, **6**, (1987), 297-302.

[2] K.R.Bhutani, On Automorphism of fuzzy graphs, *Pattern Recognition Letter*, **9**, (1989), 159-162.

[3] A.Nagoor Gani and V.T.Chandrasekaran, *A First Look at Fuzzy Graph Theory*, Allied Publishers, 2010.

[4] A.Nagoor Gani and V.T.Chandrasekaran, Free nodes and busy nodes of a fuzzy graph, *East Asian Math. J.*, **22**(2), (2006), 163-170.

[5] A. Nagoor Gani and S.R. Latha, On Irregular Fuzzy Graphs, Applied Mathematical Sciences, **6**(11), (2012), 517-523.

[6] A. Nagoor Gani and S.R. Latha, Isomorphism on Irregular Fuzzy Graphs, *International J. of Math.Sci & Engg. Appls.*, (IJMSEA), ISSN 0973-9424, **6**(III), (2012).

[7] A. Nagoor Gani and J.Malarvizhi, Isomorphism on Fuzzy Graphs, *International Journal of Computational and Mathematical Sciences*, **2**(4), (2008).

[8] A. Nagoor Gani and J. Malarvizhi, Properties of $\mu -$ complement of a fuzzy graph, *International Journal of Algorithms*, *Computing and Mathematics*, **2**(3), (August, 2009).

[9] A. Nagoor Gani and K.Radha, On Regular Fuzzy Graphs, Journal of Physical Sciences, **12**, (2010), 33-40.

[10] A. Rosenfeld, *Fuzzy graphs*, in: L.A. Zadeh, K.S. Fu, K. Tanaka and M. Shimura, (eds), *Fuzzy sets and their applications to cognitive and decision process*, Academic press, New York, 75-95, 1975.

[11] Yousef Alavi, F.R.K. Chung, Paul Erdos, R.L.Graham and Ortrud R. Oellermann, Highly Irregular Graphs, *Journal of Graph Theory*, **11**(2), (1987), 235-249.

[12] Yousef Alavi, Alfred J.Boals, Gary Chartrand, Ortrud R.Oellermann and Paul Erdos, k- path irregular graphs, *Congressus Numerantium*, **65**, (1988), 201-210.

[13] M.S. Sunitha and A.Vijayakumar, Complement of fuzzy graph, *Indian J. Pure Appl. Math*., **33**(9), (September, 2002), 1451-1464.

[14] L.A. Zadeh, Fuzzy sets, *Information and Control*, **8**, (1965), 338-353.