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Portfolio insurance strategies in a low interest rate environment: A simulation based study

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Abstract

The aim of this study is to ascertain through a simulation process how low and even negative interest rates affect the performance of different portfolio insurance (PI) methodologies and which concepts are successful in different assumed scenarios. In the past, many papers have been published providing empirical evidence on the benefits of PI strategies in different markets. However, hardly any paper focuses on the impact of low interest rates on the performance of PI strategies although interest rates are currently at an all-time low throughout the OECD. In this paper we run Monte Carlo simulations for the buy-and-hold (B&H), Constant Mix, Stop Loss, Constant Proportion Portfolio Insurance (CPPI), and Time Invariant Portfolio Protection (TIPP) strategies. We show that lower interest rates have an impact on the ranking of these strategies according to different performance measures such as Sharpe Ratio, Treynor Ratio, Sortino Ratio, or Lower Partial Moment (LPM) performance measures. B&H and Constant Mix perform relatively well in respect of the Sharpe and Treynor Ratio. However, when considering the Sortino Ratio or LPM performance measures these concepts are particularly badly affected by the reduction in interest rates, especially when it comes to negative rates. Here, the strength of the CPPI strategy becomes obvious.

JEL classification number: G11
Keywords: Portfolio insurance, Constant Proportion Portfolio Insurance, CPPI, Time Invariant Portfolio Protection, TIPP, Buy-and-Hold, Constant Mix, Stop Loss, downside protection, Monte Carlo Simulation

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1 Introduction

Portfolio insurance (PI) strategies are designed to protect portfolios against large falls by a contractually guaranteed predetermined floor through a dynamic allocation. They can be used to reduce downside risk and to participate in rising stock markets as it can be defined to guarantee a minimum level of wealth while the investor can participate in the potential gains of a reference portfolio (Hoque and Meyer-Bullerdiek, 2016, 80).

Among other dynamic versions of PI strategies that are not option-based, the Constant Proportion Portfolio Insurance (CPPI) seems to be the most popular one in the asset management industry (Dichtl and Drobetz, 2010, 41). This methodology was introduced by Perold (1986) on fixed income assets and extended by Black and Jones (1987) for equity based underlying assets. The Time Invariant Portfolio Protection (TIPP) methodology was introduced by Estep and Kritzman (1988) and modified (TIPP-M) by Meyer-Bullerdiek and Schulz (2003). Some simpler ways to hedge a risky portfolio are the Constant Mix strategy (Perold and Sharpe, 1995) and the Stop Loss strategy (Bird, Dennis and Tippett, 1988). Brennan and Schwartz (1988) pointed out that a PI strategy may be of considerable significance to portfolio managers whose investment performance is monitored periodically and to investors who need to meet liabilities in the future.

Several studies have examined the efficiency of not option based PI strategies using Monte Carlo simulation. For example, Zhu and Kavee (1988) showed that CPPI has the ability to reduce downward risk and to retain a certain part of upward gains. Cesari and Cremonini (2003) compare some dynamic strategies in different market situations. Their simulations show a dominant role of constant proportion strategies against all other portfolio insurance strategies in bear and no-trend markets.

Pain and Rand (2008) found out that on the one hand, higher levels of leverage (as defined by the multiplier) tend to increase the upside to a CPPI strategy. On the other hand, higher leverage results in more frequent underperformance and hence more variable returns. They also used a simulation under different assumptions on the volatility of the risky asset price process. A higher realised volatility leads to a lower CPPI performance.

Annaert, Van Osselaer, and Verstraete (2009) used the stochastic dominance criteria to compare PI strategies. Their results indicate that portfolio insurance strategies outperform a buy-and-hold (B&H) strategy with regard to downside protection and risk/return trade-off. However, they provide lower excess returns. According to the stochastic dominance results, these reduced returns are compensated by the lower risk so that PI strategies can be valuable alternatives to B&H investments.

Dichtl and Drobetz (2011) run Monte Carlo simulations and historical simulations for several portfolio insurance strategies. Their results reveal that the traditional portfolio insurance strategies Stop Loss, Synthetic Put, and CPPI are the preferred investment strategy for a prospect theory investor.

Pézier and Scheller (2013) show that optimal CPPI strategies are superior to optimal option based portfolio insurance (OBPI) strategies. They use a certainty equivalent return based on a two-parameter HARA utility function to compare the performance of these strategies in realistic circumstances.

In a more recent paper, Hoque and Meyer-Bullerdiek (2016) analyse the performance of different dynamic portfolio insurance methodologies for securities in the German market in different time periods by comparing them to the buy-and-hold (B&H) strategy, the stock only strategy and the bonds only strategy. Based on the Sortino ratio, the TIPP strategy turned out to deliver the best results.

The above mentioned authors analyse the performance of PI methodologies under different perspectives. None of these studies examines the impact of low (or even negative) interest rates on the performance of PI strategies including the B&H portfolio. As interest rates are currently at an all-time low throughout the OECD, investors who are willing to use a PI strategy need to be aware of how low interest rates affect the performance of this strategy. Therefore, in this paper we examine the impact of low interest rates on the performance of the B&H portfolio and of (not option based) PI strategies using Monte Carlo simulations. In order to cover the traditional PI strategies we examine the Constant Mix, Stop Loss, Constant Proportion Portfolio Insurance (CPPI), and Time Invariant Portfolio Protection (TIPP) strategies.

This paper is structured as follows: Section 2 provides an overview of the portfolio insurance strategies we consider in our simulation analysis. Section 3 shows the Monte Carlo simulation design used in our study. Our results from the simulations are presented and discussed in section 4. Section 5 summarizes the main results of the study.

2 Overview of the PI strategies considered in this study

2.1 Constant Mix

The Constant Mix strategy is a dynamic strategy without using derivatives. Its basis is a constant ratio between specific asset classes (e.g. stocks and bonds) during the investment period. In contrast to the B&H strategy the initial ratio is restored at regular intervals. In the event of a rising stock market, shares are sold, while shares are purchased in a falling stock market. This strategy is anti-cyclical in nature, which means a disadvantage (compared to B&H) when markets are continuous, either rising or falling, but an advantage in market reversals (Perold and Sharpe, 1995, 151). The Constant Mix strategy impresses with its simplicity. However, due to the regular rebalancing, this strategy can result in high transaction costs. Furthermore, it does not offer a real loss limiter. Depending on the development of stock prices, it may also occur that the predefined minimum return or the level of the guaranteed value is not met.

2.2 Stop Loss Strategy

The Stop Loss strategy is often referred to as the simplest and most widely used portfolio insurance strategy. In this strategy, the total wealth is initially invested in the risky asset.

If the portfolio value falls below the floor (or level of the guaranteed value), all assets are converted into the safe asset. The minimum accepted portfolio value can be expressed as the present value of the floor (PV_t^{Floor}) or minimum accepted portfolio value at time t, respectively (Bruns and Meyer-Bullerdiek, 2013, 226):

$$PV_t^{Floor} = F \times (1 + r_f)^{-t}, \tag{1}$$

where F is the floor, r_f is the risk free rate which is constant up to the end of the period, and t is the remaining time until the end of the period.

If the risk-free rate changes, the present value will also change. The strategy is therefore susceptible to interest rate changes over time. On the one hand, this process results in very low transaction costs because there is only one shift. On the other hand, the Stop Loss strategy is seen as strongly path-dependent. Thus, at the beginning of the period, the entire portfolio could be converted into the risky asset even though high price gains could have been achieved in the longer term. For this reason, the Stop Loss strategy is often viewed as obsolete and unsuitable in volatile environments. In addition, in the case of strong price leaps, it may happen that the final value is significantly below the minimum value because of a “too slow” reaction (Meyer-Bullerdiek and Schulz, 2004, 38).

2.3 Constant Proportion Portfolio Insurance (CPPI)

In portfolio management practice, CPPI strategies are quite popular and often used, for example in hedge funds, retail products or life-insurance products. To keep the risk exposure constant, the CPPI is invested in various proportions in a risky asset (e.g. a stock portfolio) and in a non-risky one (e.g. a risk free bond). The following equation can be used to define the amount of these assets (Hoque and Meyer-Bullerdiek, 2016, 80):

$$E = m \times C = m \times \max\left(V - PV_t^{Floor}; 0\right), \tag{2}$$

where E is the exposure which is the part of the total value that should be invested in the risky asset, m is the multiplier which represents the risk aversion of the investor with $m \geq 1$ (the greater m, the less risk averse is the investor), V is the total portfolio value, and C is the cushion which can be defined as follows:

$$C = \max\left(V - PV_t^{Floor}; 0\right) \quad (3)$$

The multiplier (which is constant over time) and the initial floor both are defined at the beginning of the period (Lee, Hsu and Chiang, 2010, 221). While the floor remains constant during the time period, the present value will change because of the decreasing remaining lifetime of the total period. After a change of the value of total assets, the amount of the risky asset can be calculated using the abovementioned equation (“exposure”). After that, the part of the risk-free asset will be allocated by the difference between the total assets and the risky asset (Meyer-Bullerdiek and Schulz, 2004, 55). In this paper, the risky asset is represented by a stock portfolio, and the non-risky asset is a risk free bond.

2.4 Time-Invariant Portfolio Protection (TIPP)

The TIPP methodology is very similar to the CPPI. According to Estep and Kritzman (1988) TIPP has the following characteristics:

(1) The portfolio can never decline below a preset floor or present value of the floor
(2) The floor is adjusted continuously to be a specified percentage of the highest value the portfolio reaches
(3) Protection is continuous and has no ending date

The floor is set as a fixed percentage of the total value of the portfolio and the strategy follows this process:

(1) Calculation of the total value of the portfolio
(2) Multiplication of the total value with the preset floor percentage rate and calculation of the present value of the floor
(3) Setting of a new floor (present value) if the result of (2) is greater than the previous floor (present value)
(4) Calculation of the cushion: $C = \max\left(V - PV_t^{Floor}; 0\right)$
(5) Calculation of the exposure: $E = m \times C$
(6) Purchase / Sell of the risky asset according to (5)

Hence, the only difference between TIPP and CPPI is the assumption in respect to the initial floor which is not constant. The preset floor will increase if the total value of the portfolio increases. In case of a decrease of the total portfolio value, this value should not be less than the insured amount. That’s why TIPP can be regarded as a more passive strategy than CPPI (Tiefeng and Rwegasira, 2006, 98).

3 Monte Carlo simulation design and performance measures

In our empirical analysis we use the Monte Carlo simulation to generate daily logarithmic stock returns. Thus, problems with data specific results can be avoided. It is assumed that these returns are normally distributed. The simulation is based upon an annual geometric stock market return of 6% and the corresponding daily logarithmic return. Furthermore we assume a stock market volatility of 20% and the corresponding daily standard deviation as the second parameter to model stock market returns. For example, the average annual geometric return of the German stock index DAX from October 1959 to October 2016 was roughly 6% and the annual return volatility about 20%. Dimson, Marsh and Staunton (2006) found in their study of equity premiums in 17 countries similar long-run annualized standard deviations, while real equity returns were more diverse in the different countries. In our simulation, the random numbers (daily logarithmic stock returns) are generated with MS Excel. For each PI strategy we transform these logarithmic returns of the risky asset into daily absolute returns.

Like many investors, we use a one year investment horizon and simulate 250 daily stock market returns (Dichtl and Drobetz, 2011). In our analysis we neglect transaction costs and use an initial investment of € 1,000,000. We assume a daily rebalancing for the considered strategies.

The CPPI and the TIPP strategies are implemented with different multipliers (m=2, m=5, m=10) whereas m=5 is often used in commercial applications (Dichtl and Drobetz, 2011, 1688). The protection level is assumed to be 90% for all PI strategies. Referring to the risk-free asset, we make a distinction between three different interest rate scenarios: We assume rates of 2%, 0% and -2% which remain constant for the whole year. That’s why, the initial present value of the floor amount to € 882,353 (2%), € 900,000 (0%) and € 918,367 (-2%).

We perform 5,000 simulations for each PI strategy including the B&H portfolio. This number can be regarded as sufficient for our study (Hagen, 2002, 201). With this simulation setup we can examine the impact of low interest rates on the performance of the PI strategies. For performance measurement purposes we calculate absolute returns of each PI strategy and use the arithmetic average of all 5,000 annual total returns for the calculation of the mean annual return.
All strategies are at first examined to what extent they fulfill the objectives of portfolio insurance concepts:

- Keep downside protection at a certain level
  (We use the lower partial moments (LPM) with a minimum return of 0% as our downside risk measure)
- Attractive return (upside participation)
- Create a distribution which is skewed to the right
- Create a convex return profile

Knowing the skewness of a distribution is important because risk-averse investors prefer a distribution that is skewed to the right. For the same mean return and the same standard deviation, distributions that are skewed to the left bear the risk of high negative extreme values (Poddig, Dichtl and Petersmeier, 2003, 141-142). The skewness (S) can be calculated as follows (Bruns and Meyer-Bullerdiek, 2013, 47):

$$S = \frac{\frac{1}{n} \cdot \sum_{i=1}^{n} (r_i - \mu)^3}{\sigma^3}, \tag{4}$$

where n is the number of random returns, r_i is the return in scenario i, μ is the mean return, and σ is the standard deviation of the returns. A normal distribution is completely symmetrical, resulting in a value for the skewness of zero. Positive values indicate that the distribution is skewed to the right.

In our calculation, we use an alternative definition of the sample skewness which is provided by major software packages (e.g. Excel) and which includes an adjustment for sample size (Doane and Seward, 2011, 7):

$$S_{Sample} = \frac{n}{(n-1) \times (n-2)} \times \sum_{i=1}^{n} \left(\frac{r_i - \mu}{\sigma} \right)^3 \tag{5}$$

Furthermore, the creation of a convex return profile is attractive for risk-averse investors. This is shown in Figure 1. In a falling stock market, the slope of the yield curve decreases and reaches a value of zero from a certain stock index level, so that no further value losses occur from here on. Conversely, the slope of the yield curve increases as the stock market moves upwards (Leoni, 2008, 251).

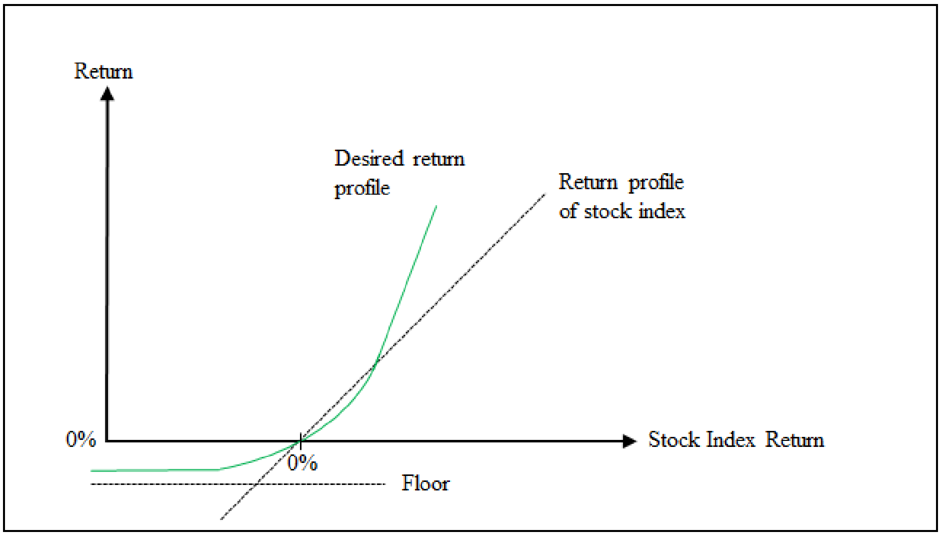

Figure 1: Convex return profile

The above mentioned criteria are also examined to which extent the PI strategies meet these objectives in the low-interest rate environment and to what extent a change in the risk-free rate affects the final result.

The outcomes of the strategies are measured with different approaches. The first one refers to the calculation of the Sharpe Ratio where the arithmetic average of the daily portfolio returns ($\bar{r}$), the risk-free rate (r_f) and the standard deviation of the portfolio returns (σ) are used (Sharpe, 1966):

$$SR = \frac{\bar{r} - r_f}{\sigma} \quad (6)$$

The second approach refers to the Treynor Ratio which differs from the Sharpe Ratio only in terms of the risk measure. The Treynor Ratio uses beta to measure the risk (Treynor, 1965):

$$TR = \frac{\bar{r} - r_f}{ß} \quad (7)$$

We use the Treynor Ratio as a performance measure since the simulated stock portfolios are based on broadly diversified portfolios. Besides, the benchmark in the form of the simulated stock returns is clearly defined, so that the use of the Treynor Ratio seems to be appropriate. In our simulation, we measure beta by using the stock only portfolio as the benchmark.

The third approach to assess the different portfolio insurance strategies is the Sortino ratio (Sortino and Price, 1994, 62; Fischer, 2010, 467):

$$\text{Sortino ratio} = \frac{\bar{r} - r_{min}}{\sqrt{LPM_2}} \qquad (8)$$

where r_{min} is the hurdle rate of return and LPM are the lower partial moments that are used as a downside risk measure.

The LPM allow for a relaxation of the normal distribution assumption that is used for the standard deviation as a risk measure. The following expression can be used to estimate LPM in practice (Bawa and Lindenberg, 1977, 191; Harlow, 1991, 30):

$$LPM_d = \sum_{t=0}^{n} p_t \times \left(r_{min} - r_t^l\right)^d \qquad (9)$$

where r_t^l is the return that is lower than r_{min}, n is the number of observations where $r_t^l < r_{min}$, p_t is the probability of r_t^l being lower than r_{min}, and d is the degree of the moment.

The exponent d indicates how different levels of negative deviations from the hurdle rate of return are weighted. In the case of d=2, large deviations are higher weighted than smaller ones compared to the case of d=1. If LPM is calculated empirically, each observed return is assigned the same probability, i.e. one divided by the total number of periods included (Poddig, Dichtl and Petersmeier, 2003, 135-136). For performance measurement purposes, a degree of the moment of 2 is appropriate (Wittrock, 1995, 132-133) which is also used in the Sortino ratio. In our simulations r_{min} is set to be zero for the LPM calculation.

The fourth approach we use to evaluate the performance is the LPM performance measure which can be defined as follows (Wittrock, 1995, 132):

$$LPM_d\text{-performance measure} = \frac{\bar{r} - r_f}{\sqrt[d]{LPM_d}} \qquad (10)$$

In contrast to the Sortino Ratio, the risk-free rate is used in the numerator instead of the predetermined minimum return.

4 Monte Carlo simulation results

4.1 Individual Results of the single strategies

In this section we present our simulation results for the B&H portfolio and for each PI strategy. For the B&H strategy, we assume that the present value of the floor is invested directly in the risk-free asset at the beginning of the period. Table 1 shows the results for this portfolio and for the stock only portfolio for comparative purposes. Please note that the average return of the stock only portfolio is higher than the assumed 6% because we take the arithmetic average of all 5,000 annual return simulations instead of the geometric average which would have been closer to 6%.

Table 1: Results of the B&H strategy

	Stocks only	**B&H**		
		r_f = 2%	**r_f = 0%**	**r_f = -2%**
Average Return	8.49%	2.76%	0.85%	-1.14%
Volatility	21.87%	2.57%	2.19%	1.78%
Number of Paths < -10%	983	0	0	0
Average Return < -10%	-18.99%	-	-	-
Median	6.01%	2.47%	0.60%	-1.35%
Skewness	0.68	0.68	0.68	0.68
LPM_0	38%	12%	38%	77%
LPM_1	4.615%	0.113%	0.461%	1.460%
LPM_2	0.861%	0.002%	0.009%	0.037%

With respect to the downside protection, the B&H strategy obviously fulfills its purpose because no default (return < -10%) occurs. However, the price of this protection can also be seen directly: An average return of 2.76% at 2% risk-free rate up to -1.14% at -2% risk-free rate is relatively low compared to a pure stock investment. Interest rate sensitivity is a key factor for the overall yield. Due to a change in interest rates of 2%, the change of the average return is slightly less than 2%. The same is true for the median. The lower returns are particularly pronounced when looking at the LPM_0, which increases from 12% (2%) to 77% (-2%). Please note that the skewness of the stock portfolio is positive although a normal distribution was assumed in the simulation. The reason for this is that we use the arithmetic average of the 5,000 annual returns while the arithmetic average of the corresponding logarithmic returns would lead to a skewness close to zero.

The return profile of the B&H strategy compared to the stock only portfolio is shown in Figure 2. It is almost linear. B&H is obviously suitable for hedging purposes, but it has weaknesses when stock returns are relatively high.

Figure 2: Return profile of the B&H strategy

Table 2 shows the results for the Constant Mix strategy and again for the stock only portfolio for comparative purposes. For the Constant Mix strategy, the present value of the floor is also used for determining the initial investment in the risk-free asset in line with the B&H strategy. Thus the different risk-free rates are also visible in the initial investment.

Table 2: Results of the Constant Mix strategy

	Stocks only	**Constant Mix**		
		r_f **= 2%**	r_f **= 0%**	r_f **= -2%**
Average Return	8.49%	2.75%	0.82%	-1.18%
Volatility	21.87%	2.40%	2.00%	1.60%
Number of Paths < -10%	983	0	0	0
Average Return < -10%	-18.99%	-	-	-
Median	6.01%	2.68%	0.77%	-1.22%
Skewness	0.68	0.14	0.13	0.12
LPM_0	38%	12%	34%	78%
LPM_1	4.615%	0.137%	0.443%	1.404%
LPM_2	0.861%	0.003%	0.009%	0.036%

Even though it formally does not provide downside protection, in our simulation it also provides a satisfactory protection. On the return side, it pays off in a similar way to the B&H approach. The average return and median are not very different. The variation in the risk-free rate is also reflected on the returns, while the mean and median changes are mostly slightly below 2%. However, a somewhat greater degree of independence from the interest rate becomes apparent through the anticyclics. Besides, the Constant Mix strategy is not suitable for creating a distribution that is skewed to the right. On the contrary, it reduces the skewness of the pure stock investment from 0.68 to 0.14 (2%), 0.13 (0%) and 0.12 (-2%). This is due to its anticyclic character and is reflected in the concave return profile (Figure 3). It thus does not lead to the convexity desired by many investors.

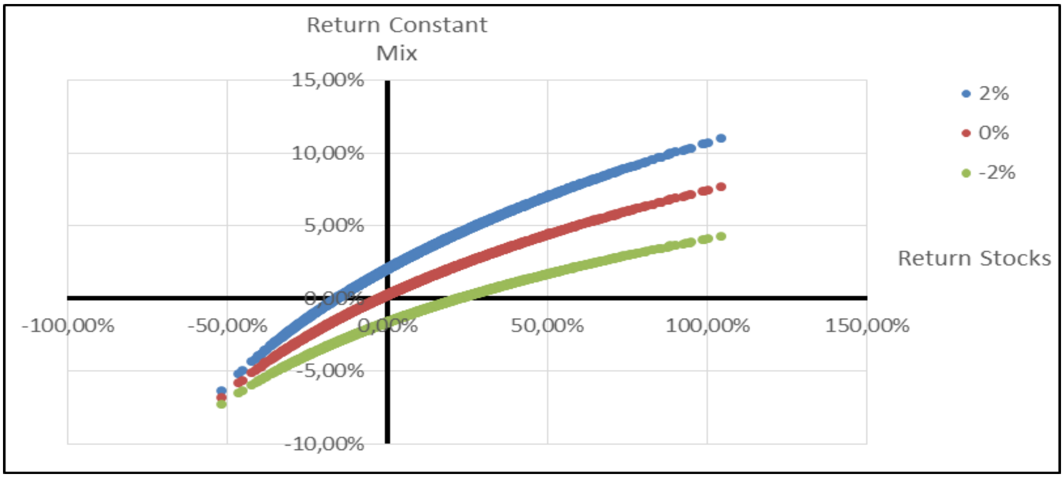

Figure 3: Return profile of the Constant Mix strategy

The Constant Mix approach thus shows good results with respect to the downside protection, but has weaknesses in the other fields.

Within the framework of the Stop Loss approach, the fair value of the floor is also used to determine the sales level. If the portfolio value reaches or drops below this level, the entire risky asset is sold and the amount is invested in the risk-free portfolio. In this case, the investor can no longer participate from any upward stock market movement. The results of this strategy are shown in Table 3.

Table 3: Results of the Stop Loss strategy

	Stocks only	Stop Loss		
		$r_f = 2\%$	$r_f = 0\%$	$r_f = -2\%$
Average Return	8.49%	6.96%	6.11%	4.98%
Volatility	21.87%	20.92%	21.03%	20.95%
Number of Paths < -10%	983	2164	2362	2596
Average Return < -10%	-18.99%	-10.66%	-10.67%	-10.66%
Median	6.01%	0.15%	-3.27%	-10.03%
Skewness	0.68	1.19	1.24	1.34
LPM_0	38%	50%	53%	57%
LPM_1	4.615%	4.857%	5.251%	5.711%
LPM_2	0.861%	0.506%	0.551%	0.601%

Obviously, the Stop Loss approach has clear weaknesses in the minimum hedge. 2,164 cases (at a 2% risk-free rate) where hedging is not fully effective, up to 2,596 cases (at a risk-free rate of -2%) appear quite high.

In our analysis we assume daily reviews. Therefore, larger price drops below the present value of the floor cannot be considered immediately. So, the average deviation values below the protection level of -10% are relatively far below -10%. The Stop Loss strategy leads to a higher number of defaults compared to the stock only portfolio which would have largely recovered in the course of the year. A further consequence is a steep rise in the proportion of the risk-free asset during the course of the year. A lower interest rate leads to higher floor present values and thus to even more entire stock sales and a higher average proportion of the risk-free asset.

In terms of yield, the Stop Loss strategy reaches pretty high average annual returns. However, considering the corresponding medians, it becomes obvious that the relatively high mean returns result from a few outliers. With 0.15%, -3.27% and -10.03% (which is even below the predefined level of protection), the medians of the distributions are significantly less than the corresponding average returns. This impression is additionally reinforced by the high values of the LPM measures.

Nevertheless, the Stop Loss is able to influence the skewness in a positive way. Furthermore, the lower the risk-free rate the higher the skewness. In addition, the Stop Loss generates a kind of convex return profile (Figure 4).

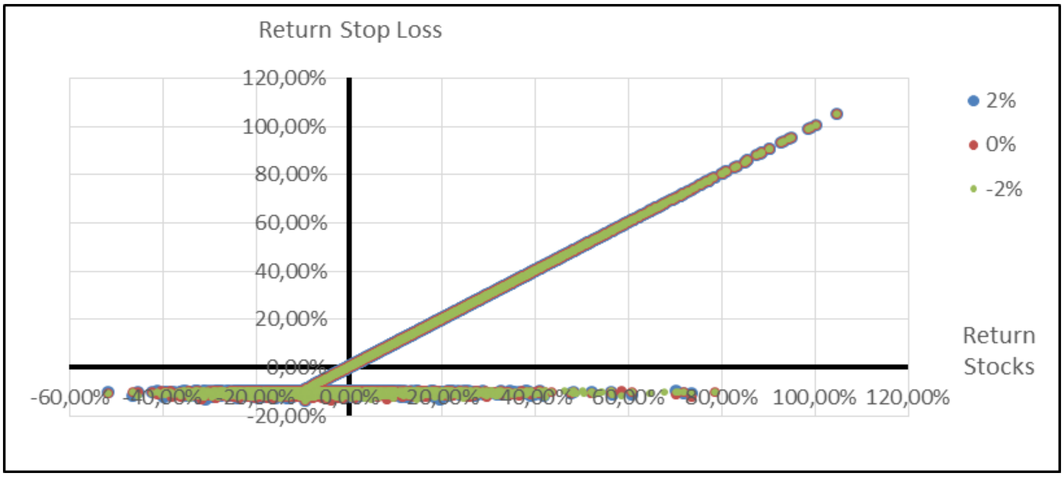

Figure 4: Return profile of the Stop Loss strategy

The results of the CPPI strategy are shown in Table 4 for different multipliers. As we take the present value of the floor for our calculations, this further strengthens the impact of an interest rate change because of the multiplier m.

Table 4a: Results of the CPPI strategy

	Stocks only	**CPPI 2**		
		r_f = 2%	**r_f = 0%**	**r_f = -2%**
Average Return	8.49%	3.57%	1.77%	-0.20%
Volatility	21.87%	5.67%	4.91%	4.09%
Number of Paths < -10%	983	0	0	0
Average Return < -10%	-18.99%	-	-	-
Median	6.01%	2.46%	0.80%	-1.00%
Skewness	0.68	1.33	1.33	1.33
LPM_0	38%	28%	42%	61%
LPM_1	4.615%	0.570%	0.963%	1.668%
LPM_2	0.861%	0.017%	0.032%	0.062%

Table 4b: Results of the CPPI strategy

	CPPI 5			**CPPI 10**		
	r_f = 2%	**r_f = 0%**	**r_f = -2%**	**r_f = 2%**	**r_f = 0%**	**r_f = -2%**
Average Return	5.81%	4.51%	2.87%	6.77%	5.91%	4.76%
Volatility	15.82%	14.86%	13.59%	19.82%	19.54%	18.99%
Number of Paths < -10%	0	0	0	0	0	0
Average Return < -10%	-	-	-	-	-	-
Median	-0.06%	-0.89%	-1.99%	-2.74%	-4.14%	-5.39%
Skewness	1.88	2.00	2.17	1.40	1.47	1.58
LPM_0	50%	54%	59%	53%	56%	59%
LPM_1	2.388%	2.625%	2.983%	4.115%	4.336%	4.608%
LPM_2	0.144%	0.161%	0.187%	0.348%	0.369%	0.393%

It should be noted that the CPPI offers a good portfolio hedge. In none of the simulations, even with different interest rates and multipliers, does it lead to a default. The average return is increased by a higher multiplier, as are the average stock proportions in the portfolio. This is not surprising because this is ultimately the purpose of the multiplier. Contrary to the mean returns, however, the medians decrease with increasing m. Obviously, the average returns are significantly caused by outliers (the maxima are strongly influenced by m). This should be taken into account when choosing m.

It can also be observed that the spreads of the average returns between the different risk-free rate scenarios decrease with increasing m. Due to the change in interest rates, the stock exposure is initially smaller but a higher m also leads to a higher proportion of the stocks and thus to a lesser dependency on the risk-free rate. This impression is reinforced by the LPM values which rise as m increases, but the LPM relative spreads between the different risk-free levels decrease as m increases. The median is different in this respect: Here, m= 5 leads to the least effect by changes in interest rates.

With respect to the skewness, the CPPI 5 (i.e. m=5) has the highest values. The CPPI also benefits from declining interest rates with regard to the skewness to the right. This effect is also highest at m=5, and is almost absent at m=2. Overall, the CPPI also reliably achieves a distribution that is skewed to the right.

The convexity increases with a higher m but also causes a greater downward spread of the CPPI values, even with the same final stock value. This is where the path-dependent character of the CPPI comes into play. This obviously increases with a higher m which is especially obvious when looking at CPPI 10 in Figure 7.

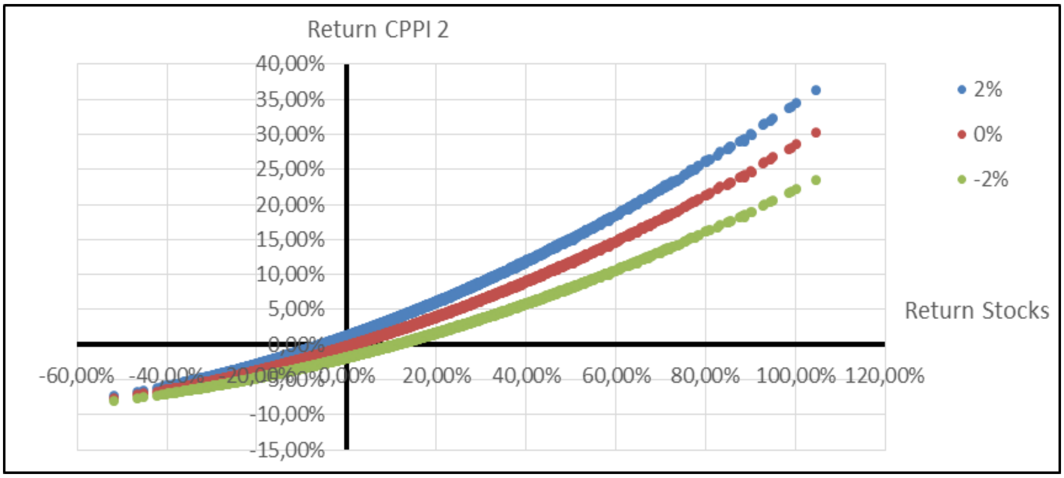

Figure 5: Return profile of CPPI 2 (i.e. m=2)

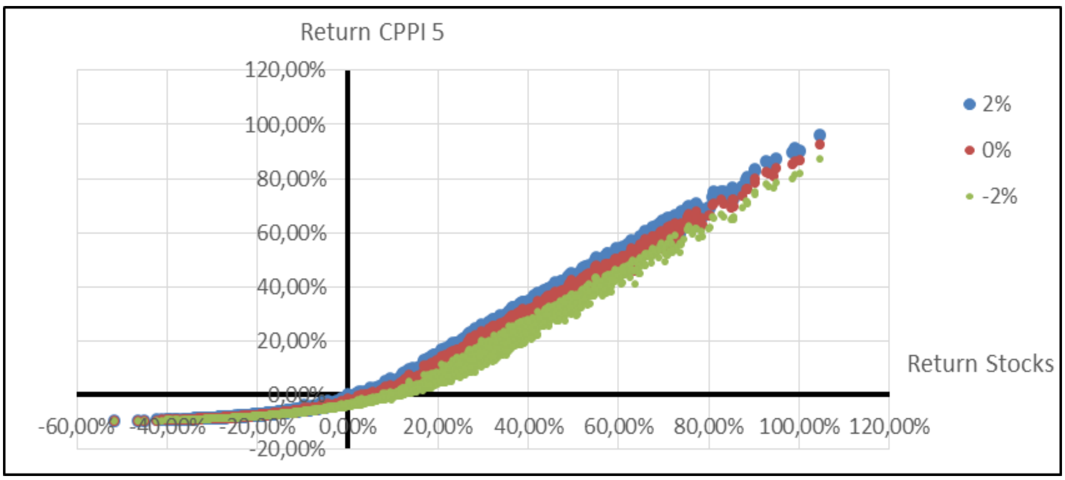

Figure 6: Return profile of CPPI 5 (i.e. m=5)

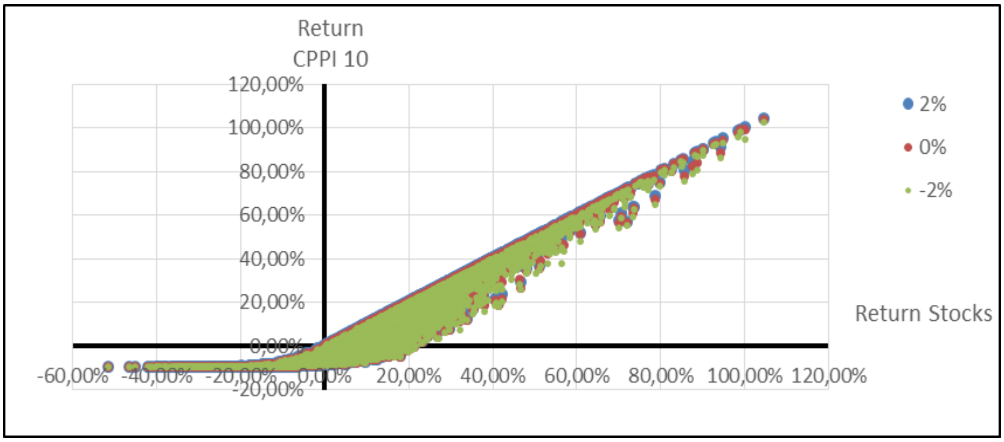

Figure 7: Return profile of CPPI 10 (i.e. m=10)

The CPPI also offers return chances depending on the multiplier, but these are bought at a higher probability of an underperformance.

The results of the TIPP strategy are shown in Table 5 for different multipliers. In line with the CPPI strategy, the present value of the floor is taken into account at the TIPP strategy.

Table 5a: results of the TIPP strategy

	Stocks only	TIPP 2		
		r_f = 2%	r_f = 0%	r_f = -2%
Average Return	8.49%	3.24%	1.41%	-0.62%
Volatility	21.87%	4.16%	3.63%	2.99%
Number of Paths < -10%	983	0	0	0
Average Return < -10%	-18.99%	-	-	-
Median	6.01%	2.93%	1.05%	-1.10%
Skewness	0.68	0.41	0.51	0.75
LPM_0	38%	23%	38%	63%
LPM_1	4.615%	0.452%	0.806%	1.557%
LPM_2	0.861%	0.014%	0.025%	0.052%

Table 5b: results of the TIPP strategy

	TIPP 5			TIPP 10		
	r_f = 2%	r_f = 0%	r_f = -2%	r_f = 2%	r_f = 0%	r_f = -2%
Average Return	4.46%	2.76%	0.70%	5.26%	3.59%	1.39%
Volatility	9.13%	7.99%	6.71%	14.42%	12.81%	10.71%
Number of Paths < -10%	0	0	0	0	0	0
Average Return < -10%	-	-	-	-	-	-
Median	2.82%	1.14%	-0.98%	1.27%	-0.12%	-1.92%
Skewness	0.93	1.02	1.22	1.55	1.68	1.88
LPM_0	37%	44%	56%	46%	50%	59%
LPM_1	1.526%	1.752%	2.218%	2.661%	2.772%	3.086%
LPM_2	0.086%	0.094%	0.115%	0.195%	0.194%	0.202%

Like B&H, Constant Mix and CPPI, the TIPP strategy also has a very good downside protection. This is not surprising as it is a more defensive version of the CPPI. The average returns are below the CPPI returns for all multipliers and risk-free rates, but the medians are above (each with one exception).

Although TIPP does not lead to returns as high as the CPPI, it results in a much more constant generation of returns. This can be explained by the increase of the floor in the case of advancing stocks. As a result of this adjustment, TIPP also receives a higher proportion of the risk-free asset than CPPI at all levels. As far as interest rate sensitivity is concerned, TIPP shows greater changes in the average return and especially the median than CPPI (each with one exception).

However, the interest rate sensitivity in relation to the median decreases with an increasing multiplier, instead of having the smallest effect as with CPPI 5. With regard to the average return, the interest rate impact doesn't change much with a rising multiplier, other than CPPI where the interest rate sensitivity decreases.

The TIPP strategy does not, however, necessarily create a distribution that is more skewed to the right than the one of the stock only portfolio. In the case of m=2 and risk-free rates of 2% and 0%, it leads to a distribution that is more skewed to the left than the one of the stock only portfolio. As the multiplier increases, however, the distribution is more skewed, so that TIPP 10 then even exceeds CPPI 10. Again, as with all the other strategies, there is also a greater tendency to a right-skewed distribution at lower interest rates.

TIPP does not create constantly a convexity. The convexity is mainly caused by the downside protection. In the case of m=2, a convex-concave curve is more likely. Only at m=5 and m=10, the typical convex curvature can be observed. This is shown in Figures 8-10.

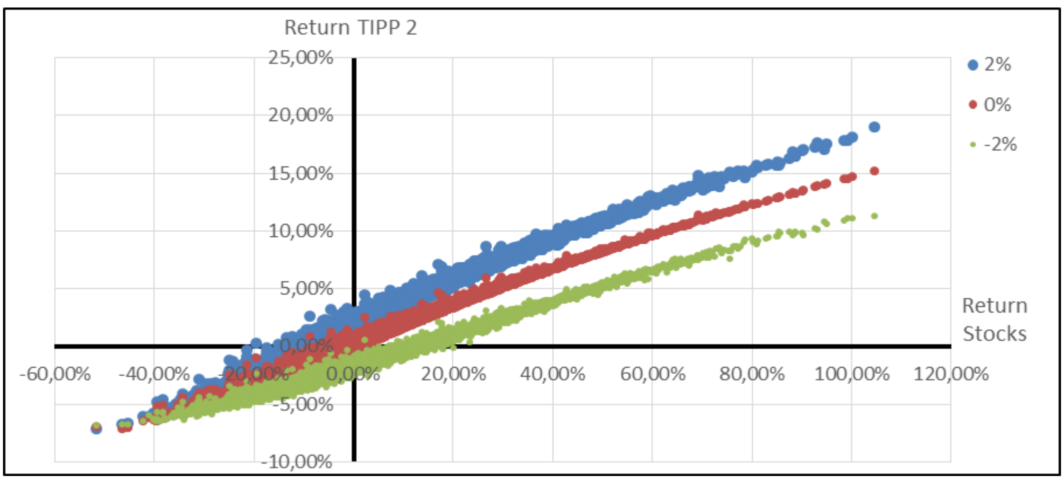

Figure 8: Return profile of TIPP 2 (i.e. m=2)

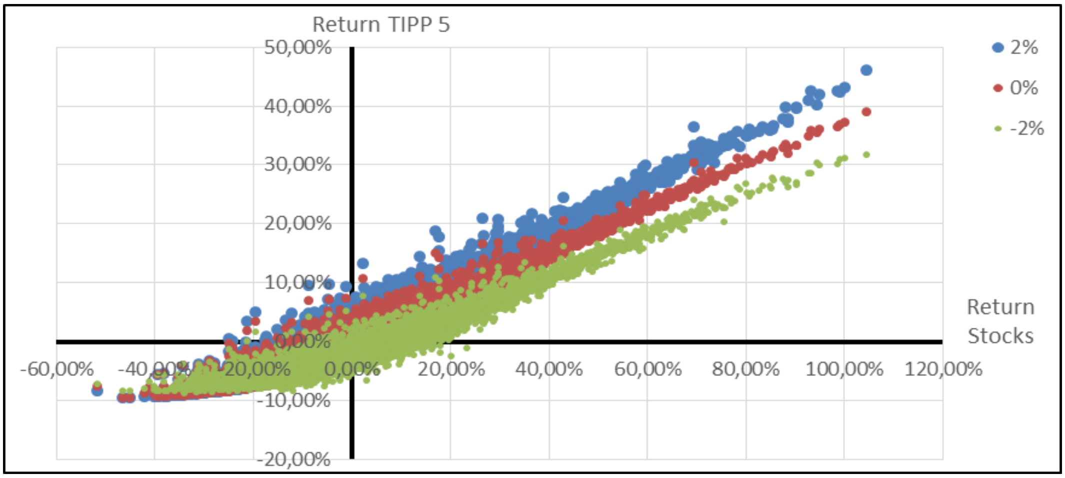

Figure 9: Return profile of TIPP 5 (i.e. m=5)

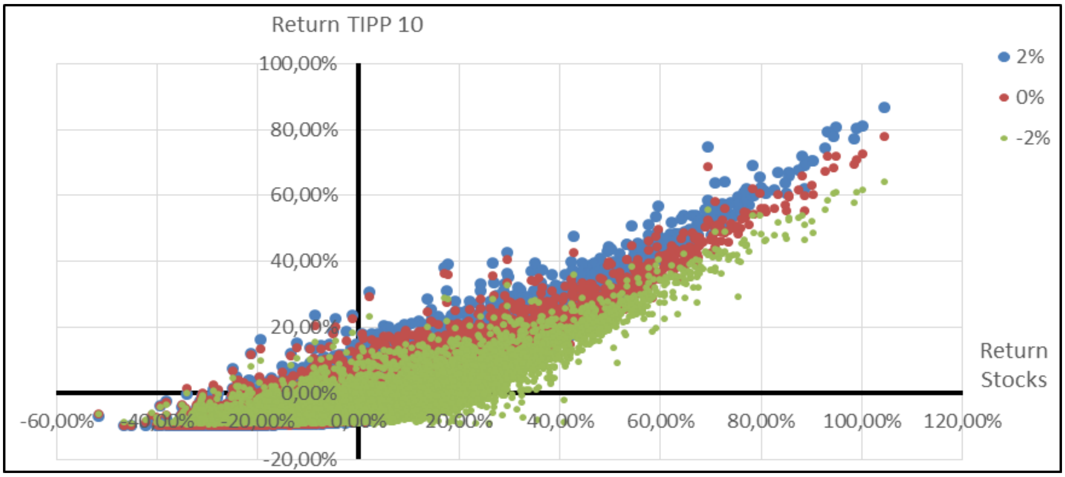

Figure 10: Return profile of TIPP 10 (i.e. m=10)

Despite the fact that the TIPP strategy appears to be more constant than the CPPI when looking at the pure key figures, a higher path dependency becomes apparent when looking at the payout profiles. The distributions are more dispersed than the ones of CPPI. Especially in the case of lower stock returns (return < 20%), the TIPP strategy can lead to higher returns than CPPI because of the increasing floor values. Furthermore, the LPM values of TIPP are lower in almost every case. Thus, TIPP can be seen as the more constant version of the CPPI. Although the returns are slightly lower, it seems to be more suitable for more defensive investors when using the same multiplier.

The above mentioned results are summarized in Table 6. With regard to the stock proportions, the values for the risk-free rates of 2% and -2% are shown. The value for 0% is between these.

Table 6: Summarized results

	Downside Protection	Upside Participation	Skewness	Convexity	Stock Proportion
B&H	✓	O	-	-	**12% - 9%**
Const. Mix	(✓) [1]	O	--	--	**12% - 8%**
Stop Loss	(-)	(+++) [2]	+	+	**78% - 68%**
CPPI 2	✓	+	+	+	**25% - 18%**
CPPI 5	✓	++	+++	++	**60% - 48%**
CPPI 10	✓	+++	++	+++	**75% - 66%**
TIPP 2	✓	+	-	O	**20% - 14%**
TIPP 5	✓	++	+	+	**39% - 27%**
TIPP 10	✓	+++	++	++	**52% - 37%**

[1] formally not given [2] if not shifted

Table 6 shows that the upside participation of the individual strategies is decisively dependent on the stock proportion. This relationship, however, refers only to the average level.

4.2 Comparative performance analysis of the strategies

The comparative analysis is performed on the basis of the presented two-dimensional performance measures. Here, too, the interest rate sensitivity of the concepts is examined. Table 7 shows a ranking based on the Sharpe Ratio which is a very frequently used performance measure.

Table 7: Sharpe Ratio results

	SR, $r_f = 2\%$	**SR, $r_f = 0\%$**	**SR, $r_f = -2\%$**
Constant Mix	0.311	0.410	0.511
B&H	0.297	0.388	0.480
TIPP 2	0.299	0.387	0.461
CPPI 2	0.277	0.359	0.440
TIPP 5	0.270	0.346	0.403
CPPI 5	0.241	0.304	0.359
CPPI 10	0.241	0.303	0.356
Stop Loss	0.237	0.291	0.333
TIPP 10	0.226	0.281	0.317

Surprisingly, the simplest researched concepts – B&H and Constant Mix – together with TIPP 2 offer the highest performance. This cannot be explained with the return on these concepts as they perform lower-than-average. Thus, the only explanation is that they are able to generate this return with a relatively low risk. This also explains why the Stop Loss strategy in this ranking is quite bad. Although it offers a relatively high average return, this return is at a relatively high risk. The same applies to CPPI and TIPP with m=10. The Sharpe ratio thus

rewards the more defensive concepts.
Interest rate changes are also reflected here. With an interest rate reduction, the Sharpe Ratio increases constantly in all approaches. However, as already shown, average return does not decrease as much as the risk-free rate, which leads to a higher excess return in the numerator.

Table 8 shows the ranking according to the Treynor Ratio.

Table 8: Treynor Ratio results

	TR, r_f = 2%	**TR, r_f = 0%**	**TR, r_f = -2%**
Constant Mix	0.068	0.090	0.113
B&H	0.065	0.085	0.105
TIPP 2	0.066	0.085	0.102
CPPI 2	0.061	0.079	0.097
TIPP 5	0.061	0.078	0.092
Stop Loss	0.058	0.073	0.087
CPPI 10	0.056	0.071	0.085
CPPI 5	0.056	0.071	0.084
TIPP 10	0.055	0.070	0.083

It can be observed that the Treynor Ratio leads to a similar ranking as the Sharpe Ratio. Again, the simple and defensive concepts perform better. Thus the TIPP is mostly preferred in direct comparison to CPPI. However, the Stop Loss strategy performs better according to the Treynor Ratio. This can be explained by its exceptionally high average return, while its beta is not significantly higher than that of CPPI 10 and TIPP 10. Interest rate fluctuations have a similar effect to the Sharpe Ratio. They lead to a higher Treynor Ratio.

The results according to the Sortino Ratio are shown in Table 9.

Table 9: Sortino Ratio results

	Sortino Ratio, r_f = 2%		**Sortino Ratio, r_f = 0%**		**Sortino Ratio, r_f = -2%**
B&H	6.788	**CPPI 5**	1.125	**CPPI 10**	0.758
Constant Mix	5.338	**CPPI 2**	0.989	**CPPI 5**	0.663
TIPP 2	2.791	**CPPI 10**	0.974	**Stop Loss**	0.642
CPPI 2	2.724	**B&H**	0.915	**TIPP 10**	0.310
CPPI 5	1.531	**TIPP 5**	0.898	**TIPP 5**	0.207
TIPP 5	1.525	**TIPP 2**	0.885	**CPPI 2**	-0.080
TIPP 10	1.191	**Constant Mix**	0.852	**TIPP 2**	-0.272
CPPI 10	1.148	**Stop Loss**	0.824	**B&H**	-0.592
Stop Loss	0.978	**TIPP 10**	0.817	**Constant Mix**	-0.624

At an interest rate of 2%, the simple, defensive concepts are ahead according to the Sortino Ratio. This can be explained by their good hedging ability. Due to its high number of paths with an average return below -10%, however, the Stop Loss performs very poor. If the risk-free rate is 0% or -2%, the results are quite different. Overall, the values of the Sortino Ratio fall with the interest rate, since the returns fall more than the LPM_2 values. CPPI 5 and CPPI 10 show their strengths, with the CPPI versions performing better than their TIPP equivalents at an interest rate level of 0% or -2%, respectively. As already shown, these concepts are only slightly interest-sensitive due to their high stock proportions. B&H, Constant Mix and TIPP 2 are performing relatively bad at lower interest rates. This reversal of the rankings is partly due to the fact that in case of the Sortino Ratio, unlike Sharpe and Treynor Ratio, the risk-free rate is not directly included in the calculation. In addition, the LPM was given a minimum return of 0%. This is also used in the numerator of the Sortino Ratio. The defensive concepts, however, show a very low return, coupled with a strong sensitivity of the returns to the risk-free rate. In the event of an interest rate change, therefore, a large portion of the simulation iterations is shifted to the range below 0%, which makes the LPM rise sharply. This shows the high interest rate sensitivity of the defensive concepts. With a risk-free rate of -2%, this impression is even strengthened.

Table 10 shows the results for the LPM_2 performance measure.

Table 10: LPM_2 performance measure results

	LPM_2 performance measure, $r_f = 2\%$		**LPM_2 performance measure, $r_f = 0\%$**		**LPM_2 performance measure, $r_f = -2\%$**
B&H	1.875	**CPPI 5**	1.125	**CPPI 5**	1.126
Const. Mix	1.450	**CPPI 2**	0.989	**CPPI 10**	1.077
CPPI 2	1.198	**CPPI 10**	0.974	**Stop Loss**	0.900
TIPP 2	1.070	**B&H**	0.915	**TIPP 5**	0.797
CPPI 5	1.004	**TIPP 5**	0.898	**TIPP 10**	0.754
TIPP 5	0.841	**TIPP 2**	0.885	**CPPI 2**	0.720
CPPI 10	0.809	**Const. Mix**	0.852	**TIPP 2**	0.604
TIPP 10	0.738	**Stop Loss**	0.824	**B&H**	0.443
Stop Loss	0.696	**TIPP 10**	0.817	**Const. Mix**	0.433

The ranking is very similar to the Sortino Ratio. Here, too, the rank of B&H falls sharply with declining interest rates. The rank of the Constant Mix strategy is getting worse the lower the interest rate. In case of lower rates, the return is more often lower than 0%. Because of its more symmetrical return distribution, high downward deviations, which can occur with the Constant Mix, are weighted more heavily using the LPM_2 performance measure.

With the LPM_1 performance measure, Constant Mix performs much better in case of a 2% and 0% risk-free rate because the weight of the LPM measure is much lower. The results for the LPM_1 performance measure are shown in Table 11.

Table 11: LPM_1 performance measure results

	LPM_1 performance measure, r_f = 2%		**LPM_1 performance measure, r_f = 0%**		**LPM_1 performance measure, r_f = -2%**
B&H	6.756	**Const. Mix**	1.853	**CPPI 5**	1.633
Const. Mix	5.428	**B&H**	1.839	**CPPI 10**	1.466
CPPI 2	2.754	**CPPI 2**	1.834	**Stop Loss**	1.221
TIPP 2	2.751	**TIPP 2**	1.745	**TIPP 5**	1.218
TIPP 5	1.613	**CPPI 5**	1.719	**TIPP 10**	1.099
CPPI 5	1.596	**TIPP 5**	1.577	**CPPI 2**	1.079
TIPP 10	1.225	**CPPI 10**	1.364	**TIPP 2**	0.885
CPPI 10	1.159	**TIPP 10**	1.296	**B&H**	0.586
Stop Loss	1.020	**Stop Loss**	1.164	**Const. Mix**	0.583

Our simulation shows that the lower the risk-free rate, the more preferable the more offensive concepts are. In principle, CPPI performs better than TIPP.

5 Conclusion

In this paper we use Monte Carlo simulation to analyse portfolio insurance strategies in a low interest environment. These strategies are designed to protect portfolios against large losses by a contractually guaranteed predetermined floor through a dynamic allocation. The goal of these strategies is to reduce downside risk and to participate in rising markets.

The analysis shows that these strategies cannot meet completely the ideal conception of investors achieving a downside protection with a full upside participation. Our simulations show that all strategies except for the Stop Loss approach deliver a satisfactory performance with regard to downside protection. The Constant Mix strategy also shows solid performance although it does not formally provide a hedge. Regarding the upside participation, however, the Stop Loss approach shows strengths together with CPPI 10 (i.e. m=10) and TIPP 10.

Here, however, the return peaks are purchased at a higher risk. In particular, CPPI offers a relatively positive risk-return relationship as shown especially by the LPM performance measure and the Sortino Ratio. At the higher risk-free rate of 2%, B&H and Constant Mix perform relatively well, while they perform consistently well in respect of the Sharpe and Treynor Ratio. However, these concepts have weaknesses when considering the performance measures that use an asymmetrical risk measure. They are particularly badly affected by the reduction in interest rates, especially when it comes to negative rates. Here the strength of more flexible concepts such as CPPI becomes obvious. By varying the multiplier, the risk/return profile can be adapted to the investor's specific requirements. The evaluation of the LPM performance measures shows in particular the strength of CPPI, with the connection of a relatively high return with low downside risks. Overall, the concepts with high average stock proportions also show strengths in the creation of a convex return profile. In particular, CPPI 10 and TIPP 10 may be mentioned in this context.

It remains to be noted that portfolio insurance concepts offer suitable return profiles for many investors in a low interest rate environment. Our simulation analysis shows strengths of the more flexible concepts, in particular CPPI.

Our study does not consider changes in interest rates during the year. Furthermore, transaction costs were not taken into account. Besides, some of the presented concepts are path-dependent. Further research work could illuminate the extent to which the times of extreme price falls or price rises influence the final result. A combination of the presented strategies with each other and an analysis to which extent these combinations can influence the performance may also be conceivable for further research.

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