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Smooth Transition Autoregressive-GARCH Model in Forecasting Non-linear Economic Time Series Data

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Abstract

The regime switching models are particularly popular in the comity of non-linear models; it is of interest to investigate regime switching models with GARCH specification. GARCH model was augmented with STAR model vis-a vis Exponential autoregressive GARCH (EAR-GARCH), Exponential smooth transition autoregressive GARCH (ESTAR- GARCH) model and Logistic smooth transition autoregressive GARCH (LSTAR-GARCH) model. The properties of the new models were derived and compared with conventional GARCH model which shows that the variance obtained for STAR-GARCH model was minimum compared to classical GARCH model, the new methodology proposed is illustrated with foreign exchange rate data from Great Britain (Pound) and Botswana (Pula) against United States of America (Dollar). It is evident that all STAR-GARCH outperformed the classical GARCH model, however, LSTAR-GARCH performed best and closely followed by ESTAR-GARCH, this is followed by EAR-GARCH. The implication is that the use of LSTAR –GARCH produces the best result; however LSTAR may be utilized in some occasions.

Keywords: GARCH models, STAR-GARCH models, EAR-GARCH. ESTAR-GARCH LSTAR-GARCH, foreign exchange data.

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1 Introduction

Non-linear time series models are increasingly becoming very popular this is because several financial assets cannot be modeled by pure linear processes. It seems to be generally accepted that many economic variables follow non-linear processes. The regime switching models are particularly popular in the committee of non-linear models, it is of interest to study regime switching models with GARCH specification, in this paper GARCH model will be augmented with STAR model vis-a vis Exponential autoregressive GARCH (EAR-GARCH), Logistic smooth transition autoregressive GARCH (STAR-GARCH) model and Exponential smooth transition autoregressive (STAR- GARCH) model.

Distinct features of GARCH model and its extension lies on the fact that they have ability to capture volatility clustering, for instance, if the shock from the previous period is high or low, large or small the values of ε_{t-1}^2 will certainly have an effect on its variance.

Smooth transition autoregressive (STAR) models are applied to time series data as an extension of autoregressive models, in order to allow for higher degree of flexibility in model parameters through a smooth transition. So also STAR models are introduced according to Terasvirta and Anderson (1992), Granger and Terasvirta (1994) and Terasvirta (1994) because of the existence of two distinct regimes, with potentially different dynamic properties and that the transition between the regimes is smooth. STAR models allow economic variables to follow a given number of regimes with switches between regimes achieved in a smooth and continuous fashion and governed by the value of a particular variable or group of variables. The transition parameter denoted by $\{s_t, \gamma, c\}$ is a slope of parameter that determines the speed of transition between the two extreme regimes with low absolute values resulting in slower transition. It should be noted that $\{s_t, \gamma, c\}$ are generated by data series. Two commonly used transition functions are the logistic autoregressive (LSTAR) model and exponential autoregressive (ESTAR) model. However, exponential autoregressive (EAR) model from where the ESTAR was generalized will be studied along with these two traditionally studied models. In the non-linear GARCH model, the conditional variance is expressed as a non linear of lagged residuals. In the STAR models, the non-linearity is introduced via either logistic or an exponential transition function. The non-linearity in this paper is linked to existence of bid-ask spread in the currencies being exchanged.

2 General Representation

We define the general representation of STAR model as:

$$y_t = \Phi_1' x_t \left(1 - G\left(y_t, \gamma, c\right)\right) + \Phi_2' x_t G\left(y_t, \gamma, c\right) + \varepsilon_t \tag{1}$$

Where $x_t = \left(1, y_{t-1}, y_{t-2}, ..., y_{t-p}\right)$, $\left(\phi_{i0}, \phi_{i1}, ..., \phi_{ip},\right)\left(i = 1, 2\right)$, ε_t is the error term distributed as independently and identically with mean zero and variance σ^2. $G\left(s_t, \gamma, c\right)$ is the transition function bounded between zero and unity and thus allowing for a smooth transition between regimes.

Now, using the lagged endogenous variable, the various forms of STAR models are as follows:
The logistic STAR model is expressed as

$$y_t = \Phi_1' y_{t-j} \left\{1 - \left((1 + \exp - \left[\gamma\left(y_{t-d} - c\right)\right)\right)^{-1}\right\} + \left\{\Phi_2' y_{t-j} \left(1 + \exp - \left[\gamma\left(y_{t-d} - c\right)\right)\right)^{-1}\right\} + \varepsilon_t$$
$$= \Phi_1' y_{t-j}\left(1 - G_t^L\right) + \Phi_2' y_{t-j} G_t^L + \varepsilon_t \quad (2)$$

The exponential – STAR model is of the form

$$y_t = \Phi_1' y_{t-j}\left(-\exp - \left[\gamma\left(y_{t-d} - c\right)^2\right) + \Phi_2' y_{t-j}\left(1 - \exp - \left[\gamma\left(y_{t-d} - c\right)^2\right) + \varepsilon_t$$
$$= \Phi_1' y_{t-j}\left(1 - G_t^E\right) + \Phi_2' y_{t-j} G_t^E + \varepsilon_t \quad (3)$$

The exponential autoregressive STAR (EAR-STAR) model is of the form

$$y_t = \Phi_1' y_{t-j}\left(1 - \exp - \left[\gamma\left(y_{t-1}^2\right)\right) + y_t = \Phi_2' y_{t-j}\left(1 - \exp - \left[\gamma\left(y_{t-1}^2\right)\right) + \varepsilon_t$$
$$= \Phi_1' y_{t-j}\left(1 - G_t^R\right) + \Phi_2' y_{t-j} G_t^R + \varepsilon_t \quad (4)$$

For large values of the parameter γ, the logistic function G_t^L converges to one when $y_{t-d} - c < 0$ when $\gamma \to 0$ the LSTAR converges to an autoregressive model of order p. The ESTAR shows slightly different patterns with respect to γ. For large values of γ, the exponential function $\left(G_t^E\right)$ converges to one for values of y_{t-d} below or above threshold parameter c. The EAR-STAR is a modified form of ESTAR with $d > 0$. Base on the aforementioned conditions, the STAR model offers the possibility to investigate the presence of non linearity in time series data which may account for the weakness of GARCH model mentioned in chapter four, and without loss of generality we can strengthen the GARCH model with STAR models by adjusting the error terms. The LSTAR GARCH model is proposed by combining equation $y_t = \sigma_t \varepsilon_t$ with equation (2), (3) and (4) to get

$$y_t = \Phi_1' y_{t-j}\left(1 - G_t^L\right) + \Phi_2' y_{t-j} G_t^L + \sigma_t \varepsilon_t \quad (5)$$
$$y_t = \Phi_1' y_{t-j}\left(1 - G_t^E\right) + \Phi_2' y_{t-j} G_t^E + \sigma_t \varepsilon_t \quad (6)$$
$$y_t = \Phi_1' y_{t-j}\left(1 - G_t^R\right) + \Phi_2' y_{t-j} G_t^R + \sigma_t \varepsilon_t \quad (7)$$

Which are EAR-GARCH (5), ESTAR-GARCH (6) and LSTAR-GARCH (7)
In general, the STAR-GARCH model proposed for the study is of the form

$$y_t = \Phi_1' y_{t-j}\left(1 - G_t^\tau\right) + \Phi_2' y_{t-j} G_t^\tau + \sigma_t \varepsilon_t \quad (8)$$

Where G_t^τ is the varying smooth transition functions defined in equations (2 through 4)

3 Properties of STAR-GARCH Model

Suppose that the general STAR-GARCH is of the form

$$y_{t(S-G)} = \underline{\phi}_1' y_{t-j}(1-G_t) + \underline{\phi}_2' y_{t-j} G_t + \sigma_t \varepsilon_t \quad = \underline{\phi}_1' y_{t-j} - \underline{\phi}_1' y_{t-j} G_t + \underline{\phi}_2' y_{t-j} G_t + \sigma_t \varepsilon_t \quad (9)$$

Assume that $\underline{\phi}_1'$ and $\underline{\phi}_2'$ have the same number of parameter such that $\phi_2' - \phi_1' = \lambda_1$ and $V_t = y_{t-j} G_t \ \forall\, j = 1, 2, ..., p$ and $Z_t = \sigma_t \varepsilon_t$, then equation (9) reduces to

$$y_{t(S-G)} = \underline{\phi}_1' y_{t-j} + \underline{\lambda}_1' V_t + Z_t \quad (10)$$

Let us assume that y_{t-j}, V_t and Z_t are independent with zero co-variances and the estimates of $\underline{\phi}_1'$ and $\underline{\lambda}_1'$ are respectively $\underline{\hat{\phi}}_1$ and $\underline{\hat{\lambda}}_1$. The mean of general STAR-GARCH model is:-

$$E(y_t) = \underline{\phi}_1' E(y_{t-j}) + \underline{\lambda}_1' E(V)_t + E(Z_t)$$

Since $E(Z_t) = 0$ and $E(y_t) = E(y_{t-j}) \ \forall$j, then the last expression is of the form

$E(y_t) - \underline{\phi}_1' E(y_t) = +\underline{\lambda}_1' E(V_t)$, which reduces to

$$E\left(y_{t(S-G)}\right) = E(y_t) = \frac{1}{1-\underline{\hat{\phi}}_1} \underline{\lambda}_1' \overline{V}_t \quad (11)$$

To derive the variance of $y_{t(S-G)}$, consider the expression for $E\left(y_t^2\right)$ as

$E\left(y_t^2\right) = \underline{\phi}_1^2 E\left(y_{t-j}^2\right) + \underline{\lambda}_1^2 E\left(V_t^2\right) + E\left(Z_t^2\right)$, this reduces to

$$E\left(y_t^2\right) = E\left(y_{t-j}^2\right) = \frac{1}{1-\underline{\hat{\phi}}_t^2}\left[\underline{\lambda}_1^2 E\left(V_t^2\right) + E\left(Z_t^2\right)\right] \quad (12)$$

From equations (11) and (12) we have

$$Var\left(y_{t(S-G)}^2\right) = \quad \frac{1}{1-\underline{\hat{\phi}}_t^2}\left[\underline{\lambda}_1^2 E\left(V_t^2\right) + \frac{\alpha_0}{1-\sum\left(\alpha_i + \beta_j\right)}\right] - \frac{1}{\left(1-\underline{\hat{\phi}}_1\right)^2}\left[\underline{\lambda}_1^2 \left(\overline{V}_t\right)^2\right] \quad (13)$$

In order to relate STAR-GARCH model with the GARCH model; if in equation (5.11) $\overline{V} = 0$ then $E\left(y_{t(S-G)}^2\right) = E(y_t)$ and the variance of $Var\left(y_{t(S-G)}^2\right)$ in equation (13) will reduce to

$$\frac{1}{1-\underline{\hat{\phi}}_t^2}\left[\underline{\lambda}_1^2 E\left(V_t^2\right) + \frac{\alpha_0}{1-\sum\left(\alpha_i + \beta_j\right)}\right]$$

4 Empirical Results/Data Analysis with Exchange Rate Data

This section examines the empirical results obtained for Smooth Transition Autoregressive GARCH models (STAR-GARCH) for four sets of exchange rates data namely British (Pounds), Japanese (Yen), Nigerian (Naira) and Batswana (Pula) against American (Dollar). Here the Parameters of Exponential autoregressive GARCH models (EAR-GARCH), Logistic smooth transition autoregressive GARCH models (LSTAR-GARCH) and that of Exponential smooth transition autoregressive GARCH (ESTAR-GARCH) models were obtained using the derived equations for all the series. The following values of variances were obtained for classical GARCH models:

Table 1: The GARCH model fitted for all series

SERIES	COEFFICIENT (S.E)			MODEL VARIANCE
	α_0	α_1	β_1	
NAIRA	3.85802 (0.34152)	1.16179 (0.52198)	-0.99980 (0.00016)	4949.20411
POUND	0.00017 (0.00005)	0.97219 (0.23370)	-0.00024 (0.02657)	0.65816
PULA	0.047613 (0.10084)	1.90362 (0.30959)	-0.91061 (0.01367)	2.14441
YEN	0.67948 (0.26140)	1.00818 (0.26247)	-0.03111 (0.12408)	5461.26025

In estimating γ and c as required in equations (5) through (7)

Estimation of γ and C:

Starting values needed for the nonlinear optimization algorithm can be obtained using two dimensional grid search over γ and c, and select those that give smallest estimator for the residual variance. The two dimensional grid give three possible values are tables2 and 3.

Table 2: Values of grid of C

SERIES	I	II	III
NAIRA	0.35•	155.76	30
POUND	0.48•	2.42	30
PULA	0.74•	7.97	30
YEN	0.74•	7.97	30

Table 3: Values of grid of γ

SERIES	I	II	III
NAIRA	0.50•	10.00	30
POUND	0.50•	10.00	30
PULA	0.50•	10.00	30
YEN	0.50•	10.00	30

In the tables (2 and 3) all the asterisk values are selected because they have minimum values and are subsequently used in equations (5)-(7). We fitted models described in equations (5)-(7) as follows:
We can now illustrate the empirical implication of these theories here under:

4.1 EARSTAR-GARCH Model for all Series

$$y_{t(S-G)} = \sigma_t\varepsilon_t + \underline{\phi}_1' y_{t-1}\left(-\exp-\left(\gamma\left(y_{t-1}^2\right)\right) + \underline{\phi}_2' y_{t-1}\left(1-\exp-\left(\gamma\left(y_{t-1}^2\right)\right)\right)\right.$$

(i) $$y_{tNaira(S-G)} = \sigma_t\varepsilon_t + \underset{(1.297588)}{5.384416*}\, y_{t-1}\left(1-G_t\right) + \underset{(0.005048)}{0.003043*}\, y_{t-1}\left(G_t\right)$$

with the variance 64.8983

(ii) $$y_{tPound(S-G)} = \sigma_t\varepsilon_t + \underset{(0.007401)}{0.097709*}\, y_{t-1}\left(1-G_t\right) + \underset{(0.003444)}{0.481528*}\, y_{t-1}\left(G_t\right)$$

with the variance 0.0037

(iii) $$y_{tPula(S-G)} = \sigma_t\varepsilon_t + \underset{(0.961958)}{9.184421*}\, y_{t-1}\left(1-G_t\right) + \underset{(0.084636)}{5.236047*}\, y_{t-1}\left(G_t\right)$$

with the variance 45.8382

(iv) $$y_{tYen(S-G)} = \sigma_t\varepsilon_t + \underset{(0.013318)}{3.397120*}\, y_{t-1}\left(1-G_t\right) - \underset{(1.425775)}{2.6550*}\, Q_t y_{t-1}\left(G_t\right)$$

with the variance 126.2495

4.2 ESTAR-GARCH for all Series

$$y_{t(S-G)} = \sigma_t\varepsilon_t + \underline{\phi}_1' y_{t-1}\left(-\exp-\left(\gamma\left(y_{t-1}-c\right)^2\right) + \underline{\phi}_2' y_{t-1}\left(1-\exp-\left(\gamma\left(y_{t-1}-c\right)^2\right)\right)\right.$$

(i) $$y_{tNaira(S-G)} = \sigma_t\varepsilon_t + \underset{(0.005034)}{0.003033*}\, y_{t-1}\left(1-G_t\right) + \underset{(1.070787)}{-4.738773*}\, y_{t-1}\left(G_t\right)$$

with the variance 64.5584

(ii) $$y_{tPound(S-G)} = \sigma_t\varepsilon_t + \underset{(0.005207)}{0.544219*}\, y_{t-1}\left(1-G_t\right) + \underset{(0.014678)}{0.899484*}\, y_{t-1}\left(G_t\right)$$

with the variance 0.0030

(iii) $$y_{tPula(S-G)} = \sigma_t\varepsilon_t + \underset{(0.083324)}{5.170756*}\, y_{t-1}\left(1-G_t\right) + \underset{(0.633186)}{1.276754*}\, y_{t-1}\left(G_t\right)$$

with the variance 43.4726

(iv) $$y_{tYen(S-G)} = \sigma_t\varepsilon_t + \underset{(0.000204)}{2.877449*}\, y_{t-1}\left(1-G_t\right) + \underset{(0.004776)}{1.014081*}\, y_{t-1}\left(G_t\right)$$

with the variance 125.6686

4.3 LSTAR-GARCH for all Series

$$y_{t(S-G)} = \sigma_t\varepsilon_t + \underline{\phi}_1' y_{t-1}\left(1-\left(1+\exp-\left(\gamma\left(y_{t-1}-c\right)\right)^{-1}\right) + \underline{\phi}_2' y_{t-1}\left(1+\exp-\left(\gamma\left(y_{t-1}-c\right)\right)^{-1}\right.\right.$$

(i) $$y_{tNaira(S-G)} = \sigma_t\varepsilon_t + \underset{(1.137383)}{26.86795*}\, y_{t-1}\left(1-G_t\right) + \underset{(0.000283)}{0.000140*}\, y_{t-1}\left(G_t\right)$$

with the variance 29.7358

(ii) $$y_{tPound(S-G)} = \sigma_t\varepsilon_t - \underset{(0.026206)}{0.585333*}\, y_{t-1}\left(1-G_t\right) + \underset{(0.014403)}{0.901189*}\, y_{t-1}\left(G_t\right)$$

with the variance 0.0001

(iii) $y_{tPula(S-G)} = \sigma_t \varepsilon_t + \underset{(0.579097)}{14.35766}{*}\, y_{t-1}\left(1-G_t\right) + \underset{(0.006820)}{0.316239}{*}\, y_{t-1}\left(G_t\right)$

with the variance 19.8816

(iv) $y_{tYen(S-G)} = \sigma_t \varepsilon_t + \underset{(0.001578)}{1.645026}{*}\, y_{t-1}\left(1-G_t\right) + \underset{(0.007704)}{1.040716}{*}\, y_{t-1}\left(G_t\right)$

with the variance 122.3990.

4.4 EAR-GARCH Model Tables for all Series

Table 4: Fitted model for EAR-GARCH series

SERIES	COEFFICIENT (SE) C(1)	C(2)	MODEL VARIANCE
NAIRA	$\underset{(1.29759)}{5.38442}$	$\underset{(0.00505)}{0.00304}$	64.8983
POUND	$\underset{(0.007401)}{0.09771}$	$\underset{(0.00344)}{0.48153}$	0.0037
PULA	$\underset{(0.96196)}{9.18442}$	$\underset{(0.08464)}{5.23605}$	45.8382
YEN	$\underset{(0.02130)}{3.39712}$	$\underset{(0.00137)}{-2.65500}$	126.2495

4.5 ESTAR-GARCH Model Tables for all Series

Table 5: Fitted model for ESTAR-GARCH series

SERIES	COEFFICIENT (SE) C(1)	C(2)	MODEL VARIANCE
NAIRA	$\underset{(0.00503)}{0.00303}$	$\underset{(1.07078)}{-4.73877}$	64.5584
POUND	$\underset{(0.00521)}{0.54422}$	$\underset{(0.01468)}{0.89948}$	0.0030
PULA	$\underset{(0.08332)}{5.17076}$	$\underset{(0.63319)}{1.27675}$	43.4726
YEN	$\underset{(0.00020)}{-2.87745}$	$\underset{(0.00478)}{-1.01408}$	125.6686

4.6 Logistic-GARCH Model Tables for all Series

Table 6: Fitted model for LSTAR-GARCH series

SERIES	COEFFICIENT (SE) C(1)	C(2)	MODEL VARIANCE
NAIRA	$\underset{(0.00503)}{0.00303}$	$\underset{(1.07078)}{-4.73877}$	29.7358
POUND	$\underset{(0.00521)}{0.54422}$	$\underset{(0.01468)}{0.89948}$	0.0001
PULA	$\underset{(0.08332)}{5.17076}$	$\underset{(0.63319)}{1.27675}$	19.8816
YEN	$\underset{(0.00020)}{-2.87745}$	$\underset{(0.00478)}{-1.01408}$	122.3990

4.7 Table Comparing the Variances of all Series with GARCH Model

Table (7) shows the variances of all STAR-GARCH models with GARCH, it is quite evident that all STAR-GARCH actually outperformed the classical GARCH model, however, the LSTAR-GARCH performed best and closely followed by ESTAR-GARCH, this is followed by EAR-GARCH, the implication of this is that for would be forecaster, the use of LSTAR-GARCH produced the best result. However, researcher can equally make do with ESTAR as its performance could be considered as well. LSTAR-GARCH is strongly recommended for optimum result.

Table 7: Variances of all series with GARCH model

SERIES	GARCH MODEL	EAR-GARCH	ESTAR-GARCH	LSTAR-GARCH
NAIRA	4949.2041	64.8983	64.5584	29.7358
POUND	0.6582	0.0037	0.0030	0.0001
PULA	2.1444	45.8382	43.4726	19.8816
YEN	5461.2603	126.2495	125.6686	122.3990

5 Conclusion

In table 7, the variances of all STAR-GARCH models with GARCH are displayed, it is quite evident that all STAR-GARCH outperformed the classical GARCH model, however, the LSTAR-GARCH performed best and closely followed by ESTAR-GARCH, this is followed by EAR-GARCH. The implication is that the use of LSTAR–GARCH produces the best result; however ESTAR may be utilized in some occasions. But LSTAR would produce optimal result.

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