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Measuring Credit Risk from annual statements - The case of Greek Banks

Eleftherios Vlachostergios[1]

Abstract

In this study we present a framework for the approximation of a commercial Bank's Credit Portfolio Risk. The proposed procedure would be particularly useful to external investors, as it is fairly simple and has minimal data and cost requirements.

The quantification of Credit Risk should incorporate:

- The additional provisions required to absorb expected future losses
- The Bank's ability to cover these losses, given its current infrastructure and business model

The above-mentioned info is sufficiently captured by the proposed BCRC index.

As an application, Credit Risk measurements for the four Greek systemic Banks are provided: National Bank of Greece, EFG Euro Bank, Alpha Bank and Piraeus Bank, for 2014-2016 period.

JEL classification numbers: G24, G32
Keywords: Credit Risk, Banking, Expected Credit losses, Capital Requirements, Risk Management, Moore-Penrose Inverse, BCRC Index

1 Relation to Current & Previous Work

Numerous studies as well as a large part of the Banking Risk industry are concerned with the application of transition matrices as a tool to measure the future

[1] National Bank of Greece, Greece

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evolution of banking portfolios, mainly employing the concept of internal or external rating grades.

A comparably large number of publications lies in the effort of analyzing a Bank's financial statements and modelling the most promising indices that point to possible failure.

The Basel Committee relies on the concept of PD, LGD and EAD parameters calculation and the subsequent Capital Adequacy Ratio comprised from the evaluation of those parameters.

EBA, ECB and SSM have developed biannual European-wide, data intensive bank tests that consist of basic and adverse macro scenarios.

Finally, the IFRS framework, has put on the map the concept of lifetime expected losses quantification and measurement.

In the current study exist concepts from all the prevailing trends in the banking practice and literature. Specifically:

- The concept of a 3-state model is deployed, constructing an accruing portfolio segment, a non-accruing one and a middle segment
- From the observation of a Bank's financial statements at different time snapshots, implied transition flows are recovered and their long-term equilibrium is examined
- The calculated transition flows are associated to the macroeconomic environment, and afterwards recalculated and accordingly weighted under possible adverse macroeconomic outcomes
- Latest available financial statement ratios describe the Bank's business model and contribute to the assessment of whether it is possible to overcome the additionally required provisions
- To conclude, this procedure transforms into a single index that is an intuitive credit risk measure

The theoretical approach just described is tested and applied to the Greek Banking system in the 2014-2016 period.

2 Methodology overview

An overview of the procedure used in order to be able to asses each Bank's credit standing, is presented next:

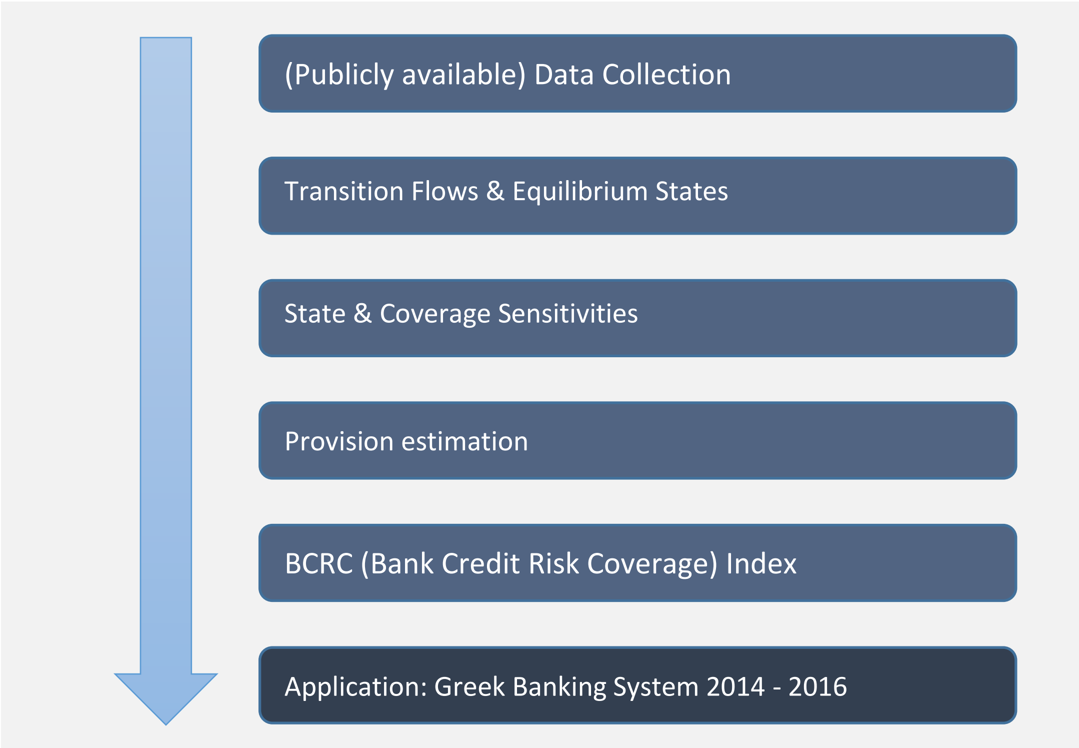

Figure 1: Theoretical Framework & application

3 Data Collection

3.1 Bank specific information

All data is collected from publicly available Bank annual reports. It should be noted that in financial institutions are not treated as groups. To obtain proper results for a multinational financial group we advise that affiliate companies are examined separately in their relevant national macro-environment.

3.1.1 Portfolios definition

The analysis is conducted on a portfolio basis and afterwards added up to comprise the Bank total additional provisions required. The portfolio definitions[2] are the ones provided in the Bank respective annual reports. The portfolios considered follow the segmentation:

[2] Government portfolio should be assigned the same sensitivity for all Banks in the same macro environment

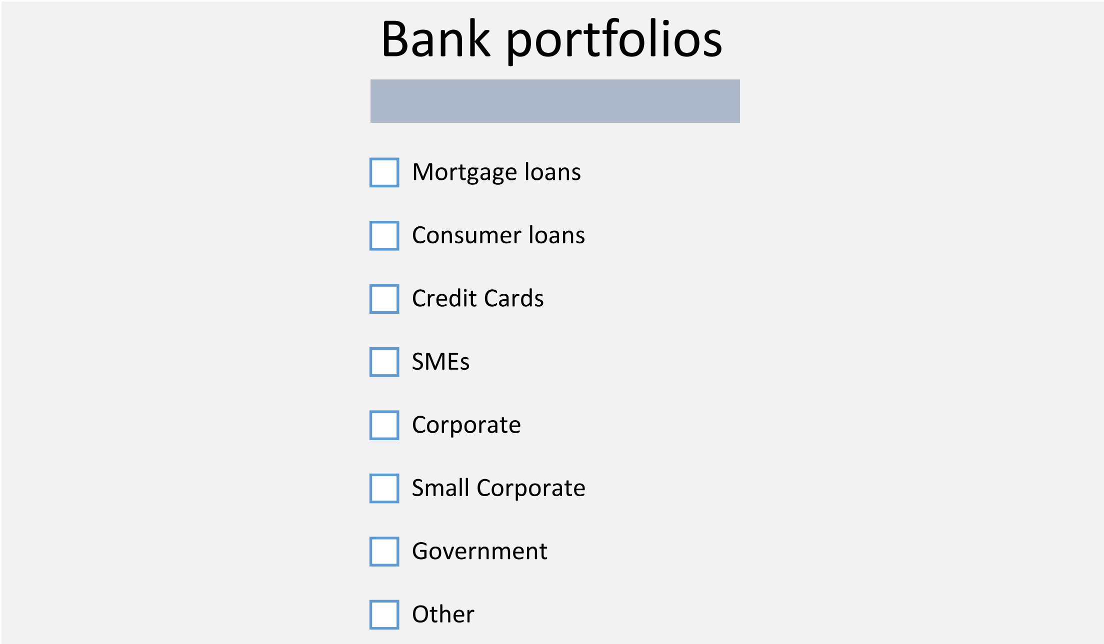

Figure 2: Bank portfolios segmentation

3.2 Economy related data

To account for sensitivities to the macro environment fluctuations, we employ the annual GDP growth, as defined in the appendix 9.4 (A_14), calculated from data provided by the corresponding National Statistical Authorities. Unemployment data series may be used as an alternative. It would be advisable not to use both since there appears to be no incremental benefit to the analysis, since the two series are highly (negatively) correlated.

4 Transition Flows & Equilibrium States

4.1 Portfolio states

The exposure of each portfolio, at a specific point in time, is divided into 3 states $S_j \quad j = 1,2,3$, according underlying philosophy that:

- The worst state, S_3 should enclose all exposures that are non-accruing and highly unlikely to return to normality.
- Intermediate state S_2 should include troubled exposures which could still return to normality. Any kind of adjustment to the initial terms of the loan signifies a troubled asset. As troubled are also considered the loans against which the Bank holds any amount of provisions (impaired loans).
- Finally, S_1 is a leftover of the above 2 states, indicating accruing loans.

We use the following definitions in order to construct three portfolio states:

$I =$ Total net amount (provisions not included) of impaired loans for the portfolio, against which provisions are held

$I_{180} =$ Net amount of impaired loans with past due over 180 days

$NI =$ Total amount of not impaired loans for the portfolio

$NI_{30-180} =$ Amount of not impaired loans with past due between 30 and 180 days

$NI_{180} =$ Amount of not impaired loans with past due over 180 days

$Restr =$ Total net amount (provisions not included) of restructured or rescheduled loans for the portfolio

The net amount at each portfolio state $S_j \quad j = 1,2,3$ is allocated sequentially, as follows:

$$S_3 = I_{180} + NI_{180}$$
$$S_2 = \max\{I + NI_{30-180}, Restr\} - S_3 \quad (R\_1)$$
$$S_1 = NI + I - (S_2 + S_1)$$

In general, provisions should be generic and characterize the total portfolio, thus total provisions amount is reassigned with exponential weights from best to worst state as described in the appendix 9.1 (A_1).

If $Prov =$ Total amount of provisions for the portfolio, as they appear on annual statements, the state amounts for each portfolio are recalculated:

$$S_1{}' = S_1 + w_1 \cdot Prov$$
$$S_2{}' = S_2 + w_2 \cdot Prov \quad (R\_2)$$
$$S_3{}' = S_3 + w_3 \cdot Prov$$

The next step is to the annual transition process among portfolio states.

4.2 State pseudo-flows estimation & equilibrium

In order to obtain transition flows among states, we need to add the time dimension. Let us symbolize

$t = 0$ Period start

$t = 1$ Period end

$S_{i,t}{}' =$ Portfolio exposures at state $i, i = 1,2,3,$ at point t

Furthermore, to compensate for any large portfolio additions or reduction effects[3] the adjustments described in the appendix 9.2 are applied.

[3] e.g. Mergers

Essentially, we need to estimate nine transition flows, from initial to period final states, as depicted below (see 9.2 (A_3) for the notation), where

f_{ij} = Amounts flow from initial portfolio state i, at start year $t = 1$ to the end of year portfolio state j at end year $t = 2$

	E_2	R_2	D_2
E_1	f_{11}	f_{12}	f_{13}
R_1	f_{21}	f_{22}	f_{23}
D_1	f_{31}	f_{32}	f_{33}

Figure 3: Flows between portfolio states

The above matrix can be analyzed into a system of 6 equations with 9 unknowns (appendix 9.3.1 (A_4)). To obtain a unique analytical solution we use the concept of the Moore-Penrose "pseudo" inverse matrix, as described in appendix section 9.3 leading to the analytical relationship (A_7).and apply, if necessary, the subsequent normalization of section 9.3.5 $(A\_8 - A\_9)$. The final result resembles a stochastic matrix, a fact that provides us with a theoretical equilibrium state as shown in 9.3.6 (A_12).

The elements f_{ij} of the quasi-stochastic matrix are labeled "pseudo-flows" as they were extracted indirectly. Nevertheless, they give a representation of reality as they match the original to the final state of the portfolio exposures.

At equilibrium the gross amount of the portfolio will be allocated initially to the three states as depicted in table A_{eq} (9.3.6 (A_12))

	E_2	R_2	D_2
$Gross\ Portfolio\ Value_1$	f_E	f_R	f_D

Figure 4: Portfolio states intermediate equilibrium table

Where

f_E = The percentage of initial gross portfolio exposure that will remain accruing (state 1)

f_R = The percentage of initial gross portfolio exposure that will remain in a "frictional" state between accruing and non-accruing (state 2)

$f_D =$ The percentage of initial gross portfolio exposure that will end up non-accruing (state 3)

With the use of a geometric progression for the purpose of provisions calculation, we end up with the long-term equilibrium 9.3.6 (A_13)

	E_2	R_2	D_2
$Gross\ Portfolio\ Value_1$	F_E	0	F_D

Figure 5: Portfolio states final equilibrium without the intermediate state

5 Provision requirements

For the calculation of provisions, it is essential to estimate the exposures that we expect to end up in state D and use the $A_{LT} = (F_E \quad F_D)$ vector, as described in 9.3.6 (A_13).

Additionally, we are obliged to take into account the possible changes in the equilibrium that we have calculated, as well as the variations of the collateral covers of the portfolio exposures.

5.1 The economy factor

The economy macro factor will be measured by the annual GDP growth g, as in 9.4 (A_14). Through the economic cycle, it is valid to assume $g \longrightarrow N(0, \sigma)$ where σ will be approximated by the standard deviation s of the standard normal distribution that best fits the observable GDP growth rate distribution.

The constructed growth rate distribution $N(0, s)$ has k intervals with g_k, p_k the central GDP growth interval value and the probability of occurrence respectively, that is to say we have k distinct expected states of the economy.

5.2 Equilibrium State Sensitivities

With the process defined in 9.4, long term equilibrium percentages are actually turned into functions of the main macro variable, GDP annual growth, enabling the calculation of extra provisions required in adverse macroeconomic conditions as concluded in (A_21).

5.3 Expected Credit Portfolio Losses

With the consideration of recoveries adjustments 9.5 and if $pf_{j,t} =$ Gross portfolio j exposures as described in 4.1 (R_2), the expected credit portfolio losses for scenario k, at time snapshot t are expressed:

$$ECL_{j,t,k} = pf_{j,t} \cdot F_{D,j,LT,k} \cdot LGD_{eq,t,k} = pf_{j,t} \cdot \left(1 - F_{E,j,LT,k}\right) \cdot LGD_{eq,t,k} \quad (R\_3)$$

The provisions to be held additionally is the value of ECL over the provisions already held (provisions value on the annual statements)

$Prov_j =$ Total amount of provisions for the portfolio, as they appear on annual statements for portfolio j at time snapshot t

$$ScenarioProvisions_{j,t,k} = ECL_{j,t,k} - Prov_{j,t} \quad (R\_4)$$

5.4 Bank provision requirement

$t =$ Currently past year, where t annual report data where the last available input

$j = 1,..,J$ The Bank portfolios

$k = 1, ... K$ Possible states of the economy, according to (5.1)

$Prov_{j,t} =$ The amount of provisions already held aside for the portfolio at time t

$pf_{j,t} =$ Portfolio amounts at time t

$LGD_{eq,t,j,k} =$ The LGD value corresponding to the economy state 9.5, for portfolio j at time snapshot t (A_27)

Total bank provisions for g_k assumed GDP growth rate, with probability p_k, are:

$$ScenarioProvisions_{t,k} = \sum_{j=1}^{J} \left(pf_{j,t} \cdot \left(1 - F_{E,j,LT,k}\right) \cdot LGD_{eq,t,j,k} - Prov_{j,t}\right) \quad (R\_5)$$

Total Bank provisions over all scenarios, are:

$$Provisions_t = \sum_{k=1}^{K} p_k \cdot max\left\{ScenarioProvisions_{t,k}, 0\right\} \quad (R\_6)$$

In section 9.7 we give an example of a hypothetical 1-portfolio Bank. Results are consistent to business logic, assigning higher portfolio provisions where required.

6 The BCRC (Bank Credit Risk Coverage) Index

The purpose of the Index construction is to assess:

- if the extra provisions requirement shock can be absorbed at all
- In how much time the extra provisions requirement shock can be absorbed smoothly by the Bank's ongoing operations

6.1 Income statement reordering

If $n = 1, ..., N$ is the number of Bank portfolios at time t

$pf_{n,t} =$ The gross exposure value of the portfolio at time t, as calculated in 3.1.1

$E_{i,t} =$ The accruing percent of the exposure value of the portfolio at time t (state 1)

$R_{i,t} =$ The troubled but still accruing percent of the exposure value of the portfolio at time t (state 2)

Historical income producing assets are:

$$IncomeProducingAssets_t = \sum_{n=1}^{N} pf_{n,t} \cdot \left(E_{i,t} + R_{i,t}\right) \quad (R\_7)$$

After expressing the income statement in a simple condensed manner 9.6, the subsequent ratios are formed, following the notation of 9.6 at time t

$$IncGen_t = \frac{IntIncome_t}{IncomeProducingAssets_t} \quad (R\_8)$$

$$grOI_t = \frac{GrOperIncome_t}{IntIncome_t} \quad (R\_9)$$

$$opexp_t = \frac{OperExp_t}{GrOperIncome_t} \quad (R\_10)$$

$$otherexp_t = \frac{OtherExp_t}{GrOperIncome_t} \quad (R\_11)$$

For the other non-recurring expenses, we assume a zero mean but use -1 standard deviation in our estimates, as a reducing factor.

6.2 BCRC Index calculation

The Bank's profitability index pi for 1€ of performing assets at time snapshot t is defined as:

$$pi_t = IncGen_t \cdot grOI_t \cdot (1 - opexp_t) \cdot \left(1 - stdev_{otherexp_t}\right) \cdot \left(1 - TaxRate^4\right) \quad (R\_12)$$

The Bank assets at equilibrium, as estimated at time t are defined by the calculated long-term equilibrium percentages for each portfolio 9.4 (A_21)

If $n = 1, \dots, N$ is the number of portfolios at time t

$pf_{n,t} =$ The exposure value of the portfolio n at time t

$F_{E,n,LT,k} =$ The long-term equilibrium percent of portfolio n that will end up accruing, assuming g_k annual GDP growth rate for next year

[4] For the purpose of application to the 4 major Greek banks a tax rate of 29% was used

$k = 1 \dots K$ Is the number of intervals for the approximated GDP annual growth rate distribution

$p_k =$ The probability that next year GDP growth will fall into interval k, which satisfies the condition

$$\sum_{k=1}^{K} p_k = 1$$

We define the following modified probabilities, following the definitions of 5.4

$$\pi_k = \begin{cases} 0 & ScenarioProvisions_{t,k} \leq 0 \\ p_k & ScenarioProvisions_{t,k} > 0 \end{cases} \qquad \pi_k{}' = \frac{\pi_k}{\sum_{k=1}^{K} \pi_k} \quad \sum_{k=1}^{K} \pi_k{}' = 1 \quad (R\_13)$$

Total bank performing assets for g_k assumed GDP growth rate, with probability p_k, are

$$ScenarioPerformingAssets_{t,k} = \sum_{n=1}^{N} pf_{n,t} \cdot F_{E,n,LT,k} \quad (R\_14)$$

Considering all possible states k

$$PerformingAssets_t = \sum_{k=1}^{K} \pi_k{}' \cdot ScenarioPerformingAssets_{t,k} \quad (R\_15)$$

The above assets will contribute to the Bank's profitability. The equilibrium profits available for extra provisions coverage, as measured at time t will be:

$$Profit_t = pi_t \cdot PerformingAssets_t \quad (R\_16)$$

In case $Profit_t = 0$ we set $Profit_t = 1€$

The total extra required provisions $Provisions_t$, were defined at 5.4. In case $Provisions_t = 0$ we set $Provisions_t = 1€$

It is time to synthesize the $BCRC_t \in [-100{,}100]$ index as an indication of the Bank's credit risk standing

$$BCRC_t = \begin{cases} max\left\{\dfrac{Provisions_t}{Profit_t}, -100\right\} & Profit_t < 0 \\ 100 - min\left\{\dfrac{Provisions_t}{Profit_t}, 100\right\} & Profit_t > 0 \end{cases} \quad (R\_17)$$

$BCRC_t < 0$

The Bank's current operating model is not sufficient to provide for any additional coverage against credit risk. The extend of credit risk exposure is revealed by the index value

$BCRC_t > 0$

The Bank appears as not adequately provisioned. Its internal ability for absorbing the extra credit risk is pointed out by the index value.

Intuitively as $BCRC_t \longrightarrow 100$ its credit risk position is minimized.

7 Application to the Greek Banking System

7.1 Data

Before presenting the methodology applied results, all related data is exhibited.

7.1.1 Bank Data

As pointed out in 3.1 all relevant data was downloaded from the corresponding Banks websites, for the period 2012-2016[5]. The reporting format of annual statements changes for the years previous to 2012, not enabling a valid portfolio segmentation according to 4.1 (R_2).

Since two equilibrium points are required in order to determine equilibrium sensitivities, credit risk was quantified for end of years 2014 – 2016.

Banks gross loan portfolios (provisions included) for the period of interest are presented below:

[5] Data collection took place in 12/2017 so 31/12/2016 was the last available date.

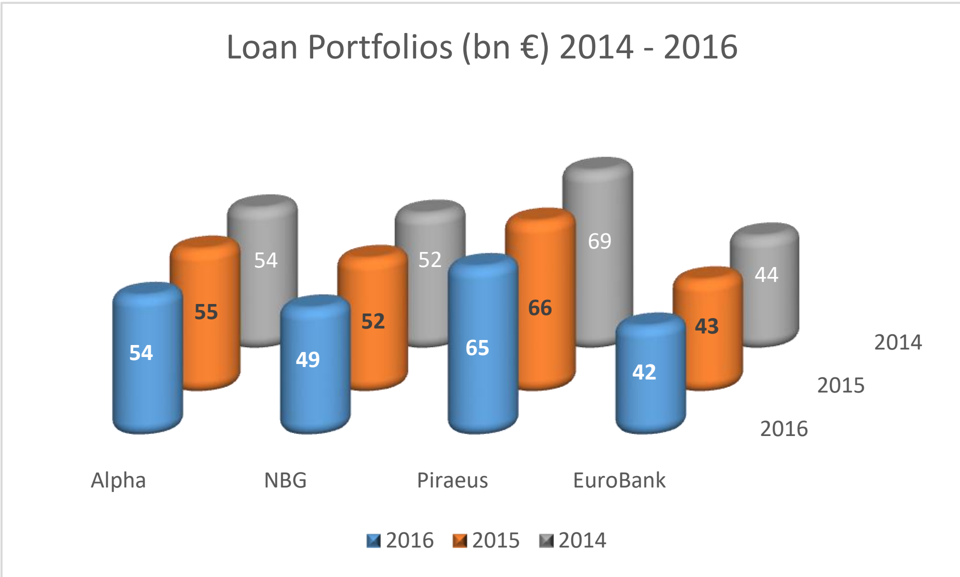

Figure 6: Gross Loan Portfolio Values evolution for major Greek Banks

7.1.2 Macroeconomic data

Macroeconomic data concerning the Greek GDP were downloaded from Hellenic Statistical Authority website. Q4 GDP values, with annual frequency, were selected for the macro factor representation. The data series is depicted on the graph below:

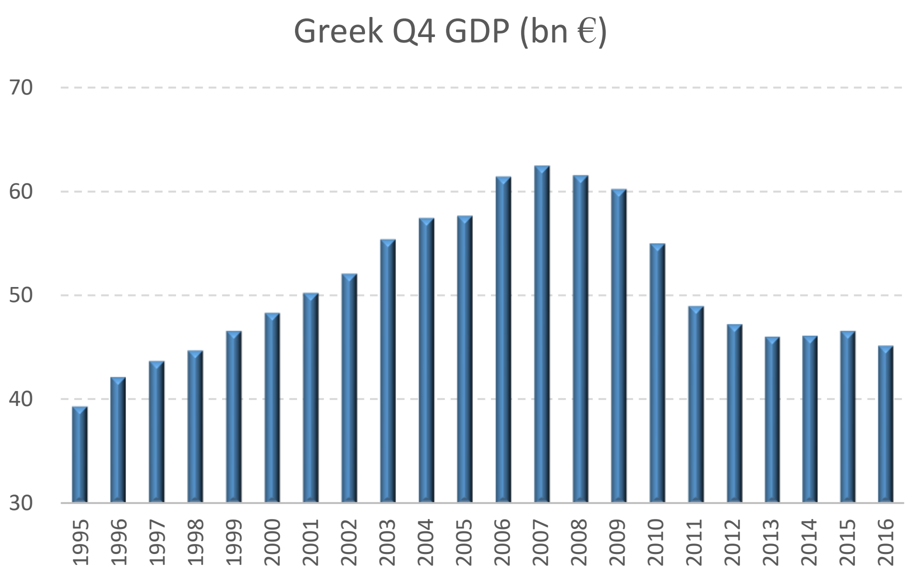

Figure 7: Greek Q4 GDP time series

Bank of Greece's Urban Real Estate Index was selected as a frame of reference that captures fluctuations of collateral coverage values.

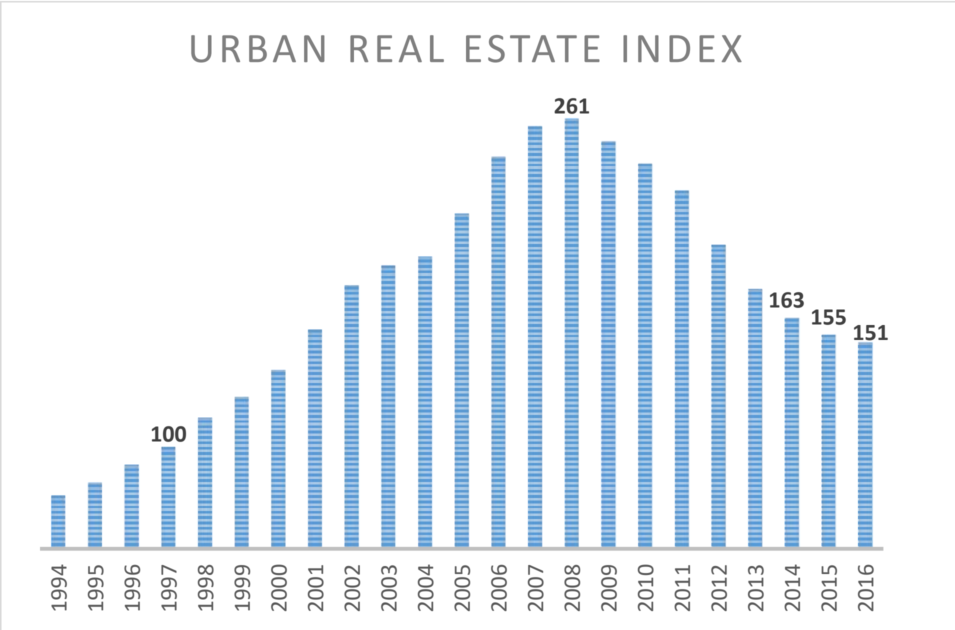

Figure 8: Bank of Greece Urban Real-Estate Index (Base Value 1997 = 100)

The Urban Real Estate Index is closely related to the GDP values. The statistical relationship between the percentage changes (notation of 9.5 (A_24)) is

t	$b_{C,t}$	$t-stat$	I_t	I_{base}	I_{max}
2014	1,428	4,46	163,28	100	261,06
2015	1,414	4,43	154,96	100	261,06
2016	1,402	4,54	151,28	100	261,06

Figure 9: Urban Real Estate Index & GDP Q4 relationship

Additionally, for scenario application, each year a zero-mean normal distribution was used (5.1) to capture the possible values of Q4 GDP growth rates. The distributions for the period 2014-2016 are presented in section 9.8.

t	$mean$	$stdev$	$Sample$
2014	0,00	0,0501	19
2015	0,00	0,0483	20
2016	0,00	0,0447	21

Figure 10: Normal distributions used in provisions calculation

In essence each of the 10 distribution intervals used represents a possible scenario for the future, at each year end 2014-2016.

7.2 Final Results

The methodology established up to this point in 2-6.2, along with the appendix 9.1-9.8 and the data specifications of the previous segment 7.1 have led to the final results, concerning the evaluation of the 4 systemic Greek Bank Institutions for the period 2014 – 2016, which are cited in amore analytic fashion appendix segment 9.9

Main results are presented here. First the time evolution of the actual amount of additional provisions estimated is exposed below.

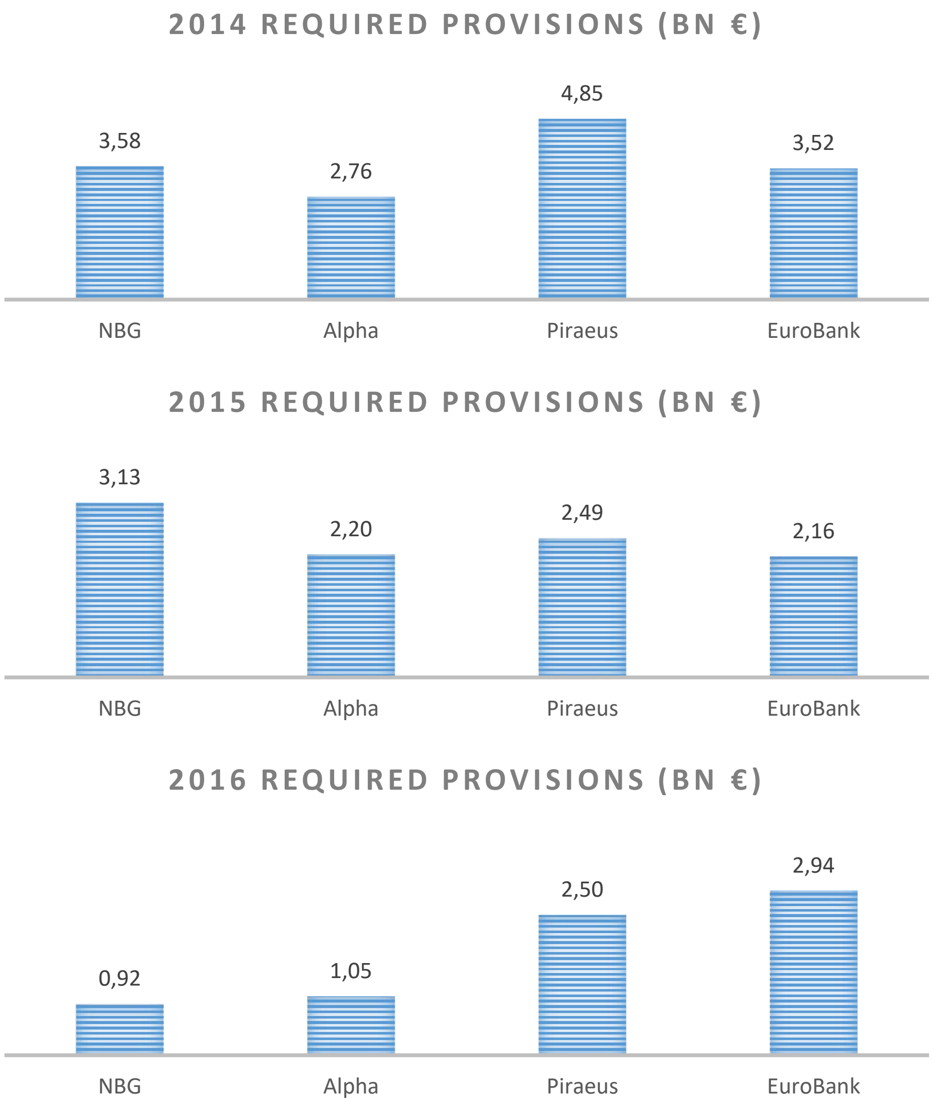

Figure 11: Additional provisions amounts estimated for major Greek Banks 2014-2016 in bn €

We should note that the absolute provisions amount should not be directly compared among banks. What is comparable is the BCRC Index Presented next in a common graph, representing the improvement and rationalization of the Greek Banking System during the period of 2014-2016.

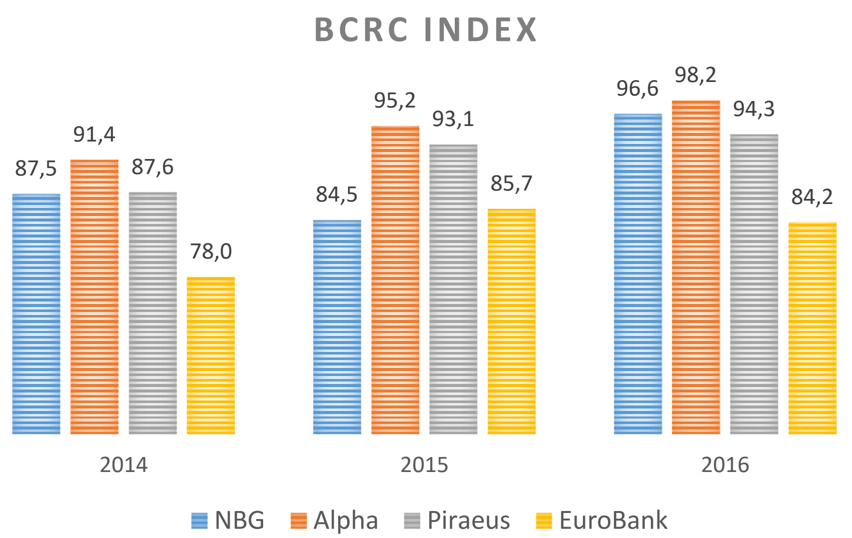

Figure 12: Credit Risk evaluation for major Greek Banks 2014-2016

8 Conclusion

In this study we examined an alternative method for measuring Credit Risk of commercial Banks that relies on publicly available info. The method results in the construction of an annually constructed index (BCRC) which encapsulates both the additional provisions requirement under stressed conditions, and the ability of a financial institution to cover this requirement undercurrent ongoing operations.

BCRC proposed index could of use to

- external investors, as it is a comparable measure among financial institutions, particularly of those operating under the same macro environment
- central authorities and policy makers, as a simple precursor credit risk measure, complementary to the data intensive controls (e.g. Asset Quality Reviews) applied in the Banking Business

The application of the methodology on the Greek Banking system during the period of 2014 – 2016 produced reasonable results, indicating and encouraging course of the post crisis banking system towards stability.

9 Appendix

9.1 Provision allocation weights

With 1 being the best loan state, the total provisions concerning the portfolio are reallocated respectively with weights:

$i =$ Loan state

$$w_i = \frac{e^i}{\sum_{j=1}^{3} e^j} \longrightarrow \begin{cases} w_1 = 0{,}09003 \\ w_2 = 0{,}24473 \\ w_3 = 0{,}66524 \end{cases} \quad (A\_1)$$

9.2 Unchanged portfolio amounts

In order to ensure the condition $\sum_{i=1}^{3} S_{i,0}{}' = \sum_{i=1}^{3} S_{i,1}{}'$ the portfolio state exposures are adjusted

Total amounts

$$S_1 = \sum_{i=1}^{3} S_{i,1}{}' \quad S_0 = \sum_{i=1}^{3} S_{i,0}{}'$$

The difference of final minus initial amount

$$d = S_1 - S_0$$

The average percentages to each stage, through the period

$$p_1 = \frac{\frac{S_{1,1}{}'}{S_1} + \frac{S_{1,0}{}'}{S_0}}{2} \quad p_2 = \frac{\frac{S_{2,1}{}'}{S_1} + \frac{S_{2,0}{}'}{S_0}}{2} \quad p_3 = \frac{\frac{S_{3,1}{}'}{S_1} + \frac{S_{3,0}{}'}{S_0}}{2}$$

The calculated adjustment amounts are

$$a_{1,1} = \begin{cases} -d \cdot p_1 & d < 0 \\ 0 & d \geq 0 \end{cases} \quad a_{2,1} = \begin{cases} -d \cdot p_2 & d < 0 \\ 0 & d \geq 0 \end{cases} \quad a_{3,1} = \begin{cases} -d \cdot p_3 & d < 0 \\ 0 & d \geq 0 \end{cases}$$

$$a_{1,0} = \begin{cases} d \cdot p_1 & d \geq 0 \\ 0 & d < 0 \end{cases} \quad a_{2,0} = \begin{cases} d \cdot p_2 & d \geq 0 \\ 0 & d < 0 \end{cases} \quad a_{3,0} = \begin{cases} d \cdot p_3 & d \geq 0 \\ 0 & d < 0 \end{cases}$$

Each state exposure is adjusted accordingly:

$$S_{i,t}{}'' = S_{i,t}{}' + a_{i,t} \quad (A\_2)$$

We rewrite the state exposure symbolisms

$$\begin{matrix} E_1 = S_{1,0}{}'' & R_1 = S_{2,0}{}'' & D_1 = S_{3,0}{}'' \\ E_2 = S_{1,1}{}'' & R_2 = S_{2,1}{}'' & D_2 = S_{3,1}{}'' \end{matrix} \quad (A\_3)$$

9.3 Estimation of pseudo – flow transitions

9.3.1 Equations formation

All initial amounts should be allocated to a stage

$$
\begin{aligned}
&f_{11}\cdot E_1 + f_{12}\cdot E_1 + f_{13}\cdot E_1 = E_1 \Rightarrow f_{11}+f_{12}+f_{13}=1\\
&f_{21}\cdot R_1 + f_{22}\cdot R_1 + f_{23}\cdot R_1 = R_1 \Rightarrow f_{21}+f_{22}+f_{23}=1 \qquad (A\_4)\\
&f_{31}\cdot D_1 + f_{32}\cdot D_1 + f_{33}\cdot D_1 = D_1 \Rightarrow f_{31}+f_{32}+f_{33}=1
\end{aligned}
$$

The final observed amounts are formed from the initial amounts

$$
\begin{aligned}
&f_{11}\cdot E_1 + f_{21}\cdot R_1 + f_{31}\cdot D_1 = E_2 \Rightarrow f_{11} + f_{21}\cdot\left(\frac{R_1}{E_1}\right) + f_{31}\cdot\left(\frac{D_1}{E_1}\right) = \frac{E_2}{E_1}\\
&f_{12}\cdot E_1 + f_{22}\cdot R_1 + f_{32}\cdot D_1 = R_2 \Rightarrow f_{12} + f_{22}\cdot\left(\frac{R_1}{E_1}\right) + f_{32}\cdot\left(\frac{D_1}{E_1}\right) = \frac{R_2}{E_1} \qquad (A\_5)\\
&f_{13}\cdot E_1 + f_{23}\cdot R_1 + f_{33}\cdot D_1 = D_2 \Rightarrow f_{13} + f_{23}\cdot\left(\frac{R_1}{E_1}\right) + f_{32}\cdot\left(\frac{D_1}{E_1}\right) = \frac{D_2}{E_1}
\end{aligned}
$$

Replacements for normalization purposes, assuming always $E_1 > 0$ value.

$$
\frac{R_1}{E_1} = a \quad \frac{D_1}{E_1} = b \quad \frac{E_2}{E_1} = c \quad \frac{R_2}{E_1} = d \quad \frac{D_2}{E_1} = e \qquad (A\_6)
$$

And in matrix notation

$$
A = \begin{pmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1\\
E_1 & 0 & 0 & R_1 & 0 & 0 & D_1 & 0 & 0\\
0 & E_1 & 0 & 0 & R_1 & 0 & 0 & D_1 & 0\\
0 & 0 & E_1 & 0 & 0 & R_1 & 0 & 0 & D_1
\end{pmatrix} \rightarrow
\begin{pmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1\\
1 & 0 & 0 & a & 0 & 0 & b & 0 & 0\\
0 & 1 & 0 & 0 & a & 0 & 0 & b & 0\\
0 & 0 & 1 & 0 & 0 & a & 0 & 0 & b
\end{pmatrix}
$$

$$
x = \begin{pmatrix} f_{11}\\ f_{12}\\ f_{13}\\ f_{21}\\ f_{22}\\ f_{23}\\ f_{31}\\ f_{32}\\ f_{33} \end{pmatrix} \qquad
B = \begin{pmatrix} 1\\ 1\\ 1\\ c\\ d\\ e \end{pmatrix}
$$

$A \cdot x = B \Rightarrow x^T \cdot A^T = B^T \Rightarrow x^T \cdot (A^T \cdot A^+) = B^T \cdot A^+ \Rightarrow x^T = B^T \cdot A^+$

$$A^T = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & a & 0 & 0 \\ 0 & 1 & 0 & 0 & a & 0 \\ 0 & 1 & 0 & 0 & 0 & a \\ 0 & 0 & 1 & b & 0 & 0 \\ 0 & 0 & 1 & 0 & b & 0 \\ 0 & 0 & 1 & 0 & 0 & b \end{pmatrix} = \begin{pmatrix} \vec{\delta}_1 & \vec{\delta}_2 & \vec{\delta}_3 & \vec{\delta}_4 & \vec{\delta}_5 & \vec{\delta}_6 \end{pmatrix}$$

9.3.2 Gram – Schmidt orthogonalization

$$\vec{u}_1 = \vec{\delta}_1 \longrightarrow \vec{q}_1 = \frac{\vec{u}_1}{|\vec{u}_1|} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{\delta}_2 = \left(\vec{\delta}_2 \bullet \vec{q}_1\right) \cdot \vec{q}_1 + \vec{u}_2 \Rightarrow \vec{u}_2 = \vec{\delta}_2 - \left(\vec{\delta}_2 \bullet \vec{q}_1\right) \cdot \vec{q}_1 = \vec{\delta}_2 \qquad \vec{q}_2 = \frac{\vec{u}_2}{|\vec{u}_2|} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{u}_3 = \vec{\delta}_3 - \left(\vec{\delta}_3 \bullet \vec{q}_1\right) \cdot \vec{q}_1 - \left(\vec{\delta}_3 \bullet \vec{q}_2\right) \cdot \vec{q}_2 = \vec{\delta}_3 \qquad \vec{q}_3 = \frac{\vec{u}_3}{|\vec{u}_3|} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$$

$$\vec{u}_4 = \vec{\delta}_4 - \left(\vec{\delta}_4 \bullet \vec{q}_1\right) \cdot \vec{q}_1 - \left(\vec{\delta}_4 \bullet \vec{q}_2\right) \cdot \vec{q}_2 - \left(\vec{\delta}_4 \bullet \vec{q}_3\right) \cdot \vec{q}_3$$

$$\vec{\delta}_4 \bullet \vec{q}_1 = \frac{1}{\sqrt{3}} \quad \vec{\delta}_4 \bullet \vec{q}_2 = \frac{a}{\sqrt{3}} \quad \vec{\delta}_4 \bullet \vec{q}_3 = \frac{b}{\sqrt{3}}$$

$$\vec{u}_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ a \\ 0 \\ 0 \\ b \\ 0 \\ 0 \end{pmatrix} - \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} - \frac{a}{\sqrt{3}} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ 0 \end{pmatrix} - \frac{b}{\sqrt{3}} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ -\frac{1}{3} \\ \frac{2 \cdot a}{3} \\ -\frac{a}{3} \\ -\frac{a}{3} \\ \frac{2 \cdot b}{3} \\ -\frac{b}{3} \\ -\frac{b}{3} \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \\ 2 \cdot a \\ -a \\ -a \\ 2 \cdot b \\ -b \\ -b \end{pmatrix}$$

$$|\vec{u}_4| = \sqrt{\frac{2}{3} \cdot (1 + a^2 + b^2)}$$

$$\vec{q}_4 = \frac{\vec{u}_4}{|\vec{u}_4|} = \frac{1}{\sqrt{6 \cdot (1 + a^2 + b^2)}} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \\ 2 \cdot a \\ -a \\ -a \\ 2 \cdot b \\ -b \\ -b \end{pmatrix}$$

$$\vec{u}_5 = \vec{\delta}_5 - (\vec{\delta}_5 \bullet \vec{q}_1) \cdot \vec{q}_1 - (\vec{\delta}_5 \bullet \vec{q}_2) \cdot \vec{q}_2 - (\vec{\delta}_5 \bullet \vec{q}_3) \cdot \vec{q}_3 - (\vec{\delta}_5 \bullet \vec{q}_4) \cdot \vec{q}_4$$

$$\vec{\delta}_5 \bullet \vec{q}_1 = \frac{1}{\sqrt{3}} \quad \vec{\delta}_5 \bullet \vec{q}_2 = \frac{a}{\sqrt{3}} \quad \vec{\delta}_5 \bullet \vec{q}_3 = \frac{b}{\sqrt{3}} \quad \vec{\delta}_5 \bullet \vec{q}_4 = -\frac{\sqrt{1 + a^2 + b^2}}{\sqrt{6}}$$

$$\vec{u}_5 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ a \\ 0 \\ 0 \\ b \\ 0 \end{pmatrix} - \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} - \frac{a}{\sqrt{3}} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ 0 \end{pmatrix} - \frac{b}{\sqrt{3}} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} + \frac{1}{6} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \\ 2 \cdot a \\ -a \\ -a \\ 2 \cdot b \\ -b \\ -b \end{pmatrix} = \frac{1}{2} \cdot \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ a \\ -a \\ 0 \\ b \\ -b \end{pmatrix}$$

$$|\vec{u}_5| = \sqrt{\frac{1}{2} \cdot (1 + a^2 + b^2)}$$

$$\vec{q}_5 = \frac{\vec{u}_5}{|\vec{u}_5|} = \frac{1}{\sqrt{2 \cdot (1 + a^2 + b^2)}} \cdot \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ a \\ -a \\ 0 \\ b \\ -b \end{pmatrix}$$

$$\vec{u}_6 = \vec{\delta}_6 - (\vec{\delta}_6 \bullet \vec{q}_1) \cdot \vec{q}_1 - (\vec{\delta}_6 \bullet \vec{q}_2) \cdot \vec{q}_2 - (\vec{\delta}_6 \bullet \vec{q}_3) \cdot \vec{q}_3 - (\vec{\delta}_6 \bullet \vec{q}_4) \cdot \vec{q}_4 - (\vec{\delta}_6 \bullet \vec{q}_5) \cdot \vec{q}_5$$

$$\vec{\delta}_6 \bullet \vec{q}_1 = \frac{1}{\sqrt{3}} \quad \vec{\delta}_6 \bullet \vec{q}_2 = \frac{a}{\sqrt{3}} \quad \vec{\delta}_6 \bullet \vec{q}_3 = \frac{b}{\sqrt{3}} \quad \vec{\delta}_6 \bullet \vec{q}_4 = -\frac{\sqrt{1+a^2+b^2}}{\sqrt{6}} \quad \vec{\delta}_6 \bullet \vec{q}_5 = -\frac{\sqrt{1+a^2+b^2}}{\sqrt{2}}$$

$$\vec{u}_6 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ a \\ 0 \\ 0 \\ b \end{pmatrix} - \frac{1}{\sqrt{3}} \cdot \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} - \frac{a}{\sqrt{3}} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ 0 \\ 0 \\ 0 \end{pmatrix} - \frac{b}{\sqrt{3}} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} + \frac{1}{6} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \\ 2 \cdot a \\ -a \\ -a \\ 2 \cdot b \\ -b \\ -b \end{pmatrix} + \frac{1}{2} \cdot \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \\ a \\ -a \\ 0 \\ b \\ -b \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

9.3.3 Moore-Penrose inverse

The normalized matrix

$Q = (\vec{q}_1 \quad \vec{q}_2 \quad \vec{q}_3 \quad \vec{q}_4 \quad \vec{q}_5)$

R triangular matrix

$$R = \begin{pmatrix} \vec{\delta}_1 \bullet \vec{q}_1 & \vec{\delta}_2 \bullet \vec{q}_1 & \vec{\delta}_3 \bullet \vec{q}_1 & \vec{\delta}_4 \bullet \vec{q}_1 & \vec{\delta}_5 \bullet \vec{q}_1 \\ 0 & \vec{\delta}_2 \bullet \vec{q}_2 & \vec{\delta}_3 \bullet \vec{q}_2 & \vec{\delta}_4 \bullet \vec{q}_2 & \vec{\delta}_5 \bullet \vec{q}_2 \\ 0 & 0 & \vec{\delta}_3 \bullet \vec{q}_3 & \vec{\delta}_4 \bullet \vec{q}_3 & \vec{\delta}_5 \bullet \vec{q}_3 \\ 0 & 0 & 0 & \vec{\delta}_4 \bullet \vec{q}_4 & \vec{\delta}_5 \bullet \vec{q}_4 \\ 0 & 0 & 0 & 0 & \vec{\delta}_5 \bullet \vec{q}_5 \end{pmatrix}$$

Set $1 + a^2 + b^2 = y$

$$R = \begin{pmatrix} \sqrt{3} & 0 & 0 & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & \sqrt{3} & 0 & a/\sqrt{3} & a/\sqrt{3} \\ 0 & 0 & \sqrt{3} & b/\sqrt{3} & b/\sqrt{3} \\ 0 & 0 & 0 & \sqrt{2\cdot y/3} & -\sqrt{y/6} \\ 0 & 0 & 0 & 0 & \sqrt{y/2} \end{pmatrix}$$

From the relationship

$R \cdot R^{-1}$ And solving the simple equations we end up with

$$R^{-1} = \begin{pmatrix} 1/\sqrt{3} & 0 & 0 & -1/\sqrt{6\cdot y} & -1/\sqrt{2\cdot y} \\ 0 & 1/\sqrt{3} & 0 & -a/\sqrt{6\cdot y} & -a/\sqrt{2\cdot y} \\ 0 & 0 & 1/\sqrt{3} & -b/\sqrt{6\cdot y} & -b/\sqrt{2\cdot y} \\ 0 & 0 & 0 & \sqrt{3/2\cdot y} & 1/\sqrt{2\cdot y} \\ 0 & 0 & 0 & 0 & \sqrt{2/y} \end{pmatrix}$$

The Moore-Penrose right inverse is given by

$A^{+} = R^{-1} \cdot Q^{T}$

$$B' = \begin{pmatrix} 1 \\ 1 \\ 1 \\ c \\ d \end{pmatrix}$$

$$A^{+} = \begin{pmatrix} (y-1)/3\cdot y & (y-1)/3\cdot y & (y+2)/3\cdot y & -a/3\cdot y & -a/3\cdot y & 2\cdot a/3\cdot y & -b/3\cdot y & -b/3\cdot y & 2\cdot b/3\cdot y \\ -a/3\cdot y & -a/3\cdot y & 2\cdot a/3\cdot y & (y-a^2)/3\cdot y & (y-a^2)/3\cdot y & (y+2\cdot a^2)/3\cdot y & -a\cdot b/3\cdot y & -a\cdot b/3\cdot y & 2\cdot a\cdot b/3\cdot y \\ -b/3\cdot y & -b/3\cdot y & 2\cdot b/3\cdot y & -a\cdot b/3\cdot y & -a\cdot b/3\cdot y & 2\cdot a\cdot b/3\cdot y & (y-b^2)/3\cdot y & (y-b^2)/3\cdot y & (y+2\cdot b^2)/3\cdot y \\ 1/y & 0 & -1/y & a/y & 0 & -a/y & b/y & 0 & -b/y \\ 0 & 1/y & -1/y & 0 & a/y & -a/y & 0 & b/y & -b/y \end{pmatrix}$$

9.3.4 Pseudo-flows analytical expression

$$x^{T} = B'^{T} \cdot A^{+}$$

The parametric expression of the results is

$$x = \begin{pmatrix} \frac{y-(1+a+b)+3\cdot c}{3\cdot y} \\ \frac{y-(1+a+b)+3\cdot d}{3\cdot y} \\ \frac{y+2\cdot(1+a+b)-3\cdot(c+d)}{3\cdot y} \\ \frac{y-a\cdot(1+a+b)+3\cdot a\cdot c}{3\cdot y} \\ \frac{y-a\cdot(1+a+b)+3\cdot a\cdot d}{3\cdot y} \\ \frac{y+2\cdot a\cdot(1+a+b)-3\cdot a\cdot(c+d)}{3\cdot y} \\ \frac{y-b\cdot(1+a+b)+3\cdot b\cdot c}{3\cdot y} \\ \frac{y-b\cdot(1+a+b)+3\cdot b\cdot d}{3\cdot y} \\ \frac{y+2\cdot b\cdot(1+a+b)-3\cdot b\cdot(c+d)}{3\cdot y} \end{pmatrix} = \begin{pmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{pmatrix} \quad (A\_7)$$

9.3.5 Pseudo-flows normalization

$$\sum_{j=1}^{3} f_{ij} = 1 \quad i = 1,2,3$$

In the case of a negative value[6] the following adjustment is applied:

In case of one negative value

$$\begin{array}{lcl} f_{ik} \geq 0 & & f_{ik}{}' = f_{ik} - f_{im} \\ f_{il} \geq 0 & \longrightarrow & f_{il} \geq 0 \\ f_{im} < 0 & & f_{im} = 0 \end{array} \qquad (A\_8)$$

In case of two negative values

$$\begin{array}{lcl} f_{ik} \geq 0 & & \\ f_{il} < 0 & \longrightarrow & \begin{cases} f_{ik}{}' = 0{,}99 \\ f_{il}{}' = 0{,}005 \\ f_{im}{}' = 0{,}005 \end{cases} \\ f_{im} < 0 & & \end{array} \qquad (A\_9)$$

6 Throughout our analysis the adjustment was performed in limited cases (central government loan portfolio)

The normalization ends up with stricter results for the Bank, in terms of required provisions.

All normalized flows will be represented as f_{ij}, to avoid further notation confusion

9.3.6 Average transition flows & equilibrium

Let us assume that with the use of the methodology described in the appendix up to now, we may calculate the annual transition flow matrix A between two points in time

However, If the time distance between is not 1 year, $\Delta t = t_{final} - t_{initial}$, then we could calculate the annual average transition matrix with the use of eigenvalue decomposition

$$A \cdot X = X \cdot \Lambda \Rightarrow A = X \cdot \Lambda \cdot X^{-1} \quad (A\_10)$$

$\Lambda =$ Matrix of eigenvalues

$X =$ Matrix of eigenvectors

So

$$A_{avg} = A^{\frac{1}{\Delta t}} = X \cdot \Lambda^{\frac{1}{\Delta t}} \cdot X^{-1} \quad (A\_11)$$

Using either a 1-year matrix, or an average matrix extracted from longer periods, we derive the long-term equilibrium matrix (which is actually a vector) with the use of eigenvectors.

$$A_{eq} = A^n = X \cdot \Lambda^n \cdot X^{-1} = (f_E \quad f_R \quad f_D) \quad n \to \infty \quad (A\_12)$$

However, for the purpose of provisions calculations and capital requirements, the f_R percentage will have to be further decomposed into E, D states (states 1 & 3)

To do so, we assume

- An infinite sequence of equilibriums
- The only way to reach D state is through R state

The previous assumptions actually result in a geometric progression as the total percent of f_R that will eventually reach D is given by

$$p_d = f_D + f_D \cdot f_R + f_D \cdot {f_R}^2 + f_D \cdot {f_R}^3 + \cdots$$

Which sums to

$$p_d = \frac{f_D}{1 - f_R}$$

The percent that will be measured as in final state E is

$$p_e = 1 - p_d = \frac{1-(f_R+f_D)}{1-f_R}$$

The long-term equilibrium for capital calculation purposes is provided by

$$A_{LT} = (F_E \quad F_D) = \left(f_E + f_R \cdot \frac{f_D}{1-f_R} \qquad f_D + f_R \cdot \frac{1-(f_R+f_D)}{1-f_R}\right) \quad (A\_13)$$

9.4 Equilibrium State Sensitivities

We define $Y_t = GDP_t$

The GDP change over a 1 year period is

$$g_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}} \quad (A\_14)$$

$F_{E,j,t} \quad F_{D,j,t} =$ The long-term equilibrium percentages, calculated with data from annual statements from year t, as described in 9.3.6, for portfolio j.

Essentially, we have one equilibrium value as $F_{E,j,t} + F_{D,j,t} = 1$. For easiness of notation we will be using F_E without portfolio or time index, in our calculations.

Since F_E is an equilibrium value, it is a cumulative value, so it is reasonable to assume that is lies on a cumulative S-shaped curve. Employing the logistic curve

$$F_E = \frac{1}{1+e^{-a-b\cdot z}} \quad (A\_15)$$

From the available data series of $\{Y_t\}$ we calculate the average and the standard deviation of the logarithmic transform $m_{lnY_t} \quad s_{lnY_t}$, and define the standardized variable

$$z_{lnY_t} = \frac{lnY_t - m_{lnY_t}}{s_{lnY_t}} \quad (A\_16)$$

In simpler terms

$$z = \frac{y-m}{s}$$

The change of the equilibrium percent is

$$\frac{dF_E}{dy} = \frac{b}{s} \cdot F_E \cdot (1-F_E) \Rightarrow b = \frac{dF_E \cdot s}{dy \cdot F_E \cdot (1-F_E)}$$

The following approximations are applied

$dF_E \approx \Delta F_E = F_{E,t} - F_{E,t-1}$

$dy = dlnY_t \approx g_{Annual}$

For the purpose of normalization and capturing cumulative effects of the macro factor we use a 5-year average for the growth rate value

$$g = g_{AVG,t} = max\left\{\sum_{t-4}^{t} \frac{g_t}{5}, 0{,}01\right\} \quad (A\_17)$$

The base sensitivity value, in full notation, for portfolio j at time t

$$b_{j,t} = \left|\frac{\left(F_{E,j,t} - F_{E,j,t-1}\right) \cdot s_{lnY_t}}{g_{AVG,t} \cdot F_{E,j,t} \cdot \left(1 - F_{E,j,t}\right)}\right| \quad (A\_18)$$

We use the absolute value as a “noise filter” and assume a symmetrical slope b for up and down movements.

If the observed value $g_{AVG,t}$ is replaced by a possible macro state value g_k then the parameter b becomes sensitive to macro environment changes.

$$b_{j,t}(g_k) = \left|\frac{\left(F_{E,j,t} - F_{E,j,t-1}\right) \cdot s_{lnY_t}}{g_k \cdot F_{E,j,t} \cdot \left(1 - F_{E,j,t}\right)}\right| \quad (A\_19)$$

$F_E = \frac{1}{1+e^{-a-b\cdot z}} \Rightarrow a = ln\frac{F_E}{1-F_E} - b \cdot z$, thus α parameter becomes, in full notation

$$a_{j,t}(g_k) = ln\left(\frac{F_{E,j,t}}{1 - F_{E,j,t}}\right) - b_{j,t}(g_k) \cdot z_{lnY_t} \quad (A\_20)$$

The projected equilibrium long term percent of accruing loans for portfolio j given a hypothetical annual growth rate value g_k will be

$$\begin{aligned} F_{E,j,LT,k} &= \frac{1}{1 + e^{-a_{j,t}(g_k) - b_{j,t}(g_k)\cdot z_{t,k}}} \\ z_{t,k} &= \frac{lnY_t + g_k - m_{lnY_t}}{s_{lnY_t}} \approx \frac{(lnY_t + dlnY_t) - m_{lnY_t}}{s_{lnY_t}} \end{aligned} \quad (A\_21)$$

9.5 Collaterals and recovery

Notation:

$C_{stmt,t} =$ The collateral values appearing on the annual reports at year t

$I_t =$ The collateral value selected index[7] calculated for year t

$I_{max} =$ The maximum selected index value, usually at the highest level of the economic cycle[8]

[7] Composite index synthesized by a Central Bank / Statistical Agency, or index of a basic collateral type e.g. Real Estate that will serve as a proxy for the whole collateral portfolio values

I_{base} = The selected index value, at base year

I_{base} = The index value we assume represents a reasonable recovery value, based on the current relative position of the economy in the economic cycle. This is a business estimate based on current conditions.

$\{g_k\}$ = The distribution of GDP growth rate values 5.1

$C_{eq,t,k}$ = The adjusted collateral value used as base at time t calculated for the g_k assumed GDP growth rate

The equilibrium adjustment coefficient is calculated as:

$$a_{0,t} = \begin{cases} \frac{I_{base} - I_t}{I_t} & I_{base} > I_t \\ 0 & I_{base} \leq I_t \end{cases} \quad (A\_22)$$

The maximum expected adjustment coefficient:

$$a_{max} = \frac{I_{max} - I_{base}}{I_{base}} \quad (A\_23)$$

Collateral cover should also be sensitive to the macro factor. Since there are observable data series for I and g (5.1) we estimate of a statistical relationship, using data up to time t, between the percentage-changes of GDP and the collateral Index.

$$u = b_{C,t} \cdot g \qquad u_t = \frac{I_t - I_{t-1}}{I_{t-1}} \quad (A\_24)$$

$b_{C,t}$ = The sensitivity of the selected index annual percentage changes to GDP annual percentage changes which will be used to increase or decrease the collateral adjustment coefficient according to the assumed annual GDP growth rate g_k

$$a_{t,k} = a_{0,t} - b_{C,t} \cdot g_k \quad (A\_25)$$

The collateral value to be used in each economy assumed state (5.1), is respectively

$$C_{eq,t,k} = C_{stmt,t} \cdot \left(1 - a_{k,t}\right) \quad (A\_26)$$

Finally, the Loss Given Default is

$$LGD_{eq,t,k} = 1 - C_{eq,t,k} \quad (A\_27)$$

The previous analysis applies on a portfolio level, since coverages are reported on a portfolio level also. For simplicity the portfolio index was not included in the notation.

[8] Depends on data availability

9.6 Adjusting the Income Statement

The format assumed for the purpose of this study is presented below.

Only interest and banking services income (core banking) is considered as gross operating income

+	Interest income
-	Interest Expense
+	Net Income from Banking Services
	Gross Operating Income

Figure 13: Operating Income rearrangement

Investment and other operating income as well as other income / expenses are integrated into non –recurring items

+	Investment & Other Operating Income
-	Other Expenses
	Nonrecurring Items[9]

Figure 14: Nonrecurring items

The condensed income statement is expressed as:

	Name	**Symbol Used 6.1**
+	Interest income	$IntIncome$
-	Interest Expense	
+	Net Income from Banking Services	
	Gross Operating Income	$GrOperIncome$
-	Operating Expense	$OperExp$
	Net Operating Income	
-	Loan Provisions Expense	
+	Nonrecurring Items	$OtherExp$
	Net Income Before Taxes	

Figure 15: Condensed income statement

[9] PSI adjustments in 2012 as well as income from acquisition of "good" bank segments in 2013 are ignored for the purpose of the study

9.7 Example of portfolio transitions

If we assume 1 portfolio with the following distribution in net amounts

	E	R	D	$Provisions$
$t = 0$	800	100	100	0

Figure 16: 1 bn € Hypothertical Portfolio Starting Values

Further we assume $LGD = 50\%$ and stable growth rate of 10% for each portfolio segment. Applying the methodology developed up to now, without considering scenarios, portfolio provisions evolve according to the following graph

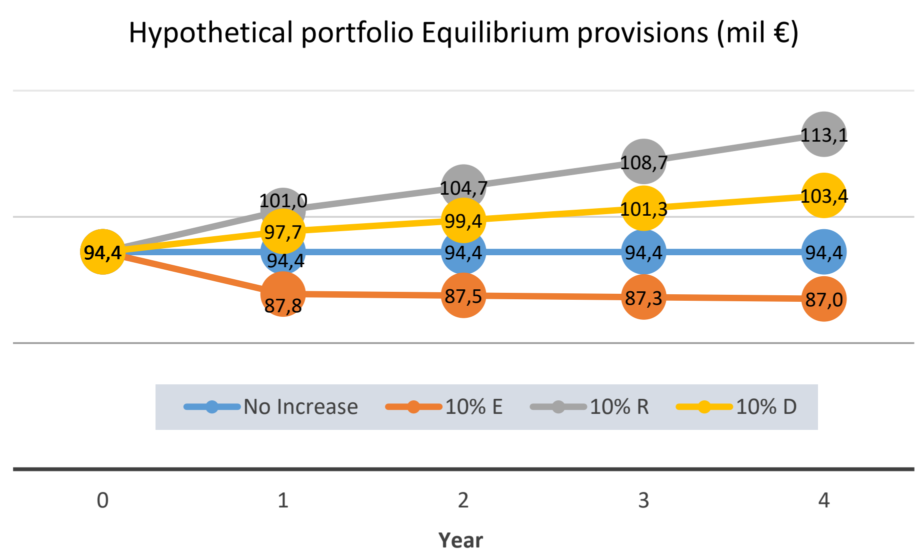

Figure 17: Hypothetical portfolio provisions evolution under segment growth assumptions

Initially stability is assumed for $[t = -1, t = 0]$ period. Then each segment E, R, D grows with 10% rate. Only in the case of E we have portfolio expansion. In the other two cases, R, D there are internal transitions to worst states.

Provisions evolve as expected in a 4-year period. Provisions amounts increase at a higher rate concerning segment R increase, compared to segment D increase, since the majority of provisions amount ($\approx 67\%$) is consumed on D segment.

9.8 Approximated GDP growth rate distributions

The normal distributions approximated in 10 intervals with least squares method, are depicted in the following graphs:

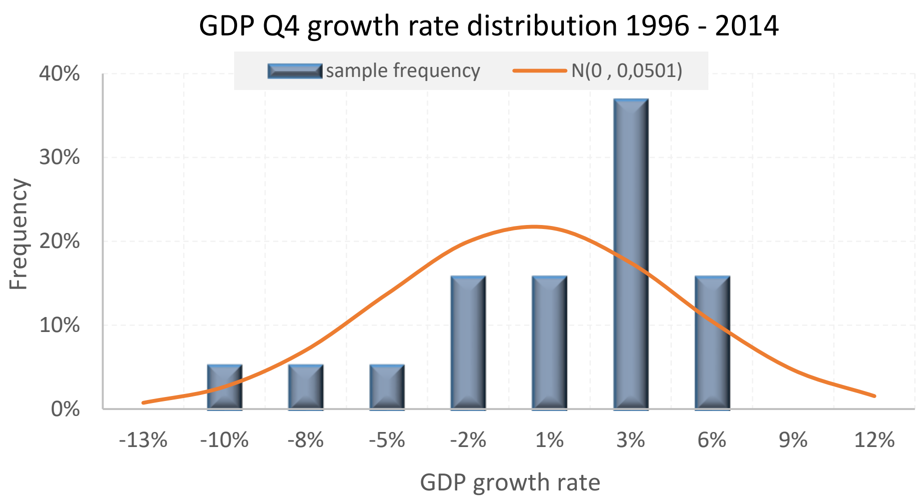

Figure 18: Approximated GDP growth rate distribution used in 2014 results

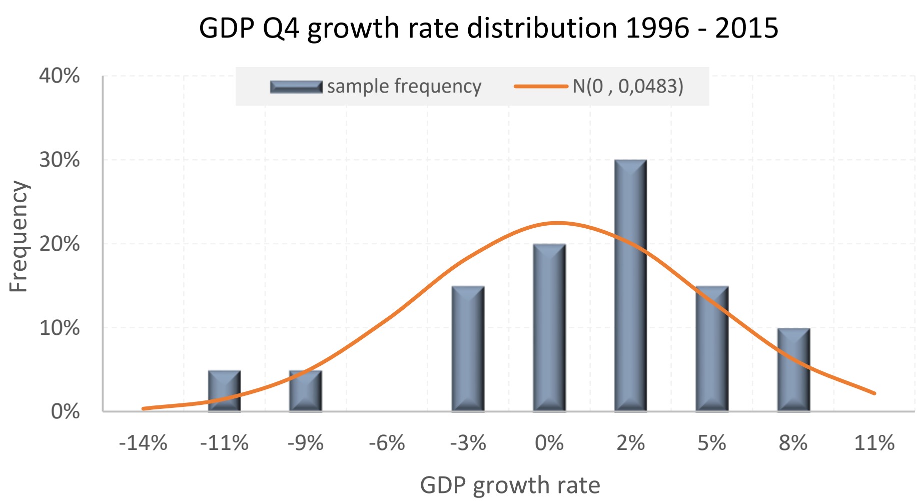

Figure 19: Approximated GDP growth rate distribution used in 2015 results

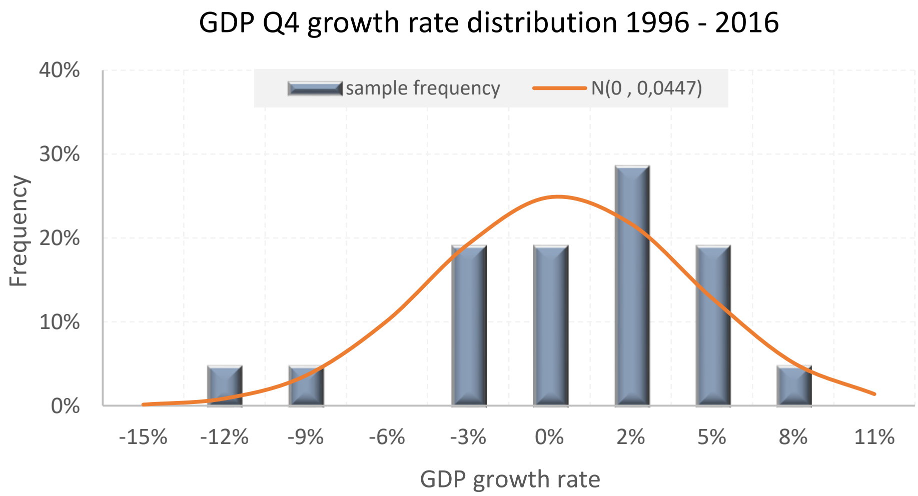

Figure 20: Approximated GDP growth rate distribution used in 2016 results

9.9 Application Results

The concentrated results of our methodology are exhibited on the next 4 tables. All relevant amounts are reported in millions €.

NBG Results	2016	2015	2014
Equilibrium performing Assets	28.522,10	31.417,48	33.863,36
AfterTax Income Available for Provisions per 1 EUR of Performing Loan Assets	0,95%	0,64%	0,84%
Calculated provisions	915,02	3.125,50	3.575,35
Profit to absorb losses	272,20	201,38	285,15
Provisions to Profit	3,36	15,52	12,54
BCRC	96,64	84,48	87,46
Equity To Net Loans after provisions subtraction	13,6%	13,1%	11,2%

Figure 21: National Bank of Greece methodology application results

Alpha Bank Results	**2016**	**2015**	**2014**
Equilibrium performing Assets	29.994,78	28.186,54	32.397,60
AfterTax Income Available for Provisions per 1 EUR of Performing Loan Assets	1,92%	1,64%	0,99%
Calculated provisions	1.048,86	2.198,44	2.757,96
Profit to absorb losses	577,13	461,12	320,14
Provisions to Profit	1,82	4,77	8,61
BCRC	98,18	95,23	91,39
Equity To Net Loans after provisions subtraction	19,1%	15,0%	9,3%

Figure 22: Alpha Bank methodology application results

Piraeus Bank Results	**2016**	**2015**	**2014**
Equilibrium performing Assets	37.560,60	37.757,75	39.980,15
AfterTax Income Available for Provisions per 1 EUR of Performing Loan Assets	1,16%	0,96%	0,98%
Calculated provisions	2.502,58	2.489,55	4.847,09
Profit to absorb losses	436,90	361,08	392,17
Provisions to Profit	5,73	6,89	12,36
BCRC	94,27	93,11	87,64
Equity To Net Loans after provisions subtraction	14,3%	14,4%	4,7%

Figure 23: Piraeus Bank methodology application results

EuroBank Results	**2016**	**2015**	**2014**
Equilibrium performing Assets	19.731,34	23.882,39	25.898,64
AfterTax Income Available for Provisions per 1 EUR of Performing Loan Assets	0,94%	0,64%	0,62%
Calculated provisions	2.937,42	2.161,91	3.518,12
Profit to absorb losses	185,98	151,68	159,67
Provisions to Profit	15,79	14,25	22,03
BCRC	84,21	85,75	77,97
Equity To Net Loans after provisions subtraction	10,1%	12,0%	5,0%

Figure 24: EFG EuroBank methodology application results

Disclaimer

The proposed methodologies reflect the author's view only and have no relation to any practices implemented in National Bank of Greece (NBG). To the best of my knowledge, up to the time the current document is written (December 2017-February 2018), there is no publication describing a similar methodology.

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