Journal of Computations & Modelling, vol.8, no.3, 2018, 17-34
ISSN: 1792-7625 (print), 1792-8850 (online)
Scienpress Ltd, 2018

Numerical estimation of phenomenological parameters for sand dune formation by solving an inverve problem

Rabi Hassane Ganda[1], **Haoua Hamidou**[2] **and Bisso Saley**[3]

Abstract

We are interested in this work to a numerical estimation of parameters intervening in the formation process of the sand dunes. They are in particular the phenomenological parameters and which influences respectively the curvature and the slope. We formulate an optimal control problem (inverse problem) and we implement several numerical methods to approach the problem. Then we make a comparative study of these numerical methods in order to choose the best for the analysis.

Keywords: Sand dunes; inverse problem; estimate of the parameters; numerical methods
Mathematics Subject Classification : 49J20; 74G15; 62P12;76M25; 97N40

[1] Department of mathematics, University Abdou Moumouni Niamey-Niger.
E-mail: rabiganda@yahoo.fr
[2] Department of mathematics, University Abdou Moumouni Niamey-Niger.
E-mail: hamidou_haoua@yahoo.com
[3] Department of mathematics, University Abdou Moumouni Niamey-Niger.
E-mail: bsaley@yahoo.fr

Article Info: *Received* : April 30, 2018. *Revised* : June 4, 2018.
Published online : August 1, 2018.

1 Introduction

Following the great noted climate changes, one attends a reactivation of the sand dunes remained motionless during thousands of years [1]. From now on, the Sahara are spread out by the wind and the dunes threaten the ecosystems. The comprehension of the characteristics of the formation of sand dunes and the phenomena related to their displacement become then an important issue. To determine the parameters concerned in the different models governing the formation of the sand dunes is a crucial step in the search for effective means fight against the stranding. The mathematical models which control the formation of the sand dunes are formed in general by systems of partial derivative equations [2,3,4] which reveal parameters unknown and inaccessible to direct measurements. To identify one or more unknown parameters controlling the formation of sand dunes generally amounts to solve an inverse problem. The resolution of this kind of problem is based on minimization of an objective functional describing the difference between the observations (measurements) and simulations (numerical calculations).

The present paper study the development of numerical approaches for estimating the parameters of the model describing the formation of sand dunes. It is therefore an inverse problem of estimation of parameters of diffusion and transport. The paper is structured as follows: in the second section we give the mathematical model which describes the formation of the sand dunes. In the third section we give a formulation of the inverse problem. The section four to six will be devoted to the numerical approximation of the inverse problem. In the section seven, we will proceed to numerical simulations followed by an analysis of the results and we finish by a conclusion.

2 Mathematical model

The mathematical model describing the formation of the sand dunes is given by the following system [2]:

$$\begin{cases} \dfrac{\partial h(x,t)}{\partial t} + \dfrac{\partial q(x,t)}{\partial x} = 0 \quad \text{if} \quad |\dfrac{\partial h(x,t)}{\partial x}| < tan(\gamma) \quad \forall \quad (x,t) \in \Omega \times [0,T] \\ \dfrac{\partial q(x,t)}{\partial x} = q_{sat}(x,t) - q(x,t) \quad \text{if} \quad h(x,t) > 0, \qquad \dfrac{\partial q(x,t)}{\partial x} = 0 \quad \text{elsewhere} \\ q_{sat}(x,t) = 1 - \alpha D \dfrac{\partial^2 h(x,t)}{\partial x^2} + \beta \dfrac{\partial h(x,t)}{\partial x} \\ h(x,t) = 0, \quad \forall \quad (x,t) \in \Gamma \times [0,T] \\ h(x,0) = h_0, \quad \forall x \in \Omega \end{cases} \tag{2.1}$$

- **h (t, x)**: Denotes the height of the dune at every point x of the space and time t;
- **q (t, x)** : Denotes the flux of sand grains transported at any point and at any time;
- q_{sat}**(t,x)**: Is saturation flux ;
- h_0**(x)** Denotes the initial condition.

When all the parameters are given, the resolution of the model can be done analytically or numerically. In the last study, we have underlined the effect of the parameters α and β [2].

3 Formulation of the inverse problem

The inverse problem of identification of parameters is written as follows:

$$\begin{cases} \min\limits_{\alpha,\beta} S(\alpha,\beta) = \min\limits_{\alpha,\beta} \displaystyle\int_0^T \int_\Omega |h(x,t,\alpha,\beta) - h_{mes}(x,t)|^2 \quad \mathrm{d}x\mathrm{d}t \\ \qquad\qquad\qquad\qquad \text{Under constraint} \\ \dfrac{\partial h(x,t)}{\partial t} - \alpha D \dfrac{\partial^2 h(x,t)}{\partial x^2} + \beta \dfrac{\partial h(x,t)}{\partial x} = f(x,t) \quad \forall \quad (x,t) \in \Omega \times [0,T] \\ h(x,t) = 0, \quad \forall (x,t) \in \Gamma \times [0,T] \\ h(x,0) = h_0, \quad \forall x \in \Omega \end{cases} \tag{3.1}$$

With:
$f(x,t) = q(x,t) - 1$
We can translate this problem of minimization in the following way: the couple of parameters $(\alpha\ \beta)$ is sought so that the height h resulting from the simulations approaches as much as possible measured height h_{mes}

3.1 Discretization

Let n be a non-zero natural integer and $\Omega = [a,b]$ the spatial domain. We define the discretization step of the domain $\Omega = [a,b]$ by $\Delta x = \frac{b-a}{n}$ and we subdivide the domain Ω into subintervals $[x_i, x_{i+1}]$ such that $x_i = a + i \times \Delta x$ for $i \in \{0, 1, \cdots, n\}$.
We also subdivide the time domain $T = [0, t_f]$ into k subintervals $[t_j, t_{j+1}]$ with k a non-zero integer and $t_j = j \times \Delta t$ for $j \in \{0, 1, \cdots, k\}$.
Thus, the functional of minimization can be written as follows:

$$S_{n,k}(\alpha, \beta, x_i, t_j) = \sum_{i=0}^{n} \sum_{j=0}^{k} [h(x_i, t_j, \alpha, \beta) - h_{mes}(x_i, t_j)]^2 \qquad (3.2)$$

We can rewrite $S_{n,k}$in the following matrix form:

$$S_{n,k}(P, x_i, t_j) = [H(P) - H_{mes}(x,t)]^T [H(P) - H_{mes}(x,t)] \qquad (3.3)$$

With:
P=(α,β) : The Parameter vector
H(P)= $(h(x_i, t_j, \alpha, \beta))_{\substack{1 \le j \le n \\ 1 \le i \le n}}$: Matrix whose components are $h(x_i, t_j, \alpha, \beta)$
$H_{mes}(x,t) = (h_{mes}(x_i, t_j))_{\substack{1 \le j \le n \\ 1 \le i \le n}}$: Matrix whose components are $h_{mes}(x_i, t_j)$
The minimum of $S_{n,k}$ is reached when its first derivate with respect to P is zero

$$\nabla S_{n,k}(P) = 0 \Longrightarrow 2[J(t,P)][H(x_i, t_j P) - H_{mes}(x_i, t_j)] = 0 \qquad (3.4)$$

With:
J(t,P): is the first derivate of h(x,t,P) with respect to parameter P, called also sensitive matrix or jacobian matrix.

$$J(t,P)=\begin{pmatrix} \frac{\partial h_1}{P_1} & \frac{\partial h_1}{P_2} & \cdots & \frac{\partial h_1}{P_m} \\ \\ \frac{\partial h_2}{P_1} & \frac{\partial h_1}{P_2} & \cdots & \frac{\partial h_2}{P_m} \\ \\ \vdots & \vdots & \cdots & \vdots \\ \\ \frac{\partial h_k}{P_1} & \frac{\partial h_k}{P_2} & \cdots & \frac{\partial h_k}{P_m} \\ \end{pmatrix} \tag{3.5}$$

4 Resolution Algorithms

4.1 Algorithm 1: Gauss-Newton

Let us suppose that the functional of minimization $S \in C^2(\mathbb{R}^n, \mathbb{R})$ the set of the functions of class twice continuously derivable.

We approach the Hessien of the functional of minimization by the formula: $H(S(P)) \approx 2[J(t,P)]^T[J(t,P)]$ [5] [6]. J(T, P) is the gradient of the functional of minimization.

Algorithm 1

Parameter Initialization $P^0 = (\alpha_{intial}, \beta_{intial})$

from r = 1 until the stopping criterion is not validated do:

Step 1: Resolution of the model with the values of P^r in order to obtain the theoretical answer of the system $h(x_i, t_j, P^r)$;

Step 2: Determination of the matrix of sensitivity $J(t, P^r)$;

Step 3: calculate the new value of the parameter P^{r+1}:

$P^{r+1} = P^r + [J(t,P^r)^T \cdot J(t,P^r)]^{-1} \cdot J(t,P^r) \cdot (H(x,t,P^r) - H_{mes}(x,t))$

Step 4: Checking of the stopping criterion

$r \leftarrow r+1$

4.2 Algorithm 2: Levenberg-Marquardt

The Levenberg-Marquardt method consists of a regularization of the matrix of the sensitivities [7-10]

Algorithm 1

Parameter Initialization $P^0 = (\alpha_{intial}, \beta_{intial})$
from r = 1 until the stopping criterion is not validated do:
Step 1: Resolution of the model with the values of P^r in order to obtain the theoretical answer of the system $h(x_i, t_j, P^r)$;
Step 2: Calculation of $S(P^r)$ and determination of the matrix of sensitivity $J(t, P^r)$;
Step 3: calculate the new value of the parameter P^{r+1}:
$P^{r+1} = P^r + [J(t, P^r)^T \cdot J(t, P^r) + \mu^r \cdot \Lambda^r]^{-1} \cdot J(t, P^r) \cdot (H(x, t, P^r)$
$-H_{mes}(x, t))$
Λ: Positive definite matrix.
μ: Positive real.
Step 4: Resolution of the model with the new values of the parameters P^{r+1} in order to obtain the new theoretical answer of the system $h(x_i, t_j, P^{r+1})$ and calculation of $S(P^{r+1})$;
Step 5: If $S(P^r) > S(P^{r+1})$, $\mu^r \leftarrow 10 \times \mu^r$ and return to step 3;
If not $\mu^r \leftarrow 0.1 \times \mu^r$
Step 6: Checking of the stopping criterion
$r \leftarrow r + 1$

---

5 Algorithm 3: The conjugate gradient

The fundamental idea of this method consists in determining the news reiterated P^{k+1} starting from the last P^k iteration by [11]:

$$P^{k+1} = P^k - \gamma^k dk \tag{5.1}$$

With:
P^k: The parameter vector estimated at the iteration k;

γ^k: The descent depth at the iteration k;
d^k: The descent direction at the iteration k;

$$d^k = \nabla S(P^k) - \beta^k d^{k-1} \tag{5.2}$$

$$\beta^k = \frac{\|\nabla S(P^k)\|}{\|\nabla S(P^{k-1})\|} and \beta^0 = 0 \tag{5.3}$$

and $\beta^0 = 0$

5.1 Problem of sensitivity

This problem consists in determining the variation $\delta h(x,t)$ of the height induced by a variation of the parameter δP. Considering the system of partial derivative equation satisfied by $h(x,t) + \eta\delta h(x,t)$ then when $\eta \longrightarrow 0$, the problem of sensitivity is given by:

- **For a variation of the parameter** $P_1 = \alpha$

$$\begin{cases} \dfrac{\partial \delta h(x,t)}{\partial t} - P_1 D \dfrac{\partial^2 \delta h(x,t)}{\partial x^2} - \delta P_1 D \dfrac{\partial^2 \delta h(x,t)}{\partial x^2} + \beta \dfrac{\partial h(x,t)}{\partial x} = 0 \\ \delta h(x,t) = 0, \quad \forall \quad (x,t) \in \Gamma \times [0,T] \\ \\ \delta h(x,0) = h_0, \quad \forall x \in \Omega \end{cases} \tag{5.4}$$

- **For a variation of the parameter** $P_2 = \beta$

$$\begin{cases} \dfrac{\partial \delta h(x,t)}{\partial t} - \alpha D \dfrac{\partial^2 \delta h(x,t)}{\partial x^2} + P_2 \dfrac{\partial \delta h(x,t)}{\partial x} + \delta P_2 \dfrac{\partial \delta h(x,t)}{\partial x} = 0 \\ \delta h(x,t) = 0, \quad \forall \quad (x,t) \in \Gamma \times [0,T] \\ \\ \delta h(x,0) = h_0, \quad \forall x \in \Omega \end{cases} \tag{5.5}$$

The descent depth γ^k is the value corresponding to the optimal pitch in the direction of descent of the new value of unknown parameters. This magnitude γ^k minimize the following criterion:

$$\gamma^k = arg[min(S(P^k - \gamma^k d^k))] \tag{5.6}$$

After developments and calculations, the depth of descent γ^k is calculated at each iteration k is obtained:

$$\gamma^k = \frac{\sum_{i=1}^{n}\sum_{j=1}^{k}[h(x,t,P^k) - h_{mes}(x_i,t_j)]\delta h(x,t,P^k)}{\sum_{i=1}^{n}\sum_{j=1}^{k}[\delta h(x,t,P^k]^2} \tag{5.7}$$

Depth of descent γ^k is calculated at each iteration according to the solution of the problem of sensitivity $\delta h(x,t,P^k)$.

6 Adjoint problem

6.1 Formulation

The goal of the adjoint problem is to obtain the expression of the gradient in order to be able to determine the direction of descent. This problem consists in building a function $\Psi(x,t)$ called ?multiplier of Lagrange? which allow to determine the expression of the gradient ?$\nabla S(P)$ of the functional of minimization S (P). The Lagrange formula is defined by [12-14]:

$$L(P,h,\Psi) = S(P,h) + R(P,h,\Psi) \tag{6.1}$$

With
$R(P,h,\Psi)$: Corresponds to the system of equation of the model multiplied by the Lagrange multiplier.
The expression of the variation of Lagrange $\delta L(P,h,\Psi)$ is:

$$\delta L(P,h,\Psi) = \frac{\partial L}{\partial P}\delta P + \frac{\partial L}{\partial h}\delta h + \frac{\partial L}{\partial \Psi}\delta\Psi \tag{6.2}$$

If we fix $\Psi(x,t)$, we have:

$$\frac{\partial L}{\partial \Psi}\delta\Psi = 0 \Longrightarrow \delta L(P,h,\Psi) = \frac{\partial L}{\partial P}\delta P + \frac{\partial L}{\partial h}\delta h \tag{6.3}$$

In order to obtain the gradient of the criterion it is necessary to choose suitably $\Psi(x,t)$. The multiplying choice of the function of Lagrange $\Psi(x,t)$ is chosen so that the following equation is satisfied:

$$\frac{\partial L}{\partial h}\delta h = 0 \Longrightarrow \delta L(P,h,\Psi) = \frac{\partial L}{\partial P}\delta P \tag{6.4}$$

Moreover, if h is solution of the equations defining the mathematical model, we have:

$$L(P,h,\Psi) = S(P) \Longrightarrow \delta L(P,h,\Psi) = \delta S(P,h) \tag{6.5}$$

6.2 Determination of the adjoint problem equations

In this part we will determine the equations of the adjoint problem for each unknown parameter.

- **For the parameter of diffusion** $P_1 = \alpha$

The Lagrange formula is defined by:

$$L(P_1,h,\Psi) = S(P_1,h) + R(P_1,h\Psi) \tag{6.6}$$

The expression of the variation of Lagrange is:

$$\delta L(P_1,h,\Psi) = \delta S(P_1,h) + \delta R(P_1,h\Psi) \tag{6.7}$$

With:

$$\delta S(P_1,h) = \int_0^t \int_\Omega [h(x_i,t,P_1 - h_{mes}(x_i,t]\delta h(x,t)\delta_D(x-x_i)\Psi dxdt \tag{6.8}$$

$$\delta R(P_1,h\Psi) = \int_0^t \int_\Omega [\frac{\partial \delta h(x,t)}{\partial t} - P_1 D\frac{\partial^2 \delta h(x,t)}{\partial x^2} - \delta P_1 D\frac{\partial^2 \delta h(x,t)}{\partial x^2} + \beta\frac{\partial h(x,t)}{\partial x}]\Psi dxdt \tag{6.9}$$

So we have:

$$\delta L(P_1,h,\Psi) = \int_0^t \int_\Omega [h(x_i,t,P_1 - h_{mes}(x_i,t]\delta h(x,t)\delta_D(x-x_i)\Psi dxdt +$$
$$\int_0^t \int_\Omega [\frac{\partial \delta h(x,t)}{\partial t} - P_1 D\frac{\partial^2 \delta h(x,t)}{\partial x^2} - \delta P_1 D\frac{\partial^2 \delta h(x,t)}{\partial x^2} + \beta\frac{\partial h(x,t)}{\partial x}]\Psi dxdt \tag{6.10}$$

We can rewrite the variation of Lagrange formula in this form:

$$\delta L(P_1, h, \Psi) = \int_0^t \int_\Omega [h(x_i, t, P_1 - h_{mes}(x_i, t]\delta h(x,t)\delta_D(x - x_i)\Psi dxdt +$$
$$\int_\Omega \delta h(x, t_f)\Psi(x, t_f)dx - \int_0^t \int_\Omega [\delta h \frac{\partial \Psi}{dt}]dxdt - \int_0^t \int_\Omega [P_1 D\delta h \frac{\partial^2 \Psi}{\partial x^2}]dxdt$$
$$- \int_0^t \int_\Omega [D\delta P_1 \frac{\partial^2 h}{\partial x^2}]\Psi dxdt + \int_0^t \int_\Omega [\beta \delta h \frac{\partial \Psi}{\partial x}]dxdt \tag{6.11}$$

The multiplier of Lagrange is chosen so that:

$$\frac{\partial L}{\partial h}\delta h = 0 \tag{6.12}$$

Which leads to the adjoint problem:

$$\begin{cases} \dfrac{\partial \Psi(x,t)}{dt} + P_1 D \dfrac{\partial^2 \Psi(x,t)}{\partial x^2} - \beta \dfrac{\partial \Psi(x,t)}{dx} = E(x,t) \\ \Psi(x, t_f) = 0 \end{cases} \tag{6.13}$$

When h is solution of the mathematical model and Ψ is solution of the associated problem, then:

$$\delta L = \delta S = \int_0^t \int_\Omega \delta P_1 \frac{\partial h}{\partial x}\Psi dxdt \tag{6.14}$$

We define the gradient of the criterion in the following way:

$$\nabla S = \int_0^t \int_\Omega \frac{\partial h}{\partial x}\Psi dxdt \tag{6.15}$$

- **For the parameter of diffusion** $P_2 = \beta$

The Lagrange formula is defined by:

$$L(P_2, h, \Psi) = S(P_2, h) + R(P_2, h\Psi) \tag{6.16}$$

The expression of the variation of Lagrange is:

$$\delta L(P_2, h, \Psi) = \delta S(P_2, h) + \delta R(P_2, h\Psi) \tag{6.17}$$

With:

$$\delta S(P_2, h) = \int_0^t \int_\Omega [h(x_i, t, P_2 - h_{mes}(x_i, t]\delta h(x,t)\delta_D(x - x_i)\Psi dxdt \tag{6.18}$$

$$\delta R(P_2, h\Psi) = \int_0^t \int_\Omega [\frac{\partial \delta h(x,t)}{\partial t} - \alpha D \frac{\partial^2 \delta h(x,t)}{\partial x^2} + P_2 \frac{\partial \delta h(x,t)}{\partial x} + \delta P_2 \frac{\partial \delta h(x,t)}{\partial x}] \Psi dxdt \tag{6.19}$$

So we have:

$$\delta L(P_2, h, \Psi) = \int_0^t \int_\Omega [h(x_i, t, P_2 - h_{mes}(x_i, t] \delta h(x,t) \delta_D(x - x_i) \Psi dxdt +$$
$$\int_0^t \int_\Omega [\frac{\partial \delta h(x,t)}{\partial t} - \alpha D \frac{\partial^2 \delta h(x,t)}{\partial x^2} + P_2 \frac{\partial \delta h(x,t)}{\partial x} + \delta P_2 \frac{\partial \delta h(x,t)}{\partial x} + \beta \frac{\partial h(x,t)}{\partial x}] \Psi dxdt \tag{6.20}$$

We can rewrite the variation of Lagrange formula in this form:

$$\begin{aligned} \delta L(P_2, h, \Psi) &= \int_0^t \int_\Omega [h(x_i, t, P_2 - h_{mes}(x_i, t] \delta h(x,t) \delta_D(x - x_i) \Psi dxdt + \\ &\int_\Omega \delta h(x, t_f) \Psi(x, t_f) dx - \int_0^t \int_\Omega [\delta h \frac{\partial \Psi}{dt}] dxdt + \int_0^t \int_\Omega [P_2 \delta h \frac{\partial \Psi}{\partial x}] dxdt \\ &- \int_0^t \int_\Omega \alpha [D \delta h \frac{\partial^2 \Psi}{\partial x^2}] dxdt + \int_0^t \int_\Omega [\delta p_2 \frac{\partial h}{\partial x}] \Psi dxdt \end{aligned} \tag{6.21}$$

The multiplier of Lagrange is chosen so that:

$$\frac{\partial L}{\partial h} \delta h = 0 \tag{6.22}$$

That implies the multiplier of Lagrange is solution of the following associated problem:

$$\begin{cases} \frac{\partial \Psi(x,t)}{dt} + \alpha D \frac{\partial^2 \Psi(x,t)}{\partial x^2} - P_2 \frac{\partial \Psi(x,t)}{dx} = E(x,t) \\ \Psi(x, t_f) = 0 \end{cases} \tag{6.23}$$

When h is solution of the mathematical model and Ψ is solution of the associated problem, then:

$$\delta L = \delta S = \int_0^t \int_\Omega \delta P_2 \frac{\partial h}{\partial x} \Psi dxdt \tag{6.24}$$

We define the gradient of the criterion in the following way:

$$\nabla S = \int_0^t \int_\Omega \frac{\partial h}{\partial x} \Psi dxdt \tag{6.25}$$

7 Numerical Simulations

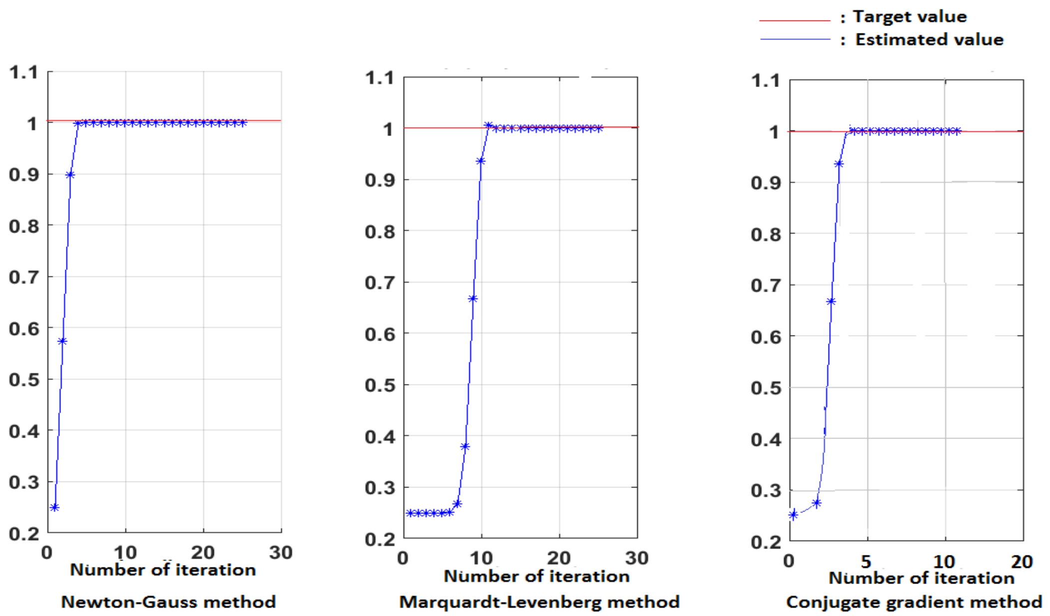

Figure 1: Evolution of the paramater α using different methods

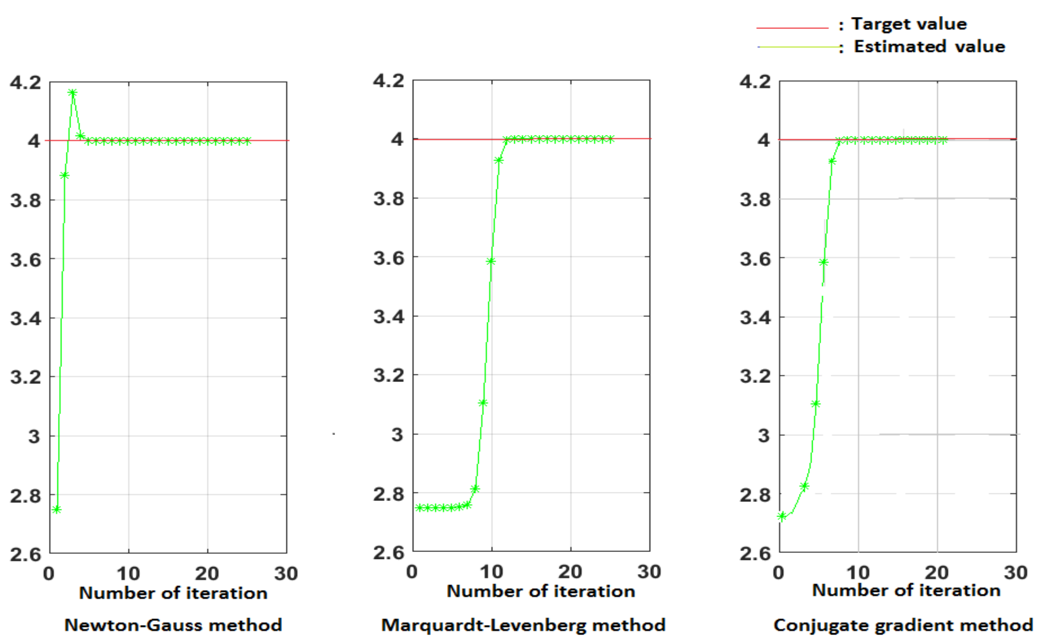

Figure 2: Evolution of the paramater β using different methods

Figures 1 and 2 describe the evolution of the parameters α and β . In this first simulation the values of initialization are taken close to the target values. The target values of the required parameters are =1 and =4 [3] and the values of initialization of the parameters and fixed respectively at 0.25 and 2.75. One notes that the various algorithms converge towards the target solutions, however this convergence does not unroll same manner. The algorithm of Gauss-Newton and the algorithm of the conjugate gradient converge after only 5 iterations while for the algorithm of Levenberg-Marquardt, one needs at least 13 iterations. Indeed, the algorithm of Levenberg-Marquardt requires a considerable computing time mainly due to the calculation of the matrices of sensitivity to each stage of the algorithm which can become petitioning important calculations.

Table 1: Error by Newton-Gauss method

Iteration	1	2	$\cdots$	6	7
Error between calculated height and measured height	36.0504	11.1731	$\cdots$	2.6168×10^{-8}	2.310×10^{-12}
Approximation error of parameter α	0.75	0.4268	$\cdots$	9.6489×10^{-10}	1.07×10^{-13}
Approximation error of parameter β	1.25	0.1191	$\cdots$	3.1197×10^{-9}	2.22×10^{-13}

Table 2: Error by Levenberg-Marquardt method

Iteration	1	2	$\cdots$	14	15
Error between calculated height and measured height	36.0504	36.0504	$\cdots$	1.6208×10^{-9}	1.6101×10^{-13}
Approximation error of parameter α	0.75	0.7499	$\cdots$	1.8868×10^{-11}	1.5×10^{-14}
Approximation error of parameter β	1.25	1.24	$\cdots$	2.0247×10^{-10}	5.4×10^{-14}

Table 3: Error by conjugate gradient

Iteration	1	2	$\cdots$	6	7
Error between calculated height and measured height	36.0504	4.5782	$\cdots$	1.3971×10^{-9}	1.5105×10^{-11}
Approximation error of parameter α	0.75	0.1037	$\cdots$	5.0726×10^{-12}	2.7936×10^{-14}
Approximation error of parameter β	1.25	0.0912	$\cdots$	8.7831×10^{-10}	2.22×10^{-14}

The results obtained, show that as the iteration increases the error between the calculated height and the measured height decreases. Indeed, the procedure of estimate of the parameters allows as well as possible to approach the required parameters which are determining in the rebuilding of the phenomenon of formation of the sand dunes.

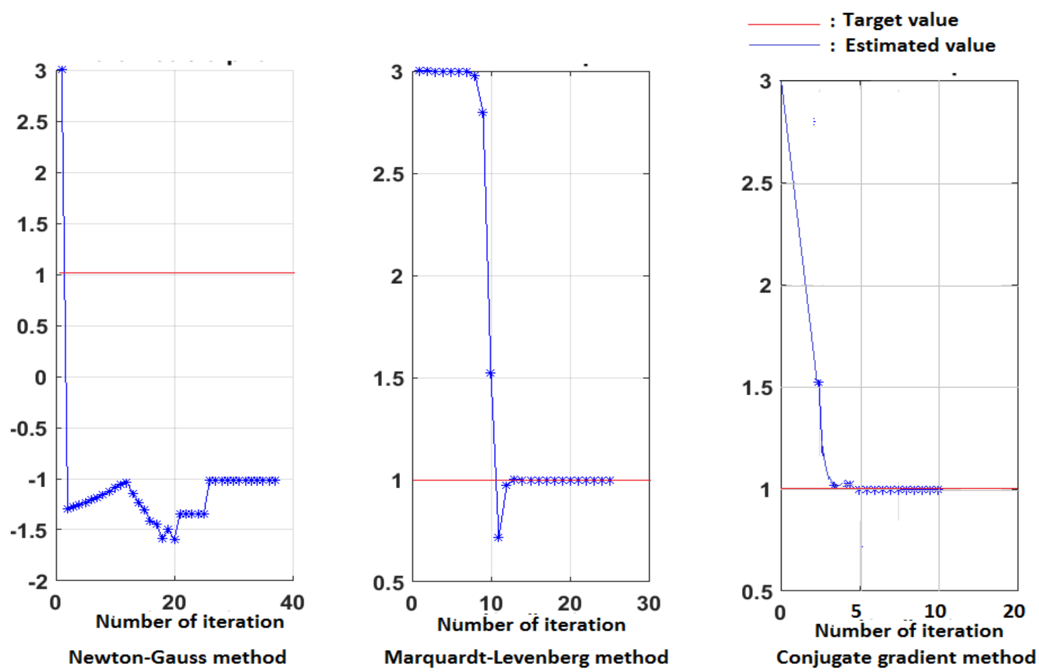

Figure 3: Evolution of the paramater α using different methods

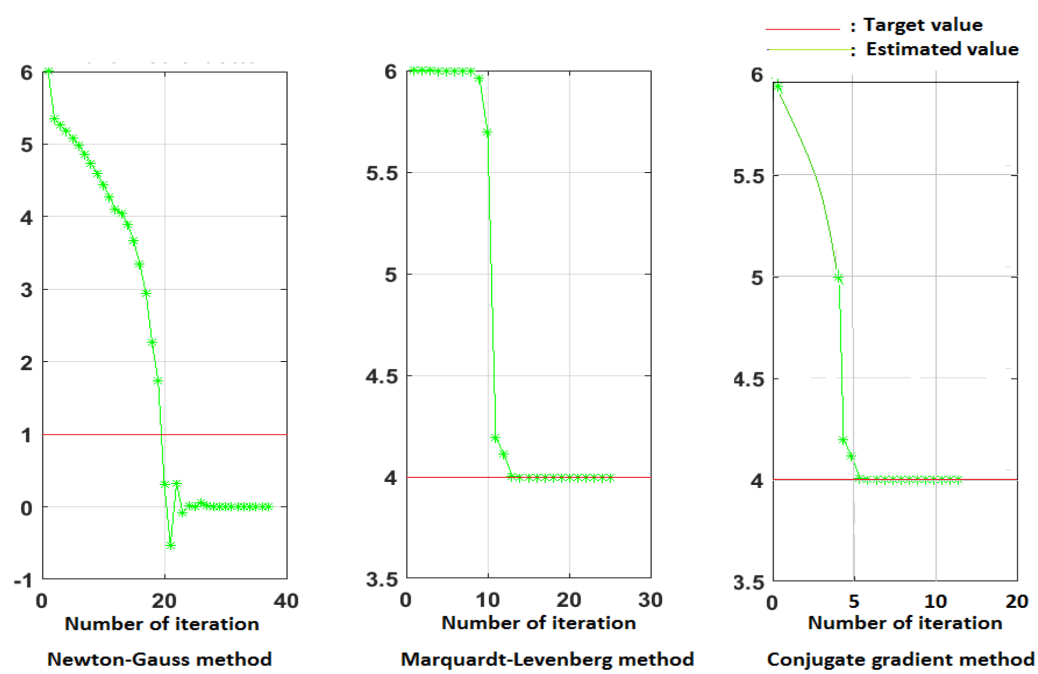

Figure 4: Evolution of the paramater β using different methods

In this second simulation, the values of initialization are taken far away from the target values. Values of initialization of the parameters and fixed respectively at 3 and 6. It is noted that the algorithm of Newton-Gauss does not convergence towards the target solutions. Indeed, the disadvantage of this method resides on the approximation of Hessien according to the gradient of the functional. While the algorithm of Levenberg-Marquardt and the conjugate gradient ensure a convergence towards the target solutions. In spite of results of convergence, the method of Levenberg-Marquardt is characterized by its slow behavior when the value of initialization is far from the required solution. The major advantage of the algorithm of conjugate gradient is that it is fast and converges independently of the initialization values.

8 Conclusion

The objective of this work is the identification of the parameters involved the model describing the formation of the sand dunes. It is an inverse problem solved by the methods of Newton-Gauss, Levenberg-Marquardt and the conjugate gradient. At first the various methods to solve the problem are presented. Simulations showed that one of the advantages of Gauss-Newtons method is that it does not require the calculation of Hessien, which makes calculation faster. Nevertheless, this method is limited because it does not ensure convergence when the initialization values are chosen far from the target solutions. The algorithm of Levenberg-Marquardt converges whatever the initialization values; but this convergence proves to be slow because of the matrices to reverse. As for the algorithm of the conjugate gradient, fast convergence makes it interesting. One of the advantages of this method is that combined with the method of the adjoint problem, it allows the exact calculation of the gradient.

References

[1] C. Bodart, A. Ozer, Apport de la télédétection dans l'étude de la remise en mouvement du sable dunaire dans la région de Gouré (sud est du Niger),

(1985), 147 - 169.

[2] H. Rabi,S. Bisso,H. Haoua, Modeling and numerical simulation of sand dunes formation., *Canadian center of science and education*, **9**(4), (2017).

[3] B. Andreotti, P. Claudin ,S.Douady., Selection of dunes shapes and velocities. Part 2: A two dimensionnal modelling, *The European Physical Journal B*, **28**, (2002), 341 - 352.

[4] K. Kroy, G. Sauermann , H.J. Herrmann., Minimal model for sand dunes., *Physical Review*, **66**, (2002).

[5] Kress R. et Lee K.M., A second degree Newton method for an inverse obstacle scattering problem, *International Journal of Thermal Sciences*, **46**, (2007), 128-138.

[6] Xiaoa X., Zhanga L. et Zhang J., A smoothing Newton method for a type of inverse semi-definite quadratic programming problem, *Journal of Computational and Applied Mathematics*, **223**, (2009), 485-498.

[7] K . Levenberg , A Method for the Solution of Certain Non-linear Problems in Least Squares, *Quart . Appl . Math .*, **2**, (1944), 164-168.

[8] D . W . Marquardt, An Algorithm for Least Squares Estimation of Non-linear Parameters, *J . Soc . Ind . Appl . Math*, **11**, (1963), 431-441.

[9] J . V . Beck , and K . J . Arnold, *Parameter Estimation in Engineering and Science*, **11**, Wiley, New York, 1977.

[10] Nasibeh Asa Golsorkhi, Hojat Ahsani Tehrani., Levenberg-Marquardt Method for Solving the Inverse Heat Transfer Problems., *Journal of mathematics and computer science*, **13**, (2009).

[11] Fletcher, Reeves, Function Minimization By Conjugate Gradients, *Computer Journal*, **7**(2), (2009), 149-154.

[12] Ozisik, N. Orlande, H.R.B., *Inverse Heat Transfer : Fundamentals and Applications.*

[13] Beddiaf S., Perez L., Autrique L. et Jolly JC.,, Function Minimization By Conjugate Gradients, *Computer Journal*, **7**(2), (2009), 149-154.

[14] Beddiaf S., Perez L., Autrique L. et Jolly JC., Parametric identification of a heating mobile source in a three dimensional geometry, *Computer Journal*, **7**(2), (2009), 149-154.