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Analyzing effect of variation of parameters on BOD and DO prediction in rivers.

Ritu Kapoor (Malik)[1], Babita Tyagi[2], Deepa Sinha[3] and Devendra Swaroop Bhargava[4]

Abstract

The role played by mathematical models in predicting DO concentration in rivers is globally known and widely accepted as a decision making tool with regard to management of water resources. The relevant literature is full of models with varied complexity. Regardless of their complexity, almost all models include some common parameters like coefficient of reaeration, rate of decay of settleable as well as dissolved BOD and initial settleable BOD. The values for these parameters as cited in literature may vary from one river to another. The objective of this chapter is to show the effect of variation (within the specified range) in the value of above stated parameters on concentration of total BOD and total DO using enhanced one dimensional model. Various changes are observed and shown graphically.

[1] Associate Professor,Starex University, Binola, Gurugram, India.
[2] Professor(Mathematics) Galgotias University, Greater Noida, U.P., India.
[3] Associate Professor (Mathematics),South Asia University, New Delhi, India.
[4] Former Professor(Environment), IIT Roorkee, AIT, Bangkok.

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1 Introduction

Water is an essential part of our life. Due to rapidly increasing urbanization and industrialization the quality of water in natural water bodies is deteriorating. This problem arises principally from the discharge of residues of human and natural activities that results in some way, in an interference of a desirable use of water. Organic matter discharge into water utilizes the DO (dissolved oxygen) of water and depletes DO in the aquatic system and hence make it difficult for the aquatic animals to survive in (oxygen-devoid) water. Also, it degrade the DO- BOD standards in rivers. The Mathematical Modelling helps in waste load allocation so that DO- BOD standards could be maintained. It also helps in keeping the ecosystem balanced. The assumption of Classical Streeter-Phelps model that advection is the only relevant transport phenomena unnecessarily restricts the model's validity in present era of modern computers. Bhargava (2, 3, 4, 5) for the first time, presented a model for accurate prediction of DO due to disposal of waste containing settleable as well as dissolved part of BOD. The model suggests a linear removal of settleable component of BOD along with simultaneous first order exponential decay of non-settleable and dissolved portion of BOD. Various one dimensional models (1, 6, 7, 8, 11, 16) have been developed to date for predicting DO conditions in river but these models are not valid in Initial period. Tyagi (19) developed a one dimensional model that takes into account the effect of both type of BOD (settleable as well as dissolved.) on river's DO but this model is applicable only after mixing length is over. Various complex two-dimensional models (9, 10, 12, 20) are available but they need a considerable amount of hydraulic data which in many cases is not available and has to be estimated. Rough estimate of parameters lead to a partial loss of accuracy gained.

An Enhanced one dimensional model proposed by Reichert and Wanner (14) that is capable for predicting solute transport in rivers for about 80% of initial period is used by Tyagi et.al.(21, 22) to predict BOD-DO conditions in rivers with large width. The model helps in accurate prediction of BOD-DO in rivers with large width in steady-state conditions.

In this work model developed by Tyagi et.al.(21,22) is used to analyze the effect of some common parameters used in relevant literature (13,15,18) which may vary from river to river to predict BOD-DO conditions in river.

2 Mathematical Model

The cross-section of the river is divided into two zones namely advective zone in the centre of river and stagnant zone along the two banks where the velocity is almost zero. A mathematical model is developed for the above stated river system based on the following assumptions:

- The entire BOD is in two forms namely settleable and dissolved forms. The dissolved part of BOD is decaying according to first order kinetics while the settle able part is being removed as per linear law.
- The size of stagnant zone is αA_T and it consists of two parts located near the two river banks while the size of advective zone is $(1 - \alpha) A_T$, where A_T is the total area of cross-section of the river and α is fraction of wetted cross-sectional area of the stagnant zone. For the sake of simplicity we are assuming $\alpha A_T = A_s$ and $(1 - \alpha) A_T = A$.

- No transverse gradient exists within any of the two zones but there is exchange of mass between the two zones (viz. advective and stagnant) which is linearly related to the difference in the respective concentration.
- In stagnant zone, only exchange of mass with the advective zone and reaeration within the stagnant zone are considered.
- In advective zone, advection, reaction and exchange of mass are considered.
- The effect of reaeration, modeled according to Henry's law, is considered in both the zones.
- The whole pollutant is being released into the advective zone only.

On the basis of above assumption, following coupled equations representing the BOD-DO mass balance equations in advective zone and stagnant zone is developed.

2.1 Advective Zone

$$L = \begin{cases} L_o\left(1 - \frac{v}{d}.\frac{x}{u}\right); x < x1 \\ 0 \qquad ; x \geq x1 \end{cases} \tag{1}$$

$$0 = -u\frac{dB_d}{dx} - kB_d - \frac{\gamma}{A}\left(B_d - B'\right) \tag{2}$$

$$0 = -u\frac{dC}{dx} - mL - kB_d - \frac{\gamma}{A}\left(C - C'\right) + k_r\left(C_s - C\right) \tag{3}$$

2.2 Stagnant Zone

$$0 = \frac{\gamma}{A_s}\left(B_d - B'\right) - kB' \tag{4}$$

$$0 = \frac{\gamma}{A_s}\left(C - C'\right) - kB' + k_r'\left(C_s' - C'\right) \tag{5}$$

Where L = concentration of settle able BOD in advective BOD; L_o= concentration of initial settle able BOD in advective zone; u = average cross-sectional velocity in the advective zone (L/T); v = settling velocity of settle able BOD in advective zone(L/T); x= distance in flow direction (L); d= depth of stream in advective zone (L);x_1 = distance in advective zone where settleable part is completely removed; $B^{/}$ = concentration of BOD in stagnant zone (M/L^3); B_d = concentration of dissolved BOD in advective zone (M/L^3); γ = exchange coefficient per unit length (L^2/T); k = decay rate of dissolved BOD (T^{-1}) ; A = cross-sectional area of the advective zone; A_s = cross-sectional area of the stagnant zone; k_r = coefficient of reaeration in advective zone (T^{-1}); m= removal rate of settle able BOD in advective zone(T^{-1}); k_r' = coefficient of reaeration in stagnant zone (T^{-1}); C = concentration of DO in advective zone (M/L^3); $C^{/}$ = concentration of DO in stagnant zone (M/L^3); C_s' = concentration of DO at saturation level in stagnant zone. (M/L^3); (T^{-1}); C_s = concentration of DO at saturation level in advective zone. (M/L^3);

The BOD in advective zone is considered in two parts settle able as well dissolved and it is evaluated separately in Eq.1 and Eq.2. Equation 1. gives the decay of settle able part of BOD while Eq.2 represents the decay of dissolved BOD with distance downstream. Eq.1 suggests that the settle able part gets removed at a distance x = x_1 after which this does not take any oxygen from the river. This distance will be longer for deeper rivers and for smaller flocculated particle size. The combined effect of both the parts of BOD on DO is given by Eq.3.

Since it is assumed that the settleable part gets settled at the outfall itself due to zero velocity in stagnant zone, there is no equation for settleable BOD in stagnant zone, however, Eq. 4 gives the concentration of dissolved part of BOD in stagnant zone and Eq.5 represents the effect of BOD on DO in stagnant zone.

The total BOD (TB) at any point x is then calculated by Eq.6 .

$$TB = (1-\alpha)B + \alpha B' \quad (6)$$

Where B is BOD in advective zone and B' is BOD in stagnant zone ref (22)

The total Dissolved oxygen (TD) at any point x is calculated by Eq.7 as follows:

$$TC = (1-\alpha)C + \alpha C' \quad (7)$$

Where C is the DO in advective zone and C' is the DO in stagnant zone (ref (22)).

2.3 Boundary Conditions

The associated boundary conditions reflecting the release of pollutant according to assumption mentioned earlier are,

$$B_d = B_{o\_d} \text{ at } x = 0 \text{ and } C = C_s \text{ at } x=0. \quad (8)$$

3 Effect of variation in parameters

The sensitivity of the proposed model is analyzed with regard to the values of various parameters namely decay rate (m) for settleable part of BOD, decay rate (k) for dissolved part of BOD, coefficient of reaeration ($\mathrm{k_r}$) and initial settleable BOD($\mathrm{L_o}$).

3.1 Decay rate (m) of settleable BOD

To analyze the sensitivity of proposed model developed by Tyagi et.at. (21, 22), a variation in decay rate (m) of settleable part of BOD is considered in the range from (10 – 5) day^{-1}. The concentration of total DO are obtained and graphs are plotted by taking

$A = 0.4m^2; A_s = 0.7m^2; k = 3\ day^{-1}; k_{r=}\ k_r^{'} = 10 day^{-1}; u =$

$2800\left(\frac{m}{d}\right); B_{o\_d} = 16\left(\frac{mg}{L}\right);\ Lo = 12\ \left(\frac{mg}{L}\right)$

and values of $m = 5,7,9\ and\ 10\ (day^{-1})$ respectively.

Figure 1 represents the effect of variation in m on concentration of total DO. It has been observed that as m increases from 5 to 10, the point of critical DO is shifted closer to the source. The recovery of DO is faster for deeper curves and magnitude of remaining DO is decreasing with increase in the value of decay rate of settleable BOD. It would require more DO from river and as a result depletion of oxygen is more consequently magnitude of total DO is less at certain point.

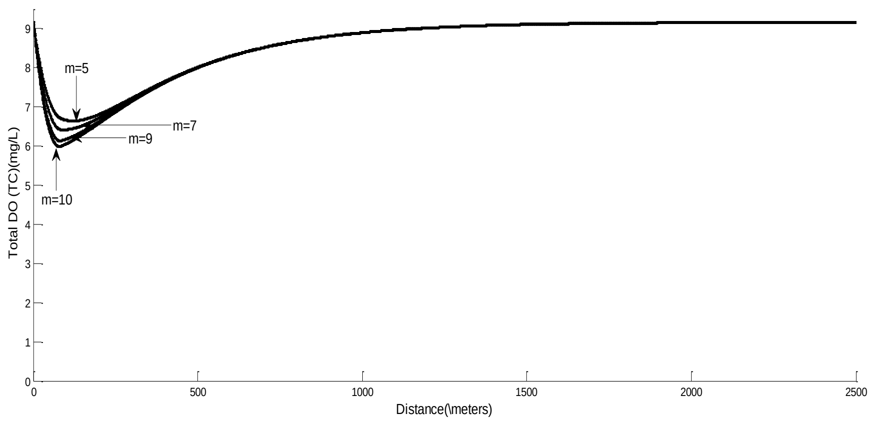

Figure 1: Concentration distributions of total DO for different values of settleable decay rate (m)

3.2 Initial input (L_o) of settleable BOD

To analyze the sensitivity of proposed model with regard to component of settleable BOD in input , a variation in value of settleable part of total BOD in initial input L_o is considered in the range (16 – 10) mg/L by taking $A = 0.4\ m^2; A_s = 0.7\ m^2; k = 3\ day^{-1}; k_r = k_r' = 10 day^{-1}; u = 2800\ \frac{m}{d}$, $m = 10\ day^{-1}$ and total initial BOD = 28 mg/L.

Figure 2 Represents the predicted values of total BOD with distance downstream taking L_o=10, 12, 14 and 16 mg/L respectively. It is observed that concentration of BOD at a particular point is decreased with increasing value of settleable BOD. Since settleable part is removed at a faster rate resulting assimilation of more BOD and consequently remaining BOD is decreased with distance downstream.

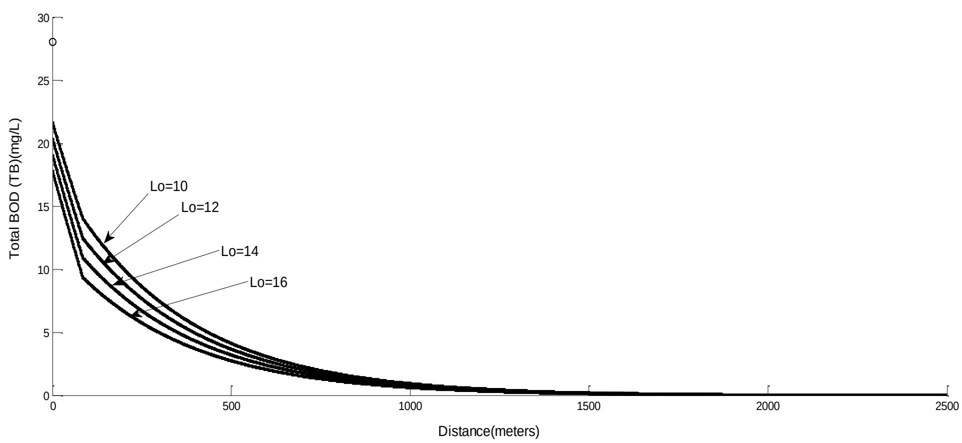

Figure 2: Concentration distribution of total BOD for different values of initial settleabl BOD (Lo)

Figure 3 shows the predicted values of total DO for different values of initial settleable BOD. It has been observed that critical point is shifted towards the source and magnitude of critical DO is increased with increased value of initial settleable BOD. Since removal of settleable BOD is at faster rate so it would require more river's DO and hence rate of oxygen depletion is more.

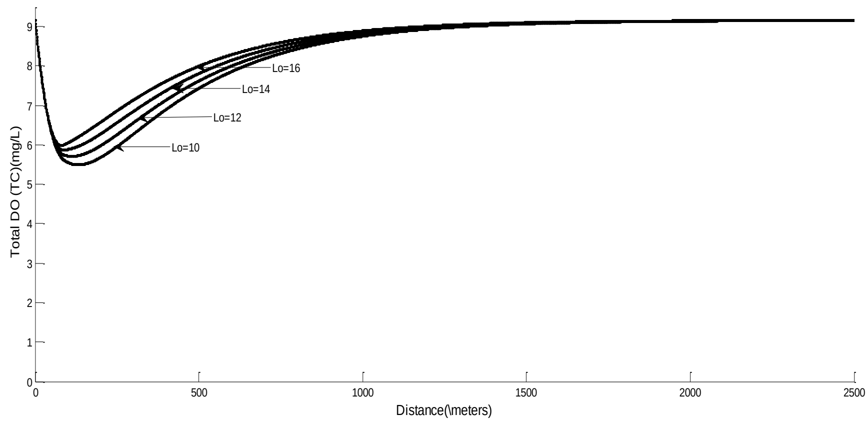

Figure 3: Concentration distribution of Total DO for different values of initial settleable BOD (Lo)

3.3 Coefficient of reaeration (k_{r}) of stagnant zone

To analyze the sensitivity of proposed model, the values of k_{r} i.e. coefficient of reaeration in stagnant zone has been changed from (9-3) day^{-1} by keeping $A = 0.4\ m^2; A_s = 0.7\ m^2; k = 3\ day^{-1}; k_r = 10 day^{-1}; u = 2800\left(\frac{m}{d}\right)$, $m = 10 day^{-1}$ and total initial BOD = 28 mg/L.

Figure 4 represents the effect of changing values of reaeration coefficient on the concentration of total DO. It has been observed that magnitude of critical DO is decreased and critical point is shifted away from the source with decreased values of k_{r}. Due to decreased rate of reaeration , recovery of DO becomes slower and thus decreasing the concentration of the remaining DO downstream.

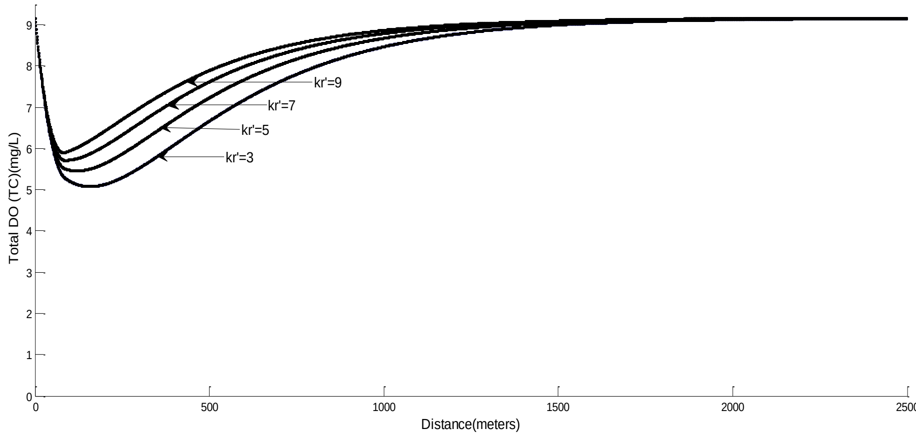

Figure 4: Concentration distribution of total DO for different values of reaeration coefficient of stagnant zone (k_r')

3.4 Decay rate (k) of dissolved BOD

To analyze the sensitivity of proposed model with regard to value of (k) in adective zone, a variation in k -values in the range (3.0 – 3.9) day^{-1} is considered. The values of other parameters are considered as follows: $A = 0.4m^2; A_s = 0.7m^2; k_r = k_r' = 10day^{-1}; u = 2800\frac{m}{d}$, $m = 10day^{-1}$ and total initial BOD = 28 mg/L.

Figure 5 represents the effect of k values on concentration of total BOD. It is observed that concentration of total BOD is decreased after a certain point with increasing value of ‘k’. This is because of faster rate of assimilation of dissolved BOD.

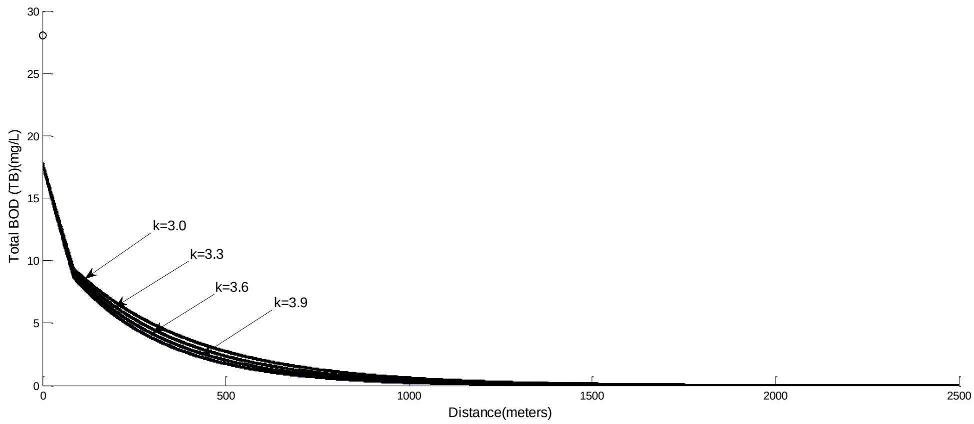

Figure 5: Concentration distribution of total BOD for different values of decay rate of dissolved BOD (k).

Figure 6 represents the effect of k values on concentration of total DO. It has been observed that critical point is shifted towards source and Sag in curve deepens with increasing value of k. The recovery of DO is faster for deeper curves and magnitude of remaining DO is decreasing with increase in k - values. Since decay rate of BOD is increased so it would require more DO from river and as a result depletion of oxygen is more thus, resulting a lower concentration of total DO upto a certain distance downstream.

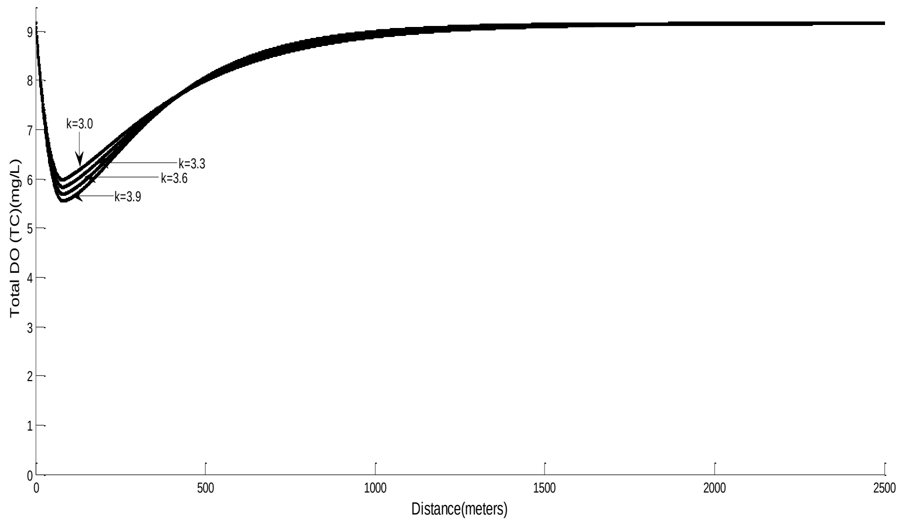

Figure 6: Concentration distribution of total DO for different values of decay rate of dissolved BOD (k).

5 Conclusion

In the presented paper sensitivity of enhanced one dimensional model for steady state is analyzed by taking variation in values of some common parameters. The observed results are shown in Figs 1- 6. It is concluded that the location as well as magnitude of critical deficit depends on values of m, k, k_r', and L_o.

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