Journal of Applied Finance & Banking, vol. 3, no. 4, 2013, 13-27
ISSN: 1792-6580 (print version), 1792-6599 (online)
Scienpress Ltd, 2013

A Prognostic Theory for the Systemic Cost of Bank Failures

Dror Parnes[1]

Abstract

We fragment the cost of bank failures within the national Deposit Insurance Fund into three components that include the resolution cost of the less-essential banks, the administrative rescue cost of the larger and more influential financial institutions, and the complete legislative bailout cost of the systemically vital banks. We develop a forecasting model that can help regulators to comprehend the expected systemic cost of future bank failures both over reasonably short-terms and through extended periods of time. The current theory can assist policy makers in better designing the reserves within the Deposit Insurance Fund and the exclusive premiums charged from banks that routinely subsidize these reserves.

JEL Classifications: G21, G22, G28
Keywords: Systemic Problem, Bank Failure, Predictive Model, Accumulated Cost, FDIC, DIF

1 Introduction

The savings and loan association crisis in the U.S. during the 1980s and 1990s resulted in the failure of 747 out of 3,234 thrift institutions and the legislative transfer of about $150 billion to depositors at these failed financial institutions. By its own estimates, the Federal Deposit Insurance Corporation (FDIC) sustained losses exceeding $36 billion to cover the 140 bank failures in 2009 alone. The FDIC has further updated its appraised cost of all U.S. bank failures in 2010 to a little more than $24 billion. On October 11, 2011, the FDIC estimated the expected total losses from bank failures at $19 billion for the five year

---

[1]Finance Department, College of Business, BSN 3127, University of South Florida, 4202 E. Fowler Ave., Tampa, FL 33620 – 5500, Phone: (813) 974 – 6357, Fax: (813) 974 – 3084.

Article Info: *Received :* March 19, 2013. *Revised :* April 11, 2013.
Published online : July 1, 2013

period 2011 through 2015. However, the FDIC admitted that its anticipated losses "are subject to considerable uncertainty."[2]

To be able to cover such enormous rates, in 2006 the Federal Deposit Insurance Reform Act unified the Bank Insurance Fund and the Saving Association Insurance Fund into a single Deposit Insurance Fund (DIF). The DIF is designed to prevent "bank runs" in times of bank insolvency by insuring the deposits of individuals up to a specified amount.[3] The DIF is required to maintain a contingent reserve to cover the projected cost of bank failures for the next twelve months, but, in addition, the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010 dictated a Designated Reserve Ratio of 1.35% of the estimated insured deposits. In several cases, however, this safety net could be insufficient, since the collapse of an exceptionally large financial institution may catch the regulatory agencies unprepared.[4] This economic setback has happened in the past and might very well happen again in the future.[5]

On the other hand, despite a common desire by regulators to be fully protected from sudden financial catastrophes, the contingent reserves within the DIF cannot be boundless. According to the Federal Deposit Insurance Reform Act, the banks themselves must shoulder the potential losses of their failures, and they are the ones that cooperatively pay the premiums for this unique insurance fund. The amount each institution is charged is based on the balance of insured deposits and on the risk level the specific bank poses to the DIF. Therefore, the FDIC cannot endlessly inflate its reserves within the DIF by over-charging the financial system with exaggerated fees, since this would add unnecessary burden on all depository institutions and in particular on the troubled ones. For that reason, the Dodd-Frank Act orders the FDIC to adopt a restoration plan should the DIF balance falls below 1.35%, and provide dividends back to the banking industry whenever the fund balance exceeds 1.50% of the estimated insured deposits. In light of this acute dilemma, a fresh diagnostic tool is needed to assist policy makers in better predicting the probable systemic cost from future bank failures.[6] We aim to contribute to this matter by proposing an original prognostic theory that approximates the accumulated future cost of bank failures to the DIF.

---

[2]According to several media sources, The FDIC acting chairman Martin Gruenberg did not reveal the underlying assumptions for the loss projections and added "As we seek to stay on track, it's important to always be mindful of the challenges we face and ongoing risks to the insurance fund."

[3]Diamond and Dybvig (1983) theoretically demonstrate that depositors normally suffer from asymmetric information concerning the specific problems of a failed bank; thus, a run on a bank can contagiously spread throughout the financial system.

[4]Hoggarth, Jackson, and Nier (2005) explain that safety nets are designed to help governments to handle crises more effectively, but they may also reduce market discipline and increase the chances for banking crises in the first place.

[5]Kaufman (2002) documents the failure of Continental Illinois National Bank in Chicago in 1984, the seventh largest bank in the U.S. at the time, as a prominent example for this systemic problem. The closure and receivership of Washington Mutual Bank in 2008, the largest thrift institution in the U.S. at that time, can also serve as an eminent paradigm for this obstacle. Harrington (2009) further discusses the decisive federal government intervention that prevented the bankruptcy of American International Group (AIG).

[6]So and Wei (2004) explain that the three key issues facing the FDIC are: (1) fair pricing of the deposit insurance fund, (2) optimal closure policy upon failure, and (3) continuous regulatory supervision.

In this article, we propose a notional model that can help regulators better assess future losses within the DIF, enhance preparations for various economic scenarios, and charge banks insurance premiums with greater accuracy. The proposed theory relies upon several stochastic variables that can and should be modified with time. Although some of these parameters are not directly observable, they can still be approximated from prior events and several additional forward-looking assumptions. We offer a rigorous analytical scheme that can estimate the expected number of failed banks as well as the projected cost to the DIF through both a relatively short-term period and over longer time horizons. However, we intentionally design the current scheme to be sufficiently general. Thus, we leave enough room for regulators to calibrate the model based on their underlying assumptions towards the future economic environment.

Our theory draws some inferences that can be methodically collected or roughly approximated, but others are open for diverse interpretations. In particular, the number of financial institutions in a given banking industry as well as their individual credit profiles can be evaluated with standard methodologies. In contrast, the first passage random time for the next systemically important failure can only be reckoned based on past experience combined with a prognostic analysis. Moreover, the comprehensive classification of which banks are too-big-to-fail and which failures mandate an administrative bailout may vary depending on the ad-hoc regulatory atmosphere. We are aware of no database that can supply either reasonable proxies or legislative judgments on these themes. Thus, we abandon any empirical effort hence our study remains a theoretical exercise. Nonetheless, our notional concept can practically serve as a decision support tool to governmental agencies by integrating presumptuous economic notions with specific credit evaluations. Altogether, the proposed model aims towards one goal: assisting the FDIC in better planning the necessary contingent reserves for the DIF, therefore soundly charging the correct insurance premiums from financial institutions.[7]

The remainder of the research is organized as follows. In Section 2, we provide a brief summary of prior studies that have examined several aspects of the resolution cost of bank failures. In Section 3, we deploy our theory. Within this segment we first derive the expected number of future bank failures, then we obtain the projected cost of bank failures over a limited term, and finally we develop a parallel prognostic tool for an extended period of time. In Section 4, we discuss policy implications rising from of our model. In section 5, we conclude.

2 Prior Literature

Numerous studies have attempted to empirically portray the complete cost structure of some famous banking catastrophes. It seems that, along the history, banking crises have carried colossal losses to dozens of nations. Nonetheless, these studies report that it is far from trivial to measure the direct impact of a banking crisis on simultaneous changes in economic output within the same region. Moreover, it is even harder to isolate the specific costs incurred by any banking crunch, because other exogenous determinants are typically

[7]The FDIC publishes a spreadsheet calculator, which illustrates the current methodology for computing deposit insurance assessment rates. According to this procedure, financial institutions are clustered into five risk categories, while each group is designated an initial base assessment rate and further adjustments for the respective unsecured debt and brokered deposits.

involved. In spite of these methodological difficulties, Caprio and Klingebiel (1996), Lindgren, Garcia, and Saal (1996), Sheng (1996), Dziobek and Pazarbaşioğlu (1998), and Caprio and Klingebiel (2002) describe major worldwide episodes of bank insolvency and associate these historic banking crises from the 1970s through the 1990s with adverse social effects by approximating their significant economic losses.

Honohan and Klingebiel (2003) describe the ruthless fiscal costs of several banking crunches around the globe. The authors find that in many countries governments spent on average more than 12% of the national Gross Domestic Product (GDP) to retrieve their financial systems, while in some developing markets these ratios exceeded 14%. In few instances including the early 1980s crises in Argentina and Chile, authorities spent much higher proportions, as much as 40-55% of their respective GDP. The authors rationalize that a significant part of the variation in the fiscal cost in these economies can be explained by ad-hoc regulatory protocols to resolving crises. In particular, administrations that offered open-ended liquidity aid, repeated partial recapitalizations, debtor bailouts, and wide safety nets for depositors typically incurred much higher losses.

Kaufman and Seelig (2005) provide profound insight on the overall cost of the savings and loan association crisis in the U.S. as well as other worldwide banking calamities. The authors illustrate the accumulated costs of these crises as a percent of some 50 nations' GDP from 1975 to 1997 and realize that except for few sporadic episodes, the relatively harsh consequences of banking crises are not materially different across industrial economies and emerging markets. Angkinand (2009) further expands this global analysis into the early 2000s.

Other researchers have tested various techniques for measuring the resolution costs of bank failures. The approach proposed by the International Monetary Fund (1998), of which the expenses of restructuring financial sectors can be divided into fiscal and quasi-fiscal costs, has been widely quoted over the years. Hoggarth, Reis, and Saporta (2002) discuss an alternative procedure for measuring the direct resolution costs to the government and then the broader costs to the welfare of the economy as captured by output losses in GDP. Bennett and Unal (2008) decompose the complete cost of bank failures into three categories: losses incurred on the disposition of the failed bank's assets, and direct and indirect expenses involved in the resolution of these failures. Bennett and Unal (2009, 2010) further contrast the cost of resolving bank failures between two commonly used systems: under the Deposit Payoffs method, where the FDIC assumes and liquidates the failed bank assets and then pays the individual depositors, and under the Purchase and Assumption method, where the FDIC leaves most of the failed bank assets in the hands of the private sector and transfers all the deposits to a potential acquirer.

Additional scholars have demonstrated the direct losses for deposit insurers realized in the failure of explicit financial institutions. The first group of articles focuses on the costs that are incurred by the specific configuration and the credit quality of the respective failed banks' asset. These studies include Bovenzi and Murton (1988), Barth, Bartholomew, and Bradley (1990), Blalock, Curry, and Elmer (1991), James (1991), Brown and Epstein (1992), Osterberg and Thomson (1994), and McDill (2004). A second set of papers also accounts for the liability structure of the failed financial institutions. These studies include Shibut (2002), Pennacchi (2005), and Schaeck (2008). Lee (2013) further derives a closed-form solution of the valuation of deposit insurance under forbearance for various banks.

The current study builds upon the preceding literature, but instead of empirically examining past banking crises, testing different techniques for measuring the resolution costs in bank failures, or even analyzing specific cost structures for individual failures of financial institutions, we present an analytical scheme that assists regulators and policy makers in forecasting the accumulated losses within the DIF. The following theory may draw inferences from those prior studies. In particular, the probability that a failure of a specific bank causes a systemic challenge, the expected time until the next systematic failure, the singular cost of a discrete bank failure, and the costs of institutional bailouts are all input variables to the model. Yet, rather than predicting the price of an isolated failure event, we focus our attention towards the aggregated losses to the Federal insurance fund throughout different time horizons.

3 The Model

The recent U.S. banking crisis has prompted several legal and financial revolutions. Among them, the "SIFI surcharge" was mandated with the Dodd-Frank Wall Street Reform and Consumer Protection Act during 2010. This regulatory chapter recognizes that the failure of systemically important financial institutions could trigger serious adverse effects on the current economic conditions and the overall financial stability; thus, it generally commands these dominant financial institutions to constantly hold extra capital. Following this viewpoint, the present theory recommends policy makers to strive for a continuous classification of three types of banks: Systemically Important Financial Institutions (SIFI), Systemically Partially Important Financial Institutions (SPIFI), and Systemically Not Important Financial Institutions (SNIFI).

By definition, the failures of SIFI are overwhelming hostile events within the banking sector and likely through other industries as well; thus, the regulator must prevent their closure beforehand. We presume that a failure of a SIFI i can spark a massive collapse among other banks, and its mandatory cost of bailout can be estimated in advance as β_i. These expected salvage costs can be approximated, for instance with the aid of the relatively new Office of Financial Research within the U.S. Department of Treasury, through a vigilant collection of relevant accounting records, meaningful business links to other financial institutions, and pertinent contractual off-balance sheet exposures. Nonetheless, since a SIFI classification is naturally preserved for few selected key banks, a conservative policy maker must deem a failure of a single SIFI as merely the tip of the iceberg, and thus project that the DIF would have to allocate more rescue funds $\mathcal{B}$ to support other troubled financial institutions as well. In this case, for the sole purpose of conservatism, we unify the administrative cost of any SIFI mandatory bailout as $\beta = Max\{\beta_i\} + \mathcal{B}$ for every $1 \leq i \leq N, \mathcal{B} \geq 0$.

Furthermore, the failure of a SPIFI is not expected to cause a colossal damage to the banking industry, yet we assume that the regulator still values its profound weight in the financial system and rescues it in advance as well. We denote the respective rescue costs of these SPIFI as λ_i, which could be substantially different from one another. Because of their relative importance to the general economy and particularly to the banking industry, the regulator bears the cost of bailing out both the SIFI and the SPIFI. However, in a financial industry with N active banks, we forecast that $\sum_{i=1}^{N} \lambda_i > \beta \stackrel{\text{def}}{=} Max\{\beta_i\} + \mathcal{B}, 1 \leq i \leq N, \mathcal{B} \geq 0$. This strict inequality implies that since there are presumably numerous SPIFI in the U.S. banking industry, the inclusive cost of saving a single SIFI and its

associated troubled banks is still cheaper than rescuing all the individual SPIFI in the system.
In contrast, by its own definition, a failure of a SNIFI does not compel any governmental bailout. A failure of a SNIFI causes at most a minor domestic disturbance with no extraneous financial repercussions. Nevertheless, the failures of these less-essential banks still carry several floating costs to the regulator. These costs usually include reimbursements on depository accounts up to the maximum allowed by the law at the time, various legal costs, and liquidation costs of some of the failed bank assets. We therefore assign κ_i to represent these isolated SNIFI failure costs to the DIF. The analysis hereafter detaches these resolution costs from the bailout costs of SIFI and SPIFI.
Altogether, our model differentiates between SIFI, SPIFI, and SNIFI by their relative size, conceivable impact on the entire banking industry, and their respective cost of failures. SIFI are typically large banks with vast influence on other financial institutions. SPIFI are commonly large depository firms but with limited effect on the global financial system. SNIFI are generally small banks with no foreseeable impact on other institutions. The cost of failures of SIFI and SPIFI is in fact the price of their bailouts, but the resolution cost of SNIFI is often derived by the FDIC with the Deposit Payoffs method or the Purchase and Assumption method. In addition, our theory considers that an administrative rescue of a SPIFI has only domestic consequences, but a bailout of a SIFI triggers a *renewal process* to the banking system, which we define in the following subsections.
To assess the systemic cost of future bank failures to the DIF, we first project the expected number of bank failures and then incorporate the pre-estimated costs associated with the three distinct groups of banks: the SIFI, the SPIFI, and the SNIFI. Nevertheless, as described hereafter, to portray a genuine scenario, we shall consider some level of ambiguity in the classification of these three clusters. Since we are not aware of any accessible database that contains tangible records for these model parameters, our investigation remains theoretical. However, the current model conveys important policy implications, of which we discuss later on.

3.1 The Number of Bank Failures before the First Collapse of a SIFI

At time $t = 0$ the financial system contains N operational banks. Each bank $1 \leq i \leq N$ has its own unique creditworthiness and accordingly its idiosyncratic expected lifetime with a respective Cumulative Distribution Function (CDF) $\Phi_i(t)$ and a matching Probability Density Function (PDF) $\phi_i(t) \stackrel{\text{def}}{=} \Phi_i(t)/dt$. Since the categorization process of a SIFI rationally depends upon the current economic cycle, the relative contribution of a specific bank within the whole financial industry, the ad-hoc regulatory environment, and the present composition of the entire banking sector, a SIFI classification may transform over time. We therefore denote $\alpha_{i,t} \in [0,1]$ as the probability that a bank is classified as a SIFI during the time $(t, t+1]$. In this setting, $\alpha_{i,t}$ also signifies the likelihood that the failure of bank i triggers a colossal damage to the banking industry, and the complement probability $\alpha_{i,t}^C \stackrel{\text{def}}{=} 1 - \alpha_{i,t}$ represents the chances that the failure of bank i has no consequential effect on the entire banking sector during the next time unit.
We now turn to assess the expected number of bank failures for a system that contains N financial institutions over the time interval $(0, t)$ with $t \leq \gamma$, where γ denotes the expected time to the first SIFI failure. Since γ is a random variable, we assign $\Psi(\gamma)$ and

$\psi(\gamma)$ to be the CDF and the PDF of the projected time to the first SIFI failure, respectively. In this case,

$$\Psi^C(t) \stackrel{\text{def}}{=} 1 - \Psi(t) = P(\gamma > t). \tag{1}$$

The current theory intentionally does not define a standard length for the time unit t. The regulator can characterize t based on its own necessities for better designing the reserves within the DIF. We can only recommend policy makers to select a long enough time horizon t such that timely modifications to the DIF remain feasible. With a general resolution of governmental bailouts, at least in theory, banks can fail more than once over a course of number of years. We designate $\mathcal{N}_i(t)$ as the number of bank i failures $(1 \leq i \leq N)$ in the time interval $(0, t)$. Therefore, the joint probability for $\mathcal{N}_i(t) = n_i$ and that all failed banks thus far are either SPIFI or SNIFI can be expressed as:

$$\begin{aligned}
&P\left(\mathcal{N}_i(t) = n_i \bigcap \text{no failed SIFI}\right) \stackrel{\text{def}}{=} \\
&P(\mathcal{N}_i(t) \geq n_i \cap \text{no failed SIFI}) - P(\mathcal{N}_i(t) \geq n_i + 1 \cap \text{no failed SIFI}) = \\
&\prod_{i=1}^{N}\left(\alpha_{i,t}^{C}\right)^{n_i}\left[\Phi_i^{*n_i}(t) - \Phi_i^{*(n_i+1)}(t)\right],
\end{aligned} \tag{2}$$

where $\Phi_i^{*n_i}(t)$ represents the *n-fold convolution* of $\Phi_i(t)$ with the universal corner solution $\Phi_i^{*0}(t) \equiv 1$.[8] In practice, n-fold convolutions are somewhat difficult to process. It is beyond the scope of this article to suggest any favorable methodology to compute n-fold convolutions however there is a vast mathematical literature on several approximation techniques to overcome this complexity including some analytical methods, a numerical method, a method of moments, and a recursive method. Tentatively, the number of bank i failures n_i $(1 \leq i \leq N)$ can vary and be anywhere from zero to infinity, thus we now obtain:

$$\Psi^C(t) = \prod_{i=1}^{N}\sum_{n_i=0}^{\infty}\left(\alpha_{i,t}^{C}\right)^{n_i}\left[\Phi_i^{*n_i}(t) - \Phi_i^{*(n_i+1)}(t)\right] = \prod_{i=1}^{N}\Psi_i^C(t), \tag{3}$$

where we define:

$$\Psi_i^C(t) \stackrel{\text{def}}{=} \sum_{n=0}^{\infty}\left(\alpha_{i,t}^{C}\right)^{n}\left[\Phi_i^{*n}(t) - \Phi_i^{*(n+1)}(t)\right] \tag{4}$$

Our next endeavor aims towards finding a reduced-form solution for the expected number of bank i failures $(1 \leq i \leq N)$ during the time interval $(0, t)$. To attain $E[\mathcal{N}_i(t)]$ we define N independent random variables γ_i accompanied by their respective survivor functions $\Psi_i^C(t)$, thus:

$$\gamma = Min\{\gamma_i, 1 \leq i \leq N\} \tag{5}$$

In essence, γ_i represents the *first passage time* for a single SIFI i failure, i.e. when only bank i can cause a systemic failure with a strictly positive $\alpha_i > 0$, but for all other banks

[8] For further explanations on this see for example Ross (1992, page 6).

$\alpha_j = 0$ for every $j \neq i, 1 \leq j \leq N$. In this context, the Inverse Gaussian distribution may serve as a legitimate bridge between theory and practical estimation. Though, when each of the N banks can trigger a systemic failure with some positive probability, the first passage time until the next SIFI failure is portrayed by equation (5). In this case, we can utilize the *Bayes rule* and obtain the following instantaneous probability:

$$P(\mathcal{N}_i(t) = n \mid t \leq \gamma \leq t + \varepsilon t) = \frac{P(\mathcal{N}_i(t)=n \cap t\leq\gamma\leq t+\varepsilon t)}{P(t\leq\gamma\leq t+\varepsilon t)} = \frac{\begin{bmatrix} P\left(\mathcal{N}_i(t)=n, t\leq\gamma_i\leq t+\varepsilon t, \gamma_j>t+\varepsilon t, 1\leq j\neq i\leq N\right) \\ +P\left(\mathcal{N}_i(t)=n, \gamma_i>t+\varepsilon t, t\leq Min\{\gamma_j\}\leq t+\varepsilon t, 1\leq j\neq i\leq N\right)+O(\varepsilon t)\end{bmatrix}}{P(t\leq\gamma\leq t+\varepsilon t)} = \frac{\alpha_{i,t}\left(\alpha_{i,t}^C\right)^n \phi_i^{*(n+1)}(t)\chi_i^C(t+\varepsilon t)\varepsilon t+\left(\alpha_{i,t}^C\right)^n\left[\Phi_i^{*n}(t+\varepsilon t)-\Phi_i^{*(n+1)}(t+\varepsilon t)\right]\xi_i(t)\varepsilon t+O(\varepsilon t)}{P(t\leq\gamma\leq t+\varepsilon t)}, \tag{6}$$

Where: O denotes the *mathematical order* (general magnitude or scale), ε signifies an *infinitesimal increment*, and we deploy the following notations in the numerator:

$$\phi_i^{*n}(t) \stackrel{\text{def}}{=} \frac{d\Phi_i^{*n}(t)}{dt}, \tag{7}$$

$$\chi_i^C(t) \stackrel{\text{def}}{=} P\left(\gamma_j > t, j \neq i, 1 \leq j \leq N\right) = \prod_{j\neq i}^{N} \Psi_j^C(t), \text{ and} \tag{8}$$

$$\xi_i(t) \stackrel{\text{def}}{=} P\left(t \leq Min\{\gamma_j\} \leq t + \varepsilon t, j \neq i, 1 \leq j \leq N\right) = \frac{-d\chi_i^C(t)}{dt} \tag{9}$$

We can further dismantle the denominator as:

$$P(t \leq \gamma \leq t + \varepsilon t) = \psi(t)\varepsilon t + O(\varepsilon t), \tag{10}$$

and acquire a more compact solution by considering that $\varepsilon t \to 0$ as:

$$P(\mathcal{N}_i(t) = n \mid \gamma = t) = \frac{\alpha_{i,t}\left(\alpha_{i,t}^C\right)^n \phi_i^{*(n+1)}(t)\chi_i^C(t)+\left(\alpha_{i,t}^C\right)^n\left[\Phi_i^{*n}(t)-\Phi_i^{*(n+1)}(t)\right]\xi_i(t)}{\psi(t)} \tag{11}$$

We can now derive the expected number of bank failures at a specific point in time as:

$$E[\mathcal{N}_i(t) \mid \gamma = t] = \sum_{n=0}^{\infty} nP(\mathcal{N}_i(t) = n \mid \gamma = t) = \frac{\nu_i(t)\chi_i^C(t)+\delta_i(t)\xi_i(t)}{\psi(t)}, \tag{12}$$

where we define two more temporary variables as:

$$\nu_i(t) \stackrel{\text{def}}{=} \alpha_{i,t} \sum_{n=1}^{\infty} n\left(\alpha_{i,t}^C\right)^n \phi_i^{*(n+1)}(t), \text{ and} \tag{13}$$

$$\delta_i(t) \stackrel{\text{def}}{=} \sum_{n=1}^{\infty} n\left(\alpha_{i,t}^C\right)^n \left[\Phi_i^{*n}(t) - \Phi_i^{*(n+1)}(t)\right] \tag{14}$$

From here we can postulate the expected number of bank failures (only SPIFI and SNIFI) before the first failure of a SIFI by using, once again, the Bayes theorem and noticing that the events $N_i(t) = n$ and $\gamma_i > t$ are independent of $\gamma_j > t, \forall j \neq i, 1 \leq j \leq N$, thus we attain:

$$P(\mathcal{N}_i(t) = n \mid \gamma > t) = \frac{P(\mathcal{N}_i(t)=n \cap \gamma>t)}{P(\gamma>t)} = \frac{P(\mathcal{N}_i(t)=n \cap \gamma_i>t)\cdot P\left(\gamma_j>t, 1\leq j\neq i\leq N\right)}{P(\gamma_i>t)\cdot P\left(\gamma_j>t, 1\leq j\neq i\leq N\right)} = \frac{\left(\alpha_{i,t}^{C}\right)^n\left[\Phi_i^{*n}(t)-\Phi_i^{*(n+1)}(t)\right]}{\Psi^C(t)}, \tag{15}$$

which finally yields the expected total number of bank failures before the first collapse of a SIFI as follows:

$$\sum_{i=1}^{N} E[\mathcal{N}_i(t) \mid \gamma > t] = \sum_{i=1}^{N}\sum_{n=0}^{\infty} nP(\mathcal{N}_i(t) = n \mid \gamma > t) = \sum_{i=1}^{N}\frac{\delta_i(t)}{\Psi^C(t)} \tag{16}$$

3.2 The Accumulated Cost of Bank Failures over a Limited Period of Time

Since the cost structure of a SNIFI failure is different from those of the other two groups of banks, hence a collapse of a SNIFI does not mandate an administrative rescue effort, at this stage of the analysis we separate the SNIFI from SPIFI and SIFI. We define $\pi(\tau)$ as the stochastic cost of SPIFI failures in the time interval $(0,\tau)$ and denote its expected value as $\Pi(\tau) \stackrel{\text{def}}{=} E[\pi(\tau)]$. We recall that after every governmental bailout of a SPIFI, the banking system is essentially at its initial state, i.e. the same N financial institutions remain operational while having the same failure probabilities as originally estimated. Therefore, conditioning on the first passage time γ for the next SIFI failure we have:

$$E[\pi(\tau) \mid \gamma = t] = \begin{cases} \sigma(t) + \Pi(\tau - t) & \text{if } t \leq \tau \\ \rho(\tau) & \text{if } t > \tau \end{cases}, \tag{17}$$

where

$$\sigma(t) \stackrel{\text{def}}{=} \sum_{i=1}^{N} \lambda_i E[\mathcal{N}_i(t) \mid \gamma = t] = \sum_{i=1}^{N} \lambda_i \frac{\nu_i(t)\chi_i^C(t)+\delta_i(t)\xi_i(t)}{\psi(t)}, \text{ and} \tag{18}$$

$$\rho(t) \stackrel{\text{def}}{=} \sum_{i=1}^{N} \lambda_i E[\mathcal{N}_i(t) \mid \gamma > t] = \sum_{i=1}^{N} \lambda_i \frac{\delta_i(t)}{\Psi^C(t)} \tag{19}$$

Furthermore, we can remove the condition $\gamma = t$ and obtain:

$$\Pi(\tau) = \int_0^{\tau}[\sigma(t) + \Pi(\tau - t)]d\Psi(t) + \rho(\tau)\Psi^C(\tau) = \int_0^{\tau}\Pi(\tau - t)d\Psi(t) + \mathrm{M}(\tau), \tag{20}$$

Where:

$$\mathrm{M}(\tau) \stackrel{\text{def}}{=} \rho(\tau)\Psi^C(\tau) + \int_0^{\tau}\sigma(t)d\Psi(t) \tag{21}$$

The purpose of rearranging equation (20) is to form a *renewal-type equation*, which can be rewritten once again by using the *Laplace transform* as[9]:

$$\Pi(\tau) = \mathrm{M}(\tau) + \int_0^{\tau}\mathrm{M}(\tau - t)d\eta(t), \tag{22}$$

where $\eta(t)$ represents a *renewal function*, which is associated with the CDF $\Psi(t)$ as:

$$\eta(t) \stackrel{\text{def}}{=} \Psi(t) + \int_0^{\tau}\eta(t - \vartheta)d\Psi(\vartheta), \tag{23}$$

[9]For more explanations on this integration see Ross (1992, page 35, Proposition 3.4).

and ϑ is a time distribution parameter. We realize that both $\eta(0) = 0$ and $\mathrm{M}(0) = 0$, thus we can now integrate by parts equation (22) and have:

$$\Pi(\tau) = \int_0^\tau [1 + \eta(\tau - t)]\mu(t)dt, \tag{24}$$

where for every $\alpha_{i,t} \in [0,1]$, $1 \leq i \leq N$, we specify the following derivative:

$$\mu(t) \stackrel{\text{def}}{=} \frac{d\mathrm{M}(t)}{dt} = \sum_{i=1}^{N} \lambda_i \frac{\alpha_{i,t}^C}{\alpha_{i,t}} \psi_i(t)\chi_i^C(t) \tag{25}$$

We recognize that a practical view of any banking sector would assign most banks low or even negligible chances to become SIFI, hence $\alpha_{i,t} \to 0$ therefore $\alpha_{i,t}^C \to 1$ for the vast majority of financial institutions. In this case:

$$\lim_{\alpha_{i,t} \to 0} \left[\frac{\psi_i(t)}{\alpha_{i,t}}\right] = \frac{d\eta_i(t)}{dt}, \tag{26}$$

where $\eta_i(t)$ is a *renewal function* associated with the idiosyncratic expected bank's lifetime CDF $\Phi_i(t)$. This outcome is a direct result of equation (4) and the fact that[10]:

$$\eta_i(t) = \sum_{n=1}^{\infty} \Phi_i^{*n}(t) \tag{27}$$

This setting dictates that the projected systemic cost to the DIF over a limited time horizon is the sum of the accumulated losses associated with SNIFI failures, the probable rescue costs for all SPIFI failures, and the expected costs of the banking system renewals as a function of the assumed SIFI failures. We therefore depict this limited-term accumulated cost as:

$$\widetilde{\Delta}(\tau) = \sum_{i=1}^{N} \Phi_i(\tau)\kappa_{i,\tau} + \Pi(\tau) + \beta\eta(\tau) =$$
$$\sum_{i=1}^{N} \Phi_i(\tau)\kappa_{i,\tau} + \int_0^\tau [1 + \eta(\tau - t)]\mu(t)dt + \beta\eta(\tau) \tag{28}$$

The first module in equation (28) captures the expected resolution cost among all SNIFI. This part is independent of any renewal process within the banking system. The second component presents the projected cost to the DIF from the failures of SPIFI. This part contains two functions: $\eta(t)$, which is defined in equation (23) and $\mu(t)$, which is uncovered in equation (25). Both of these functions together reveal the cost structure of SPIFI and its unique dependency upon the pertinent model parameters λ_i, $\alpha_{i,t}$, $\Psi(t)$, $\psi(t)$, and $\Phi_i(t)$. The third element grants the cost structure of a SIFI accompanied by the renewal function $\eta(t)$.

[10] See Ross (1992, page 32, Proposition 3.1).

3.3 The Systemic Cost of Bank Failures over a Long Period of Time

Our theory assumes that the banking system is in fact subject to a renewal process with a first passage time that can be characterized by the CDF $\Psi(t)$. Essentially, we consider that a system renewal is a direct result of any SIFI failure. Nonetheless, reality shows that while these events are rather scarce they are incredibly expensive. Actually, it can take years between any two successive SIFI failures. This rational has guided us when we defined the mandatory bailout cost for a SIFI failure as $\beta = Max\{\beta_i\} + \mathcal{B}$ for every $1 \leq i \leq N, \mathcal{B} \geq 0$, since this overall price represents not only the rescue of the specific SIFI, but also the additional necessary funds required to support the disturbed banking sector. Although this would be a rugged approximation, we can define the time length between any two consecutive SIFI failures as a random (stochastic) phase; therefore, the expected time-length of this phase becomes:

$$\omega = \int_0^\infty t d\Psi(t) \tag{29}$$

In reality, this time interval can be crudely approximated by assessing the overall macroeconomic environment, and by aggregating all the potential SIFI within the relevant financial industry, their explicit risk profiles, as well as their business interactions with other institutions. This would be merely a rough estimation, yet as long as there is at least one strictly positive $\alpha_i > 0$, $1 \leq i \leq N$, the expected time-length of a phase ω is mathematically not ill-defined, hence it is finite. In this case, we temporarily exclude the projected SNIFI failures and postulate the probable cost per phase as the sum of the failure costs of all the failed SPIFI and a single SIFI as:

$$E[cost\ per\ phase] = \beta + \int_0^\infty \sigma(t) d\Psi(t) \tag{30}$$

We can then incorporate the failure cost of the SNIFI, utilize the *renewal reward process*, and estimate the total expected cost per phase for a long period of time as[11]:

$$\widetilde{\Delta}(\tau \to \infty) = \sum_{i=1}^{N} \Phi_i(\tau)\kappa_{i,\tau} + \frac{\beta + \int_0^\infty \sigma(t) d\Psi(t)}{\omega} = \sum_{i=1}^{N} \Phi_i(\tau)\kappa_{i,\tau} + \frac{\beta + \int_0^\infty \sigma(t) d\Psi(t)}{\int_0^\infty t d\Psi(t)} \tag{31}$$

The asymptotic properties of this renewal process use the *strong law of large numbers* and universally dictate that with probability one (certainty):

$$\lim_{\tau \to \infty} \left[\frac{\rho(\tau)}{\tau}\right] = \mathcal{C} < \infty, \tag{32}$$

where $\mathcal{C}$ is some constant, i.e. this limit is determinate. In addition, since the expected time-length of a phase ω is finite, we learn that:

[11] Ross (1992, page 51) further elaborates on that. Other mathematical textbooks refer to this type of transition as the "*elementary renewal theorem for the reward process*."

$$\lim_{\tau\to\infty} \tau\Psi^C(\tau) = 0, \text{ and} \tag{33}$$
$$\lim_{\tau\to\infty} \rho(\tau)\Psi^C(\tau) = 0 \tag{34}$$

We can now integrate equation (34) with equations (21) and (25) and obtain:

$$\int_0^\infty \sigma(t)d\Psi(t) = \int_0^\infty \mu(t)dt, \tag{35}$$

and finally acquire the estimated systemic cost of all bank failures, SNIFI, SPIFI, and SIFI, over a long period of time as:

$$\widetilde{\Delta}(\tau \to \infty) = \sum_{i=1}^{N} \Phi_i(\tau)\kappa_{i,\tau} + \frac{\beta + \int_0^\infty \mu(t)dt}{\omega} = \sum_{i=1}^{N} \Phi_i(\tau)\kappa_{i,\tau} + \frac{\beta + \int_0^\infty \mu(t)dt}{\int_0^\infty t d\Psi(t)} \tag{36}$$

Similar to the short-term assessment in equation (28) the first module of equation (36) captures an independent estimation of the resolution cost of all SNIFI in the banking sector. However, over the long horizon, the second component of equation (36) jointly evaluates the expected costs to the DIF from the failures of SPIFI and SIFI. This is a direct result of the fact that over a long period of time, we can expect to observe several SIFI downfalls accompanied by multiple system renewals.

4 Policy Implications

In the prior section we have developed a dynamic model that incorporates several quantifiable parameters including the number of financial institutions N in a given banking sector, the credit profiles $\Phi_i(t)$ of the individual banks within $1 \le i \le N$, the timely likelihoods $\alpha_{i,t}$ for banks to be classified as SIFI, the first passage random time γ for the next SIFI failure along with its respective CDF $\Psi(t)$, the resolution costs κ_i of SNIFI, the rescue costs λ_i of SPIFI, the complete price of bailout β of SIFI, and the expected time length ω between any two consecutive SIFI failures. In practice, these key factors are not readily observable, yet they can be approximated based on the accumulated experience thus far, the explicit perspective of the regulatory authorities, and the prognostic assumptions towards the future economic

The proposed theory has several discernible advantages. Unfortunately, some of these features can also be considered as apparent disadvantages; it all depends on the eye of the beholder. First, the present model is stochastic hence it can be straightforwardly adjusted over time. All of the above determinants can diverge throughout the era; thus, the suggested scheme allows boundless flexibility for regulators when designing the necessary reserves of the DIF under different economic settings. This excess elasticity, on the other hand, may result with a wide range of predictions. We thus conclude that a prudent regulator would probably be better-off by incorporating some dogmatic constraints into the model. We have exercised such a conservative approach when assembling the unified failure cost of SIFI as the sum of the maximum individual losses from SIFI failures and an extra marginal cost to support other troubled financial institutions, i.e. $\beta = Max\{\beta_i\} + \mathcal{B}$ for every $1 \leq i \leq N, \mathcal{B} \geq 0$.

Second, the current framework permits policy makers to form a vigorous scenario-based analysis. With relatively little effort, regulatory agencies can modify their forward-looking assumptions with respect to their view of the forthcoming economic cycle. In particular, a primary classification of the overall macroeconomic environment including a contractionary cycle, an expansionary phase, a lasting stagnation, a moderate recovery, etc., would alter some if not all of the above model parameters thus assist in obtaining adaptable expected costs within the DIF. Regrettably, an immediate drawback rises since in most cases, an economic cycle is distinctly defined only ex-post, thus this conventional opacity could generate uneven estimations.

Third, the present scheme further admits various sensitivity analyses. Both equations (28) and (36) are the core essence of the theory. In both of these derivations there are separate cost modules for the SNIFI, i.e. $\sum_{i=1}^{N} \Phi_i(\tau)\kappa_{i,\tau}$. By design, these components are independent from any renewal process in the banking sector; thus, economists can perform sensitivity analyses of these parts based on the likely number of SNIFI in the banking system and their respective resolution costs. Furthermore, policy makers can explore the relative damage carried by this module while contrasted with the expected losses to the insurance fund encountered by the SPIFI and the SIFI. For instance, driven by their comparative sizes, it would be reasonable to assume that the singular resolution costs of SNIFI are lower than the individual rescue costs of SPIFI, i.e. $\kappa_i < \lambda_i$, yet their estimated failure probabilities as well as their inclusive quantity in any well-diversified banking industry should be higher. Therefore, the overall impact of SNIFI failures on the systemic cost within the DIF could be quite significant.

In addition, economists may study the consistency of their predictions for each cost segment and further accommodate the model over time. Presumably, the cost modules of SNIFI can be predicted with greater visibility since these institutions are more prone to fail; therefore, a long-term moving-average of their failure rates can be approximated with greater accuracy. The resolution costs of SNIFI κ_i are also relatively stable, compared to the bailout costs of other financial institutions. Conversely, the cost components of SPIFI and SIFI are more unpredictable, hence they are more sensitive to the underlying economic assumptions. This sensitivity can be further examined and reduced with more data collected over the years.

Last but not least, while equation (28) portrays the expected cost to the DIF over a short-term horizon, equation (36) presents the projected cost structure over a long period of time. Our intention is to offer regulatory agencies sovereign tools that can forecast the systemic costs of bank failures before the next SIFI collapse and over a longer era that includes several such failures. Nonetheless, we intentionally retain these two time frames undetermined. This allows regulators to further calibrate the model according to the relevant economic circumstances and over different time intervals.

5 Summary

In this study we disentangle the systemic cost of bank failures to the Federal deposit insurance fund into three modules: (1) the resolution costs associated with the likely failures of the not-so-important banks, i.e. those relatively small financial institutions that convey merely a limited regional influence within the banking system, (2) the administrative rescue costs of the relatively larger banks that exhibit only limited impact on the entire banking industry, and (3) the governmental comprehensive bailout costs of the systemically vital institutions, i.e. those highly selective banks that pose critical role within the financial sector.

To construct a predictive theory that can project the accumulated cost of bank failures to the national Deposit Insurance Fund we integrate the banks' failure probabilities with their respective insolvency costs as well as the plausible interactions among these three groups of banks. Explicitly we express these relations as the probabilities of each financial institution to become systemically important and a general renewal process over the entire banking system. We then derive two predictive tools that can help regulators to comprehend the expected total cost of future bank failures either within a moderately short-term or over longer periods of time.

We aim to develop a rigorous yet comprehensible and applied scheme that can be deployed in practice. For that purpose we evade further complications of the underlying renewal theory such as delayed renewal processes for the insolvency courses of systemically important financial institutions. This and other mathematical complexities can be added upon specific necessities in the future. The final outcome is an analytical model that can assist policy makers in better designing the reserves within the Federal deposit insurance fund as well as the exclusive premiums charged from banks to routinely sustain these reserves. This unique function exhibits vast significance both within the financial system and in the economy as a whole.

References

[1] A. P. Angkinand, Banking Regulation and the Output Cost of Banking Crises, *Journal of International Financial Markets, Institutions & Money*, **19**, (2009), 240-257.

[2] J. R. Barth, P. F. Bartholomew, and M. G. Bradley, Determinants of Thrift Institution Resolution Costs, *The Journal of Finance*, **45**, (1990), 731-754.

[3] R. L. Bennett and H. Unal, Understanding the Components of Bank failure Resolution Costs, FDIC Working Paper, (2008).

[4] R. L. Bennett and H. Unal, The Effects of Resolution-Method Choice on Resolution Costs in Bank Failures, FDIC Working Paper, (2009).

[5] R. L. Bennett and H. Unal, The Cost Effectiveness of the Private-Sector Resolution of Failed Bank Assets, FDIC Working Paper, (2010).

[6] J. Blalock, T. Curry, and P. Elmer, Resolution Costs of Thrift Failures, *FDIC Banking Review*, **4**, (1991), 15-25.

[7] J. F. Bovenzi and A. J. Murton, Resolution Costs of Bank Failures, *FDIC Banking Review*, **1**, (1988), 1-11.

[8] R. A. Brown and S. Epstein, Resolution Costs of Bank Failures: An Update of the FDIC Historical Loss Model, *FDIC Banking Review*, **5**, (1992), 1-15.

[9] G. Caprio and D. Klingebiel, Bank Insolvencies: Cross-Country Experience, The World Bank, Washington, D.C., Policy Research Working Paper No. 1620, (1996).

[10] G. Caprio and D. Klingebiel, Managing the Real and Fiscal Effects of Banking Crisis, The World Bank, Washington, D.C., Discussion Paper No. 428, (2002).

[11] D. W. Diamond and P. H. Dybvig, Bank Runs, Deposit Insurance, and Liquidity, *The Journal of Political Economy*, **91**, (1983), 401-419.
[12] C. Dziobek and C. Pazarbaşioğlu, Lessons from Systemic Bank Restructuring, International Monetary Fund, Washington D.C., Economic Issues Paper 14, (1998).
[13] S. E. Harrington, The Financial Crisis, Systemic Risk, and the Future of Insurance Regulation, *The Journal of Risk and Insurance*, **76**, (2009), 785-819.
[14] G. Hoggarth, P. Jackson, and E. Nier, Banking Crises and the Design of Safety Nets, *Journal of Banking & Finance*, **29**, (2005), 143-159.
[15] G. Hoggarth, R. Reis, and V. Saporta, Cost of Banking System Instability: Some Empirical Evidence, *Journal of Banking & Finance*, **26**, (2002), 825-855.
[16] P. Honohan and D. Klingebiel, The Fiscal Cost Implications of an Accommodating Approach to Banking Crises, *Journal of Banking & Finance*, **27**, (2003), 1539-1560.
[17] International Monetary Fund, World Economic Outlook, Chapter IV: Financial Crises, Characteristics and Indicators of Vulnerability. IMF, Washington, D.C., (1998), 74-97.
[18] C. James, The Losses Realized in Bank Failures, *The Journal of Finance*, **46**, (1991), 1223-1242.
[19] G. G. Kaufman, Too Big to Fail in U.S. Banking: What Remains?, *The Quarterly Review of Economics and Finance*, **42**, (2002), 423-436.
[20] G. G. Kaufman and S. Seelig, Minimizing Post-Resolution Costs in Bank Failures, Chapter 5 in Global Banking Issues, ed. Klein E, Nova Science Publishers, (2005).
[21] S. C. Lee, Cost of Deposit Insurance under Capital Forbearance: Basel I vs. II, *Journal of Applied Finance & Banking*, **3**(2), (2013), 39-48.
[22] C. J. Lindgren, G. G. Garcia, and M. I. Saal, Bank Soundness and Macroeconomic Policy, International Monetary Fund, Washington, D.C, (1996).
[23] K. McDill, Resolution Costs and the Business Cycle, FDIC Working Paper, (2004).
[24] W. P. Osterberg and J. B. Thomson, Underlying Determinants of Closed-Bank Resolution Costs. Federal Reserve Bank of Cleveland, Working Paper No. 9403, (1994).
[25] G. G. Pennacchi, Risk-Based Capital Standards, Deposit Insurance, and Procyclicality, *Journal of Financial Intermediation*, **14**, (2005), 432-465.
[26] S. M. Ross, Applied Probability Models with Optimization Applications, Dover Publications, Inc., New York, (1992).
[27] K. Schaeck, Bank Liability Structure, FDIC Loss, and Time to Failure: A Quantile Regression Approach, *Journal of Financial Services Research*, **33**, (2008), 163-179.
[28] A. Sheng, Bank Restructuring: Lessons from the 1980s. World Bank, (1996).
[29] L. Shibut, Should Bank Liability Structure Influence Deposit Insurance Pricing?, FDIC Working Paper, (2002).
[30] J. So and J. Z. Wei, Deposit Insurance and Forbearance under Moral Hazard, *The Journal of Risk and Insurance*, **71**, (2004), 707-735.