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Performance Based Diversification
How To Create a Multistrategy hedge fund?

Donatien Tafin Djoko[1]

Abstract

This article investigated the implications of distribution free investment strategies on constructing portfolio of individual hedge funds. The author proposes a dynamic, performance-adaptive asset allocation model that allows to optimally diversify across multiple hedge funds styles. The approach to be followed is related to the adaptive allocation of resources between elementary concurrent from the perspective of the theory of sequential investment strategies. The methodological frame gives up the common global stationary hypothesis and approximates locally, a nonstationary paradigm. The approach is then evaluated in a multivariate basis, by examining the performances of several time evolving portfolio strategies in a sample of 16 funds. Empirical experiments are conducted across 5 different hedge fund categories as classified by the Hedge Funds Research and Barclay CTAs databases. We find that the dynamic performance-adaptive allocation strategies amongst hedge funds yield superior annualized average performances, compared to various alternative benchmarks. These findings are robust to different hedge fund restriction provisions such as, lockup and redemption periods.

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[1] Institute of Statistics, University of Neuchâtel, Pierre-à-Mazel 7, 2000 Neuchâtel, Switzerland.

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1 Introduction

This article reports a methodology for constructing portfolios of hedge funds that drops the common assumption of global stationarity. Nonstationary modeling has a long history in empirical financial analysis. The reader may refer to [16], [24], and [23] for a more recent investigation. Working under nonstationarity has always been motivated by the belief that financial market microstructure is continuously changing, with direct direct consequences in asset price dynamics.

To date, portfolio managers tend to use mean-variance (MV) optimization techniques to construct optimal risk-return balanced asset allocation. However, to perform a MV allocation of resources, the manager has to provide the next-period variance-covariance matrix. One easiest way is to use the historical covariance matrix. A common extension of MV allocation is the construction of volatility timing strategy using the dynamic conditional correlations (DCC) model of [8]. In this regard, [26] focus on the case where an investor or funds of funds manager is concerned with the volatility of a portfolio of hedge fund indexes. Their work successfully extends the static MV asset allocation framework to allow for time varying volatility of returns. However, like most of the multivariate methodological frame for financial returns, the former model assumes that the volatility is homogeneous and stationary. Additionally, the standard MV portfolio construction involving hedge funds has been subject to criticisms in the empirical financial literature. The most common criticism is that MV analysis is suitable, exclusively for normally distributed returns, which is far from being a hedge fund characteristic. Empirical analyzes of hedge fund return series tend to exhibit asymmetry and excess kurtosis, which imply theoretical issues to implement a proper MV portfolio construction.

Assuming a MV criterion directly implies that the data generating process of returns is *Gaussian*, or at least that moments over the first and the second are trivial for asset allocation. Early empirical study reveals that, the MV criterion fails to approximate the expected utility in non-normality cases. In this vein, [14] express the view that systematic skewness (co-skewness with the market return) yields economic value to investors. In a recent manuscript, [17] examine the additional value of taking the complexity of time varying higher moments into the underlying distribution of returns. [17] estimate the model in a Bayesian configuration and show that higher moment investment strategies (up to order four) significantly outperform the MV strategy.

MV portfolio modeling and extensions of it are dominated by stationary long-memory, conditional framework. In this paper, I advocate an approximate nonstationary paradigm to construct a dynamic portfolio allocation of capital across individual hedge funds. Due to the unregulated status of their business model, hedge fund managers enjoy enormous flexibility in pursuing investment returns. The industry investment philosophy is time varying and evolves according to the structure of the market. Therefore, investing in hedge funds through a stationary-constructed portfolio of individual hedge fund strategies can bring to seriously underestimating the risk associated to the portfolio, with potential drawback in terms of market timing ability and risk management. The changing nature of the data generating process requires a consequent adjustment of the approximating stationary approach. The goal of the article is to investigate the economic benefits of locally stationary versus global stationary (parametric) models.

The building block of our locally stationary model is to identifying periods of time where market conditions are 'similar' to the current trading period. This specification will provide intervals of homogeneity in which an optimal allocation of resources can be decided. As a proxy of market conditions, the author intends to capitalize on the vast literature on factor-based hedge fund replication. I use an approximate version of [15]'s six factors model, to identify the variables that characterize the market at each trading period. The first step of the method consists of identifying the instances where the state of the market, i.e. the risk factor specification, was 'similar' to the current conditions. Then, only the hedge fund return information from these instances are the input to construct the current optimal allocation strategy.

The definition of the 'distance' between past and current market conditions is, of course, a crucial aspect in applying the methodology. Since we do not have *a priori* knowledge of the variables (market factors) that are instrumental in defining the state of the market, a refinement of the methodology puts in competition a number of elementary strategies. Each one of them is characterized by a proxy for the market conditions, i.e. a given set of relevant variables (and a depth of the history of these variables) whose values are used to define 'similar' or 'close' market conditions. For each one of the market condition proxy, a portfolio is constructed, based on the time instances identified by the proxy as 'similar' to those of the allocation moment. The final investment decision follows then an adaptive, time varying allocation of resources between different proxy portfolios based on their recent past performance. See Section 2 for further details.

Once homogeneous market proxies have been identified, I implemented two types of locally stationary models: A fully nonparametric and an asymptotic MV. The later is an extension of [10] kernel-based, sequential investment strategy in which the investor maximizes his wealth in the long run without any assumption of the underlying probabilistic structure of the data. The portfolio allocation is time evolving and distribution free. The former is a Markowitz-type portfolio in which a risk-adverse investor carries each time a mean-variance optimization, however without knowing the statistical characterizations of the underlying process. See [12] additional information.

Research on investment strategies with minimal assumptions on the underlying distribution driving the available assets, has been addressed by various authors such as [3], [2]. [11] has shown the existence of universal portfolios, which can achieved asymptotically, the maximum rate of growth of capital without any knowledge of the data generating process. Given the complex structure of hedge fund investment vehicles, I work with minimal assumptions on funds return's distribution. In fact, the only assumption is that monthly fund's gain factor ($(1+r_{j,t})$, where $r_{j,t}$ is the return of fund j at month t) forms a locally stationary and ergodic process. In theory, this assumption allows to the allocation strategy to reach in the long, the maximum rate of growth of capital that can be obtained knowing completely the distribution of the underlying process.

As the paper addresses the issue of dynamic portfolio allocation in a non-stationary paradigm, a comparison with equally weighted constantly rebalanced and dynamic mean-variance portfolios is implemented. The parametric, stationary dynamic MV optimization requires a statistical model to predict subsequent volatility and the conditional expected returns. The dynamic conditional volatility forecast is carried out using the [27] generalized orthogonal GARCH(1,1) model.

The main contribution of this article is directly related to the nonparametric structure of the analysis. Indeed, I move away from the parametric and conditional perspective, to a nonstationary and distribution free paradigm. This work is the first, at our knowledge, that clearly account for time variability and nonparametric design in constructing portfolios of individual hedge funds. Empirical studies are implemented in various hedge fund data sets, including the Chicago-based HFR and Barclay CTAs databases. Our findings suggest that unconditional models are able to uncover and exploit hidden past dependence structures between funds's gain factors. The model allows to incorporate time variation and is constantly out-of-sample. A clear advantage of this model is that it is completely data driven and remains computationally tractable even when several funds are incorporated.

The remainder of the paper is structured as follows. The methodology behinds the portfolio construction strategy is documented in Section 2. Section 3 describes the data used in the empirical exercise and formulate our approach for selecting a restricted basket of funds to construct portfolios of hedge funds. In Section 4 the author discusses some implementation issues and a detailed characterization of the proposed investment process. Empirical results are outlined in Section 5. The main characteristics of the portfolios constructed under several alternative optimizers are outlined. Finally, I provide some robustness assessment of core findings. A tentative conclusion is outlined in Section 6. Additional information are formulated in Appendices 7.1 and 7.2.

2 Methodology

This section describes the nonparametric portfolio allocation methodology to construct dynamic portfolios of hedge funds. The investment process consists of a performance-adaptive allocation of resources between a finite set of elementary strategies, from the perspective of the theory related to sequential investment strategies. The analysis is distribution free and the only mathematical assumption used is that the monthly fund's relatives Net Asset Value (NAV) form a locally stationary and ergodic process. Under this assumption, the asymptotic rate of growth of the constructed portfolio has a well-defined maximum, which can be achieved without knowing the data generating process. As in [10] and [12], I approximate a kernel-based allocation mechanism in order to obtain an optimal and robust growth rate of the capital, in a finite investment horizon.

2.1 Background

The goal here is to construct a multistrategy portfolio of hedge funds, implementing a dynamic allocation algorithm that optimally diversifies and maximizes in the long run the investor wealth over multiple hedge funds styles. The portfolio predictions relied only on past market information.

To crystallize the context of the analysis, consider a hedge fund universe of m managers, $i = 1, \ldots, m$, spanning an heterogeneous range of investment strategies. The vector $\mathbf{X} = (x^{(1)}, \ldots, x^{(m)}) \in \mathbb{R}^m_+$ is the vector of m nonnegative numbers representing fund's relatives NAV at a given trading period, where the jth component $x^{(j)} \geq 0$ of $\mathbf{X}$ represents the ratio between the current and previous NAV of fund's manager j $\left(x^{(j)} = NAV^{(j)}_t / NAV^{(j)}_{t-1} = 1 + r_{j,t}\right)$. Basically, $x^{(j)}$ is the coefficient by which capital allocated in fund j grows during the trading period. We consider an investment setting where an investor or fund of hedge funds (FOF) manager is allowed to allocate his capital at the beginning of each trading period[2] according to a portfolio vector $\boldsymbol{\alpha} = (\alpha^{(1)}, \ldots, \alpha^{(m)})$. The j$th$ component $\alpha^{(j)}$ of $\boldsymbol{\alpha}$ denotes the weight of the investor's capital allocated in fund j. Throughout the article, I assume the investment strategy is self-financing (no borrowing) and the proceed is fully reinvested. This specification implies a long only portfolio selection strategy with $\alpha^{(j)} \geq 0$ and $\sum_{j=1}^{m} \alpha^{(j)} = 1$.

Different hypothesis may be formulated on the path dependent process behind a time varying portfolio selection. For a more general algorithm, the $\boldsymbol{\alpha}$-portfolio vector depends on market history, characterized the sequences of funds's realized gain factors and factor model specification. The state of the market is approximate by the [15]'s six factors model representation. Let suppose that there are m risky, open for investment funds and their market

[2] We will further relax this assumption to account for hedge fund redemption restrictions such as lockup period and redemption period.

dynamic is well described by a sequence of market vector $\mathbf{X}_1, \mathbf{X}_2, \ldots \in \mathbb{R}^m_+$, where the jth component $x_t^{(j)}$ of $\mathbf{X}_t$ indicates the amount obtained, net of fee after investing an unit of capital in the jth fund at the t_{th} trading period. The factor model is described by a sequence of vector $\mathbf{Y}_1, \mathbf{Y}_2, \ldots \in \mathbb{R}^6$. To ease the notation, for $i \leq t$, $\mathbf{X}_{i,t}$ denotes the array of market vectors $(\mathbf{X}_i, \ldots, \mathbf{X}_t)$.

Therefore, at each trading period $t = 1, 2, \ldots$, we want to perform an optimal allocation of resources based on past market information $\mathcal{I}_{1,t-1} = (\mathbf{X}_{1,t-1}, \mathbf{Y}_{1,t-1})$. Then, $\boldsymbol{\alpha}_t = \boldsymbol{\alpha}(\mathcal{I}_{1,t-1})$ denotes the portfolio vector chosen by the investor in the t_{th} trading period, after observing the past behavior of the market.

Because we are primarily interested in maximizing the long tern rate of growth of investor's capital, I describe the wealth accumulation dynamic. Starting with an initial capital C_0, the end-of-period wealth of investment strategy $\mathbf{I}$, after t trading periods is

$$C_t = C_0 \prod_{i=1}^{t} \langle \boldsymbol{\alpha}_i, \mathbf{X}_i \rangle = C_0 \exp^{\sum_{i=1}^{t} \log\langle \boldsymbol{\alpha}_i, \mathbf{X}_i \rangle} = C_0 \exp^{tW_t(\mathbf{I})},$$

where $W_t(\mathbf{I})$ indicates the average growth rate

$$W_t(\mathbf{I}) = \frac{1}{t} \sum_{i=1}^{t} \log\langle \boldsymbol{\alpha}_i, \mathbf{X}_i \rangle$$

Since it is rationally to assume that the objective of any investor is to maximize his wealth, it is straightforward that the maximization of $C_t = C_t(\mathbf{I})$ is equivalent to the maximization of $W_t(\mathbf{I})$.

The present work moves away from the MV portfolio framework and concentrates on a hypothetical investor concerned to maximize his end of period capital in a dynamic optimization setting. Following [10], I adopt a fully nonparametric setting. We do not assume any parametric structure on the distribution of the sequence of hedge fund's relatives NAV. Under the mild hypothesis, [3] show that the best possible choice of the sequential investment problem is the so-called *log-optimum portfolio* $\mathbf{I}^\star = \{\alpha_t^\star(\cdot)\}$. That is, in trading period t the optimal allocation $\alpha_t^\star(\cdot)$ is such that,

$$\mathbb{E}\{\log\langle \boldsymbol{\alpha}_t^\star, \mathbf{X}_t \rangle \mid \mathcal{I}_{1,t-1}\} = \max_{\boldsymbol{\alpha}(\cdot)} \mathbb{E}\{\log\langle \boldsymbol{\alpha}_t^\star, \mathbf{X}_t \rangle \mid \mathcal{I}_{1,t-1}\}$$

No investment rule can have a faster rate of growth than the *log-optimal portfolio*, where a full knowledge of the distribution of the process is required. However, [2] shows that there exist strategies reaching a identical rate of growth without knowing the distribution of the underlying stochastic process, the so-called *universal portfolio*. Following Algoet's scheme, [10] provided a more practical and general version of the *universal portfolio* concept. For the scope of our study, I implement the so-called kernel-based portfolio optimization.

2.2 Time varying kernel-based portfolio selection

Given our interest in constructing portfolios of hedge funds relying uniquely on predictions based on past market conditions, I build a model that enables us to collect the historical sequences of data that are informative to current market states. This section introduces the kernel-based sequential investment strategy, which exploits market condition proxies to estimate the optimal portfolio weights of funds. We describe and implement the uniform kernel version. The approach is closely related to [10], who form a nonparametric *log-optimum portfolio* strategies.

Basically, the idea behind the *kernel-based* ($\mathbf{I}^K$) is quite simple and intuitive. Under a well defined set of parameters, the strategy develops a flexible algorithm to individuate different sequences of data to estimate the optimal portfolio weights by maximizing the investor wealth. The approach is completely data driven and consists firstly, to identify those months in the past where the state of the market as described by a set of factors, was 'similar' to the current conditions. Only the the hedge fund return information from the time instances that follow are then used to determine the optimal allocation of capital amongst funds.

Since we do not have *a priori* knowledge of the variables (risk factors) that are instrumental in defining the state of the market, the methodology puts in competition a number of elementary investment strategies. Each of these elementary investment strategies is characterized by a given set of relevant variables whose values are used to define 'similar' market conditions. Each elementary strategy hence proposes its own allocation, based on the time instances that it identified as 'close' to the current investment period. The final investment decision follows then an adaptive, time varying allocation of resources between these 'experts'. The allocation is based on their recent past performance. What follows provides a detailed description of the methodology that I implement in the sequel.

The working hypotheses motivating the *kernel-based* sequential algorithm are rather intuitive: First I allow the state of the market (characterized by a set of risk factors) to have an impact to the risk-return structure of hedge fund and second, the dynamics of these impacts should be similar during periods that are characterized by similar market conditions.

The $\mathbf{I}^K$ investment strategy is constructed as follows. First I introduce a data selection process which individuates periods characterized by similar market conditions. The market is identified by an approximate version of [15]'s six factors model. The similarity between market conditions characterizing two time periods is measured by the Mahalanobis distance between the values of all past approximate market vectors. Let $\mathbf{Y}_{t-1}$ be the vector of market proxy at time $t-1$ and $\mathbf{Y}_{k,t-1} = (Y_{t-k}, \ldots, Y_{t-1})$ the sequence of past k values of the market condition variable. For a given d small, define[3]

$$S_t^{(k)} = \{k < i < t : \ \| \mathbf{Y}_{t-k,i-1} - \mathbf{Y}_{t-k,t-1} \|_M \leq d\} \tag{1}$$

the set of all past time instances, following those when the market conditions as measured by the k past values of the variable $\mathbf{Y}$ were similar to the last seen vector $\mathbf{Y}_{t-k,i-1}$.

If $\mathbf{X}_i$ is the array of hedge fund gain factors that correspond to the instances $i \in S_t^{(k)}$, then for each $t > k+1$, elementary portfolios are estimated as

$$\begin{aligned}\boldsymbol{\alpha}(S_t) &= \underset{\boldsymbol{\alpha}\in\Delta_m}{\operatorname{argmax}} \prod_{\{i\in S_t^{(k)}\}} \langle \boldsymbol{\alpha} \ , \mathbf{X}_i\rangle \\ &= \underset{\boldsymbol{\alpha}\in\Delta_m}{\operatorname{argmax}} \frac{\sum\limits_{\{i\in S_{t,(k,l)}\}} \log\langle \boldsymbol{\alpha} \ , \mathbf{X}_i\rangle}{\mid S_t^{(k)} \mid}\end{aligned}$$

if $S_t^{(k)} \neq \emptyset$ and

$$\boldsymbol{\alpha}(\mathbf{X}_{1,t-1}) = \underset{\boldsymbol{\alpha}\in\Delta_m}{\operatorname{argmax}} \frac{\sum\limits_{\tau} \log\langle \boldsymbol{\alpha} \ , \mathbf{X}_{\tau,t-1}\rangle}{\tau}$$

[3] $\|\cdot\|_M$ in Equation (1) stands for the Mahalanobis distance. Also refers as the statistical distance, it takes into account the correlation the correlation between variables when computing distances. Lets $\vec{x}$ and $\vec{y}$ two random vectors of identical distribution with covariance matrix Σ, the Mahalanobis distance may be defined as $d(\vec{x},\vec{y}) = \sqrt{(\vec{x}-\vec{y})'\Sigma^{-1}(\vec{x}-\vec{y})}$.

otherwise. That is, I use the past τ length of data to estimate the portfolio vector. In this work, τ is fixed at 24 months and Δ_m is the simplex of all m-dimensional vectors with nonnegative components summing up to one.

The second step consists in aggregating elementary sequential portfolios yielded by the

- different choices of the parameter k that measure the similarity between various market conditions as well as
- different distances d_l for measuring the similarity of the market conditions.

The aggregation is based on the past performance, in terms of cumulative wealth of each elementary strategy to produce a final allocation portfolio. This further refinement is of particular importance as it addresses the issue of the choice amongst various market conditions. The sequential method puts in competition concurrent proxies and follows a performance adaptive allocation of capital based on their recent wealth. In what follows I provide a more formal description of this additional step.

Given $\{q_{k,l}\}$, the probability distribution on the set of all pairs (k, l) of positive integers such that for all k and l, $q_{k,l} > 0$ and fixing the learning parameter[4] $\eta_t > 0$, the weights are defined by:

$$w_{k,l,t} = q_{k,l} e^{\eta \log C_{t-1}^{(k,l)}}.$$

where $C_{t-1}^{(k,l)}$ is the wealth accumulated by the pair (k, l) elementary strategy up to time $t-1$ with initial investment C_0. Their normalized values are then expressed as:

$$\upsilon_{k,l,t} = \frac{w_{k,l,t}}{\sum_{k,l}^{K,L} w_{k,l,t}}. \tag{3}$$

Obviously, the weighing function $\upsilon_{k,l,t}$ must satisfy the usual constraints for combining elementary sequential strategies, namely $0 \leq \upsilon_{k,l,t} \leq 1$ and $\sum_{k,l,t} \upsilon_{k,l,t} = 1$ and each elementary strategies will receive a large weight if its past performance was relatively good.

[4]Thorough this paper, I fixed the leaning parameter η_t as $1/\sqrt{t}$. For a deep discussion on the best practical choice of η_t, see [7].

Finally, the combined portfolio $\boldsymbol{\alpha}^{hf}$ at time t is then defined by weighting the set of $K \times L$ predicted portfolios according to their past performances and the positive distribution probability $\{q_{k,l}\}$ as follows:

$$\boldsymbol{\alpha}^{hf} = \sum_{k,l}^{K,L} \upsilon_{k,l,t} \boldsymbol{\alpha}(\mathbf{X}_{1,t-1}).$$

Therefore, under the dynamic *kernel-based* strategy $\mathbf{I}^{K}$, the investor's capital accumulated at time t can be expressed after some simplification as

$$C_t = \sum_{k,l}^{K,L} q_{k,l} C_t^{(k,l)}. \tag{4}$$

where $C_t^{(k,l)}$ is the final capital arose from the strategy associated to the pair (k, l).

The key idea of combining several concurrent portfolios is simple: to improve the final investment decision. Basically, the worse a portfolio performed in the recent past, the less it will contribute to the final allocation. The final prediction is constantly updated according to the recent performances of competitors.

2.3 Markovitz-type kernel-based portfolio selection

As an alternative unconditional allocation model, I perform the [12] asymptotic MV kernel-based portfolio ($\mathbf{MV}^{K}$). The author extends the [22] MV characterization in a multi-period setting. While the approach is as in the dynamic *kernel-based* strategy $\mathbf{I}^{K}$, it differs from the fact the investor is concerned with a Markovitz-type utility function.

Just like before, for each pair (k, l) of positive integers, elementary portfolios $\tilde{\boldsymbol{\alpha}}$ are estimated as follows[5]:

$$\tilde{\boldsymbol{\alpha}}(\mathbf{X}_{1,t-1}) = \underset{\boldsymbol{\alpha}\in\Delta_m}{\operatorname{argmax}} \left((1-2\lambda) \sum_{\{i\in S_t^{(k)}\}} (\langle \boldsymbol{\alpha} \ , \ \mathbf{X}_i \rangle - 1) \right.$$

$$\left. -\lambda \sum_{\{i\in S_t^{(k)}\}} (\langle \boldsymbol{\alpha} \ , \ \mathbf{X}_i \rangle - 1)^2 + \frac{\lambda}{\mid S_{t,(k,l)} \mid} \left(\sum_{\{i\in S_t^{(k)}\}} \langle \boldsymbol{\alpha} \ , \ \mathbf{X}_i \rangle \right)^2 \right) \tag{5}$$

[5] In appendix B, I provide additional mathematical details on the kernel-based MV portfolio selections.

if $S_t^{(k)} \neq \emptyset$, and used otherwise the past 24 months of data, as it is the case in Equation (2), to estimate the optimal portfolio weights. $S_t^{(k)}$ is the aforementioned similarity set outlined in Equation (1) and λ measures the investor's coefficient of relative risk aversion.

Furthermore, $\mathbf{MV}^K$ elementary portfolios are then aggregated as in the $(\mathbf{I}^K)$ strategy according to:

$$\tilde{\boldsymbol{\alpha}}^{hf} = \sum_{k,l} \tilde{\upsilon}_{k,l,t} \tilde{\boldsymbol{\alpha}}(\mathbf{X}_{1,t-1}).$$

where $\tilde{\upsilon}_{k,l,t}$ are derived as in Equation (3). Using the same representation as in Equation (4), the wealth achieved by the $\mathbf{MV}^K$ strategy is given by

$$\tilde{C}_t = \sum_{k,l}^{K,L} q_{k,l} \tilde{C}_t^{(k,l)}.$$

2.4 Alternative Optimizers

The scope of this section is to illustrate alternative portfolio allocation strategies for comparison purpose. We compare the dynamic *kernel-based* strategy $\mathbf{I}^K$ to three different optimizers.

Our initial benchmark strategy is the equally weighed constantly rebalanced portfolio (**EWCRP**). We consider an investor who equally allocated her initial capital according to $\boldsymbol{\alpha}_0 = (1/m, \ldots, 1/m)$ and at each trading period, rebalanced her portfolio to respect this uniform allocation constraint. This naive diversification strategy does not involve any optimization and estimation errors related to the plug-in of sample means and variance-covariance matrix.

Secondly, I implement the dynamic mean-variance portfolio selection technique. Under the standard mean-variance optimization, the investor selects $\boldsymbol{\alpha}_t$ to maximize the quadratic objective function

$$\mathcal{Q}(\boldsymbol{\alpha}_t) = \boldsymbol{\alpha}_t^{'} \mu_t - \frac{\lambda}{2} \boldsymbol{\alpha}_t^{'} \Sigma_t \boldsymbol{\alpha}_t \quad (7)$$

where μ_t is the conditional mean vector of the fund gain factors, Σ_t is the conditional variance-covariance matrix, λ expresses the investor's coefficient of relative risk aversion, and $\boldsymbol{\alpha}_t$ are nonnegative and sum up to one. To solve the problem in Equation (7), I consider an investor who replaces the mean and the covariance matrix by their sample counterparts $\hat{\mu}$ and $\hat{\Sigma}$, respectively. The investor believes that the conditional expected return and covariance are time-varying and will adjust his portfolio weights accordingly. Therefore, I use an autoregressive of order one model ($AR(1)$) to estimate the fund expected returns and the [27] generalized orthogonal GARCH(1,1) model to forecast the time varying conditional covariance matrix under the assumption of a joint normal distribution. In what follows, I refer to this investment strategy[6] as **GO-GARCH(1,1)**. The **GO-GARCH(1,1)** estimation method is based on common eigenvectors of the observed data. It does not suffer of numerical convergence problem[7]. The performance of the various strategies are compared using several measures. The first measure is the wealth achieved by each investment strategy and the annualized average yield (AAY). Then I compute the so-called Sharpe ratio, in an annualized basis. I estimate this ratio using the sample mean and variance of the excess returns for each strategy considered over the monthly US treasury bill. I also report a relative performance measure based on a modified version of the Sharpe ratio (mSR) introduced by [9], which provides the measure of out-performance of a given investment strategy over an alternative with different level of risk

$$mSR = \frac{\sigma_a}{\sigma_K}(\mu_K - \mu_a), \tag{8}$$

where μ_K and σ_K, μ_a and σ_a are the annualized average yield and the annualized volatility of the $\mathbf{I}^K$ investment strategy and those of the various alternative strategies, respectively.

Considering the dynamic structure of our analysis, it is important to assess the possible effect of transaction costs. Indeed, the gain of a dynamic investment strategy may be, partially or totally offset by transaction costs related to portfolio turnover. However, there is no general consensus among academi-

[6]In appendix B, complementary details are outlined on the **GO-GARCH(1,1)** dynamic MV portfolio selections.

[7]A sample size around $1000 - 1500$ is generally assumed to be the minimum for a precise estimation of GARCH(1,1) model. In our multivariate case the limit is even higher. Given the short history and low frequency of hedge fund data, a correct estimation of GARCH(1,1) model is not guaranteed. The choice of generalized orthogonal GARCH(1,1) is motivated by the limited sample size.

cians about the right range of values in the financial industry. The issue is even more complicated for non traditional asset classes like hedge funds. For these reasons, to account for the transaction costs, I adopt the approach proposed by [13]. Since in practice it is difficult to derive the realized transaction costs, the author computes tc^{be}, the break-even transaction cost. The break-even transaction cost measures the level of transaction costs that a given investor will be indifferent to pay when selecting between the $\mathbf{I}^K$ investment strategy and the alternative allocation decisions. The break-even transaction cost between the two strategies is then defined as

$$tc^{be} = \frac{\bar{r}_p^K - \bar{r}_p^a}{tv^K - tv^a},$$

where $tv = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{m}\left|\boldsymbol{\alpha}_{j,t} - \frac{\boldsymbol{\alpha}_{j,t-1}\mathbf{X}_{j,t}}{1+r_{p,t}}\right|$ is the monthly average of the value traded in all individual hedge fund in the portfolio, $\bar{r}_p^K$ is the average return of the portfolio constructed from the strategy $\mathbf{I}^K$, and $\bar{r}_p^a$ is the average return form one the alternative optimizers. If transaction costs are a fixed proportion ε of the value traded in the portfolio, the average transaction cost in each strategy is then $\varepsilon \times tv$. Therefore, if an investor has transaction costs smaller than tc^{be}, she will prefer the $\mathbf{I}^K$ investment strategy; otherwise, the investor will be better off with one of the three alternatives. In other words, high break-even costs is synonym to low portfolio turnover rates.

Finally, I calculate out-performance rate $\mathcal{S}$, that is, the percentage of allocation periods for which the $\mathbf{I}^K$ investment strategy performs better than one of the various alternative strategies. This measure is an expression of the *kernel-based* strategy ability to capture time-varying investment opportunities by discovering significant patterns in the local structure of the data in the past.

3 Data

This section presents the data used in our empirical investigation. I begin by discussing the individual hedge funds and commodity trading advisors (CTAs) in the Hedge Fund Research (HFR) and Barclay CTAs databases. Then, the section follows presenting the market risk factor specification.

3.1 Funds and sample restrictions

A proper study of hedge fund returns requires accurately measured data. Such a quality is a primary concern in the field since hedge fund managers *voluntarily* provide information to databases and the industry lacks an uniform reporting standard. Despite these difficulties a number of commercial databases are currently providing hedge fund information both at individual fund level as well as at aggregated level (indexes).

To analyze the relevance of the proposed methodology in constructing portfolio of hedge funds, I focus on live funds and use two main sources of hedge fund data: the Hedge Fund Research (HFR), and Barclay Commodity Trading Advisors (CTAs) databases. The HFR consists of returns and basics information for individual funds and fund of hedge funds from January 1981 to September 2008, and the Barclay CTAs database contains CTAs returns and fund specific comprehensive information from January 1980 to September 2008.

The HFR database is composed of returns reported on different frequencies (mainly monthly) and additional qualitative/quantitative information such as main strategy, sub-strategy, asset under management, etc. on 5230 individual funds and 2720 Funds-of-Funds that are still active on September 30, 2008. The data provider groups individual fund data in four main categories[8] depending on their sub-strategies: Macro, Relative Value, Equity Hedge, and Event-Driven. Out of the 5230 numbers of individual funds, 940 are classified as Macro, 2796 as Equity Hedge, 961 are Relative Value, and 533 are grouped as Event-Driven funds.

The Barclay CTAs data system is widely recognized by both practitioners and academicians as the largest, most comprehensive, available Commodity Trading Advisors sample. Generally, CTAs are funds primarily trading listed commodity and financial futures contracts. The database consists of 981 reporting funds as of September 2008. CTAs, also denominated Managed futures are by no means homogeneous investment vehicles. CTAs managers employ a large range of strategies and asset classes. Combined, the two databases provide a rather completed and detailed picture of the hedge funds universe.

For the scope of our empirical analysis, I impose a set of filters on both

[8]The reader may refer to **Appendix A** for a description on HFR hedge fund classification

databases. First I selected funds that reported in U.S. dollar net of fee on a monthly basis. Then I required that each fund has at least 15 years of non-missing reported returns. Additionally, for each strategy, I extract the group of funds with the longest consecutive stretch of non-missing returns and keep those funds having at least 10 years of reported assets under management (AUM). I end up with a final sample of 429 funds, which consists of 356 funds in HFR and 73 in Barclay CTAs. All funds in the final sample are opened for new investment. Since the author is interested in constructing multistrategy portfolios of hedge funds, I finally constituted for each hedge fund category, an heterogeneous basket of 16 funds based on their average first year reported AUM. According to [19], in general Funds-of-Funds managers hold a basket of underlying funds ranging between 15 to 50. However, [20] extending the previous work of [4], shown that approximately 10 funds are sufficient to construct a diversified portfolio of hedge funds. In line with this approach and the fund of hedge funds industry practice, I choose to fix the number of funds to sixteen and implement an asset-related selection process. We ranked the funds per average AUM and for each category, selected the five lowest and the top five, three funds directly below the median, the median ranking fund, and two funds above the median. A detailed representation of the data set is described in Panel A. of Table 1. Empirical experiments are performed on 5 styles (Global Macro, Equity Hedge, Relative Value, Event-Driven, and CTAs), excluding the Funds-of-Funds category. Naturally, this data selection process creates an additional survivorship bias in the residual time series of funds. I apply our empirical investigation on different structure of the data set to provide some robustness checks of our main results. In addition, since the purpose of our empirical study is to construct portfolios of hedge funds that maximize in the long run the investor wealth, and to study the portfolios relative performances under different allocation strategies, the fund selection routine may not be a big issue.

Furthermore, for consistency, I examine the impact of hedge fund redemption restrictions on the performances of the dynamic portfolio allocation. In general, the hedge fund industry uses what is commonly called share restriction provisions[9] such as redemption, lockup, and advance notice periods. Various

[9]Hedge funds restrictions also contain provisions such side pockets, gates, redemption suspension, $\cdots$. Some of those restrictions are not reported in the databases considered in

authors have pointed the importance of redemption restrictions on hedge fund business model. Conclusive results on the subject are mixed. Some studies provide evidence that redemption restrictions are associated to excessive risk taking with fund's managers and potential costs on investors [5, 18]. Other studies, however, suggests that share restrictions are linked to higher hedge fund performance [21, 6, 1]. In general, these studies conclude that redemption restrictions affect various hedge fund characteristics such as fund flows, returns, and risk. It is then important to analyze the evolution of our dynamic allocation strategies under the complexity of hedge fund restriction provisions. Panel B of Table 1 reports average value of several characteristics for the sample of funds under investigation. along this panel, three variables capture restriction provisions: redemption period and notice period in days, and lockup period in months. Inspection of Panel B reveals a strong heterogeneity amongst hedge fund categories. Lockup periods are on average null for Barclay CTAs and Global Macro funds, with a maximum value of 8.25 months for Event-Driven. Redemption periods range between 3.75 and 170.83 days. Notice period measures are between 0 to 14.83. Over our samples, Barclay CTAs funds are far the least restrict.

3.2 Factor model specification

To define the variables that characterize the state of the market at every trading month, we approximate the [15]'s six factors model. The authors show that, this factor model specification has a significant explanatory power amongst hedge fund strategies. They advocate the use of factors[10] that correspond to the main drivers affecting the hedge fund's risk-return tradeoff:

The equity market: the S&P 500 total return.

Currencies: the US Dollar major currencies index.

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the present study. We refer the reader to **Appendix A** for a short variables definitions.

[10]Data are downloaded from various sources: S&P 500 total return from DataStream; Goldman Sachs commodity (GSCI) and the Volatility (VIX) indexes are from yahoo finance; US three months treasury bill and Moody's corporate Aa bond, Moody's corporate Baa bond and US Dollar major currencies indexes from the FED bank of St. Louis data library.

The bond market: the month end-to-month end return on the Moody's corporate Aa bond index.

Credit: the monthly change in the Moody's corporate Baa bond index less the three months treasury bill (month end-to-month end).

Commodity: the Goldman Sachs commodity index total return.

Volatility: the first difference of the end-of-month value of the CBOE volatility index.

3.3 Data summary statistics

We begin our analysis reporting some preliminary statistical characteristics on the various fund categories under our study. The data are monthly, ranging from July 1993 to December 2007 for Global Macro, December 1993 to August 2007 for Equity Hedge, December 1993 to January 2008 for Event-Driven, September 1993 to December 2007 for Relative Value, and from October 1993 to June 2008 for the Barclay CTAs category. From Panels A to E of Table 2, except in few cases, average returns are in general positive and significant. Through all categories, volatilities range between 18% and 0.1% per month. The less volatile hedge fund strategy is the Relative Value group. The highly volatile strategies are Global Macro and Barclay CTAs funds. The measure of asymmetry (skewness) is heterogeneous across individual funds and main strategies. Barclay CTAs and Macro funds are in general positively skewed, indicating that booms occur more often than crashes in those categories. In opposite, Relative Value and Event-Driven strategies are more often negatively skewed. In all investment styles, kurtosis measures are larger than 3, which is not consistent with the normality assumption. The hypothesis of normality in fund's return series is generally rejected by both the Jarque-Bera and Lilliefors tests.

Regarding time dependency in individual hedge fund performances, the natural way of testing it is the Ljung-Box statistic (ρ). We estimate the Ljung-Box statistic for returns and squared returns to assess the presence of serial correlation in the first and second moment, respectively. Taking apart Relative Value and Event Driven funds, there is no strong evidence that the series are serially correlated. However, squared returns are more often serially correlated at 95% confidence level, which indicates temporal correlation in volatilities.

4 Implementation details

In this section, we discuss some practical implementation issues. First, I focus on the *kernel-based* portfolio selections, namely, the $\mathbf{I}^K$ and the $\mathbf{MV}^K$ optimizers. As described in Section 2, elementary strategies are constructed for different proxy of market conditions, different number of past values of the proxy used in measuring the similarity of market conditions (parameter k) as well as for different values of the maximal distance between the values of these proxies (parameter $\mathrm{d}_{k,l}$). The choice of the last two parameters is as follows: I initially define the two positive integers as $l = 1, \cdots, L$ and $k = 1, \cdots, K$, fixing $L = 10$ and varying $K = 7$. Additionally I use the uniform probability distribution $\{q_{k,l}\} = 1/(K \times L)$ for the weighting scheme of elementary strategies indexed over k and l. The maximal distance $d_{k,l}$ is defined by:

$$d_{k,l}^2 = \exp((0.002 \cdot k \cdot m) + (0.02 \cdot m \cdot l)) + (k/2)$$

Then, for each k, we have $l = 10$ different values of the radius $d_{k,l}$ and an array of 10 different elementary portfolios.

For all hedge fund categories, I fix the starting point of the dynamic portfolio prediction algorithm after 12 months. The first year of the data is dedicated to the initial parametrization. Therefore, the first data selection process defined in Equation (1) uses this initial window to identify the similarity set. However, this window increases in a monthly basis with the trading period.

The final maximized wealth at time t is obtained by exponentially weighting the $K \times L$ elementary sequential strategies according to their past performances as described in Equations (4, 6).

The initial investment capital C_0 is set equal to 1. The estimation of the portfolio weights for MV dynamic strategies is implemented for different choices of the investor's relative risk aversion coefficient. That is, we set λ equal to 5, 10, and 15.

To make the *kernel-based* portfolio construction implementable, some usual hypothesis are assumed. In addition to the assumption of local stationarity and ergodicity of the market, I assume that any fund is infinitively divisible and that all funds are opened for new investment at each trading period. The dynamic allocation problem is expressed here from the view of a long only agent investing in US dollars, so the annualized average yields are expressed in US$. Last, the $\mathbf{I}^K$ and the MV (kernel-based and GO-GARCH(1,1)) investment processes are solved using the R version of the [25] DONLP2 optimization routines.

5 Empirical analysis

We now focus on the empirical analysis of the performance of the various investment strategies illustrated in the present article. This section displays some results of the nonparametric *kernel-based* portfolio selection, compared to the three alternative investment strategies. We will provide evidence of the benefit that results of switching from a dynamic mean-variance allocation of resources to a kernel-based distribution free strategy. Table 4 documents the statistics of the wealth achieved by the various trading strategies under a month-to-month portfolio construction, while Table 5 reported the results of the strategies under different set of market restrictions. The evolutions of the accumulated capital for each strategy are displayed through Figures 1 to 10 (for a relative risk aversion of $\lambda = 5$). We tested the investment strategies on 5 distinct categories of hedge funds: HFR Global Macro, Equity Hedge, Event-Driven, Relative Value and the Barclay CTAs. For each category we constructed a heterogeneous panel of 16 individual hedge funds as described in Section 3. This means that $m = 16$.

5.1 Performance analysis of the dynamic investment strategies

The estimation of the optimal portfolio allocation is carried out each month by maximizing the average rate of growth of capital. Table 4 reports the sample performance of the $\mathbf{I}^K$, $\mathbf{MV}^K$, **GO-GARCH(1,1)** and **EWCRP** strategies for the 5 categories of hedge fund. The results for the two mean-variance specifications ($\mathbf{I}^K$, **GO-GARCH(1,1)**) are outlined for $\lambda = 5, 10, 15$. As the panels reveal, the $\mathbf{I}^K$ investment strategy show superior annualized average yield (AAY) across all styles. In the absence of transaction costs, the percentage values of AAY range from 40.80% for Barclay CTAs portfolio to 23.5% in Event-Driven funds. These numbers are always significantly bigger than those yield by the three alternative strategies. As expected, the ex-post annualized standard deviations (ASD) are also bigger, which translate into a smaller Sharpe ratios (SR), specially compared to the $\mathbf{MV}^K$ portfolios (for all values of λ). Except for Barclay CTAs funds, the SR of the dynamic $\mathbf{I}^K$ strategy are bigger than one across all hedge fund styles and are equal magnitude to **GO-GARCH(1,1)** and **EWCRP** strategy values.

On a relative basis, the success rate, $\mathcal{S}$, that is, the proportion of months for which the $\mathbf{I}^K$ investment strategy has superior return than the alternative optimizers, further illustrate the attractiveness of the former strategy. Over the sample of hedge fund styles under scrutiny, the success rate is between 50% and 75.82%, a range that is consistent with an upgrade quality to capture time varying investment opportunities. This evidence suggests that the outperformance of the dynamic $\mathbf{I}^K$ strategy is not due to some specific extreme events. The modified Sharpe ratio measure, mSR, is the return that the $\mathbf{I}^K$ strategy would have earned if it had the same risk as the alternative optimizer. Across Panels A to E, mSR values are positive and bigger for the the **GO-GARCH(1,1)** and **EWCRP** strategies than for the $\mathbf{MV}^K$ strategy.

When the investor considers the dynamic changes in the variance-covariance matrix under the *kernel-based* framework ($\mathbf{MV}^K$ strategy), the SR significantly increase for all values of λ and across the 5 hedge fund categories. Indeed, the $\mathbf{MV}^K$ strategy show superior SR values. For the same coefficient of relative risk aversion, the $\mathbf{MV}^K$ strategy tends to exhibit higher average annual yield and lower annualized volatility than the **GO-GARCH(1,1)** strategy. Accordingly, their relative performance (mSR and $\mathcal{S}$) to the $\mathbf{I}^K$ strategy are in favor of $\mathbf{MV}^K$. As a consequence, in contrast to the dynamic **GO-GARCH(1,1)** portfolios, the $\mathbf{MV}^K$ portfolios benefit from volatility timing. However, one has to take with cautious the value added by MV strategies since they are subjected to estimation risk (uncertainty regarding the estimation of

$\hat{\mu}$ and $\hat{\Sigma}$).

The results documented above have been derived without transaction costs. It is clear that if we take into account transactio costs, the above presented performance will deteriorate. Table 4 also reveals the break-even transaction costs, tc^{be}, of the alternative three strategies for all levels of risk aversion. The numbers are listed in the last column entry from Panel A to E and are different across hedge fund categories and investment strategies. In general, highest tc^{be} suggests lower portfolio turnover rates for the strategy. For Global Macro, Equity Hedge, and Event-Driven funds, **GO-GARCH(1,1)** strategies show lower break-even costs than $\mathbf{MV}^K$ and **EWCRP** strategies. The results are opposite for Barclay CTAs and Relative Value funds. For the later, the tc^{be} are positive and higher with **EWCRP** strategies.

Last, but not least, Figures 1 to 10 depict the evolution of wealth accumulated for each model specification over the allocation period. One can clearly see that the $\mathbf{I}^K$ strategy significantly dominates the others. Overall, these results suggest that the time evolving *kernel-based* investment strategies provide significant performances for a risk-seeking investor who is willing to bear the risk - in terms of higher volatility. Amongst *kernel-based* strategies, the distribution free $\mathbf{I}^K$ strategy out-performs the $\mathbf{MV}^K$ counterpart. The results show the superiority of $\mathbf{I}^K$ and $\mathbf{MV}^K$ models, supporting the hypothesis that modeling hedge fund returns in a nonstationary, unconditional paradigm yields significant economic benefits.

5.2 Robustness assessment

To further evaluate the robustness of the analysis in constructing a multistrategy portfolio of hedge funds, we have implemented an additional set of more restrictive empirical experiment, the principal results of which are presented in this section. These investigations are performed on the same data, however with different specification.

In a first analysis, we assess the effect of hedge fund redemption provisions on the portfolio performance characteristics. We adjusted the dynamic allocation of resources to take into account average restriction provisions. We focus on two variables. The lockup period in months, and the redemption period which is the number of days between two consecutive redemption dates.

They vary across funds, and categories. On average, in the portfolio of 16 funds under study, the lockup period is between 0 (HFR Global Macro and Barclay CTAs) and 8.25 months (Event-Driven). Redemption period ranges between 45.08 (HFR Macro) to 170.83 days (Event-Driven). For the portfolio of HFR Global Macro and Barclay CTAs funds, with on average no lockup period, we used the full sample average (per style) restriction provisions to impose the time sequence of the investment routine. This yields 3 (2) months redemption periods and 12 (6) months lockup periods for HFR Global Macro (Barclay CTAs) category. For the three other hedge fund styles, we restrict ourself to what is displayed in Panel B. of Table 1. Therefore, the experiment is conducted in the following way: After our initial allocation of resources to the portfolio, we block this investment for the time indicates by the average lockup period. Next, the investment is dynamically re-allocated at each pre-specified redemption dates as reported in Panel B. of Table 1. Table 5 reports the absolute and relative performance for all strategies and level of risk aversion. As is evident, the performances of the various strategies are just slightly different from those obtained in the month-to-month portfolio construction. By respecting the hedge fund industry restriction provisions, the Sharpe ratios of $\mathbf{I}^K$ strategies improve for HFR Global Macro and Equity Hedge styles and are relatively unchanged for the rest. Once again, the success rate enhanced the $\mathbf{I}^K$ investment strategy, with percentage values between 48% to 69%. The risk-adjusted excess return (mSR) is always positif, with bigger values relative to **GO-GARCH(1,1)** and **EWCRP** strategies.

Secondly, we performed a data driven model evaluation method to question our fund selection process. The analysis intends to test whether different competing choices of sample constituents would lead to equivalent performance characteristics of the investment dynamics. The implemented test is an approximation of the procedure advised by Racine and Parameter (2009). The experiment is flexible and allows to defeat the inconveniences associated on relying on only one choice of data. In this article, we intend to construct the distribution of the investment strategy's true annualized average yield, by randomly testing the strategies under various structures of the data. Therefore, instead of using a fix sample of 16 funds selected according to their ranking AUM as explained in Section 3, we draw randomly a basket of funds of size 16, then perform our allocation strategies accordingly. This process is repeated 150 times, each time reassessing the performances of our dynamic portfolios across the four investment methodologies. The repeated investigation will produce vectors of length 150 of annualized performance statistics for all models under scrutiny. To discriminate between different allocation strategies, we use a paired t-test of differences in sample mean and a the Mann-Whitney-Wilcoxon test for differences in location. Some useful Boxplots are also represented to highlight dominance relationships between alternative optimizers. For what follows, we consider an application on HFR Equity hedge funds. The data set consists of 97 funds, obtained after imposing several filters as explained in Section 3. The results are reported for the coefficient of relative risk aversion, $\lambda = 5$. Observing 6 reveals that the $\mathbf{I}^K$ allocation specification is preferred to all three alternative investment strategies on the AAY basis. Both the t-test and the Mann-Whitney-Wilcoxon p-values are close to zero. From Figure 11, the median value for the distribution free approach is equal to 26.2%, which is of equal magnitude to what achieved by the sample of 16 funds ranked according to their AUM. However, on the relative performance basis expressed by the Sharpe ratio distributions in Figure 12, the **GO-GARCH(1,1)** and **EWCRP** strategies performs better than the $\mathbf{I}^K$ and $\mathbf{MV}^K$ strategies. To summarize, the $\mathbf{I}^K$ allocation higher annualized yields, at the cost of an increase in the annualized volatility.

Overall, the results of the present section are in line with Subsection 5.1. The empirical robustness analysis confirms the dominance of the *kernel-based* investment strategies. That is, this investment technique over-performs the two alternative optimizers in terms of AAY and out-performance rate ($\mathcal{S}$), while these results are mixed on a risk-adjusted basis. Additionally, this dominance relation is in general unaffected by different market and statistical restrictions. These results suggest that, across our hedge fund category and investment horizon, allocating capital on the basis of an unconditional, nonparametric modeling is likely to provide sizeable benefits in the allocation process.

6 Conclusion

In this manuscript, the performance of various allocation strategies across individual hedge funds is investigated. We particularly illustrated the time varying *kernel-based* approach to solve the optimal allocation of resources within various hedge fund styles. An empirical study is performed on HFR and Barclay CTAs databases. For the first, in our knowledge, this article analyzes the risk-return characteristics of hedge fund portfolios when relaxing distribution hypothesis. Indeed, under the *kernel-based* framework, the hypothetical investor is concerned by the long term maximization of his average capital rate of growth in a time evolving optimization setting, without knowing the underlying distribution generating the hedge fund's NAV. I investigated the benefits of approximating nonstationary data locally on a multivariate portfolio of hedge funds.

I found that the distribution free investment strategy provides reliably results. Empirical findings suggest that there are distinct benefits in performance improvement for portfolios constructed under the *kernel-based* investment strategy. On a relative basis, the proportion of months for which the strategy outperforms alternative investment approaches is high. The annualized risk-return profile, expressed by the Sharpe ratio, is sometimes undermined by the high volatility associated to dynamically investing in hedge funds. Furthermore, this strategy ignores the volatility problem and it is particularly constructed for an investor concerned by any parametric hypothesis on the distribution. I further perform various experiments designed to assess the robustness of the results. Evidences suggest that our findings are solid, and the magnitude of the performances compared to two conditional alternative optimizers is invariant.

Several extensions to this paper may be considered. Future researches might address the issue of how to incorporate a risk measure in the optimization routine without distorting the completely nonparametric structure of the methodology. Transaction costs are also ignored in this article. Additional works are necessary to fully incorporated the cost related to the time-varying allocation of capital across hedge funds. Furthermore, investors do allocate resources to hedge funds in combination with different asset classes, in order to achieve the desired performance profile. Therefore, it might be interesting to assess the return pattern of an aggregate portfolio of hedge fund and several traditional asset classes, under the *kernel-based* investment strategy.

Clearly, the core idea of this manuscript is that there exist a class of distribution free allocation strategies that are able to uncover and exploit hidden structure in the past of hedge funds data, to optimally allocate capital in a multistrategy portfolio.

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7 Appendices

7.1 Appendix A: Hedge Fund Characteristics

7.1.1 Variable Definitions

$\mathbf{NAV}_t$: The month t net asset value of a given hedge fund. As LH:04, we assume that the $\mathbf{NAV}_t$ has been adjusted to take into account all realized and non-realized capital gains, accrued dividends and interest income, capital distributions, splits and all the impacts of equalization and crystallization. The gain factor or simple gross return at time $t+1$, of a fund j is defined as: $\frac{\mathbf{NAV}_{j,t+1}}{\mathbf{NAV}_{j,t}} = (1 + r_{j,t+1})$.

Lockup Period: The minimum number of months that an investor has to wait, after his initial investment, before he can withdraw money from the fund.

Redemption Period: Number of days between two consecutive specified dates. Hedge fund investors are allowed to take back their capital only at these pre-specified intervals.

Notice Period: The advance number of days that a hedge fund investor is required to inform the fund of his willingness to withdraw his capital.

Incentive Fee (%): Also called Performance fee, it is the main source of hedge fund's profits. It is shaped as an option to provide incentives for a hedge fund manager to generate profits. Calculated as a fraction of profits generated above the high-water mark.

Management Fee (%): Designed to provide to the fund's manager enough money to cover his operating costs. It is a fraction of the fund's asset under management.

Minimum Investment($M): The minimum amount of money required by the fund to be accepted as an investor.

High Water Mark: Defines the fund's manager compensation. The clause means that the manager receives performance fee only on increases in the NAV of the fund in excess of the previous highest NAV.

Leverage: Percentage of capital borrowed by the fund to boost the potential profit of the strategy.

7.1.2 Investment Strategies

The term 'hedge funds' is a generic acronym associated to a pool of professionally managed capital. However, hedge funds are highly heterogeneous, covering a large range of investment styles with various approaches, objectives and performance characteristics. There is no consensus in the financial literature in the way to classify the wide basket of strategies implement in the hedge fund universe. For simplicity, we adopt LH:04 classification which is closed to Chicago-based Hedge Fund Research database. Hedge funds are classified into five main strategies: tactical trading, equity hedge, event-driven, relative value and funds of funds.

Tactical trading strategies: Also called directional funds, the term refers to funds that speculate in the direction of market prices of commodities, currencies, bonds and/or equities. They trade on a discretionary or directional basis. CTAs and Global macro funds belong to this category.

Equity Hedge: Indicates managers implementing long/short investment strategies in equities. The strategy is not automatically market neutral. The category can be further divided into different sub-strategies, based on manager invest target as: Sector or/and geographically focus, emerging market, dedicated short bias, and market timers.

Event-driven: As their name claimed, event-driven funds are particularly interested in situations where the investment opportunity is associated to specific corporate events such as: mergers and acquisitions, bankruptcy, recapitalization, stock buybacks. Fund's managers invest mainly on equity, debt or trade claim from those companies. Risk arbitrage and distressed securities funds dominate this group.

Relative value: They intend to profit on pricing differences between similar or related assets such as options, futures, equities and debt. The underlying assumption is that the price gap between the two similar/related securities will return to its fair value as the investment horizon growths. This category regroups strategies such as: fixed income arbitrage, convertible arbitrage, statistical arbitrage, index arbitrage and mortgage-backed securities arbitrage.

Funds of funds: This category represents funds investing in a pool of hedge funds. The investment principle relies on the assumption that combining individual funds will reduce the risk and provide a more stable return in the long run. The manager may allocate his resource within a strategy, or in multiple strategies.

7.2 Appendix B: Alternative optimizers

Asymptotic Mean Variance Portfolio Model: Following Gyo:Vaj:07 we briefly define the conditional expected value of the Markowitz-type utility function as:

$$\begin{aligned}\mathbb{E}\{U_M(\langle \boldsymbol{\alpha}(\boldsymbol{X}_{1,T-1}), \boldsymbol{X}_T\rangle, \lambda) \mid \boldsymbol{X}_{1,T-1}\} &\overset{def}{=} \mathbb{E}\{\langle \boldsymbol{\alpha}(\boldsymbol{X}_{1,T-1}), \boldsymbol{X}_T\rangle \mid \boldsymbol{X}_{1,T-1}\} \\ &-\lambda \mathbf{Var}\{\langle \boldsymbol{\alpha}(\boldsymbol{X}_{1,T-1}), \boldsymbol{X}_T\rangle \mid \boldsymbol{X}_{1,T-1}\} \quad (10)\end{aligned}$$

where $\boldsymbol{X}_T$ is the T-th day market vector, $\alpha(\boldsymbol{X}_{1,T-1})$ is the investor portfolio weight and λ is a positive constant coefficient representing the investor absolute risk aversion.

Under mild conditions, equation 10 can be expressed in as follow:

$$\begin{aligned}\mathbb{E}\{U_M(\langle \boldsymbol{\alpha}(\boldsymbol{X}_{1,T-1}), \boldsymbol{X}_T\rangle, \lambda) \mid \boldsymbol{X}_{1,T-1}\} &= (1-2\lambda)\mathbb{E}\{\langle \boldsymbol{\alpha}(\boldsymbol{X}_{1,T-1}), \boldsymbol{X}_T\rangle - 1 \mid \boldsymbol{X}_{1,T-1}\} \\ &-\lambda \mathbb{E}\{(\langle \boldsymbol{\alpha}(\boldsymbol{X}_{1,T-1}), \boldsymbol{X}_T\rangle - 1)^2 \mid \boldsymbol{X}_{1,T-1}\} \\ &+1-\lambda+\lambda \mathbb{E}^2\{\langle \boldsymbol{\alpha}(\boldsymbol{X}_{1,T-1}), \boldsymbol{X}_T\rangle \mid \boldsymbol{X}_{1,T-1}\}.\end{aligned}$$

finally, the kernel-based mean-variance portfolio is expressed as:

$$\begin{aligned}\tilde{\boldsymbol{\alpha}}(\boldsymbol{X}_{1,t-1}) = \underset{\boldsymbol{\alpha}\in\Delta_m}{\operatorname{argmax}} \Bigg(&(1-2\lambda) \sum_{\{i\in S_{t,(k,l)}\}} (\langle \boldsymbol{\alpha}\ , X_i\rangle - 1) - \lambda \sum_{\{i\in S_{t,(k,l)}\}} (\langle \boldsymbol{\alpha}\ , X_i\rangle - 1)^2 \\ &+ \frac{\lambda}{\mid S_{t,(k,l)} \mid} \Bigg(\sum_{\{i\in S_{t,(k,l)}\}} \langle \boldsymbol{\alpha}\ , X_i\rangle \Bigg)^2 \Bigg)\end{aligned}$$

where $S_{t,(k,l)}$ is the similarity set associated to each pair (k,l).

GO-GARCH Portfolio Selection: To relax the notation, let's denote by X_t the vector of m funds's gain factors at time t. We assume that the dynamics of the gain factors vector is

$$X_t = \beta_0 + \beta_1 X_{t-1} + \varepsilon_t,$$

where ε_t is the vector of unexpected returns. We estimated an $AR(1)$ model to filter out autocorrelation. Finally, I adjust the Roy:van:02 **GO-GARCH(1,1)** model for the residuals of the $AR(1)$ to estimate the time varying conditional covariance matrix.

Briefly, the fundamental assumption behind the **GO-GARCH** model is the following:

Given an observed economic process $\{x_t\}$, there exists an uncorrelated components $\{y_t\}$ such that

$$x_t = Zy_t$$

where Z is the linear map that links the observed economic process to the unobserved components is assumed to be constant and invertible. Associated to the **GO-GARCH(1,1)** process:

$$\begin{aligned} x_t &= Zy_t \; y_t \sim N(0, H_t) \\ \boldsymbol{H_t} &= diag(h_{1,t}, \cdots, h_{m,t}) \quad (13) \\ h_{i,t} &= (1 - \alpha_i - \beta_i) + \alpha_i y^2_{i,t-1} + \beta_i h_{i,t-1} \quad i = 1, \cdots, m \quad (14) \end{aligned}$$

with $\boldsymbol{H_0} = \boldsymbol{I}$ represents the unconditional covariance matrix of the components. Therefore, the conditional covariance matrix of $\{x_t\}$ are obtained by:

$$\boldsymbol{V}_t = \boldsymbol{Z H_t Z'}$$

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Table and Figure Captions

Table 1: This table presents the fund's filtering procedure across all hedge fund categories (Panel A) and summary statistics of individual hedge fund characteristics (Panel B) such as: Leverage, High Water Mark, Redemption period, Management Fee, Incentive Fee, Notice period, Lockup period, Minimum Investment.

Table 2: This table presents summary statistics on individual hedge funds gain factors, across all styles through Panel A to E: the Mean, the standard deviation [SD], the skewness, the kurtosis, the minimum [Min], the maximum [Max], the t-statistic of mean, skewness and kurtosis, the Jarque-Bera [JB Test] and Lilliefors normality test statistics, and the [JB Test] 95% P-value. The critical value of the Lilliefors test at 95% confidence level is $0.805/\sqrt{T}$, where T is the sample size.

Table 3: This table presents summary serial correlation statistics on individual hedge funds amongst all styles through Panel A to Panel E: the first-order serial correlation of gain factors [$\rho(r)$] and of squared gain factors [$\rho(r^2)$], with their respective 95% P-value.

Table 4: This table presents the annualized summary statistics on the optimal portfolio constructed across the 5 styles under scrutiny. Through Panel A to E, the results of various allocation strategies are reported. Several measures of performance are outlined: the annualized average yield [AAY], the annualized standard deviation [sdev], the Sharpe ratio [SR], the modified Sharpe ratio [mSR] as defined in Equation (8), the out-performance rate [$\mathcal{S}$], that is, the percentage of allocation periods for which the $\mathbf{I}^K$ investment strategy performs better than one of the various alternative strategies. The coefficient of relative risk aversion, λ are set equal to $5, 10$, and 15.

Table 5: This table presents the annualized summary statistics on the optimal portfolio constructed across the 5 styles under scrutiny, in the case of respecting hedge funds redemption restrictions such as Lockup period and Redemption period. Through Panel A to E, the results of various allocation strategies are reported. Several measures of performance are outlined: the annualized average yield [AAY], the annualized standard deviation [sdev], the Sharpe ratio [SR], the modified Sharpe ratio [mSR] as defined in Equation (8), the out-performance rate [$\mathcal{S}$], that is, the percentage of allocation periods for

which the $\mathbf{I}^K$ investment strategy performs better than one of the various alternative strategies. The coefficient of relative risk aversion, λ are set equal to $5, 10$, and 15.

Table 6: This table presents the results of robustness assessment of the fund selection process. We test the effect of drawing various samples of funds, on the dominance relationships between our four investment strategies. The experiment is performed on the HFR Equity hedge fund category. Test statistics on paired t-test [t-statistics] of differences in sample mean and the Mann-Whitney-Wilcoxon test [M-W tests] for differences in location are reported, with their respective 95% P-values in brackets.

Figures 1, 3, 5, 7, 9: These figures display respectively the evolution of the wealth achieved, investing in the HFR Global Macro, CTAs Barclay, HFR Equity Hedge, HFR Relative Value, and HFR Event-Driven funds under our four investment strategies. The investor's coefficient of relative risk aversion, $\lambda = 5$.

Figures 2, 4, 6, 8, 10: These figures display respectively the evolution of the wealth achieved, investing in the HFR Global Macro, CTAs Barclay, HFR Equity Hedge, HFR Relative Value, and HFR Event-Driven funds under our four investment strategies, in the case of respecting hedge funds redemption restrictions such as Lockup period and Redemption period. The investor's coefficient of relative risk aversion, $\lambda = 5$.

Figures 11, 12: These figures are Boxplots of respectively, the annualized average yield [AAY] and the annualized Sharpe ratio for the 150 samplings of HFR Equity Hedge funds. Those Boxplots intends to highlight the the dominance relationships between our four investment strategies in different structure of the data set. The coefficient of relative risk aversion, $\lambda = 5$.

Table 1: **HFR individual funds summary characteristics**

Panel A: Data Filtering Process

HF Categories	Initial	After Filtering	Ranked per AUM
	# of funds	#-of-funds	#-of-funds
HFR Macro	940	58	16
HFR Equity Hedge	2796	97	16
HFR Relative Value	961	45	16
HFR Event-Driven	533	53	16
HFR Funds of Funds	2720	103	16
Barclay CTAs	981	73	16
Total	8931	429	96

Panel B: Summary of Fund Characteristics

Main Strategy	Macro	Equity Hedge	Event-Driven	Relative Value	Barclay CTAs
Leverage (%)	92	50	50	80	94
High Water Mark (%)	83.3	66	81	73	6
Redemption period (days)	45.08	131.25	170.83	58	3.75
Management Fee(%)	2.66	1.02	1.31	1.2	2.1
Incentive Fee(%)	18.94	15.41	16.86	15.53	20.6
Notice period (days)	14.83	32.92	50	43.67	0
Lockup period (months)	0	7	8.25	1.6	0
Minimum Investment ($M)	5.24	3.33	0.82	1.89	5.84

Table 2: HFR individual funds general descriptive statistics

Panel A: Global Macro funds

Funds	Mean	SD	Skewness	Kurtosis	Min	Max	Mean t-statistic	Skewness t-statistic	Kurtosis t-statistic	JB Test	P-value P-value	Lilliefors Test
Fund1	1.02	0.11	0.24	3.67	0.71	1.34	122.92	1.27	1.82	4.91	0.09	0.09
Fund2	1.01	0.05	-0.29	4.46	0.81	1.19	252.94	-1.55	3.94	17.89	0.00	0.06
Fund3	1.02	0.08	0.62	5.55	0.71	1.32	162.69	3.33	6.87	58.34	0.00	0.12
Fund4	1.01	0.06	-0.02	2.89	0.86	1.19	209.44	-0.08	-0.30	0.09	0.95	0.04
Fund5	1.01	0.08	0.53	4.56	0.81	1.28	174.31	2.85	4.20	25.73	0.00	0.08
Fund6	1.02	0.10	0.94	5.68	0.80	1.44	135.37	5.05	7.22	77.64	0.00	0.09
Fund7	1.01	0.03	0.81	5.57	0.92	1.16	401.53	4.38	6.91	67.03	0.00	0.08
Fund8	1.01	0.05	0.30	3.57	0.86	1.15	263.80	1.63	1.54	5.01	0.08	0.07
Fund9	1.01	0.07	0.63	3.83	0.83	1.28	183.94	3.39	2.23	16.49	0.00	0.09
Fund10	1.01	0.05	-0.24	4.65	0.80	1.14	286.40	-1.29	4.44	21.35	0.00	0.05
Fund11	1.01	0.03	0.55	3.45	0.94	1.12	427.48	2.97	1.20	10.28	0.01	0.07
Fund12	1.01	0.07	-0.66	9.01	0.64	1.23	202.47	-3.54	16.19	274.58	0.00	0.07
Fund13	1.01	0.04	0.04	2.78	0.89	1.12	308.09	0.22	-0.59	0.39	0.82	0.03
Fund14	1.01	0.05	0.54	4.18	0.90	1.20	282.50	2.90	3.18	18.52	0.00	0.07
Fund15	1.02	0.10	0.23	3.21	0.76	1.29	132.04	1.25	0.56	1.86	0.39	0.04
Fund16	1.01	0.03	0.41	4.39	0.93	1.12	456.57	2.21	3.74	18.84	0.00	0.07

Panel B: Barclay CTAs funds

Funds	Mean	SD	Skewness	Kurtosis	Min	Max	Mean t-statistic	Skewness t-statistic	Kurtosis t-statistic	JB Test	P-value P-value	Lilliefors Test
Fund1	1.00	0.11	0.20	6.94	0.47	1.37	117.28	1.07	10.71	115.78	0.00	0.18
Fund2	1.03	0.18	2.33	14.59	0.60	2.20	76.33	12.67	31.47	1151.25	0.00	0.16
Fund3	1.01	0.07	0.48	4.12	0.79	1.22	202.91	2.59	3.03	15.87	0.00	0.07
Fund4	1.01	0.03	0.67	4.40	0.93	1.13	417.48	3.64	3.80	27.72	0.00	0.07
Fund5	1.01	0.04	0.56	3.85	0.91	1.15	339.37	3.05	2.32	14.67	0.00	0.08
Fund6	1.01	0.03	0.69	4.67	0.94	1.13	502.34	3.76	4.54	34.71	0.00	0.06
Fund7	1.02	0.12	1.46	9.52	0.71	1.75	109.62	7.92	17.71	376.40	0.00	0.08
Fund8	1.01	0.02	3.89	21.59	0.98	1.16	544.71	21.11	50.49	2995.14	0.00	0.27
Fund9	1.00	0.02	0.81	4.96	0.96	1.06	863.82	4.38	5.33	47.59	0.00	0.08
Fund10	1.02	0.07	0.08	2.95	0.86	1.20	203.05	0.42	-0.14	0.20	0.91	0.04
Fund11	1.01	0.10	0.05	9.30	0.52	1.48	141.40	0.27	17.11	292.90	0.00	0.08
Fund12	1.02	0.09	0.51	6.22	0.75	1.44	158.23	2.79	8.74	84.21	0.00	0.06
Fund13	1.01	0.04	0.81	4.07	0.92	1.17	311.15	4.37	2.90	27.57	0.00	0.09
Fund14	1.01	0.04	0.03	2.87	0.89	1.12	315.51	0.18	-0.34	0.15	0.93	0.03
Fund15	1.01	0.05	0.05	3.85	0.84	1.17	275.80	0.29	2.30	5.35	0.07	0.04
Fund16	1.01	0.07	0.57	3.70	0.82	1.28	180.83	3.11	1.90	13.29	0.00	0.08

Panel C: Equity Hedge funds												
Funds	Mean	SD	Skewness	Kurtosis	Min	Max	Mean t-statistic	Skewness t-statistic	Kurtosis t-statistic	JB Test	P-value P-value	Lilliefors Test
Fund1	1.02	0.06	0.80	8.00	0.78	1.31	213.71	4.22	13.11	189.70	0.00	0.11
Fund2	1.01	0.06	0.45	6.15	0.80	1.27	208.66	2.37	8.25	73.65	0.00	0.15
Fund3	1.01	0.04	-0.63	3.91	0.86	1.10	323.36	-3.28	2.38	16.43	0.00	0.05
Fund4	1.01	0.09	1.18	10.72	0.70	1.52	149.58	6.17	20.25	448.20	0.00	0.11
Fund5	1.01	0.05	-0.31	3.60	0.84	1.16	249.76	-1.64	1.58	5.21	0.07	0.06
Fund6	1.00	0.02	-0.36	3.77	0.95	1.05	806.07	-1.88	2.01	7.58	0.02	0.07
Fund7	1.01	0.06	-2.24	13.84	0.66	1.17	232.45	-11.73	28.42	945.51	0.00	0.15
Fund8	1.01	0.07	0.64	6.39	0.77	1.32	182.98	3.35	8.89	90.18	0.00	0.10
Fund9	1.01	0.05	1.26	16.62	0.81	1.35	253.70	6.60	35.72	1319.40	0.00	0.13
Fund10	1.01	0.06	-0.48	3.83	0.83	1.15	230.91	-2.51	2.19	11.10	0.00	0.06
Fund11	1.02	0.12	0.57	3.98	0.76	1.49	106.32	2.99	2.57	15.55	0.00	0.05
Fund12	1.01	0.03	0.69	12.28	0.87	1.18	429.18	3.62	24.32	604.71	0.00	0.10
Fund13	1.01	0.03	0.00	3.09	0.94	1.08	479.48	-0.02	0.23	0.05	0.97	0.05
Fund14	1.00	0.02	0.06	2.88	0.96	1.04	800.75	0.34	-0.32	0.21	0.90	0.03
Fund15	1.01	0.04	-0.65	4.13	0.85	1.10	332.23	-3.41	2.97	20.45	0.00	0.08
Fund16	1.01	0.04	-0.54	4.90	0.85	1.14	294.80	-2.85	4.99	33.04	0.00	0.07

Panel D: Event-Driven funds

Funds	Mean	SD	Skewness	Kurtosis	Min	Max	Mean t-statistic	Skewness t-statistic	Kurtosis t-statistic	JB Test	P-value P-value	Lilliefors Test
Fund1	1.01	0.05	0.18	6.50	0.80	1.25	242.76	0.97	9.31	87.55	0.00	0.10
Fund2	1.01	0.02	-2.10	11.61	0.92	1.04	838.09	-11.19	22.91	650.20	0.00	0.12
Fund3	1.01	0.01	0.52	6.52	0.98	1.05	1509.18	2.77	9.37	95.41	0.00	0.14
Fund4	1.01	0.01	-0.29	4.37	0.95	1.05	924.25	-1.54	3.65	15.73	0.00	0.07
Fund5	1.01	0.09	1.26	11.68	0.65	1.51	147.33	6.72	23.11	579.21	0.00	0.18
Fund6	1.01	0.02	0.60	5.16	0.93	1.10	553.59	3.17	5.75	43.16	0.00	0.08
Fund7	1.01	0.03	-1.46	8.70	0.86	1.06	515.28	-7.76	15.17	290.37	0.00	0.08
Fund8	1.01	0.03	-0.61	4.47	0.89	1.08	446.41	-3.23	3.91	25.76	0.00	0.04
Fund9	1.01	0.01	-0.43	4.56	0.95	1.05	919.68	-2.29	4.15	22.47	0.00	0.09
Fund10	1.01	0.03	-1.52	8.88	0.86	1.06	513.78	-8.07	15.65	309.90	0.00	0.09
Fund11	1.01	0.01	-0.18	3.43	0.97	1.05	1005.29	-0.95	1.14	2.20	0.33	0.04
Fund12	1.00	0.04	-0.48	5.58	0.86	1.12	340.37	-2.54	6.87	53.59	0.00	0.09
Fund13	1.01	0.04	0.95	8.47	0.86	1.24	296.82	5.07	14.55	237.26	0.00	0.10
Fund14	1.01	0.01	-1.03	6.29	0.97	1.03	1562.35	-5.51	8.77	107.21	0.00	0.08
Fund15	1.01	0.01	-1.17	7.46	0.93	1.04	899.26	-6.22	11.88	179.90	0.00	0.07
Fund16	1.01	0.04	-0.55	5.79	0.83	1.13	332.35	-2.91	7.43	63.60	0.00	0.08

Panel E: Relative Value funds

Funds	Mean	SD	Skewness	Kurtosis	Min	Max	Mean t-statistic	Skewness t-statistic	Kurtosis t-statistic	JB Test	P-value P-value	Lilliefors Test
Fund1	1.02	0.08	-0.97	27.75	0.40	1.50	160.44	-5.20	66.25	4415.64	0.00	0.23
Fund2	1.01	0.01	-0.18	6.06	0.96	1.06	1066.13	-0.96	8.20	68.22	0.00	0.09
Fund3	1.01	0.00	3.04	15.92	1.00	1.03	3599.15	16.26	34.58	1460.35	0.00	0.24
Fund4	1.01	0.01	-0.74	4.48	0.98	1.02	1896.05	-3.99	3.96	31.57	0.00	0.09
Fund5	1.01	0.03	-5.02	42.48	0.75	1.10	439.97	-26.87	105.69	11892.48	0.00	0.22
Fund6	1.01	0.01	-0.40	5.74	0.95	1.05	1005.59	-2.12	7.34	58.32	0.00	0.12
Fund7	1.01	0.01	-0.90	5.94	0.96	1.03	1458.86	-4.84	7.86	85.28	0.00	0.08
Fund8	1.01	0.00	1.19	6.32	1.00	1.02	6153.22	6.35	8.88	119.17	0.00	0.09
Fund9	1.00	0.02	-0.58	5.92	0.89	1.07	561.73	-3.12	7.81	70.76	0.00	0.05
Fund10	1.01	0.01	-0.21	5.69	0.98	1.04	1701.42	-1.15	7.19	52.99	0.00	0.08
Fund11	1.01	0.02	-1.07	8.38	0.94	1.06	858.61	-5.70	14.41	240.27	0.00	0.11
Fund12	1.01	0.01	0.06	4.41	0.96	1.05	1025.82	0.34	3.79	14.45	0.00	0.07
Fund13	1.01	0.01	0.06	4.66	0.96	1.05	1021.13	0.34	4.44	19.87	0.00	0.08
Fund14	1.01	0.02	-3.01	20.21	0.90	1.04	878.87	-16.14	46.07	2382.68	0.00	0.19
Fund15	1.00	0.00	-0.53	5.92	0.99	1.02	3321.75	-2.82	7.83	69.21	0.00	0.07
Fund16	1.00	0.01	-0.48	3.78	0.98	1.02	2035.89	-2.58	2.08	10.98	0.00	0.07

Table 3: Ljung-Box statistics for individual HFR hedge fund returns and squared returns

Panel A: Global Macro funds

	$\rho(r)$	P-value	$\rho(r^2)$	P-value
Fund1	14.12	0.17	42.17	0.00
Fund2	9.50	0.49	22.58	0.01
Fund3	10.80	0.37	81.07	0.00
Fund4	23.66	0.01	13.25	0.21
Fund5	12.51	0.25	65.55	0.00
Fund6	16.98	0.07	15.55	0.11
Fund7	24.05	0.01	22.54	0.01
Fund8	7.46	0.68	8.07	0.62
Fund9	26.36	0.00	14.31	0.16
Fund10	12.98	0.23	3.82	0.96
Fund11	7.29	0.70	24.92	0.01
Fund12	13.50	0.20	3.08	0.98
Fund13	15.11	0.13	22.52	0.01
Fund14	10.25	0.42	2.94	0.98
Fund15	17.02	0.07	20.25	0.03
Fund16	17.89	0.06	45.59	0.00

Panel B: Barclay CTAs funds				
	$\rho(r)$	P-value	$\rho(r^2)$	P-value
Fund1	20.63	0.02	30.33	0.00
Fund2	29.21	0.00	38.47	0.00
Fund3	5.54	0.85	9.58	0.48
Fund4	9.59	0.48	10.84	0.37
Fund5	19.82	0.03	9.13	0.52
Fund6	20.26	0.03	7.68	0.66
Fund7	8.56	0.57	2.07	1.00
Fund8	62.78	0.00	42.40	0.00
Fund9	10.54	0.39	21.16	0.02
Fund10	21.65	0.02	27.30	0.00
Fund11	11.29	0.34	1.99	1.00
Fund12	14.74	0.14	43.16	0.00
Fund13	26.36	0.00	30.12	0.00
Fund14	14.49	0.15	20.65	0.02
Fund15	14.04	0.17	22.88	0.01
Fund16	25.30	0.00	13.89	0.18

Panel C: Equity Hedge funds				
	$\rho(r)$	P-value	$\rho(r^2)$	P-value
Fund1	16.40	0.09	9.52	0.48
Fund2	13.26	0.21	72.61	0.00
Fund3	5.53	0.85	14.45	0.15
Fund4	25.58	0.00	42.44	0.00
Fund5	3.91	0.95	18.34	0.05
Fund6	22.92	0.01	15.15	0.13
Fund7	22.74	0.01	39.76	0.00
Fund8	25.16	0.01	37.88	0.00
Fund9	14.70	0.14	1.47	1.00
Fund10	6.06	0.81	23.24	0.01
Fund11	13.33	0.21	54.29	0.00
Fund12	5.64	0.84	21.55	0.02
Fund13	13.86	0.18	22.08	0.01
Fund14	6.16	0.80	13.87	0.18
Fund15	5.78	0.83	16.62	0.08
Fund16	14.50	0.15	37.12	0.00

Panel D: Event-Driven funds				
	$\rho(r)$	P-value	$\rho(r^2)$	P-value
Fund1	22.38	0.01	15.55	0.11
Fund2	24.26	0.01	21.71	0.02
Fund3	53.03	0.00	53.47	0.00
Fund4	35.94	0.00	22.45	0.01
Fund5	16.03	0.10	82.44	0.00
Fund6	38.91	0.00	22.12	0.01
Fund7	30.86	0.00	12.24	0.27
Fund8	33.44	0.00	14.79	0.14
Fund9	42.42	0.00	11.08	0.35
Fund10	29.31	0.00	10.94	0.36
Fund11	24.86	0.01	9.74	0.46
Fund12	17.66	0.06	25.05	0.01
Fund13	19.01	0.04	31.39	0.00
Fund14	41.17	0.00	11.56	0.32
Fund15	17.67	0.06	28.91	0.00
Fund16	10.76	0.38	28.69	0.00

Panel E: Relative Value funds				
	$\rho(r)$	P-value	$\rho(r^2)$	P-value
Fund1	12.99	0.22	30.11	0.00
Fund2	36.91	0.00	32.87	0.00
Fund3	9.62	0.47	9.17	0.52
Fund4	44.99	0.00	10.51	0.40
Fund5	61.25	0.00	36.42	0.00
Fund6	21.20	0.02	111.10	0.00
Fund7	63.55	0.00	6.50	0.77
Fund8	506.62	0.00	207.20	0.00
Fund9	31.02	0.00	24.61	0.01
Fund10	15.21	0.12	29.15	0.00
Fund11	40.45	0.00	54.56	0.00
Fund12	45.96	0.00	97.36	0.00
Fund13	42.25	0.00	87.17	0.00
Fund14	35.97	0.00	26.22	0.00
Fund15	79.59	0.00	67.37	0.00
Fund16	11.79	0.30	12.35	0.26

Table 4: Annualized Portfolio Performance Statistics

Panel A: Global Macro funds

$K = 7, \ \& \ L = 10$

Strategy	λ	AAY(%)	ASD(%)	ASR	mSR	$\mathcal{S}(\%)$	tc^{be}
I^K		28.58	23.86	1.20			
	5	23.44	11.07	2.12	0.41	52.26	0.46
MV^K	10	20.45	9.01	2.27	0.55	50.32	0.43
	15	18.66	8.14	2.29	0.64	49.68	0.45
	5	14.01	10.56	1.32	1.10	56.13	-0.33
GO-GARCH(1,1)	10	13.87	10.50	1.32	1.11	56.13	-0.38
	15	13.83	10.48	1.31	1.11	55.48	-0.39
EWCRP		13.67	11.72	1.16	1.24	58.06	0.16

Panel B: Barclay CTAs funds

$K = 7, \ \& \ L = 10$

Strategy	λ	AAY(%)	ASD(%)	SR	mSR	$\mathcal{S}(\%)$	tc^{be}
I^K		40.80	45.84	0.89			
	5	25.42	12.71	2.00	0.47	52.53	-3.36
MV^K	10	21.69	10.04	2.16	0.54	51.90	-4.40
	15	19.27	8.90	2.16	0.59	51.27	-4.58
	5	13.52	11.94	1.13	0.81	59.49	2.26
GO-GARCH(1,1)	10	13.36	11.85	1.12	0.81	59.49	2.07
	15	13.31	11.82	1.12	0.81	59.49	2.04
EWCRP		13.16	11.05	1.19	0.80	59.49	0.40

Panel C: Equity Hedge funds

$k = 7, \ \& \ L = 10$

Strategy	λ	AAY(%)	ASD(%)	SR	mSR	$\mathcal{S}(\%)$	tc^{be}
I^K		29.33	19.31	1.52			
	5	25.52	12.21	2.09	0.54	53.42	0.72
MV^K	10	22.39	9.63	2.33	0.72	54.79	0.80
	15	20.46	8.32	2.46	0.81	56.85	0.84
	5	11.93	7.81	1.52	1.53	65.75	-4.23
GO-GARCH(1,1)	10	11.81	7.78	1.51	1.53	65.75	-12.73
	15	11.77	7.76	1.51	1.53	65.75	-24.22
EWCRP		13.45	10.72	1.25	1.86	65.75	0.17

Panel D: Event-Driven funds

$K = 7, \ \& \ L = 10$

Strategy	λ	AAY(%)	ASD(%)	SR	mSR	$\mathcal{S}(\%)$	tc^{be}
I^K		23.49	13.77	1.68			
	5	20.14	9.04	2.78	0.71	47.02	2.56
MV^K	10	18.85	7.72	3.17	0.77	50.99	0.77
	15	18.07	7.00	3.44	0.81	52.32	0.75
	5	13.22	10.43	1.26	3.09	63.58	-0.26
GO-GARCH(1,1)	10	13.15	10.42	1.26	3.10	63.58	-0.27
	15	13.13	10.41	1.26	3.11	63.58	-0.27
EWCRP		12.72	6.35	1.99	1.46	63.58	0.12

Panel E: Relative Value funds

$K = 7, \ \& \ L = 10$

Strategy	λ	AAY(%)	ASD(%)	SR	mSR	$\mathcal{S}(\%)$	tc^{be}
I^K		25.55	23.29	1.10			
	5	21.57	10.03	2.15	0.30	52.29	-0.50
MV^K	10	19.47	6.71	2.90	0.37	54.90	-1.04
	15	18.57	5.47	3.39	0.39	55.56	-1.20
	5	10.99	4.12	2.65	0.76	75.82	-0.24
GO-GARCH(1,1)	10	10.89	4.07	2.66	0.77	75.82	-0.24
	15	10.85	4.05	2.66	0.77	75.82	-0.24
EWCRP		11.03	2.76	3.98	0.71	71.90	0.22

Table 5: Hedge Fund Redemption Restrictions and Annualized Portfolio Performance Statistics

Panel A: Global Macro funds

$K = 7, \ \& \ L = 10$

Strategy	λ	AAY(%)	ASD(%)	ASR	mSR	$\mathcal{S}$(%)	tc^{be}
I^K		29.07	23.89	1.22			
	5	21.20	11.83	1.79	0.66	50.35	1.12
MV^K	10	18.76	9.75	1.92	0.74	50.35	1.12
	15	17.22	8.90	1.93	0.80	51.05	1.17
	5	13.93	10.49	1.32	1.14	50.35	-1.57
GO-GARCH(1,1)	10	13.87	10.47	1.32	1.14	50.35	-1.61
	15	13.85	10.46	1.32	1.14	50.35	-1.60
EWCRP		14.09	11.94	1.18	1.26	48.25	0.48

Panel B: Barclay CTAs funds

$K = 7, \ \& \ L = 10$

Strategy	λ	AAY(%)	ASD(%)	SR	mSR	$\mathcal{S}$(%)	tc^{be}
I^K		38.10	44.23	0.86			
	5	23.31	14.37	1.62	0.50	49.65	38.26
MV^K	10	19.77	11.32	1.75	0.56	52.45	15.82
	15	17.59	9.84	1.79	0.60	55.24	13.99
	5	14.40	12.06	1.19	0.74	53.85	2.24
GO-GARCH(1,1)	10	14.27	11.98	1.19	0.75	53.85	2.26
	15	14.22	11.96	1.19	0.75	53.85	2.27
EWCRP		13.67	11.21	1.22	0.75	53.15	0.98

Panel C: Equity Hedge funds

$k = 7, \ \& \ L = 10$

Strategy	λ	AAY(%)	ASD(%)	SR	mSR	$\mathcal{S}$(%)	tc^{be}
I^K		31.19	19.26	1.62			
	5	25.10	12.57	2.00	0.92	57.14	1.61
MV^K	10	21.54	9.91	2.18	1.04	57.14	1.82
	15	19.42	8.61	2.26	1.11	56.39	1.83
	5	12.32	8.03	1.53	1.70	59.40	6.02
GO-GARCH(1,1)	10	12.32	8.02	1.53	1.69	58.65	5.96
	15	12.33	8.02	1.53	1.69	58.65	6.11
EWCRP		14.09	11.09	1.27	2.11	56.39	0.56

Panel D: Event-Driven funds

$K = 7, \; \& \; L = 10$

Strategy	λ	AAY(%)	ASD(%)	SR	mSR	$\mathcal{S}$(%)	tc^{be}
I^K		20.90	13.05	1.60			
	5	18.24	7.76	2.35	0.51	47.83	2.93
MV^K	10	17.00	6.30	2.70	0.58	53.62	2.23
	15	16.30	5.51	2.96	0.61	53.62	1.74
	5	12.83	10.93	1.17	3.84	54.35	-0.68
GO-GARCH(1,1)	10	12.81	10.90	1.17	3.79	54.35	-0.69
	15	12.81	10.89	1.17	3.78	54.35	-0.69
EWCRP		12.71	6.55	1.93	1.27	55.07	0.29

Panel E: Relative Value funds

$K = 7, \; \& \; L = 10$

Strategy	λ	AAY(%)	ASD(%)	SR	mSR	$\mathcal{S}$(%)	tc^{be}
I^K		21.44	26.06	0.82			
	5	18.26	13.33	1.37	0.25	53.62	-5.68
MV^K	10	17.62	8.44	2.09	0.22	55.80	9.67
	15	17.09	6.63	2.58	0.23	54.35	5.28
	5	11.29	4.44	2.53	0.47	68.84	-0.50
GO-GARCH(1,1)	10	11.23	4.36	2.56	0.47	68.84	-0.51
	15	11.20	4.33	2.57	0.48	68.84	-0.51
EWCRP		11.23	2.88	3.88	0.44	67.39	0.48

Table 6: Robustness assessment of the fund selection process. We test the effect of drawing various samples of funds, on the dominance relationships between our four investment strategies. The investigation is addressed by drawing 150 times, a set of 16 HFR Equity Hedge funds from a sample of 97. P-values (in brackets) are computed at 95% confidence level. Small P-values indicate that the distribution free allocation strategy performs better than the optimizers listed in column 1 according to the Annualized average yields.

Strategy	**t-statistics**	**M-W tests**
MV^K	**13.4**	**10714**
	(1.125658e-27)	(1.309928e-21)
EWCRP	**39.04**	**11325**
	(1.948257e-80)	(1.161420e-26)
GO-GARCH(1,1)	**43.44**	**11325**
	(8.481405e-87)	(1.161420e-26)

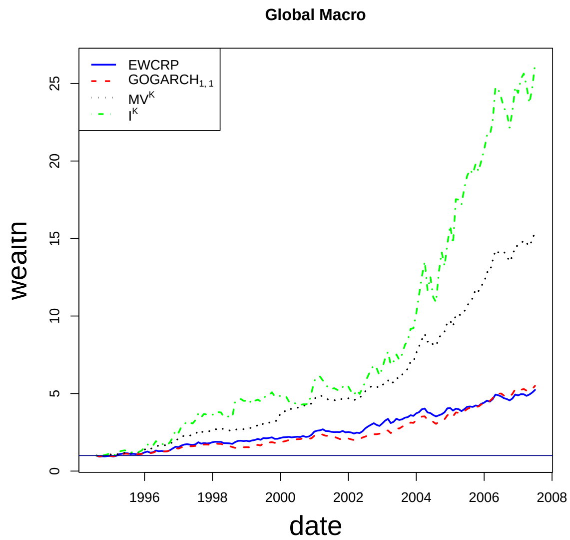

Figure 1: Wealth achieved by each investment strategy amongst 16 HFR Global Macro funds.

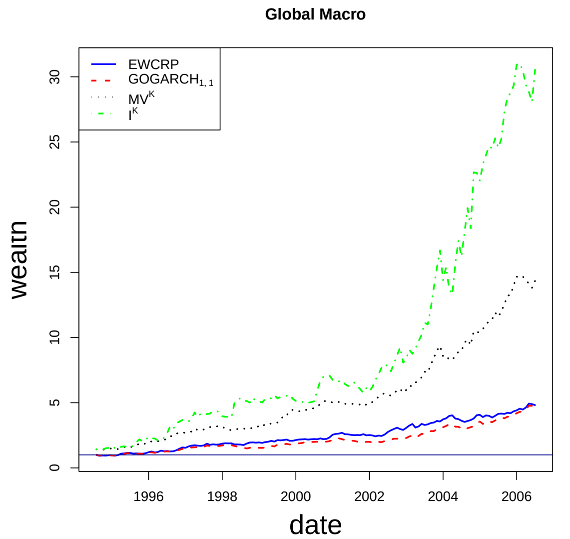

Figure 2: Wealth achieved by each investment strategy amongst 16 HFR Global Macro funds. Experiments are implemented under various hedge fund redemption restrictions.

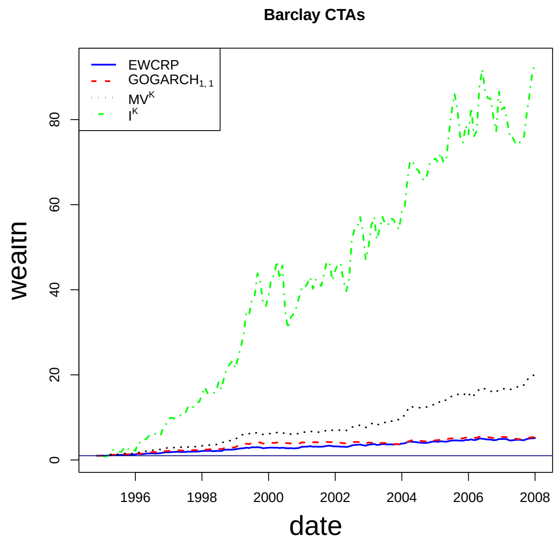

Figure 3: Wealth achieved by each investment strategy amongst 16 Barclays CTAs funds.

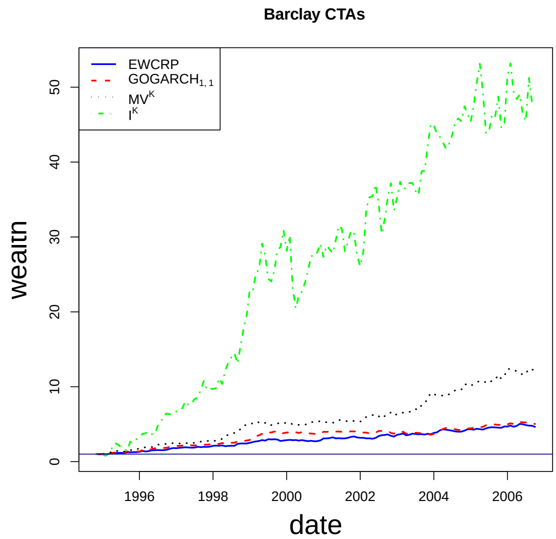

Figure 4: Wealth achieved by each investment strategy amongst 16 Barclays CTAs funds. Experiments are implemented under various hedge fund redemption restrictions.

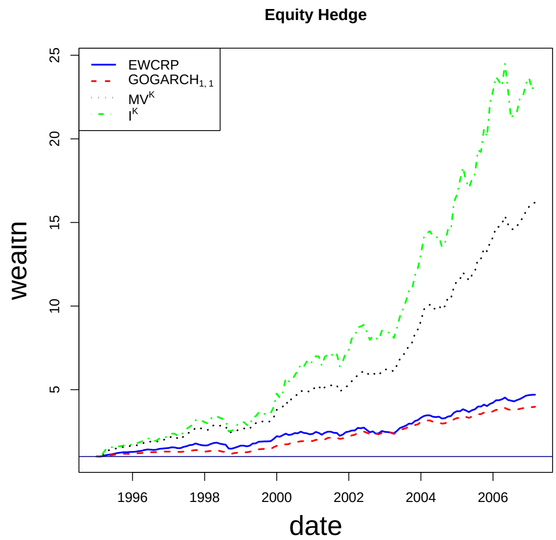

Figure 5: Wealth achieved by each investment strategy amongst 16 HFR Equity Hedge funds.

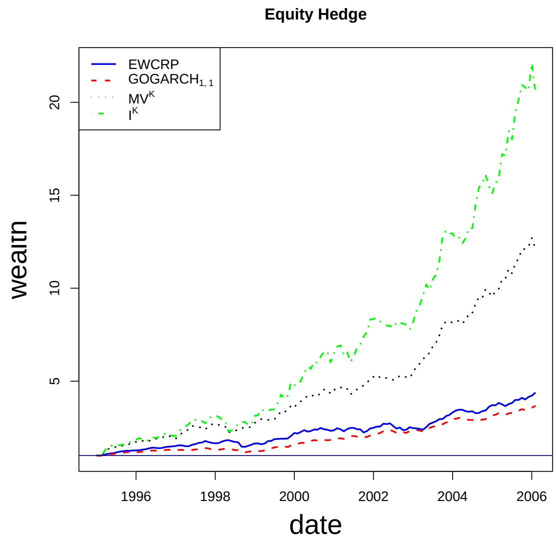

Figure 6: Wealth achieved by each investment strategy amongst 16 HFR Equity Hedge funds. Experiments are implemented under various hedge fund redemption restrictions.

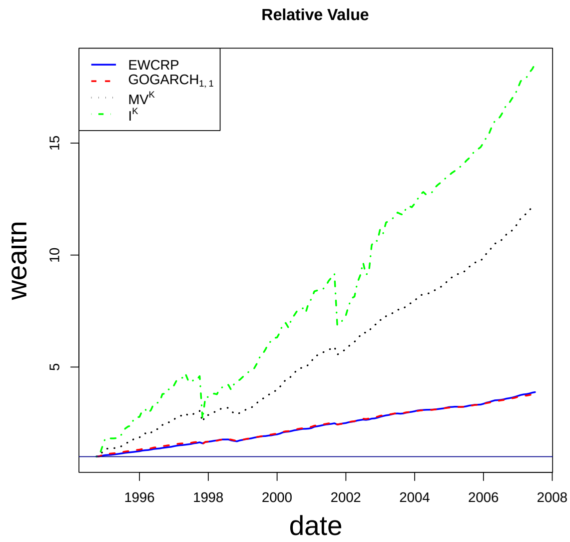

Figure 7: Wealth achieved by each investment strategy amongst 16 HFR Relative Value funds.

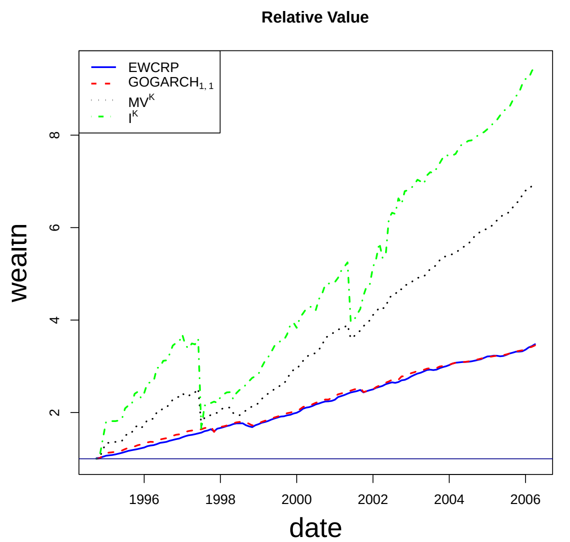

Figure 8: Wealth achieved by each investment strategy amongst 16 HFR Relative Value funds. Experiments are implemented under various hedge fund redemption restrictions.

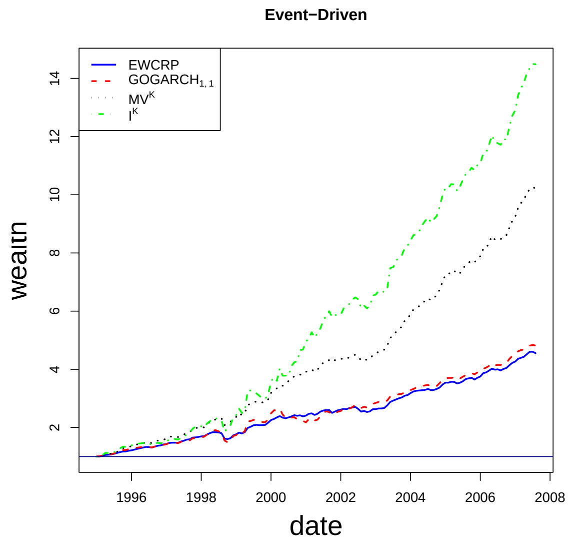

Figure 9: Wealth achieved by each investment strategy amongst 16 HFR Event-Driven funds.

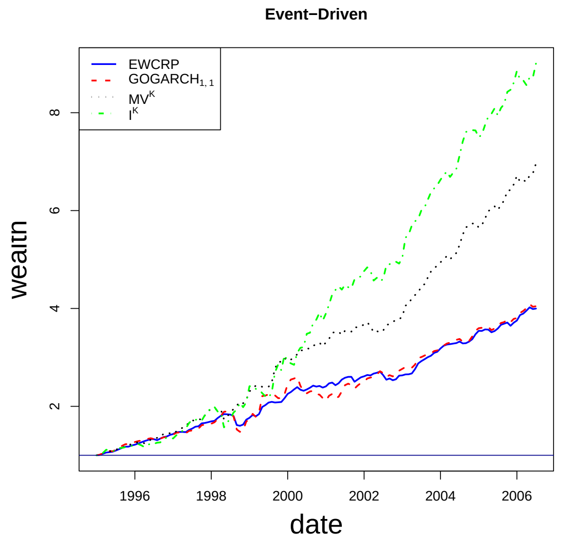

Figure 10: Wealth achieved by each investment strategy amongst 16 HFR Event-Driven funds. Experiments are implemented under various hedge fund redemption restrictions.

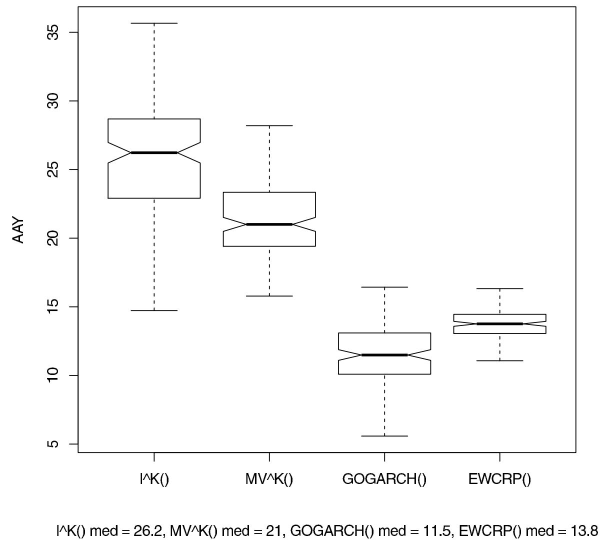

Figure 11: Boxplots of AAY for 150 resamplings HFR individual Equity Hedge funds. Median values for each strategy are displayed in the subtitle below the graphic.

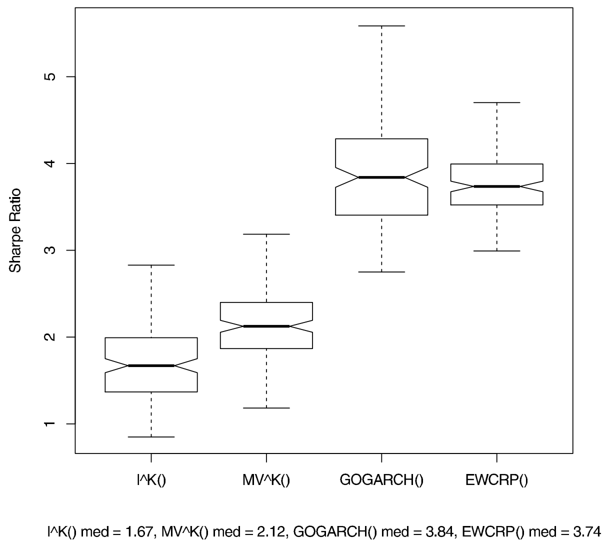

Figure 12: Boxplots of annualized Sharpe ratios for 150 resamplings HFR individual Equity Hedge funds. Median values for each strategy are displayed in the subtitle below the graphic.