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Evaluation of GARCH model Adequacy in forecasting Non-linear economic time series data

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Abstract

To date in literature, GARCH model has been described not suitable for non-linear foreign exchange series and therefore this paper proposes an Augmented GARCH model that could capture both linear and non-linear behavior of data. The properties of this new model is derived and found to have a minimum variance compared with GARCH model. We employ the use of Brock-Dechert-Scheinkman (BDS) test statistic to confirm the suitability of GARCH model on the data; the new methodology proposed is illustrated with foreign exchange rate data from Great Britain (Pound) and Botswana (Pula) against United States of America (Dollar).

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1 Introduction

The autoregressive conditional heteroscedasticity model (ARCH), introduced by Engle (1982) and its generalization GARCH, introduced by Bollerslev (1986) have been widely applied to model volatility in financial time series. These models have been useful because they are convenient representation of the persistence of variance over time despite the lack of statistical and economic theory justification (Hall et al., 1989). Several studies have investigated the adequacy of GARCH model in financial time series. Claudio and Jean (2011) used GARCH to model stock market indices and concluded that the model fails to capture the statistical structure of the market returns series for all the countries economies investigated. Lim et.al.(2005) employed the Hinich portmanteau bicorrelation test to determine the adequacy of GARCH model for eight Asian stock markets. They conclude that this model cannot provide an adequate characterization for the underlying market indices. Brooks and Hinich (1998), Liew, et.al.(2003) and Lim et.al (2004) have studied the behavior of exchange rates data using GARCH models, it was concluded that these models could not capture adequately the statistical properties of non-linearity present in the series. Besides these findings, political and financial instability that arises from period to period in most countries produces episodic non-linearities in the foreign exchange markets indices (Bonilla et.al. 2006 and Romero-Meza et.al. 2007), if the procedure utilized in the analysis of foreign exchange is not adequate it may jeopardize forecasting efficacy and lead to distortion of inference made. It therefore may be of interest to examine the statistical properties of modified

GARCH model and its suitability in the presence of non-linearities behavior of exchange rate data.

This paper examines the statistical properties of augmented GARCH model; the augmentation is performed using Bi-linear function to capture the instability of the non-linearity in the data set. We analytically compare the new model with conventional GARCH model using the model variance. The Brock-Dechert-Scheinkman test (BDS) is applied to test the adequacy of GARCH model on the series used. Guglieimo et.al (2005) have utilized this test statistic to determine the adequacy of GARCH models for capturing non-linearity in data set. The procedure involves subjecting the standardized residuals of the fitted GARCH models to BDS under the null hypothesis of GARCH sufficient characterization of the series. If the BDS test rejects the null hypothesis using appropriate critical values, then the fitted GARCH model is assumes to be inadequately characterized the data. Monthly data used in this paper covered the period of January 1975 to December 20011 (444 months). The behaviors of the series examined are as shown in figures 1a to 2b. Test for stationarity was carried out using Augmented Dickey-Fuller test and unit root test were performed.

The remaining part of this paper is organized as follows: section 2 covers the specification of augmented GARCH models, efficiency of AGM, estimation of the parameters of augmented GARCH model (AGM), properties of derived estimators of AGM, section 3, empirical illustration, identification of non-linearity status of the series with BDS test, identification of stationarity condition of series, estimation of classical GARCH and augmented GARCH models section 4 empirical comparison of models and conclusion.

2 Specification of augmented GARCH models

Literature has shown that financial time series data present volatility clustering effects, and this volatility occurs intermittently. To take care of this situation researchers make use of a conditional variance model, where the variance of the errors is allowed to change over time in an autoregressive conditional heteroskedasticity framework. Following Bollerslev (1986), the $GARCH(p,q)$ model can be represented in the following form:

Let $\{y_{(t)}\}$ be the time series of an exchange rate return, then

$$y_{(t)} = \sigma_t \varepsilon_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{1}$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$ and innovation sequence $\{\varepsilon\}_{i=-\infty}^{\infty}$ is independent and identically distributed (iid) with $E(\varepsilon_0)$=0 and $E(\varepsilon_0^2)$=1. The main idea is that σ_t^2, the conditional variance of y_t given information available up to time $t-1$ has an autoregressive structure and is positively correlated to its own recent past and to recent values of the squared return, y_t^2. This captures the idea of volatility being "persistent", large (small) values of y_t^2 are likely to be followed by large (small) values. The GARCH model formulation captures the fact that volatility is changing in time. The change corresponds to a weighted average among the long term average variance, the volatility in the previous period, and the fitted variance in the previous period as well. The model described in equation (1) is used to parameterize financial time series and in particular foreign exchange. An augmented GARCH model is an extension of the GARCH model as tool for modeling financial time series. It allows us to capture asymmetries in the conditional mean and variance of financial and economic time series by means of interactions between past shocks and volatilities. The bilinear GARCH models take into account variations between the independent variables as well as co-variations between the variables. This is very important in the study of financial market data where the covariance between independent variables may play a significant role in determining market volatility. We use AGM because we discovered that its modeling is data driven as we augment the model y_t to this error term and observe series. The inclusion of bilinear process to equation (1) will capture the non-linear behavior part of y_t, bilinear takes into account the variation within independent variables as well as co-variations between the variable. On the other hand Augmented-GARCH models (AGM) allow us to capture asymmetries in the conditional variance of financial and economic-time series by means of interactions between past shocks and volatilities; thus we postulate an augmented GARCH (AGM) as:

$$y_t = \sigma_t \varepsilon_t + \sum_{i=1}^{p}\sum_{j=i}^{q} \tau_{ij} y_{t-i}\varepsilon_{t-j} \tag{2}$$

To investigate the proportion of (2) we consider its mean and variance as follows: mean of y_t is derived using $E(y_t) = \sigma_t E(\varepsilon_t) + \sum_{i=1}^{p}\sum_{j=i}^{q} \tau_{ij} E(y_{t-i}\varepsilon_{t-j})$ as

$$E\{y_t\} = \begin{cases} 0 & \forall \mathrm{i} \neq \mathrm{j} \\ \sigma_\varepsilon^2 \sum_{i=1}^{p} \tau_{ij} & ,\forall\ \mathrm{i=j} \end{cases} \tag{3}$$

To derive the variance of y_t from the conventional expression given as:

$$Var(y_t) = E\left(y_t^2\right) - \left(E\left(y_t\right)\right)^2 \tag{4}$$

Consider an alternative representation

$$Z_t = y_t^2 - \sigma_t^2 = \sigma_t^2\left(\varepsilon_t^2 - 1\right)$$

$$y_t^2 = \sigma_t^2 + Z_t,$$

where Z_t is a martingale differences with mean zero

$$= \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + Z_t$$

$$= \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-1}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 - \sum_{j=1}^{q} \beta_j Z_{k-j} + Z_t$$

If we denote $p = \max(p,q)$, $\alpha_i = 0$ for $1 > p$ and $\beta_j = 0$ for $j > q$, then the above can be written as:

$$y_t^2 = \alpha_0 + \sum_{\mathrm{i}=1}^{\mathrm{P}} (\alpha_{\mathrm{i}} + \beta_{\mathrm{i}}) y_{t-1}^2 - \sum_{j=1}^{q} \beta_j Z_{t-j} + Z_t$$

In other words y_t^2 is an ARMA process with martingale difference innovations. Using stationarity, i.e. $E\left(y_t^2\right) = E\left(y_{t-1}^2\right)$, the unconditional variance is now easy to obtain

$$E\left(y_t^2\right) = \alpha_0 + \sum_{i=1}^{p} \left(\alpha_i + \beta_j\right) E\left(y_{t-1}^2\right) - \sum_{j=1}^{q} \beta_j E\left(Z_{t-j}\right) + E\left(Z_t\right)$$

$$\alpha_0 + E\left(y_t^2\right) \sum_{i=1}^{p} \left(\alpha_i + \beta_j\right),$$

reduces to

$$E\left(y_t^2\right) = \frac{\alpha_0}{1 - \sum_{i=1}^{p} \left(\alpha_i + \beta_j\right)} \tag{5a}$$

Also using equation 2 we have

$$y_t^2 = \sigma_t^2 \varepsilon_t^2 + \tau_I^2 y_{t-1}^2 \varepsilon_{t-1}^2$$

$$E\left(y_t^2\right) = E\left(\sigma_t^2 \varepsilon_t^2\right) + \tau_i^2 E\left(y_{t-1}^2\right)$$

$$E\left(y_t^2\right) = \frac{E\left(\sigma_t^2\right)}{\left(1 - \tau_i^2\right)} = \frac{\alpha_0}{\left(1 - \sum \left(\alpha_i + \beta_j\right)\right)\left(1 - \tau_i^2\right)} \tag{5b}$$

Using (3) and (5a and 5b) in (4) gives

$$Var(y_t) = \begin{cases} \dfrac{\alpha_0}{1-\sum\limits_{l=1}^{p}\left(\alpha_i + \beta_i\right)} & \forall\ i \neq j \\ \dfrac{\alpha_0}{\left(1-\sum\limits_{l=1}^{p}\left(\alpha_i + \beta_i\right)\right)\left(1-\tau_i^2\right)} - \Delta^2 \end{cases} \tag{6}$$

where $\Delta^2 = \sigma_\varepsilon^4 \left(\sum \tau_i\right)^2$, $\forall\ i = j$.

2.1 Efficiency of AGM

To compare the efficiency of the AGM with GARCH, we relate the variances of AGM to that of classical GARCH as follows:
The variance of AGM was derived as:

Let T_1 and T_2 be two estimators of a parametric function $k(\theta); \theta \in R^n$; is the Euclidian space. The efficiency of T_1 relative to T_2 is defined as:

$$e\{T_2 / T_1\}_1 = \frac{MSE\{T_2\}}{MSE\{T_1\}}$$

If for all $\theta, e\{T_1, \mathrm{T}_2\} \leq 1$, T_2 is more efficient than T_1, otherwise T_1 is more efficient than T_2. If T_1 and T_2 are unbiased estimators of $k(\theta)$, the efficiency of T_1 relative to T_2 is the ratio of $V(T_1)$ to $V(T_2)$ are unbiased estimators, Then the efficiency of AGM relative to GM using eguation (6) and (7) is as follows:

$$\frac{Var\left(y_{t(AGM)}\right)}{Var\left(y_{t(GM)}\right)} = \frac{\dfrac{\alpha_0}{1-\sum\limits_{i=1}^{p}(\alpha_i + \beta_i)} - \Delta^2}{\dfrac{\alpha_0}{1-\sum\limits_{i=1}^{p}(\alpha_i + \beta_i)}} = 1 - \frac{\Delta^2\left(1-\sum\left(\alpha_i + \beta_i\right)\right)}{\alpha_0} = 1 - \xi$$

where $\xi = \dfrac{\Delta^2\left(1-\sum\left(\alpha_i+\beta_i\right)\right)}{\alpha_0}$.

It can be seen that if $\xi > 1$, then AGM is more efficient than GM; besides the Δ^2 is positive and replaces the variance of AGM compared with that of GM. We shall look at empirical implications of these quantities later.

2.2 Estimation of the parameters of augmented GARCH model (AGM)

To estimate the parameters of the models in equation (2), a two stage technique is suggested as follows. The reduced form of equation (2) is:

$$y_t = \sum\sum \tau_{ij} z_{ij} + v_t \tag{7}$$

In matrix form $\underline{Y} = \underline{\tau}'\,\underline{z} + \underline{v}$, where we assume $\underline{v_t} \sim N\left(0, \sigma_j^2\right)$ and $\mathrm{E}\left(v_\mathrm{i} v_\mathrm{j}\right) = 0 \ \forall \ i \neq j$.

$$\underline{Y} = \underline{\tau}'\,\underline{z} + \underline{v} \tag{8}$$

Now, at the first stage we apply the method of MLE to obtain parameters of (1) and the second stage given independence of the parameters in model (1), we apply OLS to the reduce form (8), thus we have:

$$\hat{\tau} = \left(\underline{Z}'\underline{Z}\right)^{-'}\underline{Z}'\underline{Y} \tag{9}$$

$$E\left[\underline{\hat{\tau}}\right] = E\left[\left(\underline{Z}^1\underline{Z}\right)^{-1}\underline{Z}^1\left(\underline{Z}^1\underset{-}{\tau} + \underline{V}\right)\right] = \tau$$

and

$$Var(\underline{\hat{\tau}}) = E\left[\left(\hat{\tau}-\overline{\tau}\right)\left(\hat{\tau}-\overline{\tau}\right)'\right] = E\left(\underline{Z}^1\underline{Z}\right)'\underline{Z}^1\underline{V}\underline{V}^1\underline{Z}\left(\left(\underline{Z}^1 Z\right)^{-'}\right) = \sigma^2\left(\underline{Z}^1\underline{Z}\right)^{-'}$$

The estimates in (9) are unbiased and consistent and usual test of hypothesis can be undertaken to ascertain their significance.

2.3 Properties of derived estimators of AGM

We evaluate the properties of the derived estimator of AGM in this section based on some basic properties of statistical estimator.

2.3.1 Linearity and unbiased properties of least-squares estimators

From equation (10), we have

$$\hat{\tau} = \frac{\sum z_i Y_i}{\sum z_i^2} = \sum k_i Y_i \tag{10}$$

Such that $k_i = \frac{z_i}{\sum z_i^2}$. This shows that $\hat{\tau}$ is a linear estimator because it is a linear function of Y; actually it is a weighted average of Y_i with k_i serving as the weights. The assumptions on weights k_i, are

(i) z_i and k_i are assumed to be non-stochastic

(ii) $\sum k_i = 0$

(iii) $\sum k_i^2 = \left(\sum z_i^2\right)^{-1}$, and

(iv) $\sum k_i z_i = 1$.

These assumptions can be directly verified from the definition of k_i; for instance,

$$\sum k_i = \sum \left(\frac{z_i}{\sum z_i^2} \right) = \frac{1}{\sum z_i^2} \sum z_i \ .$$

Since for a given sample $\sum z_i^2$ is known = 0, since $\sum z_i$, sum deviation from the mean value, is always zero.

Now substitute $Y_i = \tau_1 + \tau_2 z_i + u_i$ into (10) to obtain

$$\begin{aligned} \hat{\tau} = \sum k_i \left(\tau_1 + \tau_2 z_i + u_i \right) &= \tau_1 \sum k_i + \tau_2 \sum k_i z_i + \sum k_i u_i \\ &= \tau_2 + \sum k_i u_i \end{aligned} \tag{11}$$

Now taking the expectations of (11) on both sides and noting that k_i , being non-stochastic, can be treated as constants, we obtain

$$E\left(\hat{\tau}\right)=\tau_2+\sum k_i E\left(u_i\right) = \tau_2.$$

Since $E\left(u_i\right)=0$ by OLS assumption. Therefore, $\hat{\tau}_2$ is an unbiased estimator of $\hat{\tau}_2$. Likewise it can be proved that $\hat{\tau}_1$ is also an unbiased estimator of $\hat{\tau}_1$.

2.3.2 Minimum-variance property of least-squares estimators of AGM

It was shown that the least-squares $\hat{\tau}_2$ is linear as well as unbiased (this holds for $\hat{\tau}_1$ also). To show that these estimators also have minimum variance in the class of all linear unbiased estimators, consider the least squares estimator $\hat{\tau}_2$ giving as

$$\hat{\tau}_2=\sum k_i Y_i$$

where $k_i=\dfrac{z_i-\overline{z}}{\sum\left(z_i-\overline{z}\right)^2}=\dfrac{z_i}{\sum z_i^2}$.

This shows that $\hat{\tau}_2$ is a weighted average of the $Y's$, with k_i serving as the weights.

Let us define an alternative linear estimator of $\hat{\tau}_2$ as

$$\tau_2^*=\sum w_i Y_i$$

where w_i are weights, not necessarily equal k_i. Now,

$$E\left(\tau_2^*\right)=\sum w_i E\left(Y_i\right)=\sum w_i\left(\tau_1+\tau_2 z_i\right)=\tau_1\sum w_i+\tau_2\sum w_i z_i.$$

Therefore for $\hat{\tau}_2$ to be unbiased, we must have

$$\sum w_i=0 \quad \text{and} \quad \sum w_i z_i=1.$$

Also we may write

$$\operatorname{var}\left(\tau_2^*\right)=\operatorname{var}\sum w_i Y_i=\sum w_i^2 \operatorname{var}\left(Y\right)_i=\sigma^2 w_1^2,$$

where

$$\operatorname{var}(\mathrm{Y}_i) = \operatorname{var}\ (\mathrm{u}_i) = \sigma^2$$

$$= \sigma^2 \sum \left(w_i - \frac{z_i}{\sum z_i^2} + \frac{z_i}{\sum z_i^2} \right)^2$$

$$= \sigma^2 \sum \left(w_i - \frac{z_i}{\sum z_i^2} \right)^2 + \sigma^2 \frac{\sum z_i^2}{\left(\sum z_i^2\right)^2} + 2\sigma^2 \sum \left(w_i - \frac{z_i}{\sum z_i^2} \right)\left(\frac{z_i}{\sum z_i^2} \right)$$

$$= \sigma^2 \sum \left(w_i - \frac{z_i}{\sum z_i^2} \right) + \sigma^2 \left(\frac{1}{\sum z_i^2} \right) \tag{12}$$

Equation (12) reduces to

$$\operatorname{var}\left(\tau_2^*\right) = \frac{\sigma^2}{\sum z_i^2} = \operatorname{var} \hat{\tau}_2 \tag{13}$$

By equations (10) through (13) we have shown that the derived model estimators of AGM parameters satisfy the conventional properties of estimators' vis-à-vis unbiasedness, minimum variance and best linear unbiased estimators (BLUE).

3 Empirical illustration

The exchange rate data collected for Great Britain and Republic of Botswana taking United States of America as basis for comparism is utilized for the empirical illustration of our proposed methodology. The statistical package for the data analysis in this paper is E-views. The analysis presented here focused on monthly exchange rate, of two economies, viz-a –viz developed economy represented by Great Britain and developing economy represented by Republic of Botswana, the currencies are denominated in British Pound and Botswana Pula against United States of America Dollar.

3.1 Identification of non-linearity status of the series with BDS test

The currencies exchange rates were analyzed through the use of E-view and the hypothesis was accordingly set as follows:

H_0: GARCH model is a sufficient characterization of series

H_1: H_0 is not true

In Table 1 the null hypothesis that the GARCH model is a sufficient characterization of series are rejected, pointing to the fact that this result agreed with Claudo A.B and Jean S (2011), Chris B and Hinich M.J. (2011), Claudo A.B et.al (2008), Kiang-ping lim, et al (2005), Chris B and Hinich M.J (1999) just to mention the few that GARCH is not adequate for financial time series data.

Table 1: BDS test statistic values

Series	BDS Statistic	Std. Error	z-Statistic	Normal Prob.	Bootstrap Prob.
Pound	0.525377	0.004723	111.2353	0.0000	0.0000
Pula	0.539863	0.004292	125.7829	0.0000	0.0000

3.2 Identification of a stationary condition of the series

The line graph of all the series (figures 1a and1b) indicates the non-stationarity of the series, since volatile values are evident and these do not fluctuate around a constant mean. We thus examine the first differences of the series (Figures 2a and 2b) since it has no persistent trend and its values fluctuate around a constant mean of zero.

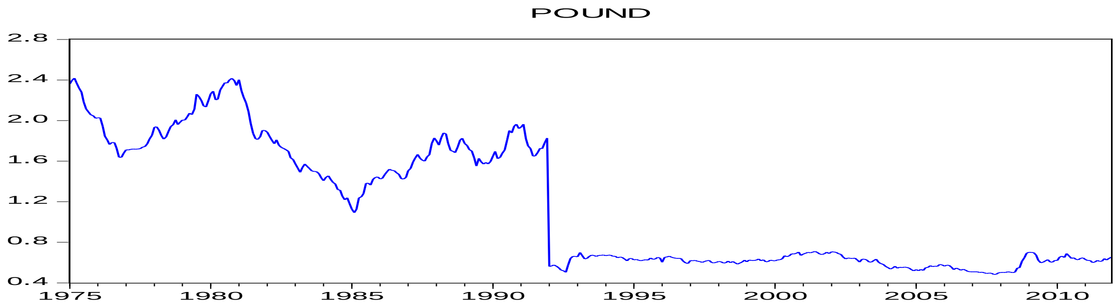

Figure 1a: Line graph of the leveled exchange rate of Dollar/pula

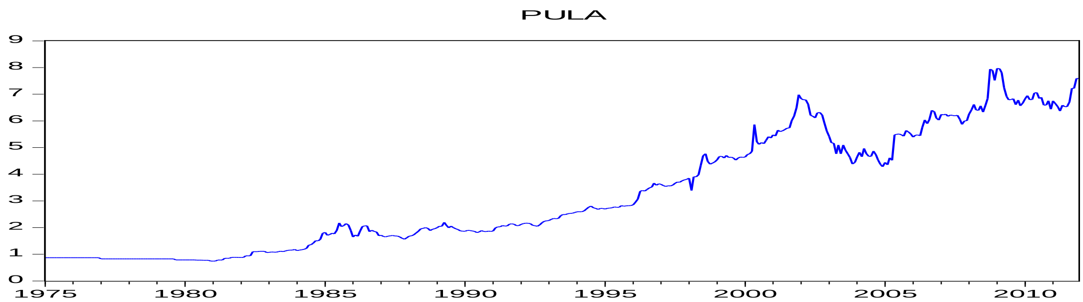

Figure 1b: Line graph of the leveled exchange rate of Dollar/pula

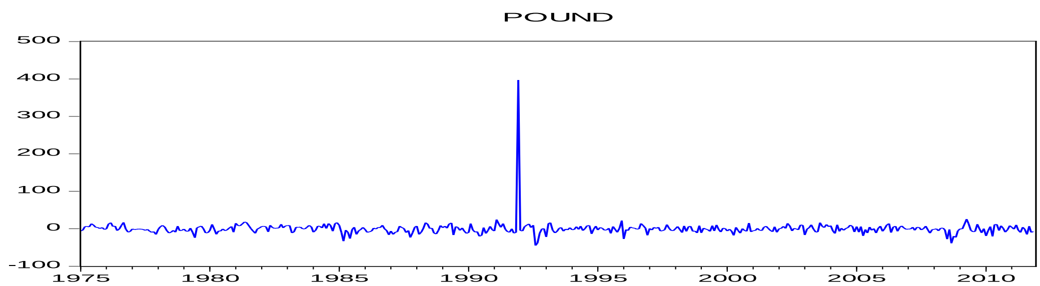

Figure 2a: Line graph of the first difference exchange rate of Dollar to Naira

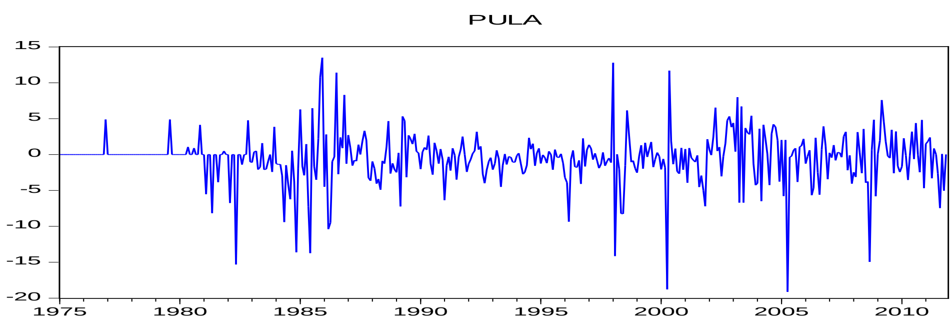

Figure 2b: Line graph of the first difference of exchange rate of Dollar/pula

The stationary condition of the series can be formally verified by using unit root test (URT) for the leveled and first differences of the series. We test for a unit root using the augmented Dickey-Fuller (ADF) statistic. At level all the series are not stationary but at first difference all series are stationary as shown in Tables (2a) and (2b) below.

Table 2a: Unit Root Test Output for the leveled for the Series

Series	ADF-Test statistic	Critical value (5%)	Mackinnon prob •
Pound	-1.8826	-2.8678	0.3405
Pula	0.2013	-2.8678	0.9725

Table 2b: Unit Root Test Output for the first difference for the Series

Series	ADF-Test statistic	Critical value (5%)	Mackinnon prob •
Pound	-19.9106	-2.8678	0.0000
Pula	-21.6683	-2.8678	0.0000

3.3 Estimation of classical GARCH model

To generate parameter estimates for the GARCH model, we used E-view to analyzed differenced data for the study as follows:

Each of the currency viz-a-viz Pound and Pula were individually analysed.

Based on tables 3a and 3b the estimated GARC(1,1) model are obtained for both Pound and Pula as follows:

$$y_{POUND/US(t)} = \sigma_t \varepsilon_t$$

where σ_t and ε_t are obtainable from the fitted model:

$$y_{POUND/US(t)} = 0.995140 y_{t-1} + \varepsilon_t$$

and (14)

$$\sigma_t^2 = 0.168022 + 0.972189\,\varepsilon_{t-1}^2 - 0.000236\left(\sigma_{t-1}^2\right)$$

$$y_{PULA/US(t)} = \sigma_t \varepsilon_t$$

where σ_t and ε_t are obtainable from the fitted model:

$$y_{PUKA/US(t)} = 1.00347 y_{t-1} + \varepsilon_t$$

and (15)

$$\sigma_t^2 = 0.47613 + 1.90366\,\varepsilon_{t-1}^2 - 0.91061\left(\sigma_{t-1}^2\right)$$

The outputs of the result are as follows:

Table 3a: GARCH model estimates for pound

Dependent Variable: POUND

Method: ML - ARCH (Marquardt) - Normal distribution

GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
DATE	0.000311	7.64E-07	407.4377	0.0000
Variance Equation				
C	0.168022	4.84E-05	3.478943	0.0005
RESID(-1)^2	0.972189	0.233697	4.160039	0.0000
GARCH(-1)	-0.000236	0.026571	-0.008892	0.9929
R-squared	0.738650	Mean dependent var		1.144981
Adjusted R-squared	0.729948	S.D. dependent var		0.615260
S.E. of regression	0.811269	Akaike info criterion		-0.589523
Sum squared resid	291.5639	Schwarz criterion		-0.552623
Log likelihood	134.8741	Hannan-Quinn criter.		-0.574971
Durbin-Watson stat	0.007310			

Table 3b: GARCH model estimate for pula

Dependent Variable: PULA

Method: ML - ARCH (Marquardt) - Normal distribution

GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
DATE	0.001026	2.75E-05	37.32744	0.0000
Variance Equation				
C	0.476132	0.100841	4.721592	0.0000
RESID(-1)^2	1.903622	0.309589	6.148864	0.0000
GARCH(-1)	-0.910605	0.013670	-66.61453	0.0000
R-squared	0.311629	Mean dependent var		3.244710
Adjusted R-squared	0.300503	S.D. dependent var		2.152726
S.E. of regression	1.464380	Akaike info criterion		3.388455
Sum squared resid	2672.195	Schwarz criterion		3.425355
Log likelihood	-748.2371	Durbin-Watson stat		0.003959

3.4 Estimation of augmented GARCH model

Estimation of parameters here was done here in two stages as the standard deviation obtained from classical GARCH was used to obtain the parameters of augmented GARCH models. The reduced form in equation (10) was estimated by making use of Bilinear (1,1) the reason for the choice of bilinear (1,1) was due to the fact that few parameters make the models to be parsimonious; from where sets of data were generated and OLS applied and the following results were obtained for the two series (Pound and Pula foreign exchange with respect to Dollar).

Table 4a: Augmented GARCH model for pound

Dependent Variable: $y_t - \sigma_t \varepsilon_t$ =ACMINFIT(POUND)

Method: Least Squares

Date: 10/09/12 Time: 14:56

Sample: 1975M01 2011M12

Included observations: 444

ACMINFIT =C(1)* $\tau_1 y_{t-1} \varepsilon_{t-1}$

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.621385	0.011382	142.4496	0.0000

R-squared	0.963201	Mean dependent var	0.524809
Adjusted R-squared	0.963201	S.D. dependent var	0.618151
S.E. of regression	0.568581	Akaike info criterion	-1.424188
Sum squared resid	6.229243	Schwarz criterion	-1.414963
Log likelihood	317.1698	Hannan-Quinn criter.	-1.420550
Durbin-Watson stat	0.505874		

Table 4a: Augmented GARCH model for pula

Method: Least Squares

Date: 10/09/12 Time: 15:18

Sample: 1 444

Included observations: 444

ACMINFIT =C(1)* $\tau_1 y_{t-1} \varepsilon_{t-1}$

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.486261	0.000736	660.2862	0.0000

R-squared	0.998666	Mean dependent var	1.199798
Adjusted R-squared	0.998666	S.D. dependent var	2.142259
S.E. of regression	1.078249	Akaike info criterion	-2.255581

Sum squared resid	2.712477	Schwarz criterion	-2.246356
Log likelihood	501.7389	Durbin-Watson stat	0.479142

By using the values generated in Table 4a the AGM fitted is

$$y_t = \sigma_t \varepsilon_t + \underset{(0.11382)}{1.621385}\, y_{t-1}\varepsilon_{t-1}$$

with variance of the model 0.01406. Also using the values generated in 4b, the AGM fitted is

$$y_t = \sigma_t \varepsilon_t + \underset{(0.000736)}{0.486261}\, y_{t-1}\varepsilon_{t-1}$$

with variance of the model 0.006123.

4 Empirical comparison of models and conclusion

Table 5 summarized the results obtained for the variances of both classical GARCH models (GM) and augmented GARCH models (AGM), this will certainly enable us to appreciate the efficiency of the new model. The implication of this is that the augmented GARCH models (AGM) is more efficient than GARCH model (GM) and this actually assert the superiority of the new model. Forecasting exchange rate is traditionally implemented using GARCH model, the shortcoming of this model is that data analyzed often exhibit some non-linearity that this model cannot captured as shown when the BDS was used to analyze the data. For its inability to capture the non-linear components of the series, the model was augmented using Bi-linear and this produced a better result than the classical GARCH model in term of their variances. For instance, the variances of classical GARCH model for Pound and Pula are 0.6582 and 2.1444 respectively while Augmented-GARCH gave 0.3233 for pound and 1.1626 for Pula in that order. The superiority of this model lies on the variance reduction. The implication of this result is that Augmented-GARCH can be used to forecast foreign exchange in these two countries more accurately and will give a desire result more than classical GARCH model.

Table 5: Variances and relative efficiencies of GM and AGM

SERIES	G.M	A.G.M	R.E.
POUND	0.6582	0.3233	0.49
PULA	2.1444	1.1626	0.54

From the fitted model we have the following table on the variance and relative efficiencies computed and the superiority of AGM over GM is evident.

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