

# Single Airport Ground-holding Model Based on the Capacity Randomness of Destination Airport

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## Abstract

Ground-holding as one of the common strategies for the current air traffic flow management, can alleviate congestion effectively and save the cost of flights waiting. Flights ground-holding strategy has a direct relationship with the landing capacity of destination airport, but the capacity of destination airport changes randomly with the affection of many factors, and moreover delay cost of different models has an important influence on the ground-holding strategy, therefore this paper considers the random changes of destination airport's capacity and delay cost of different models and establishes single airport ground random holding strategy model, and then uses genetic algorithm to simulate and verify the model. The simulation result shows that the model established is effective.

**Mathematics Subject Classification:** 90C15

**Keywords:** ground-holding, flight delays, traffic control, stochastic model

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## 1 Introduction

With the development of civil aviation, the number of airlines and the scales of operations are expanding constantly, the air traffic congestion is inevitable due to the various reasons, many scholars domestic and abroad have done a lot of research on solving this problem and proposed lots of ways to solve air traffic congestion. Nowadays ground-holding strategy is widely used, converting air-holding into ground-holding completely. But in fact, air-holding is inevitable sometimes due to a variety of reasons and most of the time is that air-holding and ground-holding happen at the same time.

Odoni discusses the ground strategies systematically [1], Terrabproposed several empirical formulas on the randomness problems of single airport [2], Andreatta, Romanin Jacur[3], Ball [4], Mukherjee [5], and so on study on single airport ground-holding problems. In domestic, the study of air traffic flow control starts late. The research on single airport ground-holding problem focuses on the following two aspects: the first one is that the establishment of the single airport ground-holding model and main single airport ground-holding models are the single airport ground-holding models based on demand randomness [6], single airport ground-holding model based on the event-driven [7], the ground-holding strategy model based on discrete system [8]. The second one is the solution of ground-holding model and the main solving algorithms are the heuristic algorithm [9], the Vogel algorithm [10], the artificial fish algorithm [11]and hybrid clustering algorithm [12] and so on.

But they do not consider the capacity randomness of the destination airport or the influence on flight waiting cost, or to solve the problem, they rely on computer simulation too much. This paper considers the randomness of destination airport

capacity, establishes the single airport ground-holding stochastic model, and uses the genetic algorithm to simulate and verify the model.

## 2 Establishment of the model

### 2.1 Assumptions and symbolic description

- 1) The scheduled departure time and landing time  $f_i$  of the flights discussed in this paper are known and its flight time is determined. In order to facilitate the problem, this paper takes no accounts of the flight navigation process and the number of flights is  $n$ . Within the time range  $[T, T + \Delta t]$ , according to the change rules of destination airport capacity, this paper will divide  $[T, T + \Delta t]$  into small periods represented by  $k$ ,  $k = 1, 2, 3, \dots, K$ , the length of the  $k$ th period is  $T_k$ .
- 2) At any period  $k$ , landing capacity of destination airport  $A$  follows the same distribution. The possible capacity value is  $q_{k,l} \in Q$  ( $q_{k,l}$  represents that the possible landing capacity of destination airport  $A$  is  $q_l$  (the  $l$ th element of set  $Q$ )( $l = 1, 2, \dots, N$ ) at any period  $k$ , the number of elements in set  $Q$  is  $N$ ), the probability is  $P_{q_{k,l}}$ .
- 3) For any flight  $F_i$ , the planned arrival time is  $p_i$  and actual arrival time is  $r_i$ .
- 4) Holding time of flight  $F_i$  is  $t_i$  and the longest holding time is  $d_i$ .

- 5) According to the destination airport capacity, this paper divides the  $k$ th period into several small periods represented by  $k_m, m = 1, 2, 3, \dots, q_{k,l}$ ; Assuming that instrumental variable is  $x_{ik_m}, x_{ik_m} = 1$  represents landing time of flight  $F_i$  is within period  $k_m$ , or  $x_{ik_m} = 0$ .
- 6) Instrumental variable  $y_{k,q_{k,l}}, y_{k,q_{k,l}} = 1$  represents that the capacity of destination airport  $Z$  within the  $k$ th period is  $q_{k,l}$ , or  $y_{k,q_{k,l}} = 0$ .
- 7) Assuming that capacity of destination airport is  $q_l, \alpha_{k,q_l}$  represents the number of ground-holding flights within  $k$ th period and  $\beta_{k,q_l}$  represents the number of air-holding flights within  $k$ th period.
- 8)  $c_{gw}$  represents the unit time ground-holding cost of the  $w$ th type of aircraft ;  
 $c_{aw}$  represents the unit time air-holding cost of the  $w$ th type of aircraft,  
 $w = 1, 2, 3, \dots, W$ .
- 9) At moment  $k$ , the set of air-holding flights is  $A_{k,q_l}$  and the set of ground-holding flights is  $G_{k,q_l}$ .
- 10) Assuming that flights do not alternate at the destination airport, so all the flights can land on the airport when capacity meets the requirements.

## 2.2 Mathematical model

$$\min z = \sum_{k=1}^K \left[ \sum_{q_{k,l} \in Q} P_{q_{k,l}} \left( \sum_{m=1}^{q_{k,l}} \sum_{F_i \in G_{k,q_l}} c_g t_i x_{ik_m} + \sum_{m=1}^{q_{k,l}} \sum_{F_i \in A_{k,q_l}} c_a t_i x_{ik_m} \right) \right] \quad (1)$$

s.t

$$t_i = \min\left\{\sum_{k=f_i+1}^j T_k x_{ik_m}, d_i\right\} \quad r_i > p_i \quad (2)$$

$$t_i = 0 \quad r_i \leq p_i \quad (3)$$

$$\alpha_{k,q_l} + \beta_{k,q_l} = n - \sum_{q_{k,l} \in Q} \sum_{j=1}^{k-1} P_{q_{k,l}} q_{j,l} \quad (4)$$

$$\alpha_{k,q_l} + \beta_{k,q_l} = \alpha_{k-1,q_l} + \beta_{k-1,q_l} - \sum_{q_{k-1,l} \in Q} P_{q_{k,l}} q_{k-1,l} \quad (5)$$

$$\sum_{q_{k,l} \in Q} P_{q_{k,l}} = 1 \quad (6)$$

$$\sum_{i \in Q} x_{ik_m} = 1 \quad (7)$$

Objective function (1) represents the minimum cost of all delay holding. Formula (2) (3) are to determine flight holding time. Formula (4) represents that within the  $k$ th period, number of ground and air holding flights is equal to the number of total planned arrival flights within  $[T, T + \Delta t]$  subtracts the number of flights which have landed at the airport before the  $k$ th period (not including  $k$ th). Formula (5) represents that within the  $k$ th period, number of ground and air holding flights is equal to the number of ground and air-holding flights within  $(k-1)$ th period subtracts the number of flights have landed at the airport within the  $(k-1)$ th period. Formula (6) represents any time, the sum of probability of sample capacity is 1 at the destination airport  $Z$ . Formula (7) represents that the plane must land at the airport.

### 3 Simulation

Table 1 represents a processed airport flight schedule from 10 a.m.to 11. When the plane is holding, the allowed periods for light aircrafts holding are less than three, the allowed periods for medium-sized aircrafts holding are less than four, the allowed periods for heavy aircrafts holding are less than five. The holding cost of planes changes with different models. To facilitate simulating and computing, the data of cost in one hour after processing is stated as following: ground-holding cost is 1.5 and air-holding cost is 3 for heavy aircrafts; ground-holding cost is 1 and air-holding cost is 2 for medium-sized aircraft; ground-holding cost is 0.8 and air-holding cost is 1.6 for light aircraft. Airport capacity changes by the influence of weather, so airport capacity refreshes every 10 minutes. Airport capacity set  $Q = \{3,4,5,6\}$  and corresponding probability set  $P = \{0.4,0.3,0.2,0.1\}$ .

Table 1: The Plan of Flight

No.	Expected landing time	Model	No.	Expected landing time	Model
1	10:00	H	18	10:30	L
2	10:00	M	19	10:30	M
3	10:00	M	20	10:35	L
4	10:05	L	21	10:35	M
5	10:05	M	22	10:35	L
6	10:05	H	23	10:40	M
7	10:10	L	24	10:40	H

8	10:10	M	25	10:45	M
9	10:10	H	26	10:45	H
10	10:15	H	27	10:45	M
11	10:15	M	28	10:50	L
12	10:20	M	29	10:50	M
13	10:20	M	30	10:55	M
14	10:25	M	31	10:55	M
15	10:25	L	32	11:00	M
16	10:25	M	33	11:00	H
17	10:30	M	34	11:00	L

Notes: H, heavy aircraft      M, medium-sized aircraft      L, light aircraft

Due to so many variables, it is difficult to get the results directly, so this paper uses genetic algorithm to reach the result through Matlab simulation, the basic idea of simulation is given in Figure 1.

The result of simulation is shown as Figure 2. Through Figure 2, we can see that the Ground –holding strategy can effectively saving the total cost of flight operation. In the example above, using the model established in this paper and the genetic algorithm, the flight landing sequence is shown in Table 2. Following the rule of first come first service strategy, the delay cost is 25.2, but after optimizing used the model established in this paper the cost is decreased to 12.1. That is to say, total cost of flight is reduced 48%. The simulation result shows that the model established is effective.

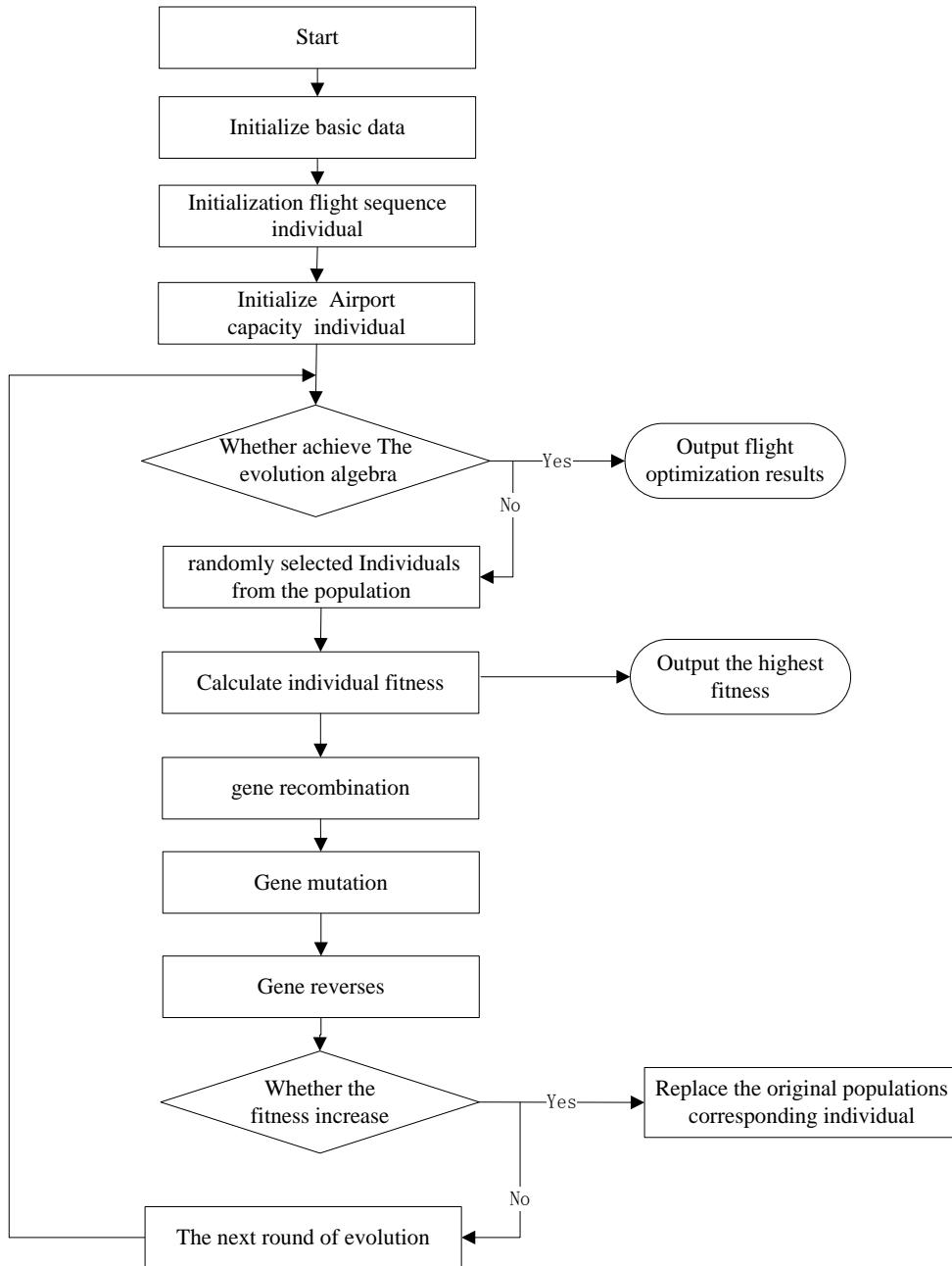


Figure 1: The flowchart of genetic algorithm for optimal solution



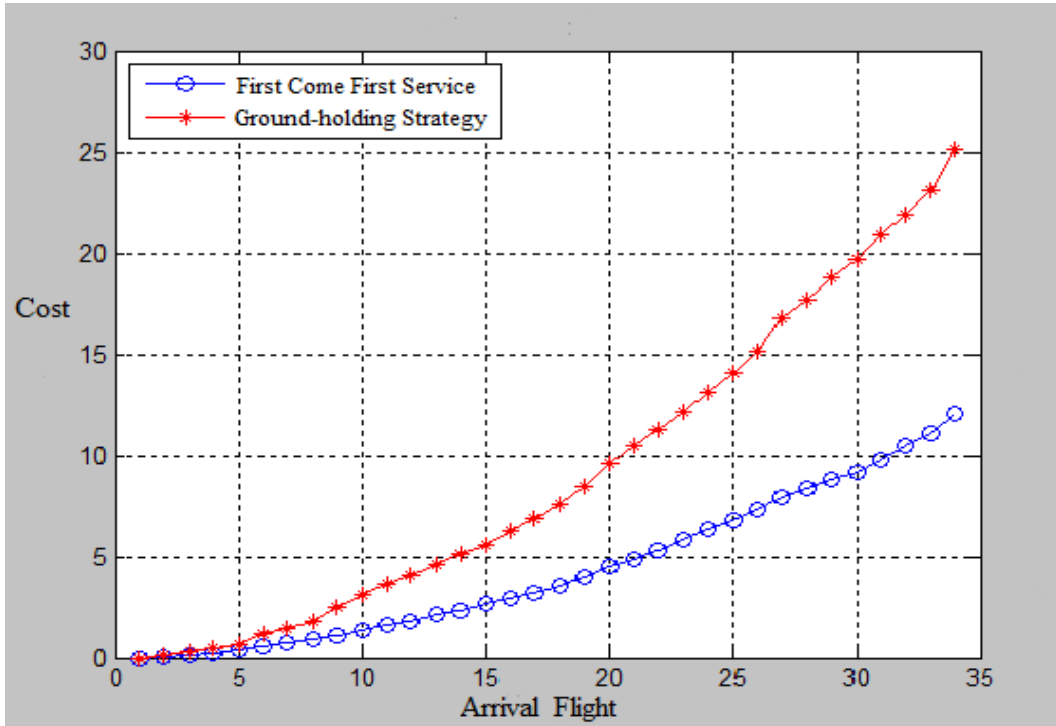


Figure 2: The simulation result of Ground-holding strategy

Table 2: Ground-holding strategy and delay cost

Type	Landing sequence	Cost
First Come First Service	1→2→3→4→5→6→7→8→9→10→11→12→13→14	25.2
	→15→16→17→18→19→20→21→22→23→24→25	
	→26→27→28→29→30→31→32→33→34	
Ground-holding Strategy	1→2→3→5→6→9→4→10→8→7→11→12→13→14	12.1
	→16→20→17→15→19→18→24→27→21→23→22	
	→26→25→31→33→32→28→29→30→34	

## 4 Conclusion

Ground-holding can solve the airspace congestion effectively and save airline operation cost, so more and more attention is focused on ground-holding strategy. This paper establishes single airport ground-holding stochastic model considering destination airport capacity stochastic changes and the flight model factor synthetically, and finally the model is verified by simulation. The model is extended to multiple destination airport ground-holding stochastic models easily.

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## References

- [1] A.R. Odoni, *The Flow Management Problem in Air Traffic Control*, Flow Control of Congested Networks, Springer Berlin Heidelberg, 1987.
- [2] M. Terrab and A.R. Odoni, Strategic flow management for air traffic control, *Operation Research*, **41**(1), (1993), 138-152.
- [3] G. Andreatta and G. Romanin-Jacur, Aircraft flow management under congestion, *Transportation Science*, **21**(4), (1987), 249-253.
- [4] M.O. Ball, Robert Hoffman and A.R. Odoni, A Stochastic Integer Program with Dual Network Structure and Its Application to the Ground Holding Problem, *Operation Research* , **51**(1), 2003, 167-171.

- [5] A. Mukherjee and M. Hansen, A dynamic stochastic model for the single airport ground holding problem. *Transportation Science*, **41**(4), (2007), 444-456.
- [6] Wang Laijun and Shi Zhongke, The Study of Optimal Modeling for Solving Demand-Stochastic GHP, *Computer Engineering and Applications*, **34**,(2004), 34-35.
- [7] Wang Lili and Shi Zhongke, To Solve Single Airports Ground-holding Problem with Genetic Algorithm, *Journal of System Simulation*, **18**(4), (2006), 894-896.
- [8] Guo Nan, Li Zhishu and Song Zhuoyang, Research and design of ground-delay policy model based on discrete event system, *Computer Applications*, **27**(11), (2007), 2626-2629.
- [9] Wang Lili and Zhang Zhaoning, A New Optimal Algorithm for the Single Airport Ground-Holding Problem, *Flight Dynamics*, **23**(2), (2005), 85-88.
- [10] Zhang Zhaoning and Lu Fei, Single Airport Ground-holding Strategy Model Solved with Vogel Method, *Journal of Transportation Engineering and Information*, **7**(4), (2009), 1-4.
- [11] Wang Fei, Xu Xiaohao and Zhang Jing, Single Airport Ground-holding Problem Optimizing Strategy Based on Artificial Fish School Algorithm, *Journal of Nanjing University of Aeronautics & Astronautics*, **41**(1), (2009), 116-120.
- [12] Wang Fei and Xu Xiaohao, Mixed clustering algorithm of airport capacity in stochastic GHP model, *Journal of Traffic and Transportation Engineering*, **11**(1), (2011), 64-68.