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Algorithm for Lease Terms, Cost and Profit

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Abstract

Most elements of the weighted average cost of capital are easy to compute. Unlike bonds, mortgages and bank loans, the cost of lease capital is never stated. Leases vary widely in application fees, down payments, deposits, prepayments and length all of which make it difficult to specify the cost of lease capital in a consistent manner. Such terms also make it difficult to compare leasing to other forms of financing. Lessors have a similar problem. The return on capital invested in lease assets is difficult to calculate. A lease with a lower monthly payment may provide greater returns than one with higher payments if terms are properly specified. A problem that both lessees and lessors have is that the time value of money functions used to compute the lease cost of capital give rise to non-linear equations. Solution of those equations is beyond the skill of most finance and accounting practitioners. This article provides a standardized framework for specifying lease terms and an algorithm for solving the resulting non-linear equations. This algorithm can be implemented using common spreadsheet software.

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1 Introduction

A significant portion of new capital expenditures are financed by leases rather than through bank loans or equity. The Equipment Leasing & Finance

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Association estimates there will be $120 billion in new equipment leases in 2018 [1]. The issue addressed in this paper is how to compute the cost of capital for financial leases. Financial leases are a substitute for loans and have minimal or no buyouts at lease end.

Most elements of the weighted average cost of capital are easy to compute. Unlike bonds, mortgages and bank loans, the cost of lease capital is never stated. Leases vary widely in application fees, down payments, deposits, prepayments and length which makes it difficult for lessees to specify the cost of lease capital in a consistent manner. The time value of money functions used in lease computations require solution of non-linear equations beyond the skill of most finance and accounting practitioners. Therefore, it is nearly impossible to compare leasing to other forms of financing.

Lessors have a similar problem. The return on lease investments is difficult to calculate. Unsophisticated lessees may opt for the lease with the lowest monthly payment. However, lower monthly payments need not result in lower returns for the lessor. A lease with a lower monthly payment may provide greater returns than one with a higher payment if terms are properly specified.

This article provides a standardized framework for specifying lease terms and an algorithm for solving the resulting non-linear equations. This algorithm can be implemented using common spreadsheet software.

The issue of tax treatment will be set aside. Every firm has a different tax strategy. That strategy can be applied after the pre-tax cost of lease capital is determined. To simplify the discussion, the pre-tax cost of lease capital will simply be called the lease cost of capital.

2 Standardized Lease Analysis Framework

One of the reasons the cost of lease capital is hard to compute is that lease terms vary so widely. Application fees, down payments, deposits, prepayments and length vary from company to company and time to time. The first step in standardizing leases is to ask how much capital the lessor is providing. This may seem like a trivial question, but it is not.

Suppose a company wants to lease a million dollars' worth of assets. Is the lessor going to provide a million dollars of capital? Not likely. Lessors may demand application fees, deposits, down payments and prepayment of one or more lease payments all of which reduce the amount of capital the lessor must provide. If a lessor takes a deposit, it will be in current dollars. When the deposit is returned, it will be in less valuable future dollars. The value of what will be returned must be discounted to present value at the lease cost of capital.

So, the first step is to determine how much capital the lessor is providing as shown in equation (1).

CP = Cost of Asset – Application Fee –Down Payment –Number of Prepayments x Monthly Payment –Deposit (1)

The present value of lease payments less the returned deposit must add up to the capital provided (CP) or the lessor will not sign the lease. To do otherwise would be foolish. The present value of lease payments may be computed using equation (2) where CP is the capital provided, Payment is the monthly payment, PVIFA is the Present Value Interest Factor of an Annuity function. LCC is the lease cost of capital, n is the lease term in months, and m is the number of prepayments.

At the end of the lease, the lessor must return the deposit. PVIF is the Present Value Interest Factor function. It discounts the returned deposit to present dollars using the discount rate LCC and the number of lease periods n.

CP = Payment x PVIFA (LCC, n-m) –Deposit x PVIF (LCC, n) (2)

The lease cost of capital, LCC, is also the lessor's yield on invested capital.

3 Solution Methodology

The capital provided by the lessor (CP) can be computed from the cost of the asset, application fee, down payment, number of prepayments, monthly payment, and deposit. This data should be available from the lease contract.

The number of periods over which the lease runs, n, should be available from the lease contract. The lease cost of capital is the discount rate which balances equation (2). Subtract CP from both sides of equation (2) giving equation (3).

0 = Payment x PVIFA (LCC, n-m) –Deposit x PVIF (LCC, n) –CP (3)

Consider a thought experiment in which the zero on the left side of equation (3) is replaced by a variable Y1 and LCC is replaced by a trial discount rate LCC1, as shown in equation (4).

Y1 = Payment x PVIFA (LCC1, n-m) –Deposit x PVIF (LCC1, n) –CP (4)

If we select a trial discount rate (LCC1) which is exactly right, Y1 will be zero. But initially, we don't know what that value is.

It is important to distinguish between the period lease cost of capital LCC1, and the annual lease cost of capital ACC1. Since lease payments are made monthly, the LCC1 is one twelfth the value of ACC1. Suppose we select an extremely high annual percentage rate for ACC1, say something greater than 100% per year. Substituting the resulting LCC1 (ACC1/12) into equation (4) the present value of the monthly payment stream and deposit would tend toward zero. Since CP is fixed and would not be affected by the discount rate, it would tend to dominate equation (4) and drive Y1 negative.

Now suppose the lease cost of capital was gradually reduced. The present value of the monthly payments and the deposit would grow. The present value of the lease payments will dominate the discounted deposit because the initial lease payments would be very lightly discounted as compared to the deposit which is discounted through the end of the lease. As a practical matter, if a lessor demanded too large a deposit, there would be no point in using a lease; a company would simply purchase the asset.

As the lease cost of capital is reduced, Y1 would eventually reach zero. When Y1 is zero, we will have discovered the lease cost of capital.

Let us continue the thought experiment by further reducing the lease cost of capital, but let's call this new lease cost of capital LCC2 which gives rise to a variable on the left of the equation called Y2. The present value of the monthly lease payments would tend to increase, as would the present value of the deposit. The present value of the monthly lease payments would still dominate the present value of the deposit. Eventually, Y2 will go positive.

Suppose a graph were created which plotted the LCC on the x-axis and Y on the y-axis as shown in Figure 1 Increasing Lease Cost of Capital.

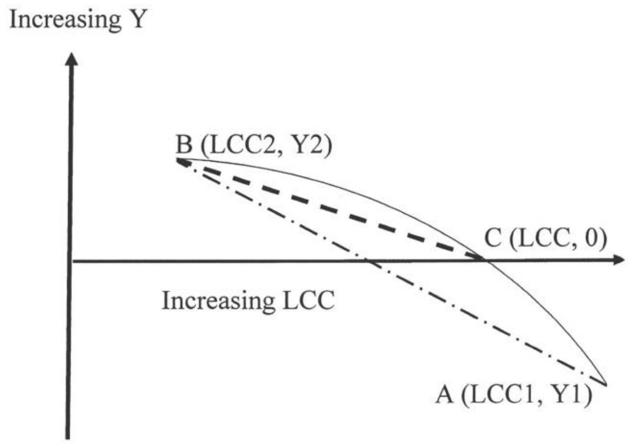

Figure 1: Increasing Lease Cost of Capital

A brute force method of solving equation (4) would be to write a computer program to increment LCC in small steps until Y1 equals zero. This is an inelegant method which requires writing customized software. There is at least one alternative method of arriving at a solution.

In Figure 1, point A represents a solution to equation (4) in which the estimated lease cost of capital, LCC1 is greater than necessary to reach zero. The coordinates of A are (LCC1, Y1). Point B represents the solution to equation (4) where the estimated lease cost of capital is too small to reach zero. The coordinates of point B are (LCC2, Y2). C is the point which drives Y to zero and the lease cost of capital is found at that point. The coordinates of C are (LCC, 0).

This is where calculus allows us to make something out of nothing. The slope of the line AB is almost the same slope as the line CB. As A converges to C, the slopes of the lines also converge. The general formula for slope shown in equation (5)

$$\text{Slope} = (y2 - y1) / (x2 - x1) \quad (5)$$

Applied to Figure 1, the slope of AB is given by equation (6)

$$\text{Slope} = (Y2 - Y1) / (LCC2 - LCC1) \quad (6)$$

Applied to Figure 1, the slope of CB is given by equation (7)

$$\text{Slope} = (Y2 - 0) / (LCC2 - LCC) \quad (7)$$

Since the slope of AB and CB is almost the same we can set the slope equations equal to each other as shown in equation (8).

$$(Y2 - Y1) / (LCC2 - LCC1) = (Y2 - 0) / (LCC2 - LCC) \quad (8)$$

Use equation (4) to compute a Y1. First select an annual percentage rate, ACC1, which is higher than any reasonable lease cost of capital. Divide that by twelve to get a period lease cost of capital LCC1. An appropriately high LCC1 will result in a Y1 which is less than zero. Next select an ACC2 that is lower than the lowest likely lease cost of capital. Divide ACC2 by twelve to get the period discount rate LCC2. Use equation (4) to compute Y2.

There is only one unknown in equation (8) and that is the lease cost of capital, LCC. We know LCC1 and LCC2 because we selected them. We know Y1 and Y2 because we computed them. Cross multiplying yields equation (9).

$$(Y2 - Y1)(LCC2 - LCC) = (Y2 - 0)(LCC2 - LCC1) \quad (9)$$

Dividing both sides by (Y2 – Y1) yields equation (10)

$$(LCC2 - LCC) = (Y2 - 0)(LCC2 - LCC1) / (Y2 - Y1) \quad (10)$$

Subtracting LCC2 from both sides and multiplying both sides by -1 yields equation (11).

$$LCC = -[(Y2 - 0)(LCC2 - LCC1) / (Y2 - Y1)] + LCC2 \quad (11)$$

In our thought experiment, we stipulated that the slope of AB is almost, but not exactly, the same as the slope of CB. That means the equation (11) provides a rough estimate of the lease cost of capital. As point A converges to point C the slopes will, in the limit, converge. At that point there is a perfect solution to the lease cost of capital. Perfect knowledge about the cost of capital is rarely needed. For example, does it make a difference whether a lease cost of capital is 14.4% as compared to 14.6%? Probably not. Nevertheless, this paper proposes a method to refine the lease cost of capital to any desired precision, and it provides a means of testing that precision.

4 Application

Suppose, a million dollars' worth of assets is leased for five years; the application fee is $25,000; the down payment is 10%; one lease payment is due at lease inception; and the deposit is 15%. Five years is 60 months so n is 60. The monthly lease payment will be explicitly stated in the lease agreement. Assume, for this example, the monthly lease payment is \$22,244.45. This is the monthly payment that would have to be made on a five-year lease if there were no application fee, down payment, deposit or prepayment and the annual percentage rate was 12%.

Using equation (1) we can compute the capital provided by the lessor.

CP = Cost of Asset – Application Fee –Down Payment (1)

–Number of Prepayments x Monthly Payment –Deposit

CP = \$1,000,000 –\$25,000 –10% x \$1,000,000

– 1 x \$22,244.45 –15% x \$1,000,000

=\$1,000,000 –\$25,000 –\$100,000 –\$22,244.45 –\$150,000

=\$702,755.55

Equation (4) contains two functions, PVIFA (LCC1, n-m), the Present Value Interest Factor for an Annuity, and PVIF (LCC1, n), the Present Value Interest Factor of a single payment to be received some time in the future.

The function PVIFA (LCC1, n-m) expands to equation (12)

PVIFA (LCC1, n-m) = 1/LCC1 –[(1/LCC1) x (1/(1+LCC2) $^{n\text{-}m}$)] (12)

The function PVIF (LCC1, n) expands to equation (13)

PVIF (LCC1, n) = 1 / (1+LCC1) ^{n} (13)

Compute Y1 using equation (4). Begin with an annual discount rate higher than any reasonable lease cost of capital. Suppose 30% per year is selected as ACC1. That would make LCC1 2.5% (30%/12).

Y1 = Payment x PVIFA (LCC1, n-m) –Deposit x PV (LCC1, n) –CP (4)

Compute the present value of the lease payments using equation (12), given a lease payment of \$22,244.45, a period discount rate of 2.5% and a number of periods, n-m of 59 (60 -1) as shown in equation (14).

Payment x PVIFA (2.5%, 59) = \$22,244.45 x [1/2.5% – (1/2.5%) x (1 / (1 +2.5%)^{59})]   (14)

= \$22,244.45 x (40 – [40 x (1/4.292)]

= \$22,244.45 x (40 – 9.320)

= \$22,244.45 x 30.680

= \$682,459.73

Compute the present value of the deposit using equation (13), the period discount rate of 2.5%, and the number of periods n of 60 as shown in equation (15). The deposit will not be returned early even though one lease payment was made at the inception of the lease.

Deposit x PVIF (2.5%, 60) = \$150,000 x (1 / (1 +2.5%)^{60})   (15)

= \$150,000 x (1 / 4.400)

= \$34,090.91

With values computed in equations (14), (15) and (1) we can use equation (4) to compute Y1 as shown in equation (16).

Y1 = \$682,459.73 – \$34,090.91 – \$702,755.55   (16)

= –\$54,368.73

We now compute a value for Y2 using an annualized lease cost of capital that is lower than any likely lease cost of capital. The algorithm presented in this paper is robust in the sense that if the actual lease cost of capital is higher or lower than initial values for ACC1 or ACC2 respectively, it will correct itself. Suppose we select the Prime Rate plus two points as a starting point for the lowest likely lease cost of capital. The Prime Rate on September 8, 2017 was 4.25%, which would make ACC2 6.25% (4.25% +2%). The period interest rate, LCC2 would be 0.521% (6.25% / 12). Following the methodology laid out in equations (12) through (16) we find the present value of least payments is \$1,127,429.35 and the present value of the deposit is \$109,831.38. The capital provided by the lessor of \$702,755.55 is unchanged. Using equation (4) we can compute Y2 as shown in equation (17).

Y2 = \$1,127,429.35 – \$109,831.38 – \$702,755.55   (17)

= \$314,842.42

We now have the information necessary to use equation (11) and get a first estimate of LCC, the lease cost of capital.

LCC = – [(Y2 –0) x (LCC2 – LCC1)] / [(Y2 – Y1)] +LCC2 (11)

= – [($314,842.42–0) x (0.521% –2.5%)] / [($314,842.42 – (–$54,368.73)] +0.521%

= – [($314,842.42 x –1.979%)] / [$369,211.15] +.521%

= – [–$6,230.73 / $369,211.15] +.521%

= – [–1.688%] +.521%

= 2.209%

Since equation (11) provides the period interest rate and there are twelve periods in a year, the annual cost of capital for this lease is 26.508% (12 x 2.209%).

This is a first estimate of the lease cost of capital. Since this is an estimate, it should be tested for accuracy. This can be done by plugging the estimated lease cost of capital into equation (4). The closer it comes to zero, the more accurate it is. However, let us first modify equation (4) so that Y1 is called EE, the Estimated Error as shown in equation (19).

EE = Payment x PVIFA (LCC, n-m) –Deposit x PVIF (LCC, n) –CP (19)

Using equations (12) through (16) and the period interest rate of 2.209% we find the present value of the monthly lease payments is $729,620.58 and the present value of the deposit is $40,443.03. The capital provided of $702,755.55 remains unchanged.

=$729,620.58 –$40,443.03 –$702,755.55

= –$13,578.00

A perfect solution would be a period lease cost of capital that drives EE to zero. The result computed above is not the close-to-zero answer that would mean the estimate of the lease cost of capital is precise. However, it does provide useful information. Since it is negative, that means that the present value of monthly lease payments has been over discounted. So, 26.508% provides an upper limit on the lease cost of capital.

The quality, Q, of the estimate is the absolute value of EE divided by the capital provided as shown in equation (20).

$$Q = EE / CP \quad (20)$$

$$= –\$13,578.00 / \$702,755.55$$

$$= –.01932 \text{ or about } 1.932\%$$

Can this estimate be improved? Yes. The more our initial selections of ACC1 and ACC2 diverge from the actual lease cost of capital ACC, the greater the error. On the other hand, the closer ACC1 and ACC2 are to the perfect answer, the smaller the error. Suppose we use our first estimate of the lease cost of capital to compute a new Y1. This new estimate for Y1 is the same as the estimate of the error, EE computed in equation (19).

Suppose we select a new ACC2 which is 2% less than our first estimate of ACC. The new ACC2 would be 24.508% (26.508% -2.0%). The period cost of capital, LCC2, would be 2.042% (24.508% / 12). Following the methodology laid out in equations (12) through (16) we find the present value of least payments is $758,830.23 and the present value of the deposit is $44,603.54. The capital provided by the lessor of $702,755.55 is unchanged. Using equation (4) we can compute Y2 as shown in equation (18).

$$Y2 = \$758,830.23 – \$44,603.54 – \$702,755.55$$

$$= \$11,471.14$$

Apply this data to equation (11) and compute a second estimate of the lease cost of capital.

$$LCC = – [[(Y2 – 0) \times (LCC2 – LCC1) / (Y2 – Y1)]] + LCC2 \quad (11)$$

$$= – [[(\$11,471.14 - 0) \times (2.042\% – 2.209\%)] / [(\$11,471.14 – (–\$13,578.00)]] + 2.042\%$$

$$= – [[(\$11,471.14 \times –.167\%)] / [\$25,049.14]] + 2.042\%$$

$$= – [\$19.157 / \$25,049.14] + 2.042\%$$

$$= – [–.076\%] + 2.042\%$$

$$= 2.118\%$$

This equates to an annualized lease cost of capital of 25.416% (2.118% x 12). This second estimate can be tested using equations (12) through (16). We find the present value of the monthly lease payments is $745,239.22 and the present value of the deposit is $42,646.78. The capital provided of $702,755.55 remains the same.

EE = \$745,239.22 –\$42,646.78–\$702,755.55

= –\$163.11

Using equation (20) we find the quality of this estimate, Q, is

$$Q = EE / CP \tag{20}$$

= –\$163.11 / \$702,755.55

= –.00023 or about 0.023%

In the unlikely event greater precision is required, the second estimate of the lease cost of capital 25.416% would become the new ACC1 giving rise to a period lease cost of capital LCC1 of 2.118%. This would provide an upper bound on the lease cost of capital. A new lower bound estimate of the lease cost of capital, ACC2, could be found by subtracting 0.5% from ACC1 giving a new annualized rate of 24.916% (25.416% -.5%). A new Y2 would be computed and a new estimate of LCC could be computed and tested.

Simulations show that the difference between the second and third estimate of the least cost of capital is on the order of a few hundredths of a percent. For the example discussed above, the second estimate of the cost of capital is 25.419% and the third estimate is 25.406%. The Q for the second estimate is 0.023% and the Q for the third estimate is 0.000%. Q equals zero is a perfect solution to the lease cost of capital.

5 Critique

The advantages of this methodology are that it does not resort to mathematics which is beyond the ability of most practitioners and one can derive as precise an answer as desired by adding more iterations to the process.

Arguably the disadvantage is that it requires a large number of computations. However, most practitioners will only do these computations once or twice to familiarize themselves with the equations. After that, they will probably use a spreadsheet to do computations.

Constructing a well-designed spreadsheet should not take more than a couple of hours. The first section of the spreadsheet should have well-labeled variable inputs. The second section should contain the lease cost of capital, EE and Q value. The third section of the spreadsheet should contain calculations for the first estimate, second and third estimates.

6 Impact of Variables

With so many variables in play, it is sometimes hard to visualize how each of them impacts the lease cost of capital. It is also difficult to visualize how they can be used by the lessor to increase the yield on invested capital without making monthly payments so high as to lose a competitive advantage as compared to other lessors.

Table 1 Five Year Lease, analyzes the impact of variables on a five-year, one-million-dollar capital lease. The monthly lease payments for this lease are $22,244.45. Table 2 Ten Year Lease, analyzes the impact of variables on a ten-year, one-million-dollar capital lease. The monthly lease payments for this lease are $14,347.09.

Table 1: Five Year Lease

This table analyzes a five-year capital lease with monthly lease payments of $22,244.45. This would be the monthly payment for a lease with no application fee, down payment, deposit or prepayment at a 12% cost of capital.

Example	Application Fee	Down Payment	Deposit	Prepayments	Lease Cost of Capital
1	$0	$0	$0	0	12.0%
2	$5,000	$0	$0	0	12.2%
3	$25,000	$0	$0	0	13.1%
3	$0	$50,000	$0	0	14.3%
4	$0	$100,000	$0	0	16.7%
6	$0	$0	$75,000	0	13.7%
7	$0	$0	$150,000	0	15.8%
8	$0	$0	$0	1	12.5%
9	$0	$0	$0	2	12.9%
10	$5,000	$50,000	$75,000	1	17.3%
11	$25,000	$100,000	$150,000	1	25.4%
12	$25,000	$100,000	$150,000	2	26.7%

As can be seen from these examples, application fees, down payments, deposits and prepayments can increase the cost of capital by as much as 122.5% ((26.7% –12.0%)/12.0%) for a five-year lease. They can increase the cost of capital for a ten-year lease by 75.0% ((21.0% –12.0%)/12.0%). The longer the lease, the more attenuated the impact of the variables discussed. For the example worked out in this paper, Table 1, item 11 has a cost of capital of 25.4%. A ten-year lease with the same application fee, down payment, deposit and prepayment has a cost of capital of 20.4%. See Table 2 item 11.

Table 2: Ten Year Lease

This table analyzes a ten-year capital lease with monthly lease payments of $14,347.09. This would be the monthly payment for a lease with no application fee, down payment, deposit or prepayment at a 12% cost of capital.

Example	Application Fee	Down Payment	Deposit	Prepayments	Lease Cost of Capital
1	$0	$0	$0	0	12.0%
2	$5,000	$0	$0	0	12.1%
3	$25,000	$0	$0	0	12.6%
4	$0	$50,000	$0	0	13.3%
5	$0	$100,000	$0	0	14.7%
6	$0	$0	$75,000	0	13.4%
7	$0	$0	$150,000	0	15.2%
8	$0	$0	$0	1	12.3%
9	$0	$0	$0	2	12.5%
10	$5,000	$50,000	$75,000	1	15.4%
11	$25,000	$100,000	$150,000	1	20.4%
12	$25,000	$100,000	$150,000	2	21.0%

7 Conclusion

The ability to compute the lease cost of capital is important so that lessees can compare it to other forms of financing. The lease cost of capital is rarely, if ever, provided by the lessor. Calculation of the lease cost of capital is complicated by the number of terms that can impact cost such as application fees, down payments, deposits and prepayments. Finding the lease cost of capital involves solutions to non-linear, time value of money equations.

Lessors have an interest in such calculations because the cost to the lessee is the return on the lessor's capital. The algorithm discussed in this paper would enable a lessor to offer a lower monthly lease payment than competitors yet realize a higher yield on invested capital.

The algorithm proposed in this paper begins by computing the capital provided by the lessor net of application fees, down payments, prepayments and deposits. It then uses data available to the lessee to estimate the lease cost of capital. That estimate may be refined to any level of precision by adding iterations to the algorithm discussed.

While the calculations may seem complicated, most practitioners will solve them once or twice and then use a spreadsheet program to compute the cost of capital. Someone familiar with spreadsheets should be able to build one in a couple of hours. A well-designed spreadsheet should be able to compute the lease cost of capital for most leases simply by varying the inputs.

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