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Empirical Investigation of MGarch Models

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Abstract

Volatility is a key parameter use in many financial applications, from derivatives valuation to asset management and risk management. Volatility measures the size of the errors made in modelling returns and other financial variables. It was discovered that, for vast classes of models, the average size of volatility is not constant but changes with time and is predictable. With the growth in the requirements of the risk management industry and the complexity of instruments that are used in finance, there has been a signicant growth in the forms of multivariate GARCH models. Multivariate ARCH/GARCH models and dynamic factor models, eventually in a Bayesian framework are the basic tools used to forecast correlations and covariances. For instance, time varying correlations are often estimated with Multivariate Garch models that are linear in squares and cross products of the data. A new class of multivariate models called dynamic conditional correlation (DCC) models proposed have the flexibility of univariate GARCH models coupled with parsimonious parametric models for the correlations. They are not linear but can often be estimated very simply with univariate or two step methods based on the likelihood function.In my paper, the general theoretical framework of GARCH models is presented in estimating the volatility in time series financial econometrics as well as i have investigated the empirical applications of the both models with respect to estimation implications. The two models which were investigated with R package are Engle's DCC MGarch and MGarch BEKK.

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1 Introduction

1.1 What is stock volatility ?

Stock volatility is the conditional standard deviation of stock returns in statistical words. The explanation behind the fact that volatility is important is that it has many applications briefly described as below :

- Option (derivative) pricing
- Risk management, e.g. value at risk (VaR)
- Asset allocation
- Interval forecasts

1.2 Properties of ARCH/GARCH Models

Since our primary interest is modeling changes in variance because the volatility across markets and assets often move together over time, ARCH & GARCH models have many useful applications which include asset pricing models, portfolio selection, hedging, Var and volatility spillover among different assets and markets and modelling the temporary dependence of second moments among variables is challenging in financial econometrics.
Main properties of ARCH/GARCH models are :

- Provides improved estimations of the local variance (volatility)
- Not necessarily concerned with better forecasts
- Can be integrated into ARMA models
- Useful in modeling financial time series

1.3 Autoregressive Conditional Heteroscedasticity

ARCH is invented by Engle (1982) in order to explain the volatility of inflation rates. In a basic ARCH (1) framework, conditional variance of a shock at time t is a function of the squares of past shocks. (Recall, h is the variance and Ɛ is a "shock," "news" or "error". Addingly, since the conditional variance needs to be nonnegative, the conditions have to be met. If $\alpha_1 = 0$, then the conditional variance is constant and is conditionally homoskedastic.A major advantage of an ARCH model is its simplicity as well as it generates *volatility clustering with heavy tails* (high kurtosis). On the other hand, weaknesses can be summarized as being restrictive and providing no satisfactory explanation as it's not sufficiently adaptive in prediction.

1.4 Garch Models

The explanation of GARCH is described as below :

- Generalized—more general than ARCH
- Autoregressive—depends on its past
- Conditional—variance depends on past info
- Heteroscedasticity—non-constant variance.

Because ARCH(p) models are difficult to estimate, and because decay very slowly, Bollerslev (1986) developed the GARCH model. GARCH models are *conditionally heteroskedastic* but have a constant *unconditional variance*. In a GARCH (1,1), the variance (h_t) is a function of an intercept (ω), a shock from the prior period (α) and the variance from the last period (β) :

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

High order Garch models are ;

1.4.1 Linear Garch Variations

a. Integrated GARCH (Engle and Bollerslev, 1986) : Phenomena is similar to integrated series in regular (ARMA-type) time-series. Integrated GARCH occurs when α+β=1. When this is the case, it means that there is a unit root in the conditional variance; past shocks do not dissipate but persist for very long periods of time.

b. GARCH in Mean (Engle, Lilien and Robbins, 1987) : There is a direct relationship between risk and return of an asset. In the mean equation, a function of the conditional variance contained is usually the standard deviation. This allows the mean of a series to depend, at least in part, on the conditional variance of the series.

1.4.2 Non-linear Garch Variations (Dozens in last 20 years)

Linear GARCH models all allow prior shocks to have a symmetric effect on h_t where as non-linear models allow for asymmetric shocks to volatility. Exponential GARCH (1,1) (EGARCH) model is developed by Nelson (1991) :

Conditional Variance :

$$\log(h_t) = \omega + \alpha_1 z_{t-1} + \gamma_1(|z_{t-1}| - \mathrm{E}[|z_{t-1}|]) + \beta_1 \log(h_{t-1})$$

where $z_t = \varepsilon_t / h_t$ and is the standardized residual. γ is the asymmetric component.

1.5 Advantages of GARCH Models Compared to ARCH Models

The main problem with an ARCH model is that it requires a large number of lags to catch the nature of the volatility, this can be problematic as it is difficult to decide how many lags to include and produces a non-parsimonious model where the non-negativity constraint could be failed. The GARCH model is usually much more parsimonious and often a GARCH (1,1) model is sufficient, this is because the GARCH model incorporates much of the information that a much larger ARCH model with large numbers of lags would contain.

2 Multivariate GARCH Models

Since the volatilities across the various markets and assets often move together over time, it becomes worthwhile in financial econometrics to model the temporary dependence of second moments among variables. Thus, we obviously can observe many useful applications such as asset pricing models, portfolio selection, hedging, VaR, and volatility spillover among different assets and markets.

Three approaches of MGarch are :

1. Direct generalization of univariate Garch Model : Exponentially weighted covariance, Diagonal VEC Model, BEKK model

2. Linear combinations of univariate Garch Model : Principal Component Garch Model, Factor Garch Model

3. Nonlinear combinations of univariate Garch Models : Constant Conditional Correlation Model, Dynamic Conditional Correlation Model

2.1 CCC Model Approaches

Bollerslev (1990) : Bollerslev assumed that the conditional correlation matrix is constant over time. It is then desirable to test this assumption by reducing the number of parameters in the estimation of Mgarch models.

Tsay (2000) : Tsay proposed a test for constant correlations.

Bera & Kim (2002) : Bera & Kim developed a test for constancy of the correlation parameters in the CCC model of Bollerslev (1990). It is an information matrix-type test that besides constant correlations examines at the same time various features of the specified model.

2.2 DCC Model

DCC model is an extension of CCC Model. The assumption of Bollerslev's (1990) model that the *conditional correlations* are *constant* may seem *unrealistic* in many empirical applications. In that respect, Tse (2000), Engle and Sheppard (2001) showed that correlations are *not constant over time*. Engle (2002) and Tse and Tsui (2002) propose a *generalization* of Bollerslev's (1990) constant conditional correlation model by making the *conditional correlation matrix time-dependent*.

DCC model calculates a current correlation between variables of interest as a function of past realizations of both the volatility between the variables & the correlations between them.

2.3 DCC MGarch Model

Conditional variance is: $H_t \equiv D_t R_t D_t$ where R_t is the time varying correlation matrix and D_t is estimated from the univariate GARCH model.

The difference between the specification of H_t in DCC model and Bollerslev's (1990) CCC model is that *Correlation*, R_t *is allowed to vary with time so that the dynamic nature of the correlation can be captured.*

In a four market DCC(1,1)-MGARCH(1,1) specification, the elements of the matrix D will take the form :

$$D_t = \begin{bmatrix} \sqrt{h_{11.t}} & 0 & 0 & 0 \\ 0 & \sqrt{h_{22.t}} & 0 & 0 \\ 0 & 0 & \sqrt{h_{33.t}} & 0 \\ 0 & 0 & 0 & \sqrt{h_{44.t}} \end{bmatrix}$$

DCC-MGARCH uses a two-stage estimation procedure: 1-Conventional univariate GARCH parameter estimation for each zero mean series 2-The residuals from the first stage are then standardized and used in the estimation of the correlation parameters in the second stage.The correlation structure is given as $R_1 = Q_t^{*-1} Q_t Q_t^{*-1}$

The covariance structure is specified by a GARCH type process as below: $Q_t = (1-\lambda_1-\mu_1)\overline{Q} + \lambda_1(\eta_{t-1}\eta_{t-1}^{'}) + \mu_1 Q_{t-1}$ where the covariance matrix is of Q_t is calculated as weighted average of $\overline{Q}$ (the unconditional covariance of the standardized residuals) $\eta_{t-1}\eta_{t-1}^{'}$ is the lagged function of the standardized residuals and Q_{t-1} is the past realization of the conditional covariance.

In DCC specification, only the first lagged realization of the covariance of the standardized residuals and the conditional covariance are used. This requires the estimation of two additional parameters, λ_1 and μ_1. Q_t^* is a diagonal matrix whose elements are the square roots of the diagonal elements of Q_t.

Hence, for a four-market specification it would take the form:

$$Q_t^* = \begin{bmatrix} \sqrt{q_{11,t}} & 0 & 0 & 0 \\ 0 & \sqrt{q_{22,t}} & 0 & 0 \\ 0 & 0 & \sqrt{q_{33,t}} & 0 \\ 0 & 0 & 0 & \sqrt{q_{44,t}} \end{bmatrix}$$

The off diagonal elements in the matrix R_t will hence take the form $\rho_{12,t} / \sqrt{q_{11,t} q_{22,t}}$ where $\rho_{12,t}$ is the conditional correlation between market 1 and market 2. If $\overline{Q}$ and $\eta_{t-1}\eta_{t-1}^{'}$ are positive definite and diagonal then Q_1 will also be positive and diagonal.

The log likelihood for the parameter estimation in the second stage is: $L = -\frac{1}{2}\sum_{t=1}^{T}(k\log(2\pi) + 2\log|D_t| + \log|R_t| + \eta_t^{'} R_t^{-1}\eta_t$ where η_t is the standardized residual derived from the first stage univariate GARCH estimation which is assumed to be i.i.d. with a mean zero a variance, R_t; $\eta_t = \varepsilon_t / \sqrt{h_t}$. R_t is also the correlation matrix of the original zero mean returns.

2.4 Advantages of DCC Models over MGarch Models

- The crucial point in MGARCH modeling is to provide a realistic but parsimonious specification of the variance matrix ensuring its positivity (Dilemma between flexibility and parsimony).
- BEKK models are flexible but require too many parameters for multiple time series of more than four elements.
- Diagonal VEC and BEKK models are much more parsimonious but very restrictive for the cross-dynamics(May be sufficient for some applications like asset pricing models).

In the contrast, Factor GARCH models allow the conditional variances and covariances to depend on the past of all variances and covariances, but they imply common persistences in all these elements. DCC models allow for different persistence between variances and correlations, but impose common persistence in the latter. They open the door to handling more than a very small number of series. (extension of the CCC model which is relatively easy to estimate.)

2.5 DCC of Engle

This model is invented by Engle by 2002 as a generalized version of the Constant Conditional Correlation (CCC) model of Bollerslev [1990]. DCC of Engle belongs to a group of multivariate models that can be seen as nonlinear combinations of univariate GARCH models. It is similar to the constant conditional correlation formulation by Bollerslev but where the correlations are allowed to vary over time.

Defining the variance-covariance matrix, H_t, as D_t is a diagonal matrix containing the conditional standard deviations on the leading diagonal and R_t is the conditional correlation matrix. Forcing R_t to be time-invariant would lead to the constant conditional correlation model of Bollerslev(1990). Numerous explicit parameterisations of R_t are possible, including an exponential smoothing approach discussed in Engle(2002). More generally, a model of the MGARCH form could be specified as

$$Q_t = S^{\text{O}}(\iota\iota' - A - B) + A^{\text{O}} u_{t-1} u'_{t-1} + B^{\text{O}} Q_{t-1}$$

Where *S* is the unconditional correlation matrix of the vector of standardized disturbances, $u_t = D_t^{-1}\varepsilon_t$ and $R_t = diag\{Q_t\}^{-1} Q_t diag\{Q_t\}^{-1}$. This specification for the intercept term simplifies estimation and reduces the number of parameters to be estimated but is not necessary. Engle (2002) also proposes a GARCH-esque formulation for dynamically modeling D_t^2.

The model may be estimated in one single stage using maximum likelihood, although this will still be a difficult exercise in the context of large systems. Consequently, Engle advocates a two-stage estimation procedure where each variable in the system is first modelled separately as a univariate GARCH process and then, in a second stage, the conditional likelihood is maximised with respect to any unknown parameters in the correlation matrix. Under some regularity conditions, estimation using this two-step procedure will be consistent but inefficient. Other DCC models are proposed by *Tse and Tsui [2002]* or *Christodoulakis and Satchell [2002]*.

2.6 DCC Model of Tsay&Tsui(2002)

$$R_t = (1-\theta_1-\theta_2)R + \theta_1\Psi_{t-1} + \theta_2 R_{t-1}$$

$$\Psi_{ij,t-1} = \frac{\sum_{m=1}^{M} \in_{i,t-m} \in_{j,t-m}}{\sqrt{\left(\sum_{m=1}^{M} \in_{i,t-m}^2\right)\left(\sum_{j=1}^{M} \in_{j,t-m}^2\right)}}$$

where:

$\theta_1, \theta_2 > 0, \theta_1 + \theta_2 < 1$, *R* is a symmetric *N x N* positive definite matrix with $\rho_{ii} = 1.\Psi_{t-1}$ is the sample correlation matrix for $(a_{t-M}, a_{t-M+1}, ..., a_{t-1})$ and R_t is a weighted average of correlation matrices (R, Ψ_{t-1}, R_{t-1}).

2.7 Drawback of the Both Models

A primary drawback of DCC models is that all conditional correlations follow the same dynamic structure. In addition, the number of parameters to be estimated equals (N +1)(N +4)/2 is large when the N is large (Bauwens et al. 2006). Therefore Engle proposes the estimation of the DCC model by two-step procedure. Finally, if the conditional variances are specified as GARCH(1,1) models then the DCC(Tsay Tsui) and DCC(Engle) models contain (N +1)(N +4)/2 parameters.

3 Empirical Investigation with DCC MGarch & MGarch BEKK Models

3.1 R Package for DCC Garch Model of Engle

In our empirical study based on the DCC Garch Modelling, we firstly obtained the index series of €/USD parity and Dow Jones. Our data consists of the index since the establishment of €/USD. The data range for the variables is **04.01.1999- 10.09.2010** with **2913 observations**.

Elementary Statistics of Index & Return Series :

Date	Eurusd	Dowjones
01.02.1999	***Min. :0.8252***	***Min. :6547***
01.02.2000	1^{st}.Qu.:1.0101	1^{st} Qu.:9779
01.02.2001	***Median:1.2144***	***Median:10483***
01.02.2002	***Mean: 1.1853***	***Mean: 10471***
01.02.2005	3^{rd} Qu.:1.3246	3^{rd} Qu.: 11036
01.02.2006	***Max. :1.5990***	***Max.: 14164***
Std deviation	***0.1954816***	***1354.852***

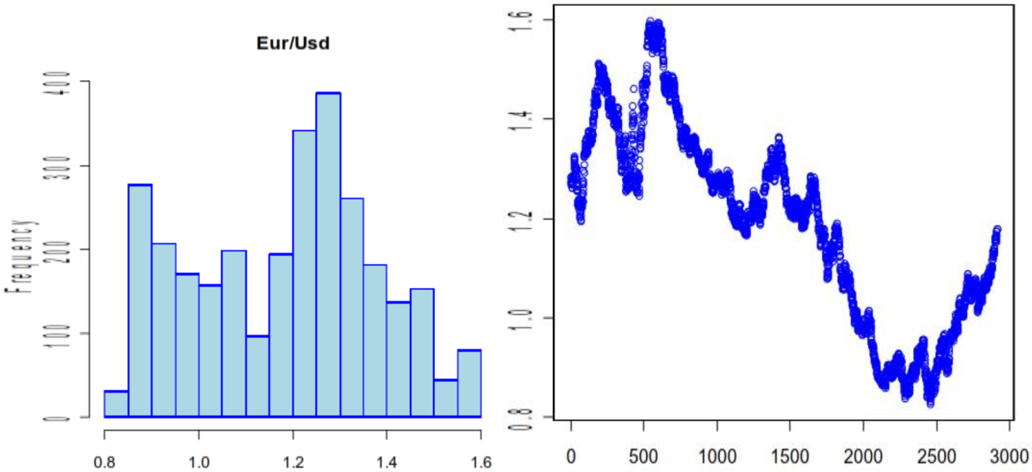

Figure 1-2 : Histogram & Trend, €/USD

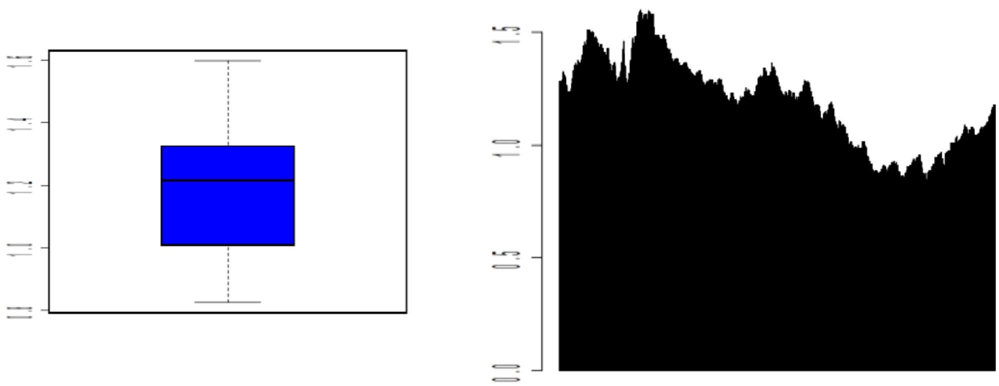

Figure 3-4: Boxplot & Barplot, €/USD

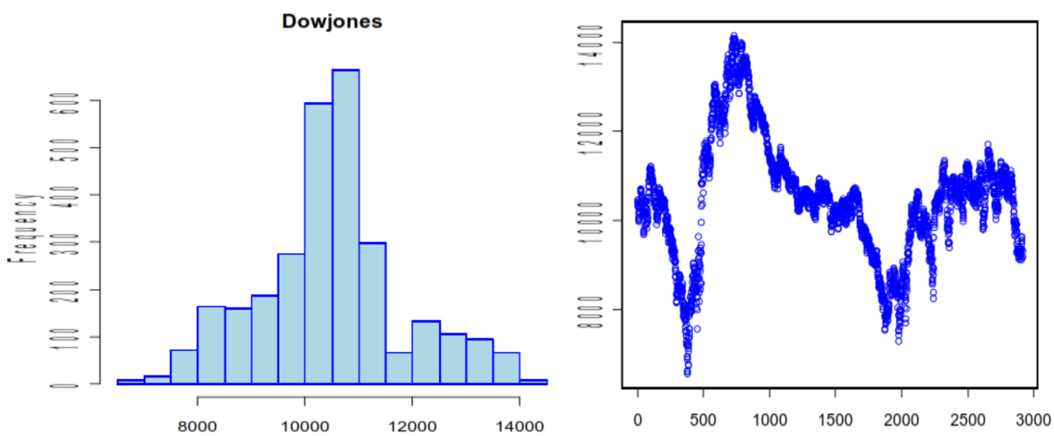

Figure 5-6 : Histogram & Trend, €/USD

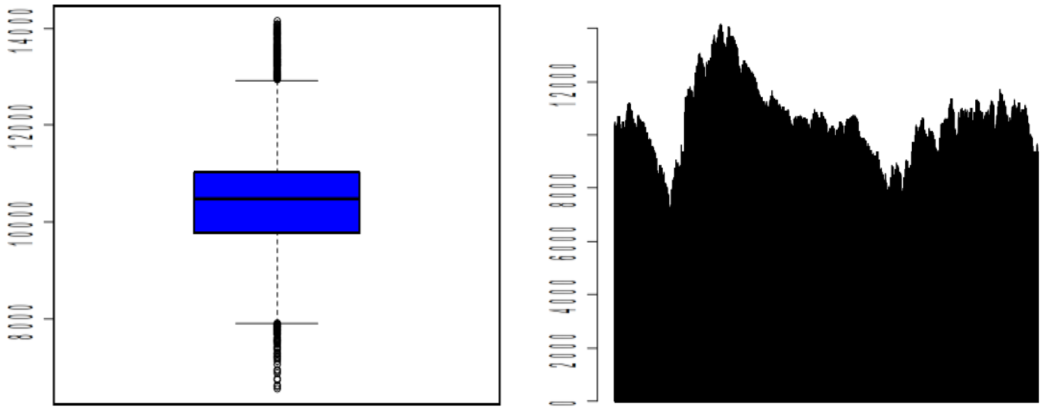

Figure 7-8: Boxplot & Barplot, €/USD

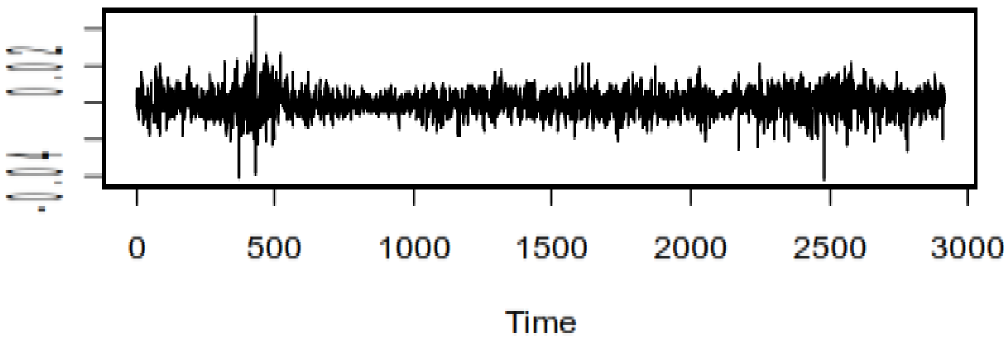

Figure 9: Log Returns, €/US

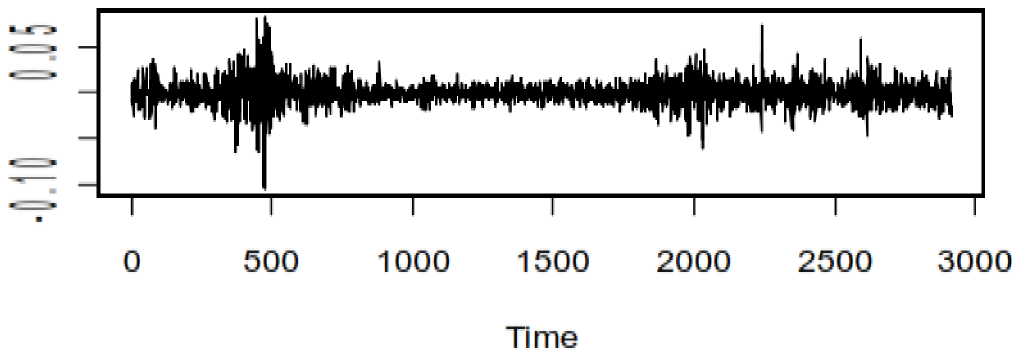

Figure 10: Log Returns, Dow Jones

3.2 Estimation a DCC Garch (1,1) Model

Total time required for DCC Garch (1,1) estimation is 3,252 seconds.

Parameter estimates and their robust standard errors:

	estimates	std.err
a1	0.0033573581	0.0005474677
a2	0.004804514	0.020184682
a3	0.001042704	0.021362648
A11	0.203297911	0.000670264
A22	0.28567690	0.02646125
	estimates	std.err
A33	0.14982199	0.0280499
B11	0.7441735309	0.0002195436
B22	0.6245203	0.0158647
B33	0.80591610	0.02070841
ddc alph	0.010276730	0.002331534
dcc beta	0.981004256	0.005542793

Rounded results are:

	Estimates	std.err
a1	0.003	0.001
a2	0.005	0.020
a3	0.001	0.021
A11	0.203	0.001
A22	0.286	0.026
A33	0.150	0.028
B11	0.744	0.000
B22	0.625	0.016
B33	0.806	0.021
dcc alpha	0.010	0.002
ddc beta	0.981	0.006

Estimation of a DCC-GARCH model by R is performed in two steps. The function dcc.estimation internally calls two other functions, dcc.estimation1 and dcc.estimation2, that carry out the first and second stage optimisation.

3.3 The Results of the First Stage Estimation

$par
[1] 0.05794271 0.06931460 0.03229092 0.45088570 0.53448751 0.38706846 0.86265493
[8] 0.79026595 0.89772830
$value
[1] -2494.872
The results of the second stage estimation
$par
[1] 0.01027673 0.98100426
$value
[1] 3922.696

3.4 The Ljung-Box Test of Autocorrelation

The Ljung-Box (LB) test statistic for serial correlations can be calculated by ljung.box.test. The LB test is often applied to squared residuals to detect evidence for ARCH effects in the time series. When this is the case, the LB test is equivalent to the McLeod and Li (1983) test. However, since Li and Mak (1994) found that the asymptotic null distribution of the McLeod and Li (1983) test statistic is not a $\chi 2$ distribution when the test is applied to the residuals of an estimated GARCH equation, the McLeod and Li (1983) test is not suitable for this purpose.

Returns on Euro/USD

Test	**stat**	**p-value**
Lag 5	5.241644	0.387107090
Lag 10	16.323310	0.090743786
Lag 15	32.000282	0.006437579
Lag 20	37.638751	0.009799530
Lag 25	48.675302	0.003093085
Lag 30	51.519416	0.008577117
Lag 35	55.293454	0.015888967
Lag 40	57.628598	0.035078579
Lag 45	68.643648	0.013136800
Lag 50	78.697371	0.005915012

Returns on Dowjones

test	**stat**	**p-value**
Lag 5	40.44241	1.215955e-07
Lag 10	51.28926	1.544369e-07
Lag 15	78.64875	1.232630e-10
Lag 20	105.57621	1.249477e-13
Lag 25	125.35354	2.491965e-15
Lag 30	129.35159	2.662721e-14
Lag 35	144.80999	2.621617e-15
Lag 40	147.56788	3.082578e-14
Lag 45	176.23872	1.957236e-17
Lag 50	184.23035	2.985796e-17

3.5 The Jarque-Bera Test of Non-normality

We compute standard and robustified skewness measures of a vector or matrix of variables. The LJB test is imple*mented by* jb.test which simultaneously returns test statistics and associated p-values for as many time series as desired.

Eur/Usd Return Series

	series 1
standard	**-0.098334945**
robust	**0.008442489**

Dowjones return series
series 1
standard -0.005726491
robust 0.040548877

In financial econometrics, it is well-known that stock returns exhibit negative skewness and large excess kurtosis which is regarded as evidence for non-normality of stock return distribution. Kim and White (2004), however, found out in the Monte Carlo simulations they conducted that the conventional measures of skewness and kurtosis are extremely sensitive to a small number of outliers, hence propose alternative measures based on quantiles that are robust against the existence of outliers. The functions ''rob.sk'' and ''rob.kr'' return both conventional and robustified measures of skewness and excess kurtosis, respectively.

3.6 Standard and Robustified Skewness Measures of a Vector or Matrix of the Variables :

Eur/Usd Return Series
series 1
standard -0.098334945
robust 0.008442489

Dow Jones return series
series 1
standard -0.005726491
robust 0.040548877

Standard and robustified excess kurtosis

Eur/Usd Return Series
series 1
standard 2.7671388
robust 0.1504004

Dow Jones return series
series 1
standard 7.1429357
robust 0.3070175

Since the difference between the Standard statistics & the robust ones are large, it implies that the conventional measures are affected by a small number of outliers.

The Jarque-Bera test of normality

Eur/Usd Return Series
series 1
test stat 9.337493e+02
p-value 1.733455e-203

Dow Jones return series
series 1

test stat	6190.628
p-value	0.000

3.7 R-Project for the Analysis of Multivariate Garch Models

Harald Schmidbauer, Vehbi Sinan Tunalioglu & Angi Rösch have presented an R package which tries to provide elementary functionality to build a synchronized multivariate time series of daily or weekly returns, on the basis of separate univariate level series, which need not be in sync (i.e., different days may be missing). The main part of the package in which diagnostic tools are also included consists of functions which permit the estimation of MGARCH-BEKK and related models, among them a novel bivariate asymmetric model which is capable of distinguishing between positive and negative returns.

3.7.1 MGarch BEKK Model

The conditional covariance matrix is defined as

$$H_t = C'C + A' \in_{t-1} \in_{t-1}^{'} A + B' H_{t-1} B$$

where:

$$A' \in_{t-1} \in_{t-1}^{'} A = ARCH\ term$$

and

$$B' H_{t-1} B = GARCH\ term$$

A significant advantage of MGarh Bekk Model is that covariance matrix must be positive definite.
With parameter matrices:

$$C = \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix}, A = \begin{pmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{pmatrix} \pi r^2$$

In a bivariate asymmetric quadratic GARCH model, the conditional covariance matrix is defined as

$$H_t = C'C + A' \in_{t-1} \in_{t-1}^{'} A + B' H_{t-1} B + S_w(\in_{t-1}).\Gamma' \in_{t-1} \in_{t-1}^{'} \Gamma$$

with an additional parameter matrix $C = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ 0 & \Gamma_{22} \end{pmatrix}$ and a weight function S_w:

$$S_w(e_1, e_2) = 0.5 - \frac{\cos\left(\frac{\pi}{4} + w\right).e_1 + \sin\left(\frac{\pi}{4} + w\right).e_2}{2\sqrt{e_1^1 + e_2^2}}$$

Both DCC & BEKK models support the hypothesis at lower probabilities of 0.1-5.0% of VaR by assuming a significance level of 5%. Positive definiteness of the conditional covariance matrix is guaranteed in both of these models, and the forecasting accuracy of the two models is equivalent on the basis of empirical excess rates. However, the BEKK model requires estimation of 24 parameters in the case of 3 variates where as DCC model involves 11 parameters.

3.7.2 R Package for MGarch BEKK Model

In an environment of scarce open-source packages for MGarch fitting, mgarchBEKK is able to simulate and estimate bivariate BEKK models, it allows for easy specification of a particular model structure, and helps in the diagnostic check of the fitted model. We applied the MGarch BEKK Model to the same data and initiated the estimation. In the application of the R package contemplated by Schmidbauer , Rösch & Tunalioglu; we obtained the data from separate sources and then combined the data sets of two variables. Return series of the variables are plotted following the combination of the data process :

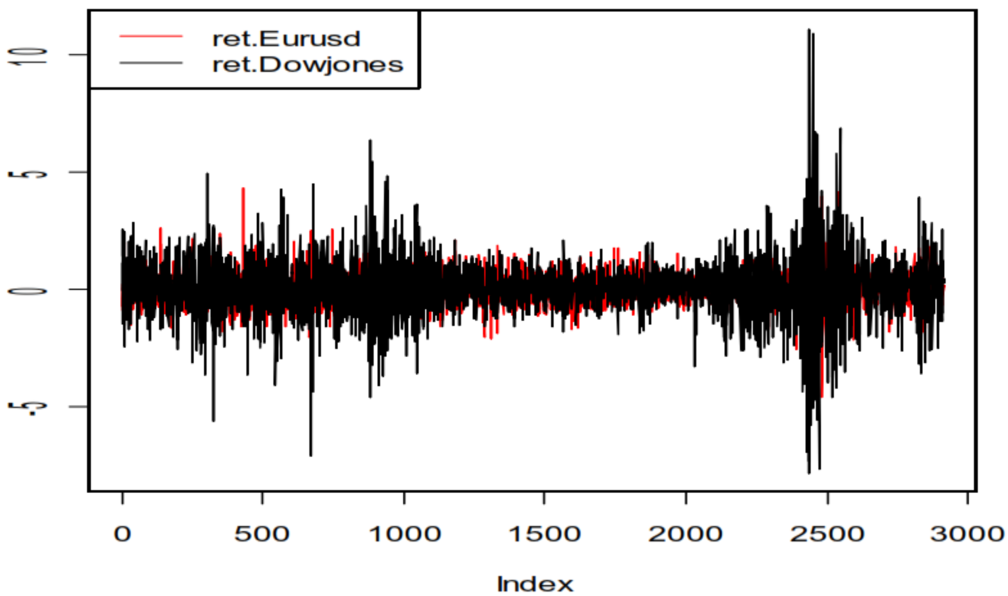

Figure 11 : Return Series

3.7.3 Unit Root Test (ADF) of the Eurusd Return Series

Dickey-Fuller = -13.5778, Lag order = 14, p-value = 0.01
Alternative hypothesis: stationary
Unit Root Test (ADF) of the Dowjones return series:
Dickey-Fuller = -13.5778, Lag order = 14, p-value = 0.01
Alternative hypothesis: stationary
Below are the plotted autocorrelation & partial autocorrelation functions of the Eurusd returns & the squared return series :

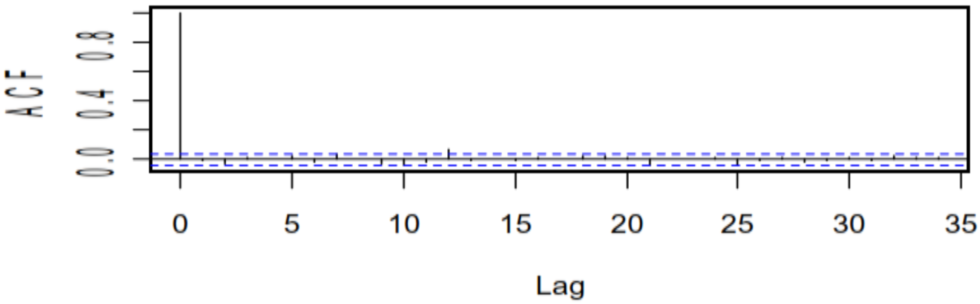

Figure 12 : ACF of Returns, €/USD

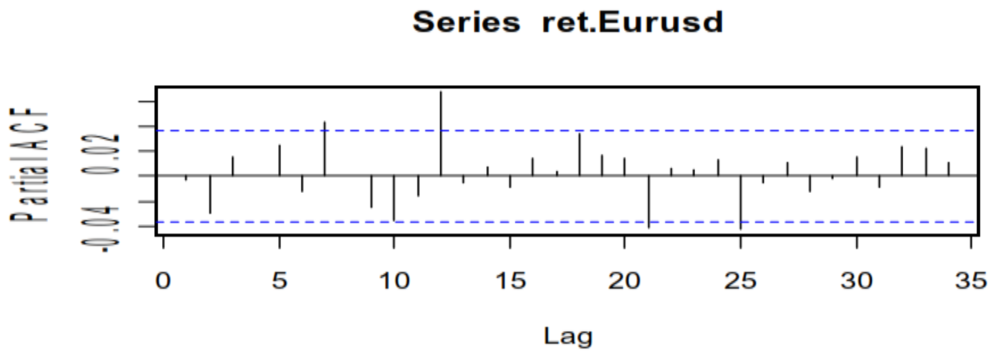

Figure 13 : Partial ACF of Returns, €/USD

The plotted autocorrelation & partial autocorrelation functions of the Dow Jones returns & the squared return series:

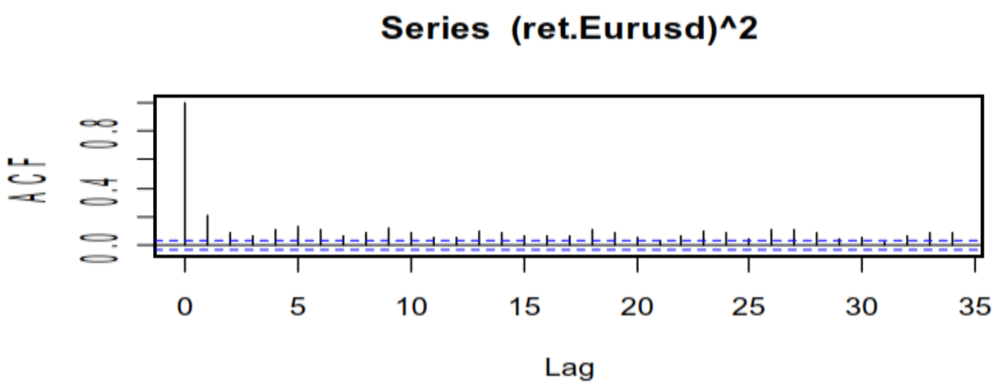

Figure 14 : ACF of Squared Returns, €/USD

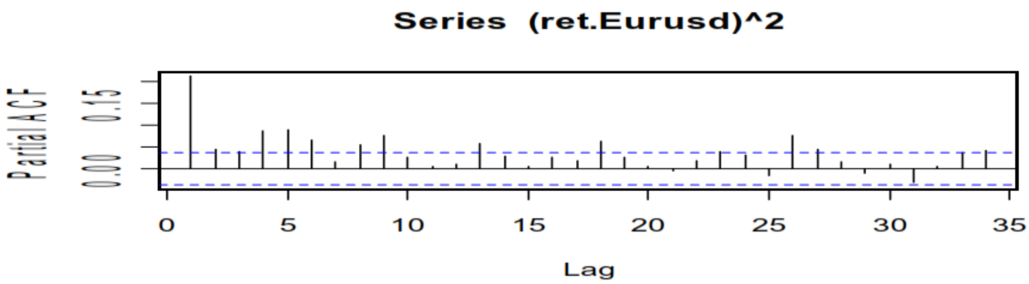

Figure 15 : Partial ACF of Squared Returns, €/USD

Significant autocorrelation properties are detected for Dowjones squared return series as well as there is partial autocorrelation in the Dowjones return series. We subtracted the arithmetic mean from each return series of (i.e. 'mean-correct') in the data, hence determine a data frame with all mean-corrected returns. The estimation results of the mean corrected data frame by MGarch BEKK Model are described as follows :

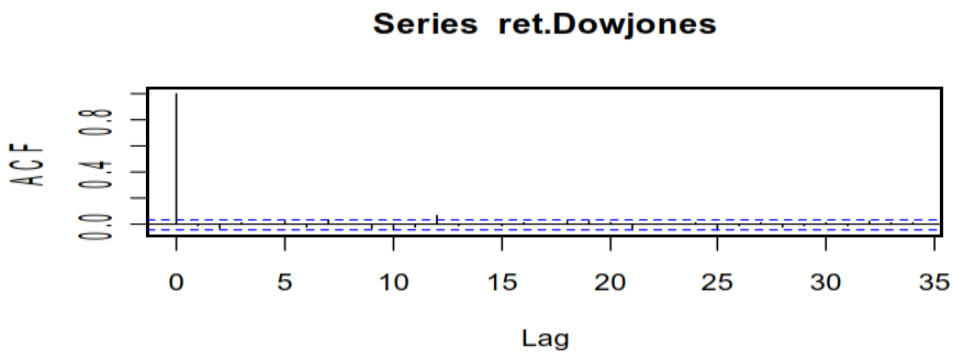

Figure 16 : ACF of Returns, Dow Jones

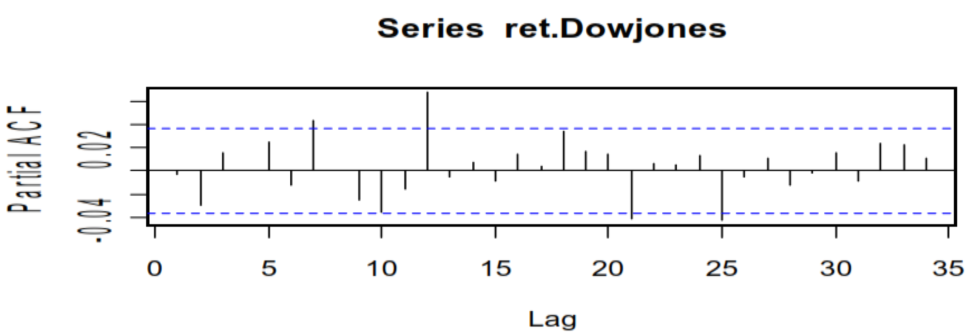

Figure 17 : Partial ACF of Returns, Dow Jones

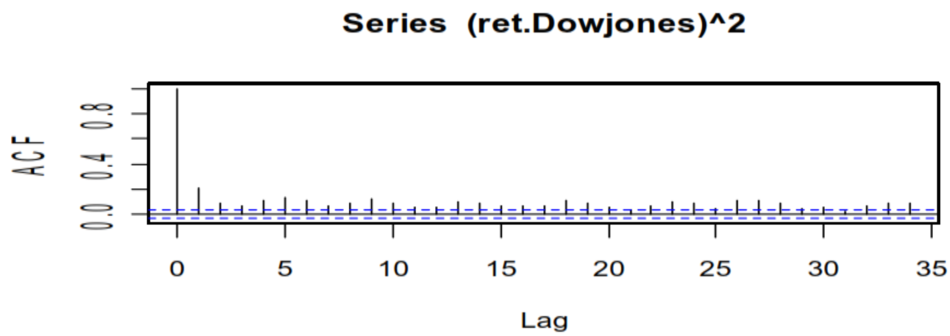

Figure 18 : ACF of Squared Returns, Dow Jones

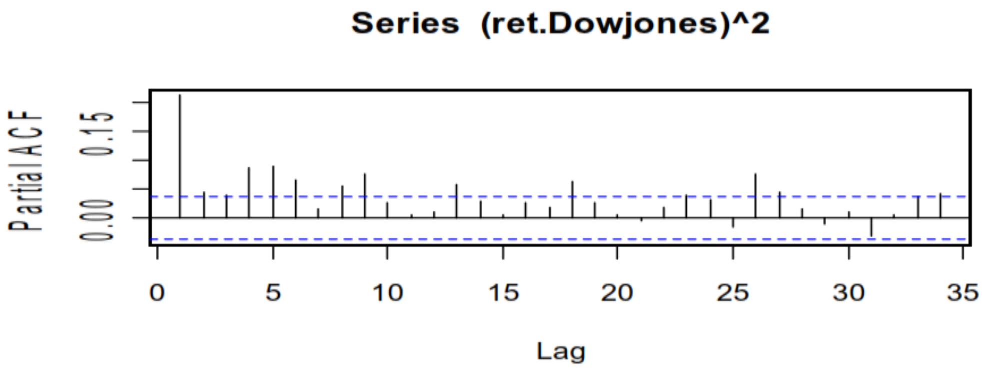

Figure 19 : Partial ACF of Squared Returns, Dow Jones

```
$`1`
[,1] [,2]
[1,] 1.103421 1.103334e+00
[2,] 0.000000 -8.036241e-06
$`2`
[,1] [,2]
[1,] -0.598543 -0.1486507
[2,] -0.598543 -1.0484455
$`3`
[,1] [,2]
[1,] -18.23163 -18.16444
[2,] 18.25462 18.18755
```

Total time required for estimation is 160.578 seconds.

3.7.4 AIC(Akaike Criterion Information) of the Model :

[1] -18542.16

The estimation results after fitting an **MGJR (i.e., baqGARCH**) to the first two columns of ret.mc where ret.mc is a dataframe with all mean-corrected returns of the variables are described as follows : (Initial values for the parameters are set at: 2 0 2 0.4 0.1 0.1 0.4 0.4 0.1 0.1 0.4 0.1 0.1 0.1 0.1 0.5)

```
$`1`
[,1] [,2]
[1,] 0.5012854 5.012910e-01
[2,] 0.0000000 2.780518e-08

$`2`
[,1] [,2]
[1,] 0.306878693 0.006879249
[2,] 0.006880917 0.306879618

$`3`
[,1] [,2]
[1,] -6.817573 -6.988166
[2,] 6.012838 6.183428

$`4`
[,1] [,2]
[1,] -0.01620412 -0.01620523
[2,] -0.01620412 -0.01620523

$`5`
[1] -5.834395
```

Through the Optimization Method ' BFGS ' (Default), the following is estimated :
1. residuals
2. correlations
3. standard deviations
4. eigenvalues

Total time required is 301.76 for the BaqGarch estimation.

4 Conclusion

The standard statistics & the robust ones being large imply that the conventional measures are affected by a small number of outliers. We also detected autocorrelations among the residuals with respect to returns on Euro/USD by Ljung-Box test. In regards to Dow Jones data, significant autocorrelation properties are detected for the squared return series as well as there is partial autocorrelation in the Dow Jones return series.

Having investigated with DCC & BEKK models, we can conclude that both models support the hypothesis at lower probabilities of 0.1-5% of Var by assuming a significance level of 5%. Positive definiteness of the conditional covariance matrix is guaranteed in both of these models as well as the forecasting accuracy of the two models is equivalent on the basis of empirical excess rates. However, the BEKK model requires estimation of 24 parameters in the case of 3 variates where as the DCC model involves 11 parameters.

In an environment of scarce open-source packages for MGarch fitting, mgarchBEKK is able to simulate and estimate bivariate BEKK models, it allows for easy specification of a particular model structure, and helps in the diagnostic check of the fitted model.

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