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Liquidity Risk and Incentive Compensation in Open-ended Funds

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Abstract

In this paper the principal-agent models between the investor and the manager of the open-ended fund are made from the new view about the liquidity risk management, and the optimal contracts and optimal policies are obtained in closed form by solving these modes. By the analysis of the optimal contract, we find that the fixed compensation of manager is the positive relationship with redemption ratio of investors and the inverse relationship with the growth ratio of total assets; the liquidation of risk assets is the positive relationship with redemption ratio and the inverse relationship with the growth ratio of total assets; the origin risk investing ratio is the positive relationship with redemption and the inverse relationship with the cumulative net growth rate. Moreover, three econometrics models are set in order to check these relations using the open-ended fund data in China, and find these relationships are fitted for the practical cases..

JEL classification numbers: G01, G21.
Keywords: Open-end fund, Liquidity risk, Incentive compensation, Principal-agent.

1 Introduction

An important character of the open-ended fund is that the investors can require the redemption of their capital by selling a number of open-ended funds when the price of the fund is very lower. When lots of the investor's redemption occur, there may be a liquidity risk, which implies that fund managers have to liquidate some risk assets to compensate the amount of redemption. As a result, it is possible that this action of this redemption make the price of fund go down and bring great losses. Therefore, the fund managers

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should consider about the probable redemption of the investors when they determine the origin investing amount of risk assets. Previous work has provided valuable insights about how liquidity affects trading behavior of market participants (e.g., Bertsimas and Lo, 1998[1]; Almgren and Chriss, 2000[2]; Huberman and Stanzl, 2005[3]). Obizhaeva and Wang(2013)[4] find that Large trades remove the existing liquidity to attract new liquidity, while small trades allow the trader to further absorb any incoming liquidity flow.

In fact, the main purpose of the fund managers using the liquidity trading strategies is to maximize their profit or utility because of the fund manager's moral hazard. The management fee is a large part of the fund manager's revenue sources, which come from a fixed proportion of net value of the fund. When the redemption rate is growing, the fund company has to liquidate assets, and the value of the fund declines, such that the management fee structure is affected by the liquidity risk of the open-ended fund. Nanda et al. (2000)[5] have proved that there are the direct relationship between management fee and liquidity and the inverse relationship between the expected revenue of fund manager and the liquidity cost in the open-ended funds.

Thus, it is important to design the optimal management fee structure in the case of existing redemption risk since the ability of controlling liquidity risk is concerned with management fee, In fact, the management fee structure is a reward structure that investors paying to managers. It mainly includes a fixed compensation and incentive compensation. The fixed compensation means some necessary expenses (such as daily consumption), and the incentive compensation is a kind of reward to encourage managers to work harder, and is connected with their performance. Designing incentive compensation is aimed at eliminating moral hazard because of a principal-agent relationship between investors and managers of the fund. The investors as principals give their capital to the managers as agents whom are responsible for specific investing management. It is invisible for the effort and investment strategies of manager since the information between managers and investors is asymmetric, which makes the managers having moral hazard and working for own profits possibly. Ou-Yang(2003)[6] proved the optimal management fee structure comprised by fixed compensation and incentive compensation including a certain proportion of fund's value and bonus or fine based on it. But one of basic assumptions in Ou-Yang' paper is no redemption, that is, he didn't consider the liquidity risk of open-end fund.

Under considering liquidity risk, this paper set the principal-agent models between the investors and managers, and study on the impact of liquidity to optimal incentive compensation contracts and optimal investment policies. Meanwhile, we check the conclusions of the theoretical models by data of China listed companies.

The paper is organized as following: the section 2 presents the theoretical analysis including basic models of open-ended funds, the incentive contract under symmetric and asymmetric information, and derives the optimal incentive contracts in the closed form. The second 3 describes the empirical models, including data analysis, test models and test results. The second 4 concludes the paper.

2 Theoretical Models and Optimal Contracts

2.1 The Basic Model

The investors delegate portfolio decisions to the fund manager because of their alleged skill in gathering superior information on movements in security prices, and can ask for redemption according to the trend of the fund price. Assume that the investor transfers some wealth V_0 by buying the open-end funds to the manager. The manager invests the capital in the financial market, where wV_0 used for investing some risk assets ($0 \le w \le 1$) and the rest part $(1-w)V_0$ used for some risk-free assets. The investment horizon is one period [0, T]. The fund manager also receives a compensation contract from the investor. This contract set the management fee as a percentage of the value of fund at the end of period V_T and consists of two components: a fixed fee that used as daily consumption of fund manager (such as wages) and performance incentive compensation，$r(V_T - b)$, where b is the incentive benchmark and r is a percentage of the portfolio's excess return. Thus the compensation contract of the fund manager is given as follows:

$$f(a,r) = a + r(V_T - b) \tag{1}$$

If some investors decide to redeem their some amount of funds S when the price goes down (or some other reasons) at moment T, the manager have to liquidate some risk-free assets for paying the redemption money. Because the cost of liquidating risk-free assets is low and won't bring down the price of risk assets, so, the manager will liquidate all the amount of risk-free assets, that is:

$$(1-w)V_0(1+\delta)^T \tag{2}$$

where δ is the yield of risk-free assets, such as bank interest rates. Usually, the proportion of risk-free assets is small because of its low return, and its liquidation couldn't satisfy the redemption requirements, so manager must liquidate some risk assets, assuming that amount of liquidated risk assets is y at the time *T*, which may cause the price movement. Assume that the price of risk assets follows an arithmetic Brownian motion:

$$P_T = P_0 + \sigma T^{1/2}\xi + \mu T - \alpha y \tag{3}$$

where P_0 is the price of risk assets at time 0, σ is the volatility rate, μ is the drift rate, and ξ is a random variable with a standard normal distribution, $0 < \alpha < 1$ is the number of stock price movements caused by each unit of stock for sale. It reflects the shock to the price by the trade strategy. The smaller α is, the better liquidity is, and vice versa. The amount of risk assets for sale is:

$$yP_T = y(P_0 + \sigma T^{1/2}\xi + \mu T - \alpha y) \tag{4}$$

So, the total amount of liquidation is:

$$(1-w)V_0(1+\delta)^T + yP_T \tag{5}$$

Its expected value is equal to the sum of redemption:

$$S = E[(1-w)V_0(1+\delta)^T + yP_T] \tag{6}$$

The equation (6) is called an expected redemption equation.
The value of fund at moment T is:

$$V_T = (\frac{wV_0}{P_0} - y)P_T = (wN - y)P_T \tag{7}$$

where $N = V_0 / P_0$. The management fee at the end of the period is:

$$A(w, y) = f(a, r) \tag{8}$$

The profit of investors without redemption is:

$$h(a, r) = V_T - f(a, r) \tag{9}$$

The performance of funds is visible, and investors could pay for the compensation based on it. Yet the investment ability of manager (i.e. choosing time ability, stock picking ability and management ability of liquidity risk, etc) is unverifiable, so, because of these, the investors couldn't design an appropriate compensation contract. This problem makes the fund managers work for their own goals but not investors. That is, the managers and investors are information asymmetric, which causes moral hazard of the fund manager.
In order to compare the differences of incentive contracts under symmetric and asymmetric information, respectively, we analyze the designing of each optimal incentive contract.

2.2 The Optimal Incentive Contract under Symmetric Information

The information symmetric means the work ability of the fund manager is visible, like investment ability and effort, and goals of the managers and investors are corresponding. There are no moral hazard from managers and no adverse selection from investors. Under the condition of the fund managers' participation, investors will choose the optimal compensation to maximize their expected utility.
The premise of participation of the fund manager is that the expected compensation from the fund management is not less than the total sum of the lowest living guarantee and the compensation got from doing other works. Assuming that the total sum mentioned before is A_0 and $E[A(w, y)]$ is the expected compensation of the fund manager, the participation constraint of the fund manager is:

$$E[A(w, y)] - h(e) \geq A_0 \tag{10}$$

where e is the fund manager's effort, $h(e)$ is a utility function of fund manager's effort e.

Assuming that investors are risk-aversion, their utility function is a negative exponential utility function,

$$U_P(h(a,r)) = 1 - \exp\{-Rh(a,r)\} \tag{11}$$

where R is the risk-aversion coefficient of investors, and $\exp\{\cdot\}$ is an exponent function. Because the random variable ξ follows a normal distribution, by equation (3)-(9), $h(a,r)$ is also a normal distribution. So, the expected utility is:

$$\begin{aligned} &E[U_P(h(a,r))] = 1 - E[\exp\{-Rh(a,r)\}] \\ &= 1 - \exp\{-RE(h(a,r)) + \frac{R^2}{2}Var(h(a,r))\} \end{aligned} \tag{12}$$

According to the monotone property of the exponent function, to maximize $E[U_P(h(a,r))]$ is equal to maximize the following function:

$$Z(a,r) \equiv E(h(a,r)) - \frac{R}{2}Var(h(a,r)) \tag{13}$$

In summary, the problem of designing the optimal contract is expressed as the following optimization model

$$P_I^1 \quad \begin{cases} \max\limits_{w,y,a,r} \ Z(a,r) \\ \text{ s.t. } E[A(w,y)] - h(e) \geq A_0 \\ S = E[(1-w)V_0(1+\delta)^T + yP_T] \\ 0 \leq w \leq 1, y \geq 0, r \geq 0. \end{cases}.$$

In order to solve P_I^1, we simplify the objective function and constraint function, and obtain the closed-form solution. From the equation (10), the participation constraint is equal to the inequality constraint:

$$a + r((wN - y)(P_0 + \mu T - \alpha y) - b) \geq A_0 + h(e) \tag{14}$$

From the equation (3) of P_T, the expected redemption equation is equal to the following equation:

$$S = (1-w)V_0(1+\delta)^T + y(P_0 + \mu T - \alpha y) \tag{15}$$

From the equation (9), the expected profit and variance of the investment without redemption are:

$$E(h(a,r)) = rb - a + (1-r)(wN - y)(P_0 + \mu T - \alpha y) \tag{16}$$

and

$$Var(h(a,r)) = (1-r)^2(wN - y)^2\sigma^2 T \tag{17}$$

respectively. By solving P_I^1, the optimal policy is given in the following proposition 1, its proof is in the appendix.

Proposition 1 *Let* $n = P_0 + \mu T$, $m = S - V_0(1+\delta)^T$, $\Delta = a_2^2 - 4a_1a_3 \ge 0$,
$a_2 = 4\alpha nN - 2\alpha V_0(1+\delta)^T$, $a_3 = -3\alpha^2 N$,
$a_1 = (n+\alpha N)V_0(1+\delta)^T - \alpha NS - Nn^2$, *then the optimal policy of investors are*

$$w^{*} = \frac{(\sqrt{\Delta}-a_2)(2a_3n - a_2 + \sqrt{\Delta})}{4V_0(1+\delta)^T a_3^2} - \frac{m}{V_0(1+\delta)^T},$$

$$a^{*} = A_0 + b + h(e) - (w^{*}N - y^{*})(n - \alpha y^{*}),$$

$$y^{*} = \frac{-a_2 + \sqrt{\Delta}}{2a_3}, \quad r^{*} = 1.$$

We called the optimal contract (a^{*}, r^{*}) in proposition 1 first-order optimality. According to proposition 1, we get the following results about the optimal policy and the relationship between the optimal compensation of the fund manager and liquidity of the fund. The positive relationship between the optimal liquidation y^{*} and α^{*} means that the larger α^{*} is, the worse liquidity, and the more liquidation y^{*}; the inverse relationship between the optimal origin risk investing amount w^{*} and α^{*} means the larger α^{*} is, the worse liquid, and less investment ratio of risk assets; the direct relationship between fixed compensation α^{*} and α means the larger liquidity is, the more compensation managers get.

2.3 The Optimal Incentive Contract under Asymmetric Information

Assuming the managers are risk-neural, how to design the optimal investing ratio w and the liquidation of risk assets y to maximum their expected utility, is equal to solve the following optimization model

$$P_A \begin{cases} \max\limits_{w,y} E[A(w,y)] \\ \quad \text{s.t. } S = E[(1-w)V_0(1+\delta)^T + yP_T]\cdot \\ \quad 0 \le w \le 1, y \ge 0. \end{cases}$$

So, from the K-K-T condition of P_A, the optimal closed-form satisfying the following proposition 2, its proof is in the appendix.

Proposition 2 *If* $\bar{\Delta} = b_2^2 + 4b_3b_1 \ge 0$, *and* $b_1 = \alpha rNS - (\alpha rN + rn)V_0(1+\delta)^T - rn^2N$, $b_2 = 4\alpha rnN + 2r\alpha V_0(1+\delta)^T$, $b_3 = 3\alpha^2 rN$, *then the optimal policy of fund manager is*

$$\bar{y} = \frac{b_2 + \sqrt{\bar{\Delta}}}{2b_3} \tag{18}$$

$$\bar{w} = \frac{V_0(1+\delta)^T - S}{V_0(1+\delta)^T} + \frac{n(b_2+\sqrt{\Delta})}{2b_3V_0(1+\delta)^T} + \frac{\alpha\left(b_2+\sqrt{\Delta}\right)^2}{4b_3^2V_0(1+\delta)^T} \tag{19}$$

According to Proposition 2, the relationship between the optimal policy of the fund manager and liquidity risk of the fund are as follow.

The positive relationship between the optimal liquidation of risk assets $\bar{y}$ and α means the more liquidity risk of redemption is, the less liquidation amount of the risk asset will be. The main reason is that the declination of the value of funds will reduce the management fee, so managers have to lessen the liquidation for protecting their profit. The direct relationship between the optimal ratio of origin investing risk assets w and α means the more liquidity is, the larger ratio of risk assets will be.

In asymmetric, the investors design the optimal contract (a,r) to maximize the expected utility under the participation constraint and incentive compatibility constraint, that is, to solve the following optimization model:

$$P_I^2 \begin{cases} \max\limits_{a,r} E[U_P(h(a,r))] \\ \text{s.t. } E[A(w,y)] - h(e) \geq A_0 \\ (w,y) \in Arg\max\{P_A\} \end{cases}$$

From the closed-from solution of P_I^2, we can get the optimal contract satisfying the following Proposition3, which proving process is in the appendix.

Proposition 3 *If the optimal policy are $\bar{y}$ and $\bar{w}$ and satisfy the equation (18) and (19), the optimal contract is:*

$\bar{a} = A_0 + b + h(e) - (\bar{w}N - \bar{y})(n - \alpha\bar{y})$, $\bar{r} = 1$.

The optimal contract ($\bar{a}$, $\bar{r}$) in Proposition 3 is called a second-order optimality. From the expression of the optimal fixed compensation $\bar{a}$, we can see the positive relationship between the fixed compensation and liquidity risk. That implies that the more liquid is, the more compensation managers could get. It's the same to the result of symmetric information.

3 Empirical Models

3.1 Data

The objects of this paper are to find the relationship between the liquidation of risk assets and the compensation of open-ended funds' manager. The total samples data are open-ended funds coming from the Shenzhen Stock Exchange and the Shanghai Stock Exchange, in which we select the data meeting the following requirements: First, these funds which started before September 2007, that is, them should be operated over 2 years; second, the size of fund is over two billion at present. At last, we select 24 funds from the total samples data for our research.

3.2 Variable Description

3.2.1 The liquidity risk indicator

Using α which is mentioned above means the number of stock price movements caused by each unit of stock for sale. That is:

$$\alpha = \frac{\text{price movements of stock}}{\text{amount of stock for sale}},$$

which can reflect the shock to the price by trade strategy and the quality of liquidity. The smaller α is, the better liquidity will be, and vice versa. So, we can use α to describe the liquidity risk.

Most of researches in this literature used the changing ratio of redemption to describe the liquidity. The formula of the rate of redemption is:

$$\text{redemption rate} = \frac{\text{current redemption shares}}{\text{origin redemption shares}}.$$

3.2.2 The independent variables

The product of redemption and risk assets ratio in January 1, 2009 to June 30 is used to stand for the redemption of risk assets. The sum of the origin fund total shares and current purchase shares represents the current amount without redemption. The quotient of the product and the sum is the liquidation ratio of risk assets. The formula is:

$$\text{Liquidation ratio of risk assets} = \frac{\text{current redemption} \times \text{current ratio of risk assets}}{\text{origin shares} + \text{current purchase shares}}$$

The origin risk investing ratio is the proportion of stock in assets.

As mentioned before, the fixed compensation is the management fee—the management compensation paid for fund manager, for some necessary expenses during the operation. The management fee is usually a certain proportion of net value of the fund and got from fund assets. Now, the fixed compensation ratio in this paper is the quotient of fixed compensation and current profit of the fund. That is, the ratio of fixed compensation is given by:

$$\text{The fixed compensation ratio} = \frac{\text{fixed compensation}}{\text{current operation profit}}$$

3.2.3 The control variables

(1) The growth rate of the cumulative net value M

The cumulative net growth rate is increasing or decreasing percentage of net value in a period of time (including dividend). It is the sum of current net value of fund and all the dividends. It can appraise the performance of the fund and display the accumulative earnings from it established. Combined with the operation time, the rate can reflect the feats of fund more intuitively and comprehensively. So it could embody the true capability exactly. Usually, the more cumulative net value the funds get, the better performance they have. The formula is:

$$\text{The cumulative net growth rate} = (\text{net growth rate of the last year}+1) \times (\text{current net growth}+1)-1$$

(2) The growth ratio of total assets N

The growth ratio of total assets is the rate of throughput of total assets this year in total assets at the beginning of the year. It can reflect the scale of growth. The higher it is, the faster expending speed the fund will be. The formula is:

$$N=\frac{\text{the net total assets this year}}{\text{total assets at the beginning of the year}}$$

3.3 The Test Model

The management fee is drawn from the net value of fund in a certain rate in domestic. So, the manager may invest the high-risk securities to increase the net value of funds in order to increase the management fee. That means that it is positive correlation between the fixed compensation of managers and liquidity indicator. To prove the relationship between the liquidation of risk assets, origin risk investing ratio, fixed compensation of managers and liquidity indicator, we set the regression models as follows:

$$a = b_1 + c_1 \times \gamma + d_1 \times N + \varepsilon_1$$

$$y = b_2 + c_2 \times \gamma + d_2 \times N + \varepsilon_2$$

$$w = b_3 + c_3 \times \gamma + d_3 \times M + \varepsilon_3$$

where, γ is the liquidity indicator, b_i is the interception term, c_i is the variable coefficient, y is the liquidity ratio of risk assets, w is the origin risk investing ratio, a is the fixed compensation ratio of fund managers, ε_i is the residual. And the statistical description of these variables is showed in Table 1.

Table 1: The statistical description of variables

The Variables	Num.	Max	Min	Average	S.d.
the liquidity indicator (the redemption ratio)	48	0.457	0.047	0.160	0.109
the fixed compensation ratio of fund managers	48	0.028	0.004	0.020	0.004
the liquidity ratio of risk assets	48	0.145	0.022	0.072	0.036
the origin risk investing ratio	24	0.908	0.238	0.637	0.238
The cumulative net growth rate	24	4.415	-0.277	1.220	1.311
The growth ratio of total assets	24	0.575	-0.328	0.028	0.184

The statistics in the table 1 are the counting results of the variables. There are some points that should be noticed: first, the gap between max and min of the redemption ratio is more than 0.4. That means the difference of redemption ratio is over 40%, so the discrepancy is large; second, it is obvious that the difference between the cumulative net growth rate reached 4.6 and standard deviation is large. It demonstrates that there are great differences between investing and management ability of different managers, which will be embodied in the designing of origin risk investing ratio; third, the max of the origin risk investing is 90.8%, and the min is 23.8%. The ratio is affected by national policies and also can reflect the investing policy and risk preference of managers. It has close relationship with liquidity.

3.4 The Empirical Results and Analysis

All the data of indicators mentioned above is got from the semi-annual report of funds in June 30, 2009 (excepting the origin ratio of risk investing). The regression results are showed in table 2.

Table 2: Regression Results of Multi-Factor Models

Independent variables	Dependent variables	coefficient	t	P	R^2	F	DW	P
	γ	0.0118	1.9636	0.063				
a	N	-0.0129	-3.6222	0.0016	0.401	7.04	2.038	0.005
	γ	0.2914	7.4108	<0.0001				
y	N	-0.0493	-2.1094	0.0471	0.723	27.46	1.036	<0.0001
w	γ	0.8374	1.5071	0.1467	0.225	3.05	1.968	0.069
	M	-0.1142	0.0463	0.0224				

From the results, we can see that the population regression is well, which demonstrates the independent variables can explain the dependent variables partly. From the results in Table 2, we can get some results.

The fixed compensation of manager is positive related with redemption ratio of investors and inverse related with the growth ratio of total assets. The management fee is drawn from net value of funds at a certain proportion and period. The redemption ratio increase, while the profit of funds will decrease. According to the formula of the fixed compensation, the proportion of the fixed compensation will be more, and the ratio will be larger. It's corresponding to the results of theoretical models and commonsense. Similarly, it's easy to understand the reason of the inverse relationship with the growth ratio of total assets. The population regression results are that the coefficient of determination is 0.4, F is 7.04, t is obvious at significant level 0.5%, and DW is nearly 2 which means that do not exist the autocorrelation. The population regression is well.

The liquidation of risk assets is positive related with redemption ratio and inverse related with the growth ratio of total assets. Obviously, the more redemption is, the more liquidation needed. When the liquidation of risk-free assets wouldn't satisfy the demand from investors, the managers should liquidate the risk assets, and they are the positive relationship. Yet risk assets are liquidated partly, the total assets will decrease and growth rate will also descend. So, they are the inverse relationship. From table 2, we can see that the population regression results are very good. The coefficient of determination is 0.723, F is 27.46, t is remarkable, but the only inadequacy is the DW value which means that the models may exist first-order autocorrelation. There would be some trend in funds. If it is declining, the current liquidation will be affected by this trend and there must be some tendency.

The origin risk investing ratio is positive related with redemption and inverse related with the cumulative net growth rate. The origin ratio can reflect the investing ability of managers, and the cumulative net growth rate also can measure the management ability and the performance of funds during the operation. So it is used as the control variable. The population regression results are not as good as the first two variables because the funds were established in different economics and politics backgrounds. But it is worth mentioning that 83.74% of every unit change in W can be explained by one unit change in R.

4 Conclusions

From the view of the risk management of open-ended funds, this paper analyses the principle-agent relationship between the investors and fund managers and provides a new understanding of the compensation contract for managers. First, we divide the relationship between the investors and fund managers into two kinds— symmetric information and asymmetric information, and then set both principal-agent models, respectively. At last, we get the optimal policy of fund managers and the closed forms of the first-order optimal contract and the second-order optimal contract by solving the models. Through analyzing these optimal contracts, we gain three theoretical conclusions as followings: the redemption ratio has positive relationships with the fixed compensation ratio, the liquidation of risk assets and the origin risk investing ratio, respectively. In order to test the rationality of the conclusions, we set up models and get samples data from the Shenzhen Stock Exchange and the Shanghai Stock Exchange. For the three theoretical results, we do empirical tests and find that the theoretical relationships are corresponding to the practical cases.

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Appendix

Proof of proposition 1

From the equations (12)-(15), the Lagrange function of P_I^1 is

$$L(a,r,w,y,l,u)=$$
$$[rb-a+(1-r)(wN-y)(P_0+\mu T-\alpha y)]$$
$$-\frac{R}{2}[(1-r)^2(wN-y)^2\sigma^2 T]$$
$$+l(a+r((wN-y)(P_0+\mu T-\alpha y)-b)-h(e)-A_0),$$
$$+u(S-[(1-w)V_0(1+\delta)^T+y(P_0+\mu T-\alpha y)])$$

where l , u , k and q are multipliers. Take partial derivative to variables and multipliers, and let the equations be 0, we get

$$\frac{\partial L}{\partial y}=P_0+\mu T+\alpha wN-(2\alpha y+u(P_0+\mu T-2\alpha y))=0 \quad \text{(A1)}$$

$$\frac{\partial \mathrm{L}}{\partial w}=N(P_0+\mu T-\alpha y)-uV_0(1+\delta)^T=0 \quad \text{(A2)}$$

$$\frac{\partial L}{\partial r}=R[(1-r)(wN-y)^2\sigma^2 T]=0 \quad \text{(A3)}$$

$$\frac{\partial L}{\partial a}=-1+l=0 \quad \text{(A4)}$$

$$\frac{\partial L}{\partial u}=S-[(1-w)V_0(1+\delta)^T+y(P_0+\mu T-\alpha y)]=0 \quad \text{(A5)}$$

$$\frac{\partial L}{\partial l}=(a+r((wN-y)(P_0+\mu T-\alpha y)-b)-e-A_0=0 , \quad \text{(A6)}$$

So, from (A1) , one has

$$P_0+\mu T+\alpha wN=2\alpha y+u(P_0+\mu T-2\alpha y) \quad \text{(A7)}$$

From (A2), we have

$$u=\frac{N}{V_0(1+\delta)^T}(P_0+\mu T-\alpha y) \quad \text{(A8)}$$

From (A5), we get

$$S-[(1-w)V_0(1+\delta)^T+y(P_0+\mu T-\alpha y)]=0. \quad \text{(A9)}$$

Taking (A8) into (A7), and combining with (A9), we can get an equation about y

$$a_1+a_2y+a_3y^2=0, \quad \text{(A10)}$$

where $a_1=(n+\alpha N)V_0(1+\delta)^T-\alpha NS-Nn^2$, $a_2=4\alpha nN-2\alpha V_0(1+\delta)^T$, $a_3=-3\alpha^2 N$, $n=P_0+\mu T$.

If $\Delta=a_2^2-4a_1a_3\geq 0$, and $y\geq 0$, the solution of (A10) is

$$y^{*}=\frac{-a_2+\sqrt{a_2^2-4a_1a_3}}{2a_3}. \quad \text{(A11)}$$

Then substituting (A11) into (A9), we have

$$w^{*} = \frac{(\sqrt{\Delta} - a_2)(2a_3 n - a_2 + \sqrt{\Delta})}{4V_0(1+\delta)^T a_3^2} - \frac{m}{V_0(1+\delta)^T} \quad , \tag{A12}$$

where $m = S - V_0(1+\delta)^T$.

Because of $wN - y \neq 0$, it can get $r^{*} = 1$ from (A3). Substituting (A11) and (A12) into (A6), we get

$$a^{*} = A_0 + b + h(e) - (w^{*}N - y^{*})(n - \alpha y^{*}) . \tag{A13}$$

It's the end of proof.

The proof of proposition 2

The Lagrange function of P_A is

$$\begin{aligned} & L_2(w, y, \ z) = \\ & a + r((wN - y)(P_0 + \mu T - \alpha y) - b) \\ & + z(S - (1-w)V_0(1+\delta)^T \\ & + y(P_0 + \mu T - \alpha y)) \end{aligned} ,$$

where z is a multiplier. Take partial derivative of $L_2(w, y)$ with respect to variables w, y and z respectively, and let everyone equal to 0, we have

$$\begin{aligned} \frac{\partial L_2}{\partial y} & = -r(P_0 + \mu T - \alpha y) - \alpha r(wN - y) \\ & + z((P_0 + \mu T - \alpha y) - \alpha y) = 0 \end{aligned} \tag{B1}$$

$$\frac{\partial L_2}{\partial w} = rN(P_0 + \mu T - \alpha y) + zV_0(1+\delta)^T = 0 \tag{B2}$$

$$\frac{\partial L_2}{\partial z} = S - (1-w)V_0(1+\delta)^T + y(P_0 + \mu T - \alpha y) = 0 \tag{B3}$$

From (B2), we have

$$z = -\frac{rN(n - \alpha y)}{V_0(1+\delta)^T} \tag{B4}$$

Let $n = P_0 + \mu T$, and substituting (B4) into (B1). Then the equation can be simplified as

$$-rn + 2r\alpha y - \alpha rwN - \frac{rN(n - \alpha y)}{V_0(1+\delta)^T}(n - 2\alpha y) = 0 \tag{B5}$$

Let

$\vartheta_1 = \alpha rNS - (\alpha rN + rn)V_0(1+\delta)^T$,

$\vartheta_2 = 2r\alpha V_0(1+\delta)^T$,

and simplify (B5), we have

$$a_3 y^2 - a_2 y - a_1 = 0 \tag{B6}$$

where $a_1 = \vartheta_1 - rn^2 N, a_2 = 4\alpha rnN + \vartheta_2, a_3 = 3\alpha^2 rN$ When $\overline{\Delta} = b_2^2 + 4b_3 b_1 \geq 0$, the solution of (B6) is

$$\bar{y} = \frac{b_2 + \sqrt{\bar{\Delta}}}{2b_3} \tag{B7}$$

Substituting (B6) into (B3)

$$\bar{w} = \frac{V_0(1+\delta)^T - S}{V_0(1+\delta)^T} + \frac{n(b_2 + \sqrt{\bar{\Delta}})}{2b_3 V_0(1+\delta)^T}$$

$$+ \frac{\alpha\left(b_2 + \sqrt{\bar{\Delta}}\right)^2}{4b_3^2 V_0(1+\delta)^T}$$

It's the end of proof.

The proof of proposition 3

The Lagrange function of P_I^2 is

$$L_3(a, r, \bar{w}, \bar{y}, \lambda)$$

$$= [rb - a + (1-r)(\bar{w}N - \bar{y})(P_0 + \mu T - \alpha\bar{y})$$

$$-\frac{R}{2}[(1-r)^2(\bar{w}N - \bar{y})^2\sigma^2 T]$$

$$+\lambda(a + r((\bar{w}N - \bar{y})(P_0 + \mu T - \alpha\bar{y}) - b)$$

$$-h(e) - A_0)$$

where λ is the multiplier. Take partial derivative of $L_3(a, r, \bar{w}, \bar{y}, \lambda)$ about the variables a, r and the multiplier λ ,respectively, and let everyone equal to 0, we have

$$\frac{\partial L_3}{\partial a} = -1 + l = 0 \tag{C1}$$

$$\frac{\partial L_3}{\partial r} = R(1-r)(wN - y)^2\sigma^2 T \ = 0 \tag{C2}$$

$$\frac{\partial L_3}{\partial \lambda} = a + (wN - y)(P_0 + \mu T - \alpha y) \tag{C3}$$

$$-(A_0 + b + h(e)) = 0$$

Because of $wN - y \neq 0$ and the equation (C2) , we get $\bar{r} = 1$.Additional, according to (C3), we have

$$\bar{a} = A_0 + b + h(e) - (\bar{w}N - \bar{y})(n - \alpha\bar{y}) . \tag{C4}$$

It's the end of proof.