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# Modelling Stratified Flow in an Inclined Flow Channel with a Bend

**P.K. Saina[1], A.W. Manyonge[2], J.K. Kimaiyo[3] and J.S. Maremwa[4]**

**Abstract**

Multi-phase flows modelling in wellbore has always been a problem to the petroleum and many other industries. In the petroleum industry, correlations are presently in use, most of which are obsolete as their application to field data has generated results with unacceptable errors. Most of the existing models proposed to correct these anomalies have always ignored the mass transfer between phases. This has created serious doubt to the predictive capability of these models. In this study, a one-dimensional transient state model of multi-phase fluid flow in an inclined well has been developed. The model is solved numerically to predict the pressure drop as the flow passes through an inclined wellbore and a bend. Results show that the higher the inclination angle the higher the rate of pressure decline in the flow of the fluid to the surface, also a dramatically high pressure drop was observed when the flow passes through a bend. This unexpected high pressure resulting from the change in flow regime from stratified layered flow to slug flow in bend can result in a counter flow of formation fluid back into the reser-

[1] Department of mathematics and Computer Science, Universisty of Eldoret, Kenya.
[2] Department of Pure and Applied Mathematics, Maseno University, Kenya.
[3] Department of Mathematics, Moi University, Kenya.
[4] Department of Mathematics and Computer Science, University of Eldoret, Kenya.

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voir in a low pressure reservoir thereby reducing the fractional recovery from such a reservoir.

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# Nomenclature

| | |
|---|---|
| $A$- | Cross-sectional area, $ft^2$ |
| $g$- | Acceleration due to gravity, $ft/sec^2$ |
| $J$- | Specific productivity index, $ft^2$/(psi.sec) |
| $s$- | Source term |
| $S$- | Wall soaked area |
| $u$- | Velocity, ft/sec |
| $\theta$ - | Well inclination to the horizontal |
| $\eta$- | Fluid fraction |
| $f$- | Friction factor |
| $P$- | Pressure in wellbore, psia |
| $q$- | Flow rate, ft3/sec |
| $sl$- | Source term of liquid, ibm/(sec-$ft^3$) |
| $x$- | Axial flow direction |
| $V$- | Volume, $ft^3$ |
| $\rho$- | Density, ibm/$ft^3$ |
| $\tau$- | Well friction |

# subscripts

| | |
|---|---|
| $g$- | Gas |
| $l$- | interface between the upper and lower layer |
| $m$- | Mixture |
| $o$- | oil |
| $s$- | Slip |
| $w$- | water |

# 1 Introduction

The search for more economic recovery methods, particularly for marginal offshore oilfields, sometimes in deep water has resulted in decision to produced low pressure wells thereby producing two-phase fluid under different conditions and wellbore geometry. Optimization of the design and successful operation of two-phase well systems requires a substantial knowledge of the behavior characteristics of such flows. Various studies have shown that at present no single theory or correlation can satisfactorily predict the characteristics of two-phase gas-liquid flow in a well over a wide range of conditions. Gas -liquid flow is more complex phenomenon than single-phase flow primarily because the distribution of the two phases is normally unknown and difficult to specify quantitatively. Unfortunately, the present design methods used within the industry are obsolete compared to the single phase correlations. Some are based on empirical correlations derived from small scale laboratory systems with small diameter pipes and using simple test fluids such as air and water at low pressure. Extrapolation of these correlations to large diameter, high pressure oil and gas pipelines usually result in unacceptable errors. In this paper, stratified gas-liquid flow with mass transfer within phases in inclined well is considered. Investigation is carried out on the two-phase flow across a bend to determine the instability that occurs in such situation.

A wide variety of flow-pattern regimes have been proposed for horizontal and inclined flows. A reasonable large amount of data of flow pattern have been accumulated and reported in the literature [1, 2, 3, 4, 5]. The majority of data are for air-water flows in relatively narrow-bore tubes. The usual approach for determining the flow regime has been to collect the data for flow rates and fluid properties and to visually observe the flow pattern through transparent pipe sections. Then a search is undertaken for a way to map the data in a two dimensional plot by locating transition boundaries between regimes. [6] have presented a theoretical model for predicting flow regime transition in two-phase gas-liquid flow. The theory predicts the effect on transition boundaries of pipe size, fluid properties and angle of inclination. The mechanisms for transition are based on physical concepts and are fully predictive in that no flow regime data are used in their development. Their developed criterion for predicting the onset of slug formation is however a subject of research. Also the model failed to account for mass transfer between phases. [7] modelled the non-equilibrium flow assuming a non-isothermal flow situation but other assumptions made in the paper made the predictive capability of their developed model error full. Based on these anomalies a model is proposed in this paper to correct the identified errors in the models reviewed in the literature.

# 2 Mathematical formulation

## 2.1 Assumptions of the proposed multi-phase model

Here we enumerate the assumptions that are used in the proposed model:

1. The loss in density by liquid and its dissolved gas-phase due to evolved gas is gained by the free gas-phase and gas volume increase in the free gas-phase due to pressure decrease in the dissolved liquid-phase
2. Slippage exists between the liquid and the free gas
3. Inflowing reservoir fluids accelerate to the mean stream velocity instantaneously
4. The flow is layered flow with the gas-phase flowing above the liquid-phase in the well-bore

## 2.2 Proposed Mathematical model

The one-dimensional continuity equations for liquid (oil and water with dissolved gas) and gas phases are given by the expressions

$$\frac{\partial}{\partial t}(\rho_l \eta_l) + \frac{\partial}{\partial x}(\rho_l \eta_l u_l) = \Delta s_l \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_g \eta_g) + \frac{\partial}{\partial x}(\rho_g \eta_g u_g) = \Delta s_g \tag{2}$$

where

$$\Delta s_l = \frac{\Delta \rho_g u_l S_i}{A} - \frac{\Delta \rho_l u_l S_i}{A} \tag{3}$$

$$\Delta s_g = \frac{\Delta \rho_g u_g S_i}{A} - \frac{\Delta \rho_l u_g S_i}{A} \tag{4}$$

This is for a liquid which is slightly compressible, for example water and most oils. The compressibility as defined by [8] is given by

$$c = -\frac{1}{V}\frac{dV}{dP} \tag{5}$$

where $V$ is the volume of any fixed amount of fluid, this is equivalent to the relation

$$\frac{d\rho_l}{dP} = c\rho_l \tag{6}$$

For slightly compressible liquids, $c$ may be taken as a constant over the pressure range of interest (small $c$ can be of order of $10^{-5}$). Integrating equation (6) gives,

$$\rho_l = \rho_l^* e^{(P-P^*)} \tag{7}$$

where $\rho_l^*$ is the density of liquid (water and oil) with the dissolved gas at pressure $P^*$. Since,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \tag{8}$$

For small x, we take

$$e^x = 1 + x \tag{9}$$

Hence

$$\rho_l = \rho_l^*[1 + c(P - P^*)] \tag{10}$$

For small change in pressure ($\Delta P$)

$$\rho_l = \rho_l^*[1 + c(\Delta P)] \tag{11}$$

$$\rho_l - \rho_l^* = \rho_l^* c\Delta P \tag{12}$$

$$\Delta \rho_l = \rho_l^* c\Delta P \tag{13}$$

For a region of free gas, then from the gas laws we have

$$\frac{P}{\rho_g} = k = \text{constant} \tag{14}$$

$$\frac{P^*}{\rho_g^*} = \frac{P}{\rho_g} \tag{15}$$

where $\rho_g^*$ is the density of the free gas at pressure $P^*$. Then given that,

$$P = P^* + \Delta P \tag{16}$$

and

$$\rho_g = rho_g^* + \Delta\rho_g \tag{17}$$

Then it follows that,

$$\Delta\rho_g \rho_g^* \ln(\frac{P}{P^*}) \tag{18}$$

Therefore, from equations (3) and (4)

$$\Delta s_l = \rho_g^* \ln(\frac{P}{P^*})\frac{u_l S_i}{A} - \frac{\rho_l^* c \Delta P u_l S_i}{A} \tag{19}$$

and

$$\Delta s_g = -\rho_g^* \ln(\frac{P}{P^*})\frac{u_g S_i}{A} + \frac{\rho_l^* c \Delta P u_g S_i}{A} \tag{20}$$

where $\Delta P = P^* - P$. Now

$$\eta_l + \eta_g = 1 \tag{21}$$

where $\eta$ is the phase volume fraction. The mixture velocity is defined as:

$$u_m = u_l \eta_l + u_g \eta_g \tag{22}$$

where $u$ stands for the phase velocities. Then defining slip velocity $u_s$ as,

$$u_s = u_l + u_g \tag{23}$$

Therefore, the phase velocities are:

$$u_l = -u_s(1 - \eta_l) + u_m \tag{24}$$

and

$$u_g = u_s\eta_l) + u_m \tag{25}$$

Considering the above equations, the continuity equations for oil and gas phases can be written using, $\eta_l$ and $u_s$:

$$\frac{\partial \eta_l}{\partial t} + \frac{\partial}{\partial x}(\eta_l(1-\eta_l)u_s + \eta_l u_m) = \rho_g^* \ln(\frac{P}{P^*})\frac{u_l S_i}{A} - \frac{\rho_l^* c\Delta P u_l S_i}{A} \tag{26}$$

$$\frac{\partial(1-\eta_l)}{\partial t} - \frac{\partial}{\partial x}(\eta_l(1-\eta_l)u_s - (1-\eta_l)u_m) = -\rho_g^* \ln(\frac{P}{P^*})\frac{u_g S_i}{A} + \frac{\rho_l^* c\Delta P u_g S_i}{A} \tag{27}$$

In order to calculate the pressure drop, the momentum equation for liquid and gas-phases are written as:

$$\frac{\partial(\rho_l\eta_l u_l)}{\partial t} + \frac{\partial(\rho_l\eta_l u_l^2)}{\partial x} + \eta_l\frac{\partial P}{\partial x} = -\tau_l\frac{S_{ol}}{A} - \tau_i\frac{S_i}{A} - \rho_l\eta_l g\sin\theta +$$

$$[\rho_g^* \ln(\frac{P}{P^*})u_l^2\frac{S_i}{A} - \rho_l^* c\Delta P u_g u_l\frac{S_i}{A}]\sin\theta \tag{28}$$

$$\frac{\partial(\rho_g\eta_l u_g)}{\partial t} + \frac{\partial(\rho_g\eta_l u_g^2)}{\partial x} + \eta_g\frac{\partial P}{\partial x} = -\tau_g\frac{S_g}{A} - \tau_i\frac{S_i}{A} - \rho_g\eta_g g\sin\theta +$$

$$[\rho_g^* \ln(\frac{P}{P^*})u_l u_g\frac{S_i}{A} - \rho_l^* c\Delta P u_g^2\frac{S_i}{A}]\sin\theta \tag{29}$$

The fluid flow rate in the wellbore would be affected by the fluid influx from the reservoir. The mass influx rate of water, oil and gas from the reservoir per unit volume of wellbore can be derived as equations (30) to (32) respectively.

$$S_w = \frac{\rho_w J_w(P_{\text{res}} - P)}{A} \tag{30}$$

$$S_o = \frac{\rho_o J_o(P_{\text{res}} - P)}{A} \tag{31}$$

$$S_g = \frac{\rho_g J_g(P_{\text{res}} - P)}{A} \tag{32}$$

$J_w$, $J_o$ and $J_g$ are the productivity index of water, oil and gas respectively. Productivity index is the volumetric inflow rate of fluid from the reservoir into the wellbore per unit pressure drop between the reservoir and the wellbore unit length. The liquid mass influx rate from the reservoir is given by,

$$S_l = S_w + S_o \tag{33}$$

and the total mass influx rate from the reservoir to the wellbore is given as

$$S_l = S_w + S_l + S_g \tag{34}$$

The total pressure drop can then be obtained by taking the sum of these two momentum balances which gives:

$$\frac{\partial(\rho_l\eta_l u_l + \rho_g\eta_g u_g)}{\partial t} + \frac{\partial(\rho_l\eta_l u_l^2 + \rho_g\eta_g u_g^2)}{\partial t} = -\tau_g\frac{S_g}{A} - \tau_l\frac{S_o}{A} - (\rho_l\eta_l + \rho_g\eta_g)g\sin\theta +$$

$$[\rho_l^* \ln(\frac{P}{P^*})u_l^2\frac{S_i}{A} - \rho_l^* c\Delta P u_g u_l\frac{S_i}{A} - \rho_g^* \ln(\frac{P}{P^*})u_l u_g\frac{S_i}{A} + \rho_l^* c\Delta P u_g^2\frac{S_i}{A}]\sin\theta \tag{35}$$

$\tau$, $S$, $A$ and $\theta$ represents wall friction, wall soaked area, pipe cross-sectional area and pipe inclination to the horizontal. The wall friction is taken as,

$$\tau_l = \frac{1}{2}f\rho_l u_l^2 \tag{36}$$

and

$$\tau_g = \frac{1}{2}f\rho_g u_g^2 \tag{37}$$

where $\rho_l$ and $\rho_g$ are the liquid and gas density respectively and $f$ is the friction factor.

### 2.2.1 Boundary conditions

The gas and liquid inflow rates from the reservoir to the wellbore must be specified. The back pressure at the well head must also be specified.

### 2.2.2 Initial conditions

The Inflowing reservoir fluids are assumed to accelerate to the mean stream velocity instantaneously. Hence, the stable flow condition is assumed to be achieved. Therefore, the pressure and velocity distribution and properties of the fluid are calculated and set as initial condition of the flow model.

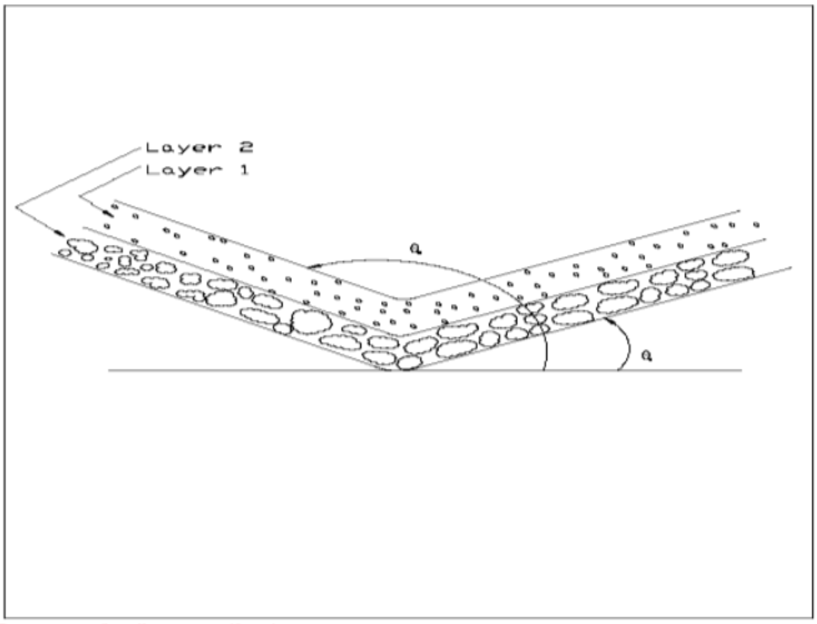




Figure 1: The schematic diagram of the bend through which the fluid flows in the wellore

# 3 Method Solution

The Crowe's [9] method for two-phase flow which is a modification of the numerical solution scheme called SIMPLE developed by Patakar [10] for single-phase flow was employed with some modification to facilitate the convergence of the numerical solution.

## 3.1 Results validation

The proposed model results were validated with field publish data listed in Table 1, and comparing with empirical correlations of other wells data obtained was done using the commonly used Duns and Ross and Hagedorn and Brown correlations in the oil industry. This is through the computation of the average liquid hold up in the tubing. The Average Absolute Deviation (AAD) values of the comparison are listed in Table 2.

The $AAD$ is defined as

$$AAD = (\frac{1}{n_d}) \sum_{i=1}^{n_d} \mid \Delta P_E - \Delta P_M \mid \qquad (38)$$

where, $\Delta PE$ = change in pressure from the field data $\Delta PM$ = predicted pressure change.

Table 1: Physical conditions and flow rates of petroleum wells studied

| Well No. | Oil rate | GRO | $H_20$ cut (%) | Oil gravity ($^0$API) | Measd Depth (ft) | Well head pres (psi) | Flow string Dia. (in) | Measd $\Delta P$ ($psi$) |
|---|---|---|---|---|---|---|---|---|
| 1 | 325 | 4025 | 30 | 10.5 | 4630 | 300 | 8.76 | 750 |
| 2 | 1245 | 5260 | 12 | 11.7 | 4545 | 250 | 25 | 16.7 |
| 3 | 1065 | 1435 | 23 | 11.4 | 3720 | 520 | - | 12.5 |
| 4 | 1300 | 230 | - | - | 4355 | 700 | - | 18.2 |
| 5 | 1250 | 957 | - | - | 4175 | 350 | - | 550 |
| 6 | 855 | 185 | - | - | 4065 | 580 | - | 800 |
| 7 | 1020 | 235 | 12 | 16.3 | 4160 | 620 | - | 650 |
| 8 | 1965 | 450 | - | 15.7 | 4487 | 700 | - | 350 |
| 9 | 2700 | 985 | 5 | 13.2 | 4505 | 450 | - | 430 |
| 10 | 968 | 1500 | 20 | 10.5 | 4400 | 350 | - | 830 |

The reduction in the error for the oil sample from ten different wells as shown in Table 2 is remarkable in reference to the AAD value of 0.7 from the application of the proposed model compared to $AAD$ value of 1.9 and 2.4 returned by Hagedorn and Brown's correlation and Duns and Ros's correlation, respectively.

Table 2: Average Absolute Deviations (AAD) between measured and predicted pressure drops

| | Proposed Method | Duns & Ros Method | Hagedorn & Brown Method |
|---|---|---|---|
| AAD | 0.7 | 2.4 | 1.9 |

# 4 Results and Discussion

Figure 2 shows the liquid fraction at different sections of the wellbore in an inclined well at different inclination angles. It was observed that the liquid fraction gradient increases with angle of inclination to the horizontal; this is due to the fact that as the angle of inclination increases the pressure gradient increases and consequently the rate of gas loss from the liquid increases. Hence for a fixed liquid fraction at the wellhead, the corresponding bottom hole liquid fraction increases with the angle of inclination to the horizontal.

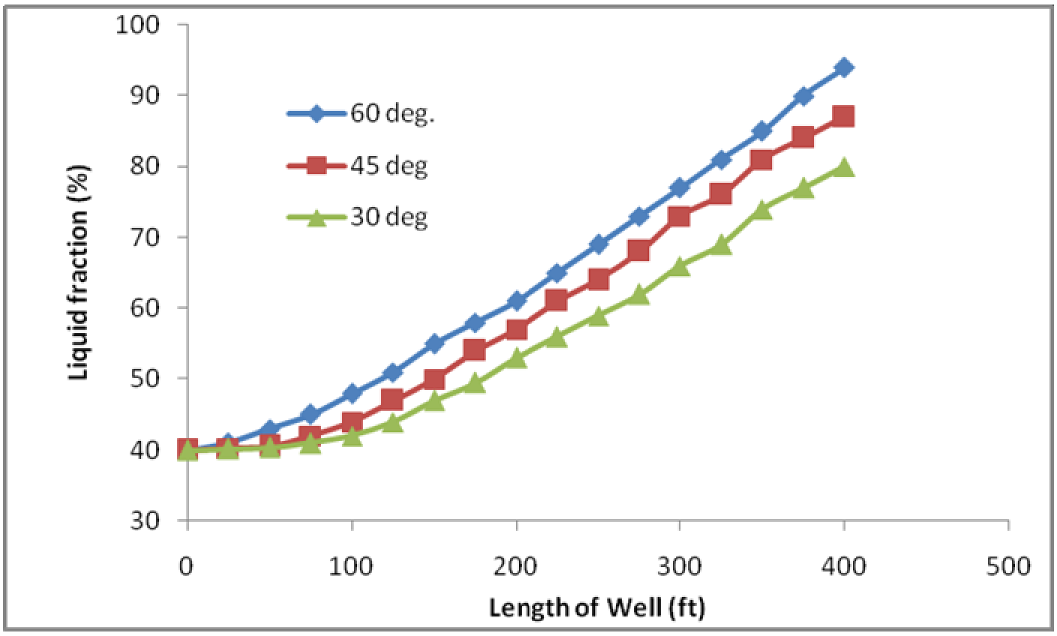


Figure 2: The Liquid Fraction at different sections in the Wellbore at different inclination angle

Figure 3 shows the time dependent pressure variations before and after a bend in the wellbore. It was observed that the upstream pressure of the bend experiences a peak when the flow passes the bend. It was observed in the early period that the pressure is drastically reduced when the flow passes through a bend, this confirmed the momentary slug flow observed during the multi-phase experiment, that is the jerk experience when the flow pass through a bend. This confirmed the theory that Kelvin-Helrholtz instability is a basic mechanism of slug formation. This instability, based on the analysis

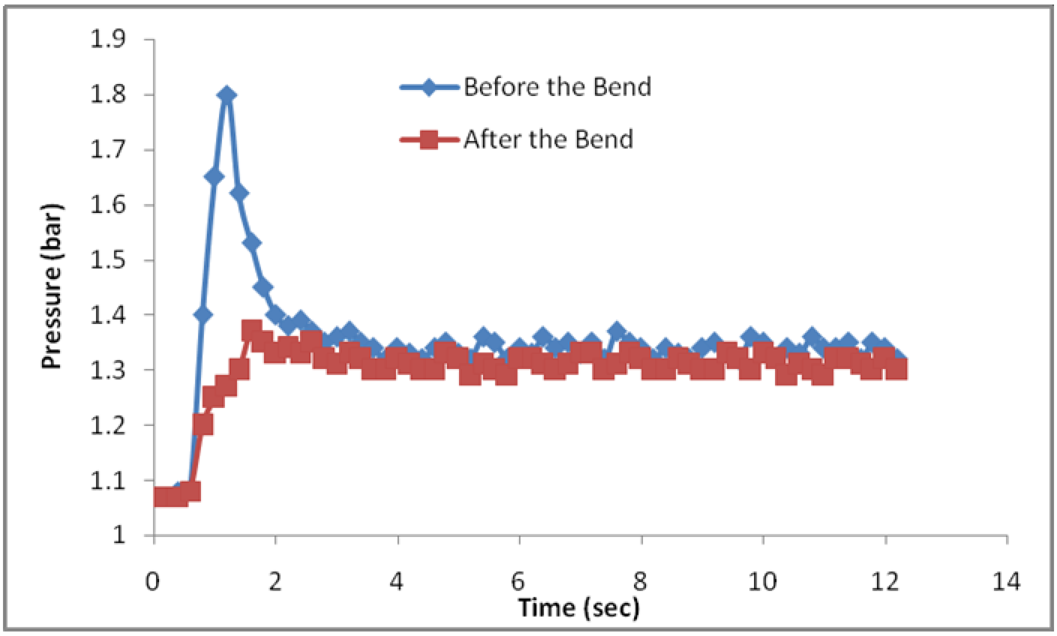


Figure 3: Time Dependent Pressure variation before and after a bend

of small sinusoidal perturbations of an interface between fluids, occur when the suction effect due to pressure variation over a wave becomes sufficiently large to overcome the stabilizing effect of gravity [11]. It should be noted that such pressure declines are complex and gas-liquid and liquid-liquid interactions might have great contributions on the flow behavior, hence, a proper solution for this kind of flow round a bend might be tracking scheme which has been used for simulation of slug flow [12]. In the alternative, the flow regime should be replaced with slug flow regime when the simulation of layered multi-phase flow around a bend is to be performed. This kind of abrupt pressure decline, can lead to counter-flow of well fluid back into the reservoir in a low pressure well/reservoir thereby reducing the recovery of oil and gas from such reservoir.

Figure 4 shows the results of the sensitivity analysis of the experienced pressures and the liquid mass influx rates at different sections of the well; it shows the dependency of the pressure on the liquid influx rate at different sections of the wellbore at the angle of inclination of 30o. It was observed that the higher the liquid influx rate the higher the pressure gradient needed to produce a specified back pressure at the wellhead. If the fluid properties were to be unchanged, higher liquid influx rate will produce higher bottom hole and wellhead pressure when compared to those determined from low liquid influx rate.

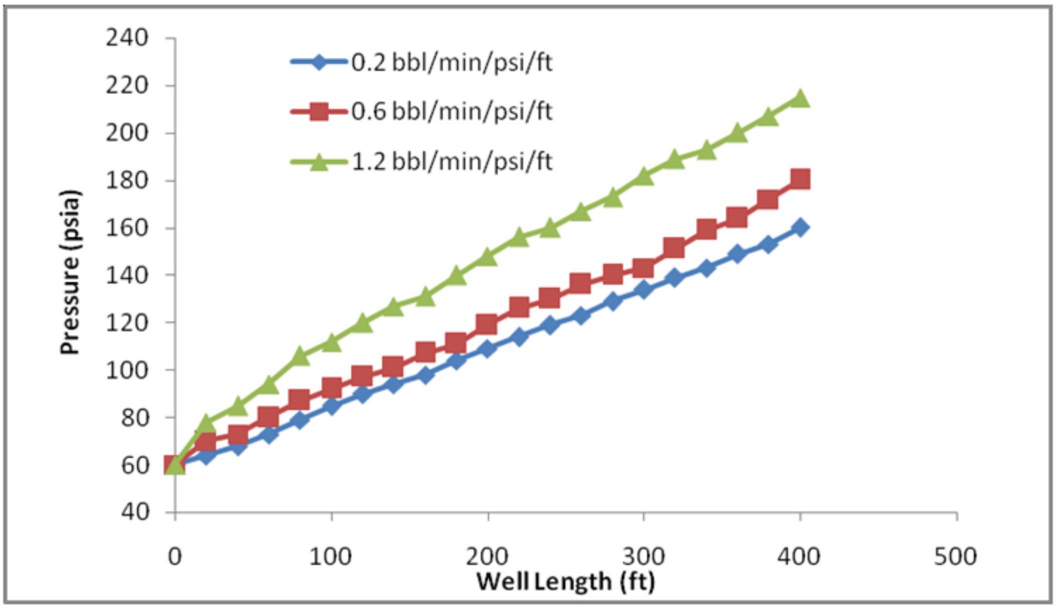


Figure 4: Pressure profile variation at different position in the well under different liquid influx rate

# 5 Conclusion

A one dimensional multi-phase model that considered mass transfer between gas and liquid has been developed. Sensitivity analysis of the model successfully predicts the instability that occurs when the gas-liquid flow passes through a bend. This is due to the transition of flow regime from stratified flow to slug flow which is accompanied by instability based on small sinusoidal perturbations of an interface between the fluids as a result of the suction effect overcoming the effect of gravity. This pressure reduction can cause a counter-flow of wellbore fluid back into the reservoir in a low pressure well there by reducing the fractional recovery from such reservoir.

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