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Multiplicative Sarima Modelling Of Nigerian Monthly Crude Oil Domestic Production

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Abstract

A realization of monthly Nigerian crude oil domestic production, NODP, from January 2006 to August 2012, is analyzed. The time plot reveals a negative trend between 2006 and 2009 and a positive trend from 2009 to 2012. Twelve-month differencing yields a series, SDNODP, with an overall positive trend. Non-seasonal differencing of SDNODP yields a series, DSDNODP, with an overall horizontal trend. The correlogram of DSDNODP reveals a seasonality of period 12 months and the involvement of a seasonal moving average component of order one. The significant spikes of the autocorrelation function at lags 1 and 12 suggests an autocorrelaton structure of a $(0, 1, 1)x(0, 1, 1)_{12}$ SARIMA model. This is hereby proposed, fitted and found to be adequate using a variety of arguments.

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1 Introduction

Crude oil is currently the mainstay of the Nigerian economy. Modelling Nigerian crude oil data has therefore engaged the attention of many researchers, a few of whom are Etuk[1, 2], Bolton[3], King *et al.*[4] and Salisu and Fasanya[5]. Many economic time series data exhibit some seasonality even though they are also known to be volatile. For such a series seasonal autoregressive integrated moving average (SARIMA) models could be used.

SARIMA models were proposed by Box and Jenkins[6]. Extensively discussed in the literature are theoretical properties and practical applications of such models. Efforts have been made to highlight the relative merits of the models. A few of the authors that have contributed extensively in this regard are Priestley[7], Madesen[8], Boubaker[9], Surhatono[10], and Etuk[11].

The data for this work is from the Data and Statistics publication of the Central Bank of Nigeria website www.cenbank.org. The crude oil production data which is expressed in million barrels per day, is in two categories: Exports and Domestic Production. The Domestic Production quota as opposed to the Exports quota is for domestic consumption. It is the purpose of this work to propose and fit an adequate multiplicative SARIMA model to monthly crude oil domestic production of Nigeria.

2 Materials and Methods

2.1 Sarima Modelling

A stationary time series $\{X_t\}$ is said to follow *an autoregressive moving average model of orders p and q*, denoted by ARMA(p, q), if it satisfies the following difference equation

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \ldots - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \ldots + \beta_q \varepsilon_{t-q} \qquad (1)$$

or

$$A(L)X_t = B(L)\varepsilon_t \qquad (2)$$

where $A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \ldots -\alpha_p L^p$ and $B(L) = 1 + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \ldots + \beta_q \varepsilon_{t-q}$ and L is the backshift operator defined by $L^k X_t = X_{t-k}$. Here $\{e_t\}$ is a white noise process. For stationarity and invertibility the zeros of A(L) and those of B(L) must be outside the unit circle respectively.

Let $\nabla^d X_t$ be the d^{th} difference of X_t, where $\nabla = 1 – L$. If non-stationary X_t is replaced by $\nabla^d X_t$ (where d is the least positive integer for which the difference is stationary) in (1) and the model is referred to as *an autoregressive integrated moving average model of orders p, d and q*. This is denoted by ARIMA(p, d, q).

If the time series $\{X_t\}$ exhibits stationarity of period s it could be modeled by a SARIMA model. $\{X_t\}$ is said to follow a *multiplicative (p, d, q)x(P, D, Q)$_s$ SARIMA model* if

$$A(L)\Phi(L^s)\nabla^d \nabla^D{}_s X_t = B(L)\Theta(L^s)\varepsilon_t \qquad (3)$$

where

$$\Phi(L) = 1 + \phi_1 L + \phi_2 L^2 + \ldots + \phi_P L^P \qquad (4)$$

$$\Theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \ldots + \theta_Q L^Q \qquad (5)$$

and the coefficients ϕ’s and θ’s are constants such that the zeros of (4) and (5) are outside the unit circle, for stationarity and invertibilty respectively.

2.2 Model Estimation

To estimate the model (3) order determination has to be done first. That is, the parameters p, d, q, s, P, D and Q must first be estimated. The parameter p being the non-seasonal autoregressive order should correspond with the cut-off point of the partial autocorrelation function (PACF). On the other hand, q being the non-seasonal moving average order is estimated by the cut-off point of the autocorrelation function, ACF. For a seasonal series of period s, the ACF shows a significant spike at lag s. If the spike is negative then a seasonal movong average component is suggestive; if positive, a seasonal autoregressive component is suggestive. D is the seasonal order of differencing necessary to achieve stationarity. Traditionally D = d = 1. It is important to note that an autocorrelation is said to be statistically significant if it is outside the range $\pm 2/\sqrt{n}$ where n is the series length.

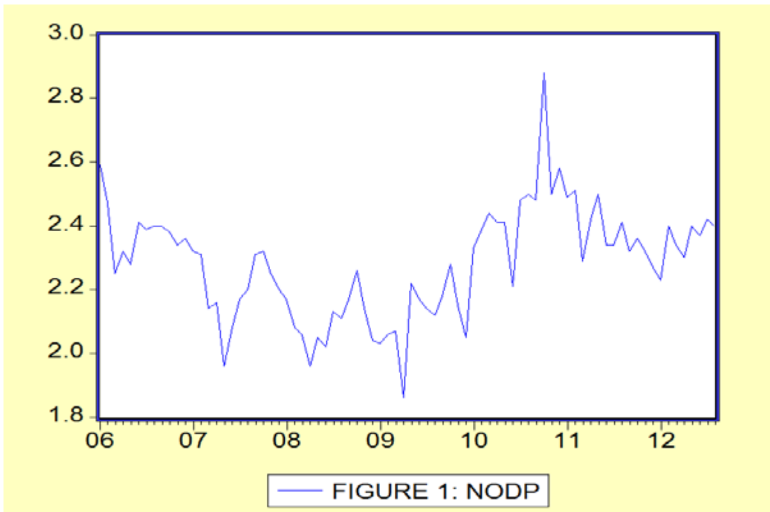
The coefficients α, β, ϕ and θ are estimated by an optimization criterion like the least squares technique, the maximum likelihood technique, etc. The statistical/econometric software Eviews which is used for this work is based on the least error sum of squares technique.

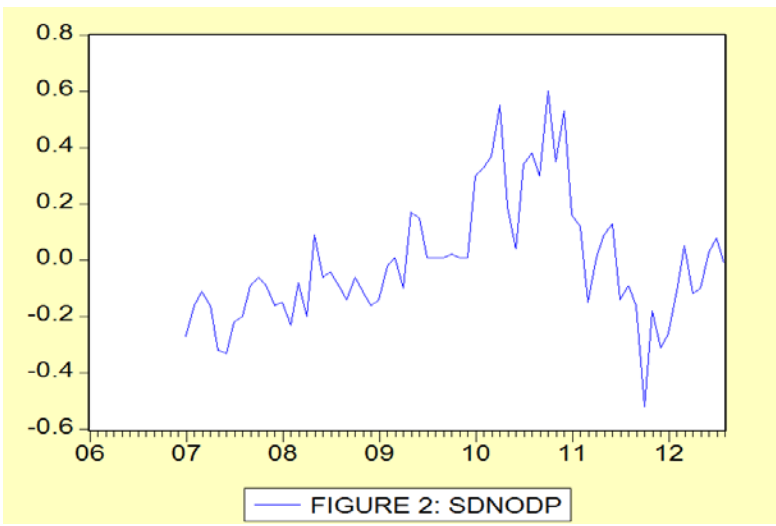
2.3 Diagnostic Checking

A fitted model should be tested for goodness-of-fit to the data. Some analyses of the model. Assuming the model is adequate, the residuals should be uncorrelated and follow a normal distribution with zero mean.

3 Results and Discussion

The time plot of the realization NDOP in Figure 1 shows a slightly negative trend between 2006 and 2009 and a positive one thereafter. Seasonal (i.e. twelve-month) differencing once produces a series SDNDOP with an overall positive trend (Figure 2). Non-seasonal differencing of SDNDOP produces a series DSDNDOP with an overall horizontal trend (Figure 3) and an ACF with significant negative spikes at lags 1 and 12 (Figure 4). The spike at lag 12 shows that the series DSDNODP is seasonal of period 12 and that a seasonal moving average





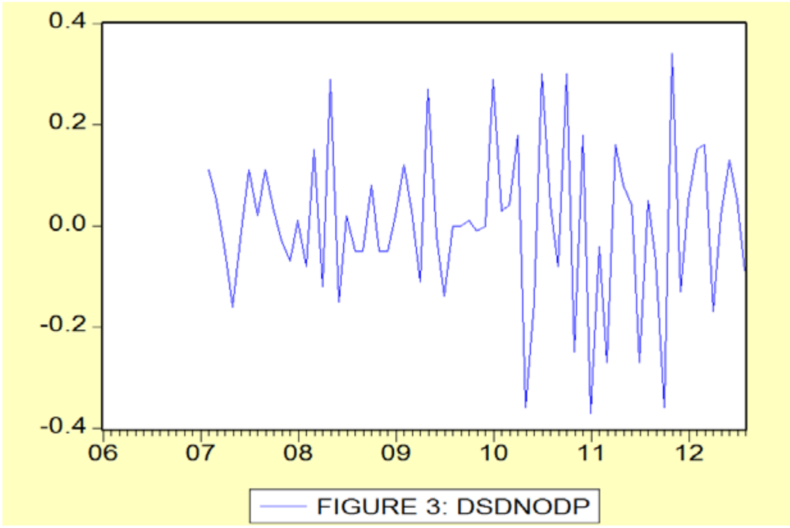


FIGURE 3: DSDNODP

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.336	-0.336	7.8939	0.005
		2	0.033	-0.090	7.9697	0.019
		3	-0.014	-0.036	7.9833	0.046
		4	-0.045	-0.066	8.1286	0.087
		5	-0.216	-0.290	11.601	0.041
		6	0.115	-0.085	12.609	0.050
		7	0.048	0.051	12.790	0.077
		8	0.073	0.119	13.203	0.105
		9	0.077	0.138	13.679	0.134
		10	0.012	0.066	13.691	0.188
		11	0.047	0.158	13.872	0.240
		12	-0.379	-0.320	25.965	0.011
		13	0.124	-0.113	27.274	0.011
		14	0.005	0.031	27.277	0.018
		15	-0.078	-0.100	27.824	0.023
		16	0.048	-0.112	28.029	0.031
		17	0.115	-0.124	29.254	0.032
		18	-0.186	-0.215	32.521	0.019
		19	0.130	0.048	34.148	0.018
		20	0.007	0.155	34.153	0.025
		21	-0.137	0.008	36.031	0.022
		22	-0.004	-0.056	36.033	0.030
		23	0.124	0.134	37.659	0.028
		24	-0.067	-0.033	38.146	0.033
		25	-0.083	-0.157	38.906	0.038
		26	0.051	-0.076	39.194	0.047
		27	0.085	0.030	40.032	0.051
		28	-0.048	-0.009	40.304	0.062

Figure 4: Correlogram of DSDNODP

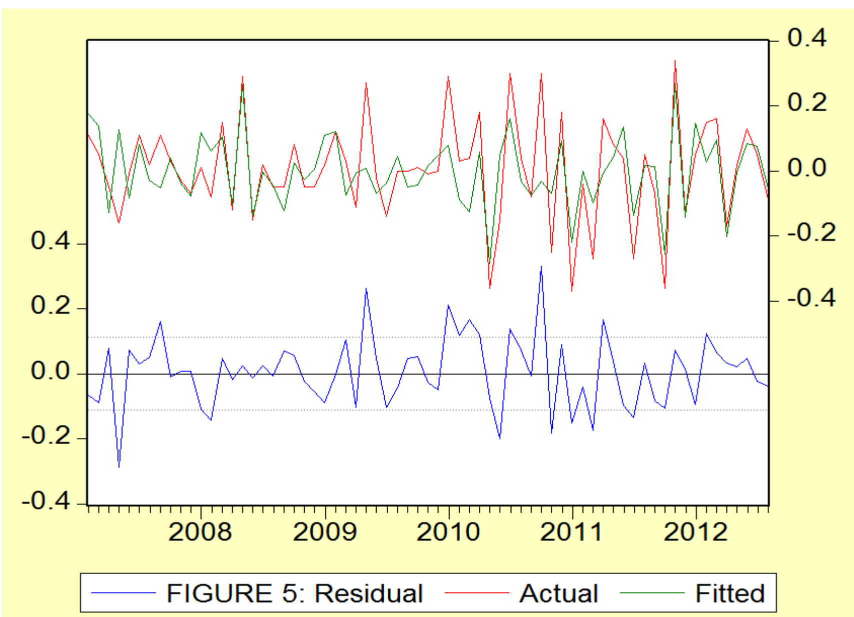
Table 1: Model Estimation

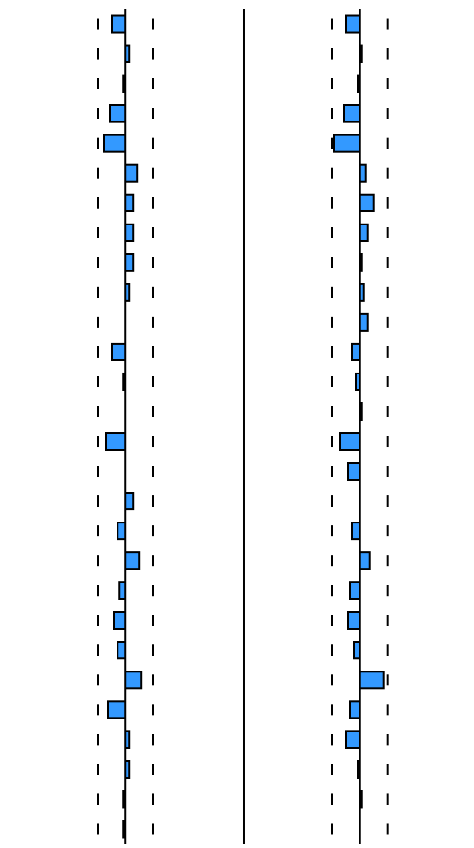
Dependent Variable: DSDNODP
Method: Least Squares
Date: 02/09/13 Time: 11:15
Sample(adjusted): 2007:02 2012:08
Included observations: 67 after adjusting endpoints
Convergence achieved after 67 iterations
Backcast: 2006:01 2007:01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.317378	0.107714	-2.946500	0.0045
MA(12)	-0.844173	0.055647	-15.17024	0.0000
MA(13)	0.247092	0.122723	2.013413	0.0483

R-squared	0.507915	Mean dependent var	0.003881
Adjusted R-squared	0.492537	S.D. dependent var	0.156166
S.E. of regression	0.111247	Akaike info criterion	-1.510386
Sum squared resid	0.792056	Schwarz criterion	-1.411669
Log likelihood	53.59795	F-statistic	33.02937
Durbin-Watson stat	2.247500	Prob(F-statistic)	0.000000

Inverted MA Roots	.99	.86+.49i	.86 -.49i	.50 -.85i
	.50+.85i	.29	.00 -.99i	.00+.99i
	-.49 -.85i	-.49+.85i	-.85 -.49i	-.85+.49i
	-.98			





	AC	PAC	Q-Stat	Prob
1	-0.131	-0.131	1.2018	
2	0.036	0.019	1.2948	
3	-0.016	-0.009	1.3143	
4	-0.138	-0.144	2.7054	0.100
5	-0.186	-0.230	5.2818	0.071
6	0.100	0.053	6.0458	0.109
7	0.081	0.121	6.5546	0.161
8	0.059	0.058	6.8241	0.234
9	0.065	0.014	7.1606	0.306
10	0.037	0.034	7.2713	0.401
11	-0.002	0.080	7.2717	0.508
12	-0.125	-0.073	8.5824	0.477
13	-0.018	-0.041	8.6102	0.569
14	0.000	0.011	8.6102	0.658
15	-0.167	-0.177	11.078	0.522
16	0.005	-0.103	11.081	0.604
17	0.063	-0.002	11.443	0.651
18	-0.062	-0.058	11.802	0.694
19	0.131	0.090	13.443	0.640
20	-0.054	-0.090	13.732	0.686
21	-0.103	-0.114	14.807	0.675
22	-0.065	-0.043	15.239	0.707
23	0.139	0.206	17.270	0.635
24	-0.164	-0.091	20.150	0.512
25	0.027	-0.120	20.228	0.569
26	0.026	-0.021	20.304	0.623
27	-0.016	0.017	20.335	0.678
28	-0.024	0.002	20.403	0.725

Figure 6: Correlogram of the Residuals

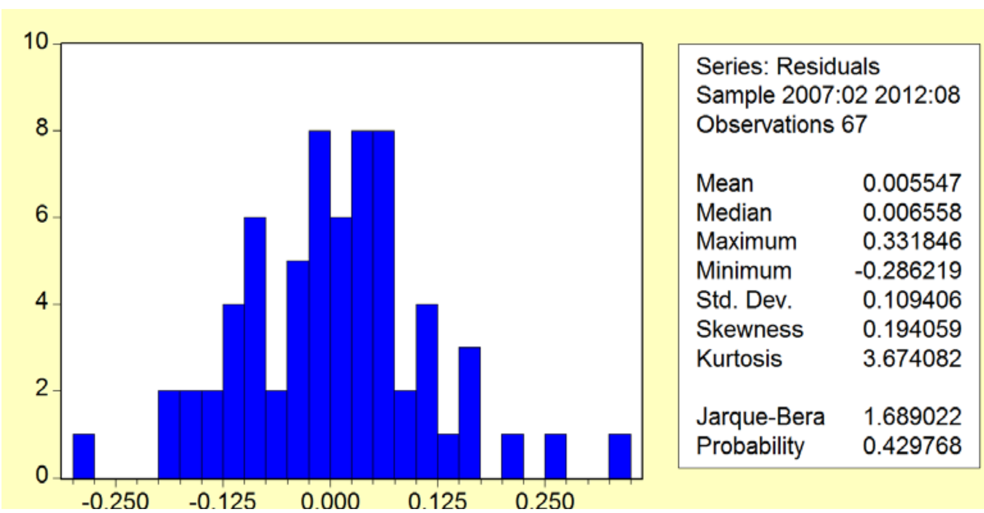


Figure 7: Histogram of the Residuals

component is involved. Moreover a $(0, 1, 1)x(0, 1, 1)_{12}$ SARIMA model is suggestive.

Estimation of the model in Table 1 yields:

$$DSDNDOP_t + 0.3174\varepsilon_{t-1} + 0.8442\varepsilon_{t-12} - 0.2471\varepsilon_{t-13} = \varepsilon_t \quad (6)$$

$$(\pm 0.1077) \quad (\pm 0.0556) \quad (\pm 0.1227)$$

It may be observed that all the coefficients of the model are stastically significant. The model, with an R^2 value of 51%, explains as high as 0.51 of the variation in DSDNODP. There is a close agreement between the fitted model and the data (See Figure 5). The correlogram of the residuals in Figure 6 shows that the residuals are uncorrelated. The histogram of the residuals in Figure 7 shows that the residuals have zero mean and follow a Gaussian distribution. All these are indications that the model (6) is adequate.

4 Conclusion

It has been shown that Nigerian Crude Oil Domestic Production follows a $(0, 1, 1)x(0, 1, 1)_{12}$ SARIMA model. It has been shown to be adequate by many approaches.

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