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Subclasses of Analytic Functions with Respect to Symmetric and Conjugate Points

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Abstract

In this paper, we introduce new subclasses of analytic functions with respect to other points. The coefficient estimates for these classes are obtained.

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1 Introduction

Let U be the class of functions which are analytic and univalent in the open unit disc $D = \{z : |z| < 1\}$ given by

$$w(z) = z + \sum_{k=1}^{n} b_k z^k$$

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and satisfying the conditions

$$w(0)=0, |w(z)|<1, \quad z \in D.$$

Let S denote the class of functions f which are analytic and univalent in D of the form

$$f(z)=z+\sum_{n=2}^{\infty} a_n z^n. \tag{1}$$

Also let S_S^* be the subclass of S consisting of functions given by (1) satisfying

$$Re\left\{\frac{zf'(z)}{f(z)-f(-z)}\right\}>0, \quad z \in D.$$

These functions are called starlike with respect to symmetric points and were introduced by Sakaguchi in 1959. Ashwah and Thomas in [2] introduced another class namely the class S_C^* consisting of functions starlike with respect to conjugate points.

Let S_C^* be the subclass of S consisting of functions given by (1) and satisfying the condition

$$Re\left\{\frac{zf'(z)}{f(z)+\overline{f(\overline{z})}}\right\}>0, \quad z \in D.$$

Motivated by S_S^*, many authors discussed the following class C_S of functions convex with respect to symmetric points and its subclasses.

Let C_S be the subclass of S consisting of functions given by (1) and satisfying the condition

$$Re\left\{\frac{(zf'(z))'}{(f(z)-f(-z))'}\right\}>0, \quad z \in D.$$

In terms of subordination, Goel and Mehrok in 1982 introduced a subclass of S_S^*, denoted by $S_S^*(A,B)$.

Let $S_S^*(A,B)$ be the class of functions of the form (1) and satisfying the condition

$$\frac{2zf'(z)}{f(z)-f(-z)} \prec \frac{1+Az}{1+Bz}, \quad -1 \leq B < A \leq 1, z \in D.$$

Also let $S_C^*(A,B)$ be the class of functions of the form (1) and satisfying the condition

$$\frac{2zf'(z)}{(f(z)+\overline{f(\overline{z})})} \prec \frac{1+Az}{1+Bz}, \quad -1 \leq B < A \leq 1, z \in D.$$

Let $C_S(A,B)$ be the class of functions of the form (1) and satisfying the condition

$$\frac{2(zf'(z))'}{(f(z)-f(-z))'} \prec \frac{1+Az}{1+Bz}, \quad -1 \le B < A \le 1, z \in D.$$

Also let $C_C(A,B)$ be the class of functions of the form (1) and satisfying the condition

$$\frac{2(zf'(z))'}{(f(z)+\overline{f(\overline{z})})'} \prec \frac{1+Az}{1+Bz}, \quad -1 \le B < A \le 1, z \in D.$$

In this paper, we introduce the class $M_S(\rho,\mu,A,B)$ consisting of analytic functions f of the form (1) and satisfying

$$\frac{2[\rho\mu z^3 f'''(z)+(2\rho\mu+\rho-\mu)z^2 f''(z)+zf'(z)]}{\rho\mu z^2[f''(z)-f''(-z)]+(\rho-\mu)z[f'(z)+f'(-z)]+(1-\rho+\mu)[f(z)-f(-z)]} \prec \frac{1+Az}{1+Bz}$$

$$-1 \le B < A \le 1, 0 \le \mu \le \rho \le 1, z \in D.$$

We note that $M_S(0,0,A,B) = S_S^*(A,B)$ and $M_S(1,0,A,B) = C_S(A,B)$. Also introduce the class $M_C(\rho,\mu,A,B)$ consisting of analytic functions f of the form (1) and satisfying

$$\frac{2[\rho\mu z^3 f'''(z)+(2\rho\mu+\rho-\mu)z^2 f''(z)+zf'(z)]}{\rho\mu z^2(f(z)+\overline{f(\overline{z})})''+(\rho-\mu)z(f(z)+\overline{f(\overline{z})})'+(1-\rho+\mu)(f(z)+\overline{f(\overline{z})})} \prec \frac{1+Az}{1+Bz}$$

$$-1 \le B < A \le 1, 0 \le \mu \le \rho \le 1, z \in D.$$

Note that $M_C(0,0,A,B) = S_C^*(A,B)$ and $M_C(1,0,A,B) = C_C(A,B)$.

By definition of subordination it follows that $f \in M_S(\rho,\mu,A,B)$ if and only if

$$\frac{2[\rho\mu z^3 f'''(z)+(2\rho\mu+\rho-\mu)z^2 f''(z)+zf'(z)]}{\rho\mu z^2[f''(z)-f''(-z)]+(\rho-\mu)z[f'(z)+f'(-z)]+(1-\rho+\mu)[f(z)-f(-z)]} = \frac{1+Aw(z)}{1+Bw(z)} = p(z), \tag{2}$$

$w \in U$ and that $f \in M_C(\rho,\mu,A,B)$ if and only if

$$\frac{2[\rho\mu z^3 f'''(z)+(2\rho\mu+\rho-\mu)z^2 f''(z)+zf'(z)]}{\rho\mu z^2(f(z)+\overline{f(\overline{z})})''+(\rho-\mu)z(f(z)+\overline{f(\overline{z})})'+(1-\rho+\mu)(f(z)+\overline{f(\overline{z})})} = \frac{1+Aw(z)}{1+Bw(z)} = p(z), \quad w \in U \tag{3}$$

where

$$p(z) = 1 + \sum_{n=1}^{\infty} p_n z^n \tag{4}$$

We study the classes $M_S(\rho, \mu, A, B)$ and $M_C(\rho, \mu, A, B)$, the coefficient estimates for functions belonging to these classes are obtained. We also need the following lemma for proving our results.

Lemma 1.1. *[3] If $p(z)$ is given by (4) then*

$$|p_n| \leq A - B, \quad n = 1, 2, 3, \ldots. \tag{5}$$

2 Main Results

In this section, we give the coefficient inequalities for the classes $M_S(\rho, \mu, A, B)$ and $M_C(\rho, \mu, A, B)$.

Theorem 2.1. *Let $f \in M_S(\rho, \mu, A, B)$. Then for $n \geq 1$, $0 \leq \mu \leq \rho \leq 1$*

$$|a_{2n}| \leq \frac{(A-B)}{2^n n![(2n-1)(2n)\rho\mu + (2n-1)(\rho-\mu) + 1]} \prod_{j=1}^{n-1} (A - B + 2j) \tag{6}$$

$$|a_{2n+1}| \leq \frac{(A-B)}{2^n n![(2n+1)(2n)\rho\mu + (2n)(\rho-\mu) + 1]} \prod_{j=1}^{n-1} (A - B + 2j) \tag{7}$$

Proof. From (2) and (4), we have

$$\begin{aligned}
&\big\{\rho\mu[6a_3z^3 + 24a_4z^4 + \cdots + (2n)(2n-1)(2n-2)a_{2n}z^{2n} + \cdots]\\
&+ (2\rho\mu + \rho - \mu)[2a_2z^2 + 6a_3z^3 + \cdots + (2n-1)(2n)a_{2n}z^{2n} + \cdots]\\
&+[z + 2a_2z^2 + 3a_3z^3 + \cdots + 2na_{2n}z^{2n} + \cdots]\big\}\\
&= \big\{\rho\mu\big[6a_3z^3 + 20a_5z^5 + \cdots + (2n-1)(2n-2)a_{2n-1}z^{2n-1}\\
&+ (2n+1)(2n)a_{2n+1}z^{2n+1} + \cdots\big]\\
&+ (\rho - \mu)[z + 3a_3z^3 + \cdots + (2n-1)a_{2n-1}z^{2n-1} + (2n+1)a_{2n+1}z^{2n+1} + \cdots]\\
&+(1 - \rho + \mu)[z + a_3z^3 + \cdots + a_{2n-1}z^{2n-1} + a_{2n+1}z^{2n+1} + \cdots]\big\}\\
&\big\{1 + p_1z + p_2z^2 + \cdots + p_{2n-1}z^{2n-1} + p_{2n}z^{2n} + \cdots\big\}
\end{aligned}$$

Equating the coefficients of like powers of z, we have

$$2a_2[2\rho\mu + (\rho - \mu) + 1] = p_1, \quad 2a_3[6\rho\mu + 2(\rho - \mu) + 1] = p_2 \tag{8}$$

$$\left.\begin{aligned} 4a_4[12\rho\mu + 3(\rho - \mu) + 1] &= p_3 + a_3 p_1[6\rho\mu + 2(\rho - \mu) + 1] \\ 4a_5[20\rho\mu + 4(\rho - \mu) + 1] &= p_4 + a_3 p_2[6\rho\mu + 2(\rho - \mu) + 1] \end{aligned}\right\} \tag{9}$$

$$\begin{aligned} &2na_{2n}[(2n-1)(2n)\rho\mu + (2n-1)(\rho - \mu) + 1] \\ &= p_{2n-1} + p_{2n-3}a_3[6\rho\mu + 2(\rho - \mu) + 1] + \cdots \\ &\qquad + p_1 a_{2n-1}[(2n-1)(2n-2)\rho\mu + (2n-2)(\rho - \mu) + 1] \end{aligned} \tag{10}$$

$$\begin{aligned} &(2n)a_{2n+1}[(2n+1)(2n)\rho\mu + 2n(\rho - \mu) + 1] \\ &= p_{2n} + p_{2n-2}a_3[6\rho\mu + 2(\rho - \mu) + 1] + \cdots \\ &\qquad + p_2 a_{2n-1}[(2n-1)(2n-2)\rho\mu + (2n-2)(\rho - \mu) + 1] \end{aligned} \tag{11}$$

Using Lemma 1.1 and (8), we get

$$|a_2| \leq \frac{(A-B)}{2[2\rho\mu + (\rho - \mu) + 1]}, \quad |a_3| \leq \frac{(A-B)}{2[6\rho\mu + 2(\rho - \mu) + 1]} \tag{12}$$

Again by applying (11) and followed by Lemma 1.1, we get from (9)

$$|a_4| \leq \frac{(A-B)(A-B+2)}{(2)(4)[12\rho\mu + 3(\rho - \mu) + 1]}, \quad |a_5| \leq \frac{(A-B)(A-B+2)}{(2)(4)[20\rho\mu + 4(\rho - \mu) + 1]}$$

It follows that (6) and (7) hold for $n = 1, 2$. We prove (6) using induction.

Equation (10) in conjunction with Lemma 1.1 yield

$$\begin{aligned} |a_{2n}| \leq\ & \frac{(A-B)}{2n[(2n-1)(2n)\rho\mu + (2n-1)(\rho - \mu) + 1]} \\ & \left[1 + \sum_{k=1}^{n-1} [(2k+1)(2k)\rho\mu + 2k(\rho - \mu) + 1]|a_{2k+1}|\right] \end{aligned} \tag{13}$$

We assume that (6) holds for $k = 3, 4, \ldots, (n-1)$. Then from (13), we obtain

$$\begin{aligned} |a_{2n}| \leq\ & \frac{(A-B)}{2n[(2n-1)(2n)\rho\mu + (2n-1)(\rho - \mu) + 1]} \cdot \\ & \cdot \left[1 + \sum_{k=1}^{n-1} \frac{(A-B)}{2^k k!} \prod_{j=1}^{k-1} (A - B + 2j)\right] \end{aligned} \tag{14}$$

In order to complete the proof, it is sufficient to show that

$$\frac{(A-B)}{2m[(2m-1)(2m)\rho\mu+(2m-1)(\rho-\mu)+1]}\cdot \cdot\left[1+\sum_{k=1}^{m-1}\frac{(A-B)}{2^k k!}\prod_{j=1}^{k-1}(A-B+2j)\right] = \frac{(A-B)}{2^m m![(2m-1)(2m)\rho\mu+(2m-1)(\rho-\mu)+1]}\prod_{j=1}^{m-1}(A-B+2j), \quad (15)$$

$m=3,4,\ldots,n$.
(15) is valid for $m=3$.

Let us suppose that (15) is true for all m, $3<m\leq(n-1)$. Then from (14)

$$\begin{aligned}
&\frac{(A-B)}{2n[(2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1]}\cdot \\
&\quad\cdot\left[1+\sum_{k=1}^{n-1}\frac{(A-B)}{2^k k!}\prod_{j=1}^{k-1}(A-B+2j)\right] \\
&= \frac{(n-1)}{n}\cdot\frac{(A-B)}{2(n-1)[(2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1]}\cdot \\
&\quad\cdot\left[1+\sum_{k=1}^{n-2}\frac{(A-B)}{2^k k!}\prod_{j=1}^{k-1}(A-B+2j)\right] \\
&\quad+\frac{(A-B)}{2n[(2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1]}\;\frac{(A-B)}{2^{n-1}(n-1)!}\cdot \\
&\quad\cdot\prod_{j=1}^{n-2}(A-B+2j) \\
&= \frac{(n-1)}{n}\cdot\frac{(A-B)}{2^{n-1}(n-1)![(2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1]}\cdot \\
&\quad\cdot\prod_{j=1}^{n-2}(A-B+2j) \\
&\quad+\frac{(A-B)}{2n[(2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1]}\;\frac{(A-B)}{2^{n-1}(n-1)!}\cdot \\
&\quad\cdot\prod_{j=1}^{n-2}(A-B+2j)
\end{aligned}$$

$$= \frac{(A-B)}{2^{n-1}(n-1)![(2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1]} \cdot$$
$$\cdot \prod_{j=1}^{n-2}(A-B+2j)(A-B+2(n-1))$$
$$= \frac{(A-B)}{2^{n}n![(2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1]} \prod_{j=1}^{n-1}(A-B+2j)$$

Thus (15) holds for $m = n$ and hence (6) follows.
Similarly we can prove (7). □

Theorem 2.2. *Let $f \in M_C(\rho,\mu,A,B)$. Then for $n \geq 1$, $0 \leq \mu \leq \rho \leq 1$*

$$|a_{2n}| \leq \frac{(A-B)}{(2n-1)![(2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1]} \prod_{j=1}^{2n-2}(A-B+j) \quad (16)$$

$$|a_{2n+1}| \leq \frac{(A-B)}{(2n)![(2n+1)(2n)\rho\mu+(2n)(\rho-\mu)+1]} \prod_{j=1}^{2n-1}(A-B+j) \quad (17)$$

Proof. From (3) and (4), we have

$$\begin{aligned}
&\{\rho\mu[6a_3z^3+24a_4z^4+\cdots+(2n)(2n-1)(2n-2)a_{2n}z^{2n}+\cdots]\\
&+(2\rho\mu+\rho-\mu)[2a_2z^2+6a_3z^3+\cdots+(2n-1)(2n)a_{2n}z^{2n}+\cdots]\\
&+[z+2a_2z^2+3a_3z^3+\cdots+2na_{2n}z^{2n}+\cdots]\}\\
&=\{\rho\mu[2a_2z^2+6a_3z^3+\cdots+(2n-1)(2n)a_{2n}z^{2n}+\cdots]\\
&+(\rho-\mu)[z+2a_2z^2+\cdots+2na_{2n}z^{2n}+\cdots]\\
&+(1-\rho+\mu)[z+a_2z^2+\cdots+a_{2n}z^{2n}+\cdots]\}\\
&\{1+p_1z+p_2z^2+\cdots+p_{2n}z^{2n}+\cdots\}
\end{aligned}$$

Equating the coefficients of like powers of z, we have

$$a_2(2\rho\mu+(\rho-\mu)+1)=p_1,\ \ 2a_3(6\rho\mu+2(\rho-\mu)+1)=p_2+a_2p_1(2\rho\mu+(\rho-\mu)+1) \quad (18)$$

$$3a_4(12\rho\mu+3(\rho-\mu)+1)=p_2+a_2p_2(2\rho\mu+(\rho-\mu)+1)+a_3p_1(6\rho\mu+2(\rho-\mu)+1) \quad (19)$$

$$\begin{aligned}4a_5(20\rho\mu + 4(\rho-\mu)+1) &= p_4 + a_2p_3(2\rho\mu + (\rho-\mu)+1)\\ &\quad + a_3p_2(6\rho\mu + 2(\rho-\mu)+1)\\ &\quad + a_4p_1(12\rho\mu + 3(\rho-\mu)+1) \qquad (20)\end{aligned}$$

$$\begin{aligned}&(2n-1)a_{2n}((2n-1)(2n)\rho\mu + (2n-1)(\rho-\mu)+1)\\ &\quad = p_{2n-1} + a_2p_{2n-2}(2\rho\mu + (\rho-\mu)+1)\\ &\qquad + \cdots + a_{2n-1}p_1((2n-2)(2n-1)\rho\mu + (2n-2)(\rho-\mu)+1) \qquad (21)\end{aligned}$$

$$\begin{aligned}&(2n)a_{2n+1}((2n+1)(2n)\rho\mu + (2n)(\rho-\mu)+1)\\ &\quad = p_{2n} + a_2p_{2n-1}(2\rho\mu + (\rho-\mu)+1) + \cdots\\ &\qquad + a_{2n}p_1((2n)(2n-1)\rho\mu + (2n-1)(\rho-\mu)+1) \qquad (22)\end{aligned}$$

By using Lemma 1.1 and (18), we get

$$|a_2| \le \frac{(A-B)}{[2\rho\mu + (\rho-\mu)+1]}, \quad |a_3| \le \frac{(A-B)(A-B+1)}{2(6\rho\mu + 2(\rho-\mu)+1)} \qquad (23)$$

Again by applying (23) and followed by Lemma 5, we get from (19) and (20), we have

$$|a_4| \le \frac{(A-B)(A-B+1)(A-B+2)}{(2)(3)(12\rho\mu + 3(\rho-\mu)+1)}$$

$$|a_5| \le \frac{(A-B)(A-B+1)(A-B+2)(A-B+3)}{(2)(3)(4)(20\rho\mu + 4(\rho-\mu)+1)}$$

It follows that (16) hold for $n = 1, 2$. We now prove (16) using induction. Equation (21) in conjunction with Lemma 1.1 yield

$$\begin{aligned}|a_{2n}| &\le \frac{(A-B)}{(2n-1)[(2n-1)(2n)\rho\mu + (2n-1)(\rho-\mu)+1]}\\ &\quad \times \left[1 + \sum_{k=1}^{n-1}|a_{2k}| + \sum_{k=1}^{n-1}|a_{2k+1}|\right] \qquad (24)\end{aligned}$$

We assume that (16) holds for $k = 3, 4, \ldots, (n-1)$. Then from (24), we obtain

$$\begin{aligned}|a_{2n}| &\le \frac{(A-B)}{(2n-1)[(2n-1)(2n)\rho\mu + (2n-1)(\rho-\mu)+1]}\\ &\quad \times \left[1 + \sum_{k=1}^{n-1}\frac{A-B}{(2k-1)!}\prod_{j=1}^{2k-2}(A-B+j) + \sum_{k=1}^{n-1}\frac{(A-B)}{(2k)!}\prod_{j=1}^{2k-1}(A-B+j)\right] \qquad (25)\end{aligned}$$

In order to complete the proof, it is sufficient to show that

$$\frac{(A-B)}{(2m-1)[(2m-1)(2m)\rho\mu+(2m-1)(\rho-\mu)+1]}\cdot$$
$$\cdot\left[1+\sum_{k=1}^{m-1}\frac{A-B}{(2k-1)!}\prod_{j=1}^{2k-2}(A-B+j)+\sum_{k=1}^{m-1}\frac{(A-B)}{(2k)!}\prod_{j=1}^{2k-1}(A-B+j)\right]$$
$$=\frac{(A-B)}{(2m-1)!((2m-1)(2m)\rho\mu+(2m-1)(\rho-\mu)+1)}\cdot$$
$$\cdot\prod_{j=1}^{2m-2}(A-B+j), \tag{26}$$

$m=3,4,5,\ldots,n$. (3.21) is valid for $m=3$.
Let us suppose that (3.21) is true for all m, $3<m\leq(n-1)$. Then from (25)

$$\frac{(A-B)}{(2n-1)[(2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1]}\cdot$$
$$\cdot\left[1+\sum_{k=1}^{n-1}\frac{A-B}{(2k-1)!}\prod_{j=1}^{2k-2}(A-B+j)+\sum_{k=1}^{n-1}\frac{(A-B)}{(2k)!}\prod_{j=1}^{2k-1}(A-B+j)\right]$$
$$=\frac{(2n-3)}{(2n-1)}\frac{(A-B)}{(2(n-1)-1)((2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1)}\cdot$$
$$\cdot\left[1+\sum_{k=1}^{n-2}\frac{A-B}{(2k-1)!}\prod_{j=1}^{2k-2}(A-B+j)+\sum_{k=1}^{n-2}\frac{(A-B)}{(2k)!}\prod_{j=1}^{2k-1}(A-B+j)\right]$$
$$+\frac{(A-B)}{(2n-1)((2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1)}\cdot$$
$$\cdot\left[\frac{A-B}{(2(n-1)-1)!}\prod_{j=1}^{2n-4}(A-B+j)\right]$$
$$+\frac{(A-B)}{(2n-1)((2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1)}\frac{A-B}{(2(n-1))!}\prod_{j=1}^{2n-3}(A-B+j)$$
$$=\frac{(2n-3)}{(2n-1)}\frac{(A-B)}{(2(n-1)-1)!((2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1)}\prod_{j=1}^{2n-4}(A-B+j)$$
$$+\frac{(A-B)}{(2n-1)((2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1)}$$
$$\cdot\frac{A-B}{(2(n-1)-1)!}\prod_{j=1}^{2n-4}(A-B+j)$$

$$= \frac{(A-B)}{(2n-1)(2(n-1)-1)!((2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1)} \cdot \\ \cdot \prod_{j=1}^{2n-3}(A-B+j) \\ + \frac{(A-B)}{(2n-1)((2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1)} \frac{A-B}{(2(n-1))!} \cdot \\ \cdot \prod_{j=1}^{2n-3}(A-B+j) \\ = \frac{(A-B)}{(2n-1)!((2n-1)(2n)\rho\mu+(2n-1)(\rho-\mu)+1)} \prod_{j=1}^{2n-2}(A-B+j)$$

Thus (26) holds for $m = n$ and hence (16) follows. Similarly we can prove (17). □

On specializing the values of ρ, μ in Theorem 2.1 and 2.2, we get the following.

Remark 2.3. *In Theorem 2.1, if we set $\mu = 0$ and $\rho = 0$, we get starlike functions with respect to symmetric points and if we set $\mu = 0$ and $\rho = 1$, we get convex functions with respect to symmetric points.*

Remark 2.4. *In Theorem 2.2, if we set $\mu = 0$ and $\rho = 0$, we get starlike functions with respect to conjugate points and if we set $\mu = 0$ and $\rho = 1$, we get convex functions with respect to conjugate points. For other values of μ and ρ, the transition is smooth.*

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