

# **Mean-Reverting-Ebit-Based Stock Option Evaluation: Theory and Practice**

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## **Abstract**

This paper examines the derivation of a capital-structure EBIT-based call option expression with the El Ibrami and Naciri [2012] equity value as the underlying asset. The model's PDE and ODE are similar to Black-Scholes but have widely different and non-constant coefficients. An empirical analysis of the new model is conducted to measure its performance, using the last close price of the evaluated stock options and the Black-Scholes values as benchmarks. The results show that the author's model is robust, whereas the Black-Scholes model overestimates the stock options.

**JEL classification numbers:** F47, G12, G13, G21, G22, G23, G24.

**Keywords:** ODE, PDE, Simulation, Stock Option, Structural Models.

## **1 Introduction**

Structural models are seldom used to analyze stock options. Instead, the literature seems to be replete with research on index derivatives. Whereas these studies seem to be divided between theory and empirical analysis, we think that the only structural model used for stock options analysis is Prucyk and Toft [1997]. The authors used the Leland [1994] equity expression as the underlying asset of their stock option value, but applied the Black-Scholes stock option formula to determine this expression, a strategy that seems inappropriate given that Black-Scholes is not a capital structure model.

In fact, market models used to determine stock option values fail to consider the accounting information included in financial statements. The expressions obtained by using such models do not lead to the determination of the fair value of derivative financial instruments. That being said, the EBIT value should be used to determine the right value of financial instruments, as advocated by financial theory. This variable should follow a

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mean reverting process. Accordingly, we use the El Ibrami and Naciri [2012] model to derive a stock option's PDE and expression.

Contrary to Prucyk and Toft's [1997] analysis, we derive this expression by using Ito's lemma, Taylor expansion and risk neutrality of a hedged portfolio. The ODE coefficients are not constant, and the correlation between the stock option evaluated and its underlying asset is impossible to obtain analytically. Using a Newton-Raphson optimizer, we numerically determine the implied coupon associated with the El Ibrami and Naciri [2012] equity value. The same implied coupon is used in a simulation to determine the stock option's value.

An empirical analysis allows us to measure the model's performance in evaluating stock options. As benchmarks, we use the last close price of the listed stock options and the Black-Scholes value for unlisted options. The analysis will proceed as follows: Section 2 contains a literature review, Section 3 explains the methodology, Section 4 discusses the stock option expression derivation, and Section 5 presents the empirical analysis. The conclusion is presented in Section 6.

## 2 Literature Review

As stated previously, Toft and Prucyk [1997] were the only authors to determine a stock option value based on structural models, using the Leland [1994] equity expression as the underlying asset and the Black-Scholes [1973] formula for obtaining a stock option equation. They warned that financial leverage and debt covenants could invalidate the use of the Black-Scholes model due to the implied volatility bias. This bias becomes more pronounced in the case of a high leverage or protected debt, which reduces the out-of-the-money stock options' value. In addition, short-term debt produces additional costs. This finding corroborates Leland's suggestion to use long-term debt to evaluate the underlying asset.

Toft and Prucyk [1997] used structural endogenous volatility when deriving their stochastic equation and thereby demonstrated that stock price is a positive function of endogenous volatility, owing to the convexity of the equity-value curve. This model also assumed an endogenous bankruptcy, which causes a bias in the Black-Scholes implied volatility in the case of a pronounced short term-long term debt ratio.

The authors demonstrated that protected debt for companies that pay both coupons and dividends produces a concave relationship between equity and the value of the company, which weakens the theory of real options as an alternative solution for evaluating equity. In addition, endogenous bankruptcy is the only way to obtain a convex relationship between the two parameters. Hence, equity should be considered as a call option written on the value of the company, which avoids an overestimation of the company caused by exogenous bankruptcy, triggering pronounced volatility. The authors obtained a closed form solution for their PDE, which allowed them to determine the value of a call and put, written on the Leland [1994] equity value, using the Black-Scholes formula.

Perrakis and Ryan [1984] explained that the Black-Scholes model serves to estimate the value of a stock option written on equity, but not its fair value. In the same vein, Constantinides, Jackwerth and Perrakis [2005] found that the Black-Scholes model is useful only in complete markets with constant volatility and in transactions that do not follow jump processes; hence, the Black-Scholes frictionless market model is only a special case within option pricing valuation models.

Using linear programming to derive the upper and lower bounds of Perrakis and Ryan, Ritchken [1985] estimated an option value by using an objective function optimizer, and obtained its bounds by maximizing and minimizing the function. The upper bound thus obtained was similar to Perrakis and Ryan, but the lower bound was obtained only in the case of a deep-in-the-money option.

For his part, Levy [1985] argued that transaction costs and taxes do not hinder option evaluation unless portfolios are continuously adjusted by investors. His model thus offers affordable solutions for discrete transactions and market frictions. Similar to Perrakis and Ryan [1984] and Ritchken [1985], the author obtained a range of values containing the equilibrium price.

Ritchken and Kuo [1988] added that Perrakis and Ryan bounds require more information for their use than Black-Scholes does, but that unlike the Black-Scholes and Binomial models, these bounds do not require consideration of continuous hedge hypotheses, which makes them applicable to both negotiated and non-negotiated contingent claims. The authors also obtained upper bounds similar to Perrakis and Ryan by using a multinomial model; however, their lower bounds were tighter.

Some studies highlighted options valuation empirically, mainly by using equilibrium models and market indexes as underlying assets. Bakshi, Kapadia and Madan [2003] used a sample of 358,851 call and put options quoted in the S&P 100 Index and written on the 30 most liquid securities listed on the Chicago Stock Exchange. Covering the period from January 1991 to December 1995, their analysis emphasized risk neutrality distribution skew causes and how they impact options price. The authors demonstrated that investor risk aversion influences the distribution's kurtosis and skewness coefficient. The distribution skew is affected by systematic and specific risk. However, they also demonstrated that stock options have less implied volatility than index options because of their more negative skewness. As a result, they concluded that stock options can be more accurately evaluated than index options.

Bollen and Whaley [2004] considered that this implied volatility increases or decreases according to the investor's position in the index, a condition that helps achieve price equilibrium. The authors used bid and ask prices of the S&P 500's implied volatility between June 1988 and 2000 and the implied volatility of the 20 most active stock options in the index between January 1995 and January 2000. They assumed that investors who preferred the index would invest in put options while those inclined toward stock options would choose call options. These assumptions would aid in understanding the difference between the two forms of implied volatility. The results show that index implied volatility is greater than stock option volatility, a condition that drives up index option costs and increases the skewness of their asymmetric profits curve. The authors concluded that the resulting implied volatility bias should not be attributed solely to the complexity of the unobservable underlying asset process, but also to the arbitrageurs' limited capacity to absorb losses, which pushes prices up. Hence, depending on investor activity, market maintainers readjust these prices, a strategy that helps them understand price dynamics and establish the risk premium to correct the implied volatility bias.

According to Driessen, Maenhout and Vilkov [2009], the options price bias is attributable to the correlation risk reducing the diversification effect. They compared daily quoted prices of options on S&P 100 stocks with prices quoted between January 1996 and December 2003. They selected these particular options because they were markedly continuously active during that period and had a short expiry and low implied volatility. The authors broke down market risk into systematic and correlation risk, and

demonstrated that index options are more expensive than stock options. They concluded that the price spread can be explained only by correlation risk, which is rewarded by a negative risk premium representing the diversification cost; as a result, the CAPM can be used to determine the fair value of stock options, but not that of index options, due to correlation risk.

Duan and Wei [2009] reasoned that a systematically high risk level triggers high implied volatility levels as well as a pronounced trend of implied volatility. The spread between two stock option prices can be explained only by the systematic risk proportion of their respective underlying assets. To conduct their study, the authors analyzed daily S&P 100 index options data and the 30 largest stocks on that index from January 1991 to December 1995. Similar to Bakshi, Kapadia and Madan, the authors concluded that the difference between real and risk-neutral distributions was due to systematic risk that can be measured by systematic variance scaled by total variance. The authors also demonstrated that implied volatility behavior is explained by systematic risk in the sense that a negative risk premium produces a high implied volatility level, but also high skewness and kurtosis coefficients. Disagreeing with Bollen and Whaley, who assumed that net buying pressure influences the slope and level of implied volatility, the authors pointed out that these assertions were not demonstrated and that only index implied volatility were considered, rather than all types of options.

Goyal and Saretto [2009] addressed the same issue by investigating the difference between realized and at-the-money<sup>2</sup> implied volatility. They conducted their study on one-month-expiry daily options data covering January 1996 to December 2006 in order to obtain a homogenous sample of liquid options. After eliminating illiquid options, non-continuous bid-ask data and data in which bid values exceeded ask values, they obtained an evaluation sample of 4, 244 stocks used as underlying assets for 75,627 call and put stock options. The analysis shows that high returns were due to the spread between realized and implied volatility. The bias observed in the volatility, accompanied by the kurtosis and skewness coefficients biases, explains the unfair price of the options evaluated. The high levels of return observed are not attributable solely to the correlation between the option and its underlying asset, but also to the misspecification of the models used to make the estimation. They concluded that both historical and implied volatility may be used to determine future volatility, thanks to the pertinent and non-contradictory information they contain, if well-specified evaluation models are used.

The body of literature reviewed was composed mainly of theoretical and empirical studies. Some research used equilibrium models, while others used structural models. Most of the studies centered on options with market indexes as underlying assets. As well, very few researchers used structural models to compare stock and index options. Our objective was to derive the fair value expression of stock options using a structural model and the El Ibrami and Naciri [2012] equity as the underlying asset to make the derivation. This strategy would be convenient for companies whose state variable follows a mean-reverting process. As suggested by the foregoing authors, the state variable should be the EBIT. We used the dynamic equation of EN to obtain the PDE followed by a derivative instrument, and then solved the ordinary equation to determine the expression of our call option. This model avoids the implied volatility problem and market speculations, and is used to determine a stock's fair value using financial statements and

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<sup>2</sup>The authors use at-the-money options in order to avoid the volatility smile.

accounting information.

This procedure was used in an attempt to establish a bridge between accounting information and the market. Nowadays, the two worlds have become parallel, which explains the problems of speculation and financial fiascos that quickly result in company and government bankruptcies. Volatility, the most important determinant of an option, should be measured endogenously by using the model's structural expression. This technique avoids the volatility smile and skewness and kurtosis coefficient problems, and helps determine the option's fair value.

### **3 Methodology**

Our analysis begins with a description of our theoretical framework. The El Ibrami and Naciri [2012] equity expression was used as the stock option underlying asset. The PDE and ODE of the foregoing derivative instruments were derived, followed by their expressions. The new stock options evaluation model was then used to perform an empirical analysis and measure the model's performance. Our initial sample was composed of the 300 largest Canadian companies listed on the Toronto Stock Exchange between 2006 and 2010 and with the potential for being listed on the Montreal Derivatives Exchange, sorted by revenue<sup>3</sup>.

Since the model is useful for mean-reverting EBIT companies, the significance level of the companies' EBIT parameters was measured using quarterly data for the period specified. The final evaluation sample was composed of companies with a 5% mean-reverting EBIT and speed of mean-reversion significance level. The model allows users to endogenously generate stock options volatility.

The empirical study used the nearest expiry of the at-the-money or the nearest to-the-money call options listed on the Montreal Derivatives Exchange during the period coinciding with the 2010 financial statement dates. The mean spread between these observed data and the theoretical results were calculated numerically by using 10,000 paths simulation. However, the Black-Scholes model was used as a benchmark for unlisted stock options. Given our belief that the Black-Scholes model overestimates the value of stock options because of the volatility smile, results with the model were compared to those yielded by our structural model for the evaluation of the entire sample. As robustness tests, linear regressions and paired-samples tests were used to compare the theoretical and benchmark values.

### **4 Model**

As discussed earlier, the El Ibrami and Naciri [2012] equity expression was used as the underlying asset. According to the authors, this expression may be presented as follows:

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<sup>3</sup>Arnott, Hsu and Moore (2005) showed that revenue accurately indicates a company's basic characteristics and could be used to measure its true value.

$$E = V - (1 - \tau) \frac{C}{r} + \left[ (1 - \tau) \frac{C}{r} - V_B \right] * \left[ e^{\left( \frac{2k\theta V_1}{\sigma^2} \right) \left( \frac{1}{V - V_2} - \frac{1}{V_B - V_2} \right)} \right] \left[ \frac{V - V_2}{V_B - V_2} \right]^\gamma$$

where:

$k$  represents the mean-reversion speed;

$\theta$  represents the EBIT mean reversion;

$\sigma$  represents asset volatility;

$V_B$  represents the value of the company at bankruptcy;

$\tau$  represents the tax rate;

$C$  represents the consolidated coupon and;

$$\gamma = \frac{(1 + \frac{2k}{\sigma^2}) - \sqrt{(1 + \frac{2k}{\sigma^2})^2 + \frac{8r}{\sigma^2}}}{2};$$

$$V = (1 - \tau) \left[ \frac{\theta k}{r(k + r)} + \frac{x}{(k + r)} \right] = v_1 x + v_2.$$

However, bankruptcy is exogenous; hence "E" depends only on "V". The first and second derivatives of "E" with regard to "V" are determined as follows:

$$\frac{\partial E}{\partial V} = 1 + \left[ (1 - \tau) \frac{C}{r} - V_B \right] * \left[ -\frac{2k\theta V_1}{\sigma^2 (V - V_2)^2} + \frac{\gamma}{(V - V_2)} \right] * \left[ \frac{V - V_2}{V_B - V_2} \right]^\gamma * e^{\frac{2k\theta V_1}{\sigma^2} \left( \frac{1}{V - V_2} - \frac{1}{V_B - V_2} \right)} \quad (1)$$

and

$$\frac{\partial^2 E}{\partial V^2} = \left[ (1 - \tau) \frac{C}{r} - V_B \right] * [\Omega + \Psi + \Delta] \quad (2)$$

where:

$$\Omega = \left[ \frac{4k\theta V_1}{\sigma^2 (V - V_2)^3} - \frac{\gamma}{(V - V_2)^2} \right] * \left[ \frac{V - V_2}{V_B - V_2} \right]^\gamma * e^{\frac{2k\theta V_1}{\sigma^2} \left( \frac{1}{V - V_2} - \frac{1}{V_B - V_2} \right)} \quad (3)$$

$$\Psi = \left[ -\frac{2k\theta V_1}{\sigma^2 (V - V_2)^2} + \frac{\gamma}{(V - V_2)} \right] * \frac{\gamma}{(V_B - V_2)^\gamma} [V - V_2]^{-1} * e^{\frac{2k\theta V_1}{\sigma^2} \left( \frac{1}{V - V_2} - \frac{1}{V_B - V_2} \right)} \quad (4)$$

$$\Delta = \left[ -\frac{2k\theta V_1}{\sigma^2 (V - V_2)^2} + \frac{\gamma}{(V - V_2)} \right] * \left[ \frac{V - V_2}{V_B - V_2} \right]^\gamma * \frac{-1}{(V - V_2)^2} * \frac{2k\theta V_1}{\sigma^2} * e^{\frac{2k\theta V_1}{\sigma^2} \left( \frac{1}{V - V_2} - \frac{1}{V_B - V_2} \right)} \quad (5)$$

By applying ito's lemma, the following PDE is obtained for equity:

$$dE = \left( \frac{\partial E}{\partial t} + \frac{\partial E}{\partial V} (k(V\theta - V) + V_2) + \frac{1}{2} \frac{\partial^2 E}{\partial V^2} \sigma^2 (V - V_2)^2 \right) dt + \frac{\partial E}{\partial V} \sigma (V - V_2) dz \quad (6)$$

Assuming perpetuity because of the company's solvency, equation (6) becomes:

$$dE = \left( \frac{\partial E}{\partial V} (k(V_1\theta - V) + V_2) + \frac{1}{2} \frac{\partial^2 E}{\partial V^2} \sigma^2 (V - V_2)^2 \right) dt + \frac{\partial E}{\partial V} \sigma (V - V_2) dz \quad (7)$$

To simplify, let:

$$a(E) = \left( \frac{\partial E}{\partial V} (k(V_1\theta - V) + V_2) + \frac{1}{2} \frac{\partial^2 E}{\partial V^2} \sigma^2 (V - V_2)^2 \right) \quad (8)$$

and

$$b(E) = \frac{\partial E}{\partial V} \sigma (V - V_2) \quad (9)$$

Substituting these values in equation (7), the latter becomes:

$$dE = a(E)dt + b(E)dz \quad (10)$$

By applying Ito's lemma to derivative instrument "G", which depends on "E" and "t", we obtain:

$$dG = \left( \frac{\partial G}{\partial t} + \frac{\partial G}{\partial E} a + \frac{1}{2} \frac{\partial^2 G}{\partial E^2} b^2 \right) dt + \frac{\partial G}{\partial E} b dz \quad (11)$$

Now assuming two financial instruments "G<sub>1</sub>" and "G<sub>2</sub>" that depend on "E" and fulfill the conditions of the following dynamic equation:

$$dG = Gmdt + GsdW \quad (12)$$

Assuming a hedged portfolio consisting of s<sub>2</sub>G<sub>2</sub> "G<sub>1</sub>" assets and s<sub>1</sub>G<sub>1</sub> "G<sub>2</sub>" assets and considering perpetuity and risk neutrality, we obtain:

$$\frac{\partial G}{\partial E} a + \frac{1}{2} \frac{\partial^2 G}{\partial E^2} b^2 = rG \quad (13)$$

By replacing a(E) and b(E) by their respective values in expression (13), the latter becomes:

$$\frac{\partial G}{\partial E} \left( \frac{\partial E}{\partial V} (k(V_1\theta - V) + V_2) + \frac{1}{2} \frac{\partial^2 E}{\partial V^2} \sigma^2 (V - V_2)^2 \right) + \frac{1}{2} \frac{\partial^2 G}{\partial E^2} \left( \frac{\partial E}{\partial V} \sigma (V - V_2) \right)^2 = rG \quad (14)$$

Where:

$$\frac{\partial E}{\partial V} = 1 + \left[ (1 - \tau) \frac{C}{r} - V_B \right] * \left[ -\frac{2k\theta V_1}{\sigma^2 (V - V_2)^2} + \frac{\gamma}{(V - V_2)} \right] * \left[ \frac{V - V_2}{V_B - V_2} \right]^\gamma * e^{\frac{2k\theta V_1}{\sigma^2} * \left( \frac{1}{V - V_2} - \frac{1}{V_B - V_2} \right)} \quad (15)$$

and

$$\frac{\partial^2 E}{\partial V^2} = \left[ (1 - \tau) \frac{C}{r} - V_B \right] * [\Omega + \Psi + \Delta] \quad (16)$$

and

$$\Omega = \left[ \frac{4k\theta V_1}{\sigma^2(V-V_2)^3} - \frac{\gamma}{(V-V_2)^2} \right] * \left[ \frac{V-V_2}{V_B-V_2} \right]^\gamma * e^{\frac{2k\theta V_1}{\sigma^2}} * \left( \frac{1}{V-V_2} - \frac{1}{V_B-V_2} \right) \quad (17)$$

$$\Psi = \left[ -\frac{2k\theta V_1}{\sigma^2(V-V_2)^2} + \frac{\gamma}{(V-V_2)} \right] * \frac{\gamma}{(V_B-V_2)^\gamma} [V-V_2]^{\gamma-1} * e^{\frac{2k\theta V_1}{\sigma^2}} * \left( \frac{1}{V-V_2} - \frac{1}{V_B-V_2} \right) \quad (18)$$

$$\Delta = \left[ -\frac{2k\theta V_1}{\sigma^2(V-V_2)^2} + \frac{\gamma}{(V-V_2)} \right] * \left[ \frac{V-V_2}{V_B-V_2} \right]^\gamma * \frac{-1}{(V-V_2)^2} * \frac{2k\theta V_1}{\sigma^2} * e^{\frac{2k\theta V_1}{\sigma^2}} * \left( \frac{1}{V-V_2} - \frac{1}{V_B-V_2} \right) \quad (19)$$

Equation (14) represents the ODE fulfilled by a call option written on the El Ibrami and Naciri [2012] equity. The coefficients of the expression are not constant. Only a numerical solution helps estimate the value of the stock option by using the proposed formula.

## 5 Empirical Analysis

Among the 300 companies composing our initial sample, only 62 presented statistically significant mean-reverting characteristics. We used this final sample to conduct our empirical analysis. Among the 62 companies sorted, 30 were listed on the Montreal Derivatives Exchange at their 2010 financial statement date. As a first step, we compared the theoretical results to their corresponding last close prices. We also used the same sub-sample to perform a comparison with the Black-Scholes model.

In the second step, we compared the theoretical values of the 32 unlisted companies to their corresponding values calculated by using the Black-Scholes model again. Finally, we conducted a third comparison between the two models by using the entire evaluation sample data. Table 1 below shows the results of the comparisons.

Table 1: Mean Spread between Theoretical Values and Benchmarks

Mean Spread <sup>4</sup>			
Stock Options	Number of Companies	Theoretical Value vs. Last-Close-Price	Theoretical Value vs. B&S Value
Listed	30	-2.89%	-23.40%
Unlisted	32	n/a	-38.02%
Entire Sample	62	n/a	-30.95%

<sup>4</sup> Spread=(Theoretical Value – Benchmark)/Benchmark



As shown in Table 1, the model estimates the listed call options correctly, but the Black-Scholes model overestimates the same sub-sample by about 23%. As expected, the volatility smile causes this overestimation, weakening the model's findings. According to the results, the model overestimates the value of the 32 unlisted stock options by about 38% and the entire evaluation sample by about 31%, which confirms our findings about existing stock options. We surmise that the spread between the results obtained by using the two models would be more pronounced for long term expiry and deep in-the-money or out-of-the-money stock options<sup>5</sup>.

As robustness tests, a paired-samples mean comparison was performed by analyzing the difference between the theoretical and observed data for listed options first. A second comparison was performed with the same sub-sample, using the Black-Scholes model as a benchmark. The results obtained by the two models were compared for unlisted companies. Lastly, the two models were compared, using all the companies in the evaluation sample. For all the comparisons made, the null hypothesis predicts a lack of significant mean differences between the paired-samples, while the complementary hypothesis seeks for significant differences between the same paired-samples. A 1% level of significance was used for all the comparisons we made. The results of such comparisons are given in Table 2.

Table 2: Paired Samples Test

		Paired Differences 99% Confidence Interval for Difference			
		Theoretical Value vs. Last-Close-Price		Theoretical Value vs. B&S Value	
Stock Options	Number of Companies	<i>p</i> -value	<i>t</i> -statistic	<i>p</i> -value	<i>t</i> -statistic
Listed	30	27%	1.126	0.6%	2.958
Unlisted	32		n/a	0.000%	- 4.493
Entire Sample	62		n/a	0.000%	- 5.291

The results presented in Table 2, along with those obtained previously, show that there is no statistically significant difference between the values obtained by our model and their corresponding last-close-prices. The paired-samples mean comparison between our theoretical values and Black-Scholes values shows a statistically significant difference, as observed previously. The comparison between the two models used for the unlisted companies also corroborates these results. These conclusions are consistent with those obtained for the entire evaluation sample.

<sup>5</sup>This assertion is beyond the scope of our paper.

The paired-samples tests conducted show that there is no statistical difference between our results and the market values, but that our model and Black-Scholes yield significantly different results. To measure the magnitude of this difference, we performed linear regressions of our model's theoretical values on those obtained by Black-Scholes. Table 3 presents these results.

Table 3: Linear Regression

Theoretical Value vs. B&S Value						
Stock Options	Number of Companies	Coefficient	R-squared	Adj. R-squared	p-value	t-statistic
Listed	30	0.602	0.847	0.717	0.000	8.268
Unlisted	32	0.467	0.915	0.837	0.000	11.536
Entire Sample	62	0.463	0.844	0.712	0.000	12.181

Table 3 shows that all the coefficients are positive and statistically significant, and have lower than one values. The R-Squared and Adjusted R-Squared are all high and significant, indicating that, unlike our model, Black-Scholes overestimates the value of the stock options for the three regressions performed. The results obtained earlier are therefore corroborated.

## 6 Conclusion

In this paper we developed a new model of stock option evaluation. The underlying asset of the stock option derived was the El Ibrami and Naciri [2012] equity model. Financial statements for mean-reverting EBIT-based companies were used. The model also uses structural data and mean-reverting EBIT as the state variable. The initial sample consisted of the 300 largest Canadian companies listed on the Toronto Stock Exchange, sorted by revenue. The companies with statistically significant mean-reverting parameters were retained, resulting in a final sample of 62 companies. Only 30 companies in the sample were listed on the Montreal Derivatives Exchange during the evaluation period.

Hence, to measure the model's performance in evaluating stock options, its values were compared to the stock options' last close price at the financial statement date. The results show a very minor spread between the theoretical and market values. The same sub-sample was used to measure the spread between the Black-Scholes model and ours in order to check for the volatility smile effect. The Black-Scholes model overestimated stock options values by about 23%, compared to our model. The two models were compared again by using an unlisted stock options sub-sample. The results corroborated those obtained for the listed stock options sub-sample, with the Black-Scholes model overestimating values by 38%.

Lastly, we performed another comparison between the two models using the entire data of the evaluation sample. The Black-Scholes model overestimated the stock options value by

about 30%, compared to our model's results. Robustness was tested using paired-sample comparisons. No significant differences were noted between our theoretical results and market values, whereas significant differences were found between our model and the Black-Scholes model, for both listed and unlisted companies.

As a post hoc test, linear regressions of our theoretical values were conducted on the Black-Scholes values. All the results were significant, leading to the conclusion that the Black-Scholes model overestimates results because the volatility smile invalidates the model's findings.

The model we derived can be used to determine CEOs' and employees' stock options-based compensation, a private company's call option value or how to quote stock options about to be listed on a Derivatives Exchange. It can also be used to measure the efficiency of the stock options market by making it possible to compare the intrinsic value of stock options to the last close price or quotes. However, the model is useful only for mean-reverting companies that have EBIT as a state variable. In addition, it presents a call option expression and does not take into account frictions. Further analysis will help determine a put option equation using other state variables and taking into consideration market incompleteness.

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