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Embedding Hamiltonian Cycles in the Extended OTIS-n-Cube Topology

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Abstract

This paper introduces theoretical and practical study on embedding Hamiltonian cycle in the Extended OTIS-*n*-Cube. A generalized Algorithm is also presented for embedding Hamiltonian cycle in the Extended OTIS-*n*-Cube. The recently proposed network has many good topological features such as regular degree, semantic structure, low diameter, and ability to embed graphs and cycles. Embedding Hamiltonian cycle is an important characteristic for any topology due to the usefulness of undertaking different types of broadcasting messages within interconnection networks. The proposed algorithm is capable to form a Hamiltonian cycle starting from any node in the network. Examples are presented on different network sizes showing complete paths of Hamiltonian cycles.

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1 Introduction

In the last decade, there has been an increasing interest in a class of interconnection networks called Optical Transpose Interconnection Systems "OTIS-networks" [4, 21, 24, 27]. Marsden *et al* were the first to propose the OTIS-networks [16]. Extensive studies and modeling results for the OTIS have been reported in [8, 9, 15, 29]. The achievable terabit throughput at a reasonable cost makes the OTIS a strong competitor to the electronic alternatives [5, 13, 16, 18]. These encouraging findings prompt the need for further testing of the suitability of the OTIS for real-world parallel applications.

The advantage of using the OTIS as optoelectronic architecture lies in its ability to manoeuvre the fact that free space optical communication is superior in terms of speed and power consumption when the connection distance is more than a few millimetres [13]. In the OTIS, shorter (intra-chip) communication is realized by electronic interconnects while longer (inter-chip) communication is realized by free space interconnects. In our topology, the hypercube; or cube for short; has been used for its attractive properties [17, 20, 23].

OTIS technology processors are partitioned into groups, where each group is realized on a separate chip with electronic inter-processor connects. Processors on separate chips are interconnected through free space interconnects. The philosophy behind this separation is to utilize the benefits of both the optical and electronic technologies.

Processors within a group are connected by a certain interconnecting topology, while transposing group and processor indexes achieve inter-group links. Using *n*-cube as a factor network will yield the OTIS-*n*-Cube in denoting this network.

OTIS-*n*-Cube is basically constructed by "multiplying" a cube topology by itself. The set of vertices is equal to the Cartesian product on the set of vertices in the factor cube network. The set of edges E in the OTIS-*n*-Cube consists of two subsets, one is from the factor cube, called *cube*-type edges, and the other subset contains the *transpose* edges. The OTIS approach suggests implementing *cube*-type edges by electronic links since they involve intra-chip short links and implementing transpose edges by free space optics. Throughout this paper the terms "*electronic move*" and the "*OTIS move*" (or "*optical move*") will be used to refer to data transmission based on electronic and optical technologies, respectively.

Although the OTIS-*n*-Cube network has many attractive topological properties it suffers from having limited optical links between the different groups. When source and destination nodes are in two different groups, the fact that only one optical link connects two distinguished groups directly create a congestion problem to most of the shortest paths that have to pass through this particular optical link. Furthermore, alternative paths are too long compared to the short path because they have to be routed via a third group which required passing via two optical links in addition to the electronic moves in each group to reach the destination.

The Extended OTIS-*n*-Cube is a proposed interconnection network based on the "OTIS-n-Cube" network [1, 2]. In [1] we proposed the new topology and presented the topological properties of the network; e.g size, regularity, and diameter. In [2], we presented a fault tolerant routing algorithm using unsafety vectors for the new topology. Recently, the initial idea of embedding a Hamiltonian cycle in the Extended OTIS Cube is proposed in [3].

Embedding of topologies with regular structure and also irregular structure has been broadly investigated in the literature, e.g [6, 10, 11, 25]. Embedding structures and other topologies is one of the key features of interest in interconnection networks. The load of an embedding is the maximum number of nodes in a graph assigned to any node in the embedded graph. We are interested in this research only in one-to-one mappings to embed a Hamiltonian cycle, so the load of any embedding is one [28].

In the mathematical field of graph theory, a Hamiltonian path is a path in an undirected graph which visits each node exactly once. A Hamiltonian cycle is a cycle in an undirected graph which visits each node exactly once and also returns to the starting node. Determining whether such paths and cycles exist in graphs is the Hamiltonian path problem [11, 12, 25].

The Hamiltonian path seeks whether there is a route in a directed network from a beginning node to an ending node, visiting each node exactly once. The Hamiltonian path problem is NP complete, achieving astonishing computational complexity. This challenge has inspired researchers to broaden the definition of computer computations. The Hamiltonian problem arises in many real world applications including DNA applications [25].

This paper proposes a theoretical study on the routing properties in general and embedding Hamiltonian cycle in specific for the Extended OTIS-*n*-Cube due to its attractive properties. Section 2 presents notations and preliminary definitions. Section 3 describes the Extended OTIS-*n*-Cube topology. Details of embedding a Hamiltonian cycle in the Extended OTIS-*n*-Cube topology will be discussed in section 4. Section 5 concludes the paper.

2 Notations and Definitions

The *n*-dimensional undirected graph binary *n*-cube is one of the well known networks which have been used in real life systems [14, 17, 19, 22].

Definition 1: The undirected graph *n*-cube with 2^n vertices, representing nodes, which are labeled by the 2^n binary digits of length *n*. The binary system consists of two bits; 0 and 1. Two nodes are connected by a direct edge if, and only if, their labels differ in exactly one bit position.

The Extended OTIS-*n*-Cube is constructed by "multiplying" a cube topology by itself. The vertex set is equal to the *Cartesian* product on the original vertex set in the factor cube network. The initial step is similar to OTIS-*n*-Cube construction;

this is why we named it Extended OTIS-*n*-Cube.

Definition 2: Let $\langle g_1, p_1 \rangle$ be group and processor addresses of a node in an Extended OTIS-*n*-Cube labelled as series of bits $\langle x_n \ldots x_2 x_1 \rangle$, $\langle y_n \ldots y_2 y_1 \rangle$ consequently where each bit is either 0 or 1. A node $\langle g_2, p_2 \rangle$ is called *an opposite of* node $\langle g_1, p_1 \rangle$ if and only if they differ only in the first bit position of g_1 and g_2 labels, and also in the first bit position of p_1 and p_2 labels. They differ only in x_1 and y_1, e.g. node $\langle 00, 00 \rangle$ is an opposite node of $\langle 01, 01 \rangle$. The edge between two opposite nodes is called and opposite edge.

Definition 3: The two nodes $\langle g_1, p_1 \rangle$ and $\langle g_2, p_2 \rangle$ are connected via a transpose edge if and only if $g_1 = p_2$ and $g_2 = p_1$.

The edge set consists of electronic edges from the factor network and two new types of edges called the transpose and opposite edges, both types of transpose and opposite edges are considered optical edges. The formal definition of the Extended OTIS-*n*-Cube is given below.

Definition 4: Let $n\text{-}cube = (V_0, E_0)$ be an undirected graph representing an *n*-cube network where n is the cube degree. The Extended OTIS-*n*-Cube = (V, E) network is represented by an undirected graph obtained from *n-cube* as follows $V = \{\langle g, p \rangle \mid g, p \in V_0\}$ and $E = \{(\langle g, p_1 \rangle, \langle g, p_2 \rangle) \mid \text{if } (p_1, p_2) \in E_0\} \cup \{(\langle g, p \rangle, \langle p, g \rangle) \mid g, p \in V_0\} \cup \{(\langle g, g \rangle, \langle p, p \rangle) \mid g, p \in V_0 \cap g \textit{ is an opposite of } p\}$.

Definition 5: Let $d(p, g)$ be the number of bit positions differ between p and g labels. The shortest path between the two nodes $\langle g_1, p_1 \rangle$ and $\langle g_2, p_2 \rangle$ contains an odd number of optical moves if $d(p_1, g_2) + d(p_2, g_1) + 1 \le d(p_1, p_2) + d(g_1, g_2) + 2$, otherwise it contains an even number of optical moves [7, 21].

Definition 6: A path in a topology is a sequence of distinct edges so that there is an edge joining successive nodes, starting at the first node and ending at the last node.

Definition 7: A cycle (or circuit) is a path where there is an edge joining the first and last nodes of this path.

Definition 8: A Hamiltonian path in a topology is a path that contains every node of the network exactly once.

Definition 9: A Hamiltonian cycle is a Hamiltonian path with an edge from the last node of the path to the first node. Hamiltonian cycles are useful in interconnection networks as they can be used to easily undertake many-to-many broadcasts [26].

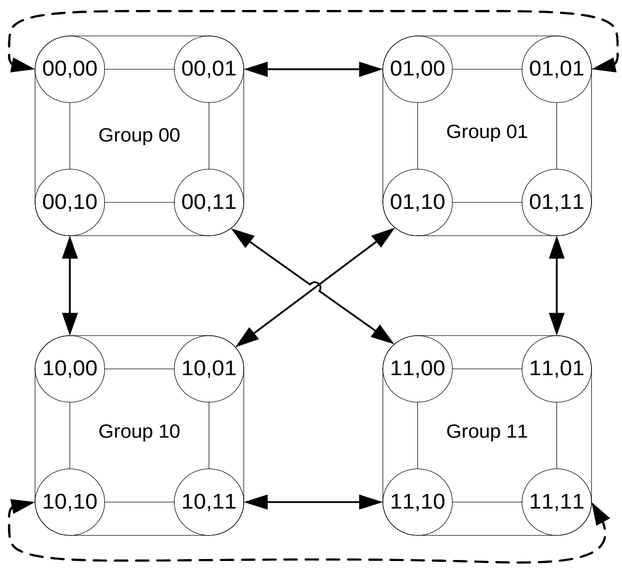

Figure 1: 16-processor Extended OTIS-*2*-cube

3 The Extended OTIS-n-Cube Graph Structure

In the Extended OTIS-*n*-Cube, the address of a node u = ⟨g, p⟩ from V is composed of two components. Figure 1 shows a 16 processor Extended OTIS-2-Cube, solid arrows represent transpose edges while dashes arrows represent opposite edges. The notation ⟨*g*, *p*⟩ is used to refer to the group and processor addresses respectively, two nodes $\langle g_1, p_1\rangle$ and $\langle g_1, p_2\rangle$ are connected by a direct edge if one of the following cases occurs:

1- If $g_1 = g_2$ and $(p_1, p_2)\in E_0$ where E_0 is the set of edges in n-cube network, in this case the two nodes are connected by an electronic edge if their labels differ only by one bit position.

2- If $g_1 = p_2$ and $p_1 = g_2$, in this case the two nodes are connected by a transpose edge.

3- If $g_1 = p_1$, $g_2 = p_2$, and g_1 is an opposite of g_2, then the two nodes are connected by an opposite edge.

The distance in the Extended OTIS-n-Cube is defined as the shortest path between any two nodes, $\langle g_1, p_1\rangle$ and $\langle g_2, p_2\rangle$, and this path involves one of the following forms:

i- When $g_1 = g_2$ then the path involves only electronic moves from source node to the destination node.

ii- When g_1 is opposite of g_2, and if the number of optical moves is an odd number of moves, then the paths can be compressed into a shorter path of the form:

$\langle g_1, p_1\rangle \xrightarrow{E} \langle g_1, g_1\rangle \xrightarrow{O} \langle g_2, g_2\rangle \xrightarrow{E} \langle g_2, p_2\rangle$

or $\langle g_1, p_1\rangle \xrightarrow{E} \langle g_1, g_2\rangle \xrightarrow{O} \langle g_2, g_1\rangle \xrightarrow{E} \langle g_2, p_2\rangle$; whichever is shorter, where the symbols O and E stand for optical and electronic moves respectively.

iii- When $p_{2op} = g_1$or $p_{1op} = g_2$, and the path involves an odd number of optical moves. In this case the paths can be compressed into a shorter path of $d(p_1, g_2) + d(p_2, g_1) + 1$ or one of the following two cases:

- $\langle g_1, p_1\rangle \xrightarrow{E} \langle g_1, g_1\rangle \xrightarrow{O} \langle p_2, p_2\rangle \xrightarrow{E} \langle p_2, g_2\rangle \xrightarrow{O} \langle g_2, p_2\rangle$ if $p_{2op} = g_{1.}$

- $\langle g_1, p_1\rangle \xrightarrow{O} \langle p_1, g_1\rangle \xrightarrow{E} \langle p_1, p_1\rangle \xrightarrow{O} \langle g_2, g_2\rangle \xrightarrow{E} \langle g_2, p_2\rangle$ if $p_{1op} = g_2$, where *op* means opposite.

iv- When $g_1 \neq g_2$ and if the number of optical moves is an even number of moves, then the paths can be compressed into a shorter path of the form:
$\langle g_1, p_1\rangle \xrightarrow{E} \langle g_1, p_2\rangle \xrightarrow{O} \langle p_2, g_1\rangle \xrightarrow{E} \langle p_2, g_2\rangle \xrightarrow{O} \langle g_2, p_2\rangle$

v- When $g_1 \neq g_2$, and the path involves an odd number of optical moves. In this case the paths can be compressed into a shorter path of the form:
$\langle g_1, p_1\rangle \xrightarrow{E} \langle g_1, g_2\rangle \xrightarrow{O} \langle g_2, g_1\rangle \xrightarrow{E} \langle g_2, p_2\rangle$.

4 Hamiltonian Cycle Structure in the Extended OTIS-n-Cube

This section presents a Hamiltonian cycle structure within the recently proposed Extended OTIS-*n*-Cube interconnection topology. First, we introduce some routing topological properties of the Extended OTIS-*n*-Cube which are needed to show the Hamiltonian cycle formation in this topology.

Theorem 1. If the cube factor degree is n, then any node in the Extended OTIS-*n*-Cube is regular and the node degree is $n+1$.

Proof. Every node has n electronic edges based on the properties of the *n*-cube factor. Also every node; $\langle g, p\rangle$; has an additional optical edge based on the Extended OTIS-*n*-Cube topology rule: $\{(\langle g, p\rangle, \langle p, g\rangle) \mid g, p \in V_0\} \cup \{(\langle g, g\rangle, \langle p, p\rangle) \mid g, p \in V_0 \cap g \textit{ is an opposite of } p\}$

so if $g = p$ *then* $\langle g, p\rangle \xrightarrow{O} \langle g_{\mathrm{op}}, g_{\mathrm{op}}\rangle$ else $\langle g, p\rangle \xrightarrow{O} \langle p, g\rangle$.

Since every node has an n number of electronic, in addition to one optical edge, then by definition the topology is regular.

Theorem 2. Let $\langle g_1, p_1\rangle$ and $\langle g_2, p_2\rangle$ be two different nodes in the Extended OTIS-*n*-Cube. The length of shortest path from the source node $\langle g_1, p_1\rangle$ to the destination node $\langle g_2, p_2\rangle$ is defined mutually exclusive as in the following order:

$$Length = \begin{cases} d(p_1, p_2) & if\ g_1 = g_2 \\ d(p_1, g_1) + d(g_{1op}, p_2) + 1 & if\ g_1 = g_{2op} and\ it\ is \leq d(p_1, g_2) + d(g_1, p_2) + 1 \\ d(p_1, p_2) + d(g_1, g_2) & if\ (g_1 = p_1 or\ g_2 = p_2) and\ d(g_{1op}, g_2) < d(g_1, g_2) \\ d(p_1, p_2) + d(g_1, g_2)) & if\ g_1 = p_{2op}\ or\ g_2 = p_{1op}\ and\ it\ is \leq d(p_1, g_2) + d(g_1, p_2) + 1 \\ \min(d(p_1, g_2) + d(p_2, g_1) + 1, d(p_1, p_2) + d(g_1, g_2) + 2) & Otherwise \end{cases}$$

Where $d(p_1, p_2)$ is the number of bit positions differ between p_1 and p_2 labels.

Proof. By following one of the five possible paths shown in sections; *i, ii, iii, iv,* and *v*. The length of the shortest path between the nodes $\langle g_1, p_1\rangle$ and $\langle g_2, p_2\rangle$ can be as follows:

If both nodes are in the same group then the shortest path is guaranteed by generating electronic moves toward the destination; $d(p_1, p_2)$.

- If $g_1 = g_{2op}$ and $d(p_1, g_1) + d(g_{1op}, p_2) \leq d(p_1, g_2) + d(g_1, p_2)$ it means that one optical move is needed to move toward the destination group via a group opposite edge otherwise minimal path must contains a transpose edge which will be explained in the next points. To reach the destination, some electronic moves might be needed first at one source group to reach $\langle g_1, g_1\rangle$ then one optical move to reach the destination group; finally other electronic moves at the destination group might be needed to reach the destination node.

- If $p_1 = p_2$, $g_1 = g_2$, and $d(g_{1op}, g_2) < d(g_1, g_2)$ it means that two optical moves in addition to some electronic moves are needed to reach the destination group through an intermediate group $g_{1\,op}$. One of the two optical moves is an opposite move. First an opposite move is required to reach $\langle g_{1op}, p_{1op}\rangle$, and then some electronic moves to reach $\langle g_{1op}, g_2\rangle$, then an optical move to reach $\langle g_2, g_{1op}\rangle$, and finally other electronic moves to reach the destination node $\langle g_2, p_2\rangle$ at minimal distance. It's worth it to mention that all diameter distances are considered under this category

- If $g_1 = p_{2op}$ *or* $g_2 = p_{1op}$ and $d(p_1, p_2) + d(g_1, g_2)) \leq d(p_1, g_2) + d(g_1, p_2) + 1$, it means that two optical moves are needed to reach the destination group through an intermediate group equal to p_{1op}if $p_{1op} = g_2$or equal to p_2 if $p_{2op} = g_1$. This requires some electronic moves to perform the two optical moves, and finally to reach the destination node at minimal distance.
- Otherwise we choose the shortest path based on the factor optical moves [7].

Theorem 3. The Extended OTIS-*n*-Cube graph is Hamiltonian.

Proof. Hamiltonian is a cycle in an undirected graph which visits each node exactly once and also returns to the starting node. In each group, there are 2^n nodes which are connected via the factor network topology, we can visit all local nodes by exchanging a bit position of the current node to make a move to the next node, and this bit position is selected in a sequential order on the *n* positions of the process address. This process is performed 2^n-1 times at each group to visit the 2^n local nodes. If we follow the same concept on the group addresses then we can verify the visiting of all 2^n groups. The only difference is that there are two types of optical moves, opposite and transpose.

We can construct such a cycle based on the following algorithm:

Algorithm HamiltonianRouting

{ Let node $<g_s, p_s>$ be the starting node;

Let $<g_c, p_c> = <g_s, p_s>$ // current node

for Groups=1 to 2^n do

{ *for loop*= 0 to *n*-1 do

if g_c xor $2^{loop} \neq$ Already visited Group

{ $N_g = g_c$ xor 2^{loop} // N_g is next group

exit for loop }

if Groups=2^n then $N_g = g_s$

if $N_g = g_{c\ opposite}$

{ *if* Groups=1 *then*

visit only local nodes of a path from $<g_s, p_s>$ to $<g_s, g_s>$

else

visit all 2^n-1 local nodes from $<g_c, p_c>$ to $<g_c, g_c>$

Make an opposite optical move from $<g_c, g_c>$ to $<g_{c\ opposite}, g_{c\ opposite}>$

}

else

{ *if* Groups=1 *then*

visit only local nodes of a path from $<g_s, p_s>$ to $<g_s, N_g>$

else

visit all 2^n-1 local nodes from $<g_c, p_c>$ to $<g_c, N_g>$

Make a transpose optical move from $<g_c, N_g>$ to $<N_g, g_c>$

}

}*// for Groups*

Finally, visit the unvisited local nodes from $<g_s, N_g>$ to $<g_s, p_s>$ // a complete Hamiltonian cycle

The 2^n-1 factor moves at each of the 2^n visited groups from the first node $<g_s, p_s>$ towards a potential neighboring node $<g_c, p_c>$ is done by complementing the i^{th} bit in the factor label, where $1 <= i <= n$. This sequential order is repeated again to visit all local nodes of a group by increasing *i* by 1 modulus *n*. The same perspective is done among the group addresses to visit all groups. The algorithm starts the permutation from the first position; *i*=1; to conduct an opposite move if the opposite group has not been visited yet.

In the following examples, the dots represent 2^n-1 factor moves of the corresponding nodes within each group; every arrow represents an optical move.

Example 1: Hamiltonian cycle within an Extended OTIS-*2*-Cube topology, Figure 2 shows a representation of such a Hamiltonian cycle. The starting node is <00,01>. The cycle starts by visiting all of the local nodes at the first group towards <g_s, $p_{s\ opposite}$> based on the cube routing properties, Then through an optical move to the second group and so on. The final group to be visited before returning back to the starting group is the $p_{s\ opposite}$ group

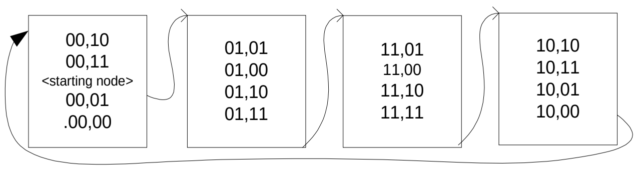

Figure 2: A Hamiltonian cycle in an Extended OTIS-*2*-Cube

Example 2: Presenting a Hamiltonian cycle within an Extended OTIS-*3*-Cube graph, Figures 3 and 4 show that the algorithm is capable to form Hamiltonian cycles regardless of the starting node. There is no precise condition on the starting node in the algorithm.

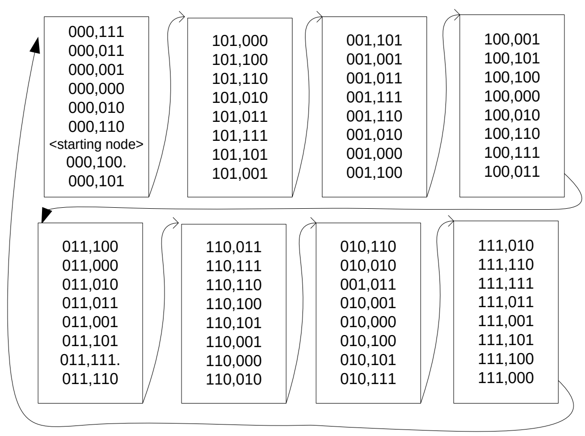

Figure 3: A Hamiltonian Cycle in an Extended OTIS-*3*-Cube starting at node <000,100>

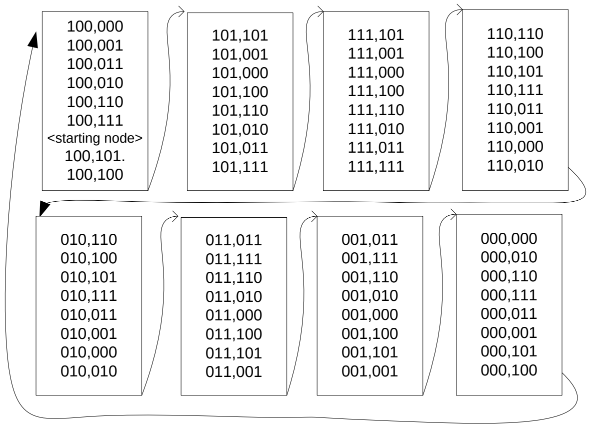

Figure 4: A Hamiltonian Cycle in an Extended OTIS-*3*-Cube starting at node <100,101>

We can state from the above two cases that the algorithm is capable to build a Hamiltonian cycle from any starting node using both opposite and transitive moves.

Example 3: A Hamiltonian cycle within an Extended OTIS-*4*-Cube graph, Figure 5 shows a representation of such a Hamiltonian cycle where the starting node is <0000,0001>.

To present a complete path cycle, figure 6 shows such a cycle in the Extended OTIS-*3*-Cube topology graph where the bold arrows represent this complete Hamiltonian cycle path starting from node <000,001>. The reader may follow the number of each arrow to observe how this cycle has been formulated.

Theorem 4. If a Hamiltonian cycle contains opposite links then the number of opposite links must be even. A Hamiltonian cycle can't contain odd number of opposite links**.**

Proof. To complete a Hamiltonian cycle in an extended OTIS- n-Cube, all 2^n groups of the network have to be visited one and only one time. This is done by exchanging the permutations of the group label in a certain order to guarantee exchanging all the n bits of the label. This order is accomplished by performing optical moves to visit the groups. An optical move is either an opposite or a transpose move.

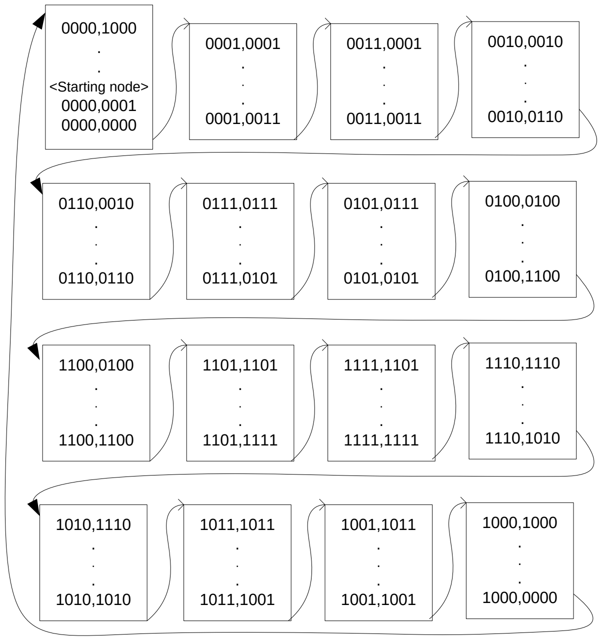

Figure 5: A Hamiltonian Cycle in an Extended OTIS-*4*-Cube

When a transpose move occurs then a permutation on the group label is done by exchanging a group label with its processor label. Performing a transpose move after visiting all local nodes at each group will lead to performing the 2^n permutations. At every time an opposite move occurs, the permutation order will be affected, to sort out this influence and go back to the order, another opposite group must occur. So there is always an even number of opposite moves in a Hamiltonian cycle.

Example 4. To show that a Hamiltonian cycle must contain an even number of opposite links by using extended edges, Figure 7 shows A Hamiltonian cycle with opposite links in an Extended OTIS-*3*-Cube. There are 4 opposite links within the Hamiltonian cycle. Figures 5 and 6 also contain 4 and 8 opposite links consequently.

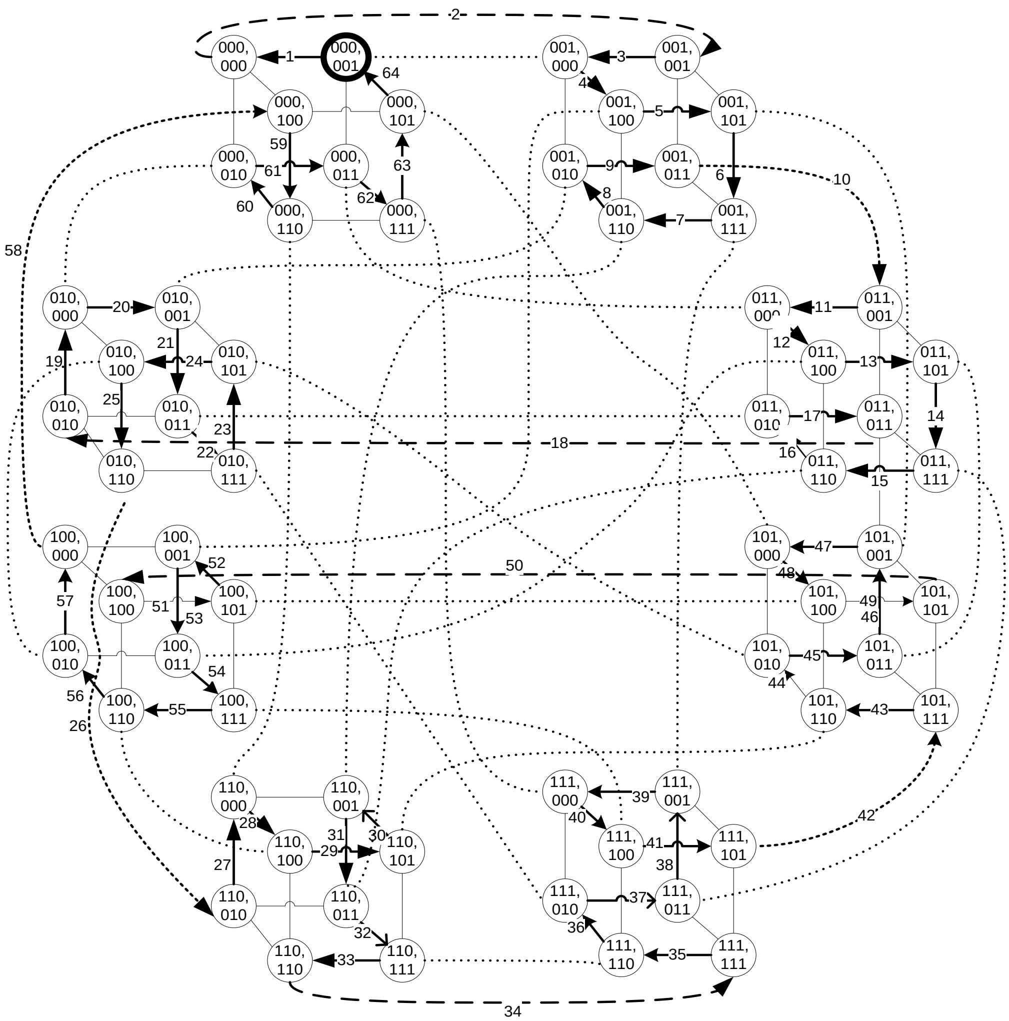

Figure 6: Extended OTIS-3-Cube

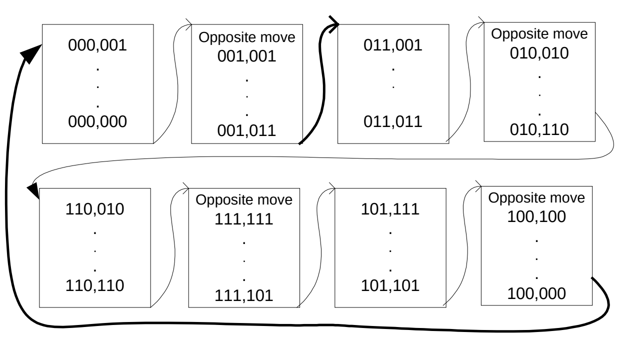

Figure 7: A Hamiltonian cycle contains opposite links

5 Conclusion

This paper presented a theoretical study on embedding Hamiltonian cycle in the Extended OTIS-*n*-Cube. Embedding a Hamiltonian cycle is an important property for any topology due to the usefulness of undertaking many-to-many broadcast messages within interconnection networks. The paper proposed a generalized algorithm to form a Hamiltonian cycle in the extended OTIS-*n*-Cube interconnection network. We also showed that the algorithm is capable to form a Hamiltonian cycle starting from any node in the network. Examples are presented on different network sizes to show complete paths of Hamiltonian cycles. Finally some related theoretical theorems were also presented in this paper.

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