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An Estimation Error Corrected Sharpe Ratio Using Bootstrap Resampling

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Abstract

The Sharpe ratio is a common financial performance measure that represents the optimal risk versus return of an investment portfolio, also defined as the slope of the capital market line within the mean-variance Markowitz efficient frontier. Obtaining sample point and confidence interval estimates for this metric is challenging due to both its dynamic nature and issues surrounding its statistical properties. Given the importance of obtaining robust determinations of risk versus return within financial portfolios, the purpose of the current research was to improve the statistical estimation error associated with Sharpe's ratio, offering an approach to point and confidence interval estimation which employs bootstrap resampling and computational intelligence. This work also extends prior studies by minimizing the ratio's statistical estimation error first by incorporating the common assumption that the ratio's loss function is the squared error and second by correcting for overestimation through an approach that recognizes that the

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negative covariance between the variables representing the estimate of the Sharpe ratio and the standard deviation can be used for corrective purposes. Results of an accompanying empirical simulation study indicated improved relative efficiency of point estimates and the coverage probability, coverage error, length, and relative bias of confidence intervals.

JEL classification numbers: C15, G11
Keywords: Sharpe ratio, Bootstrap resampling, Relative efficiency of a point estimator, Coverage error of confidence interval estimation

1 Introduction: Sharpe Ratio Estimation Error

The Sharpe's ratio [11, 12] for portfolio performance has become an established approach among analysts to assess financial portfolios within investment strategies. A ratio of the portfolio's excess expected return relative to a risk-free return divided by the standard deviation of the asset return distribution, the Sharpe ratio represents the slope of the capital market line within the mean-variance Markowitz efficient frontier [8, 9]. Christie [1], among others, remarked that investors frequently use Sharpe ratios to suggest evidence of portfolio superiority. Importantly, however, caution has been issued against an overreliance upon this metric to provide guidance for decision making, as it may provide statistically indistinguishable results due to estimation error even if obtained via large data samples. More specifically, the requirements for calculating expected returns and standard deviation are measured with error, hence subjecting the Sharpe ratio to estimation error itself. From a mathematical stance, the sampling distribution of the ratio may be difficult to determine due to the presence of the random denominator that defines the ratio. Despite challenges in its estimation, the measure continues to be an important tool in the comparative assessment of financial portfolio performance.

A number of generalizations of the Sharpe ratio have been developed since its initial publication that seek to adjust several factors including autocorrelation, skew and kurtosis (e.g., [10]), and non-normal distributions (e.g., [4, 6]). Despite developments of this nature, Lo [7] commented that relatively limited empirical work has focused upon the statistical properties of the Sharpe ratio itself. In one notable study, Jobson and Korkie [5] developed a test of the difference between Sharpe ratios utilizing the Delta method, but subsequently found that the procedure lacked statistical power; robust methods to test statistical differences in the presence of sampling error continue to be under investigated. In other important work, Lo [7] presented statistical distributions of the ratio using standard asymptotic theory under numerous assumptions, also illustrating conditions wherein the ratio may be markedly overestimated. By building upon the concept that the denominator of the Sharpe ratio is random and presents challenges to the ratio's overall estimation, Vinod and Morey [13, 14] proposed a modification that sought to control for estimation error by capturing the standard deviation of the Sharpe ratio via a bootstrap approach, termed the Double Sharpe ratio. Notably, this latter work also emphasized the notion that sampling error contributes substantially to estimation error when considering expected returns and volatilities in financial portfolios.

Given the aforementioned, the purpose of the current research was to improve the statistical estimation error and overestimation associated with calculations of the Sharpe ratio, offering a methodology for both point estimates and confidence intervals that utilize implicit bootstrap resampling and computational intelligence while adding explicit analytic control via two central lemmas. The first lemma of this estimation error correction (EEC) minimizes the statistical estimation error under the common assumption that the loss function is defined as the squared error. The second lemma corrects for overestimation by identifying that the negative covariance between two variables representing the estimate of the Sharpe ratio and the standard deviation can be used for corrective purposes. In presenting the findings of the proposed estimation error correction approach, an empirical simulation study was also conducted to present improvements in statistical estimation errors after considering the known limitations of normal distributions within financial data. To allow for more direct comparisons, the current empirical simulation study built upon the parameters presented by Vinod and Morey [13, 14] in their analysis of financial data from 30 large historical growth mutual funds in terms of overall assets managed.

2 A Proposed Estimation Error Corrected (EEC) Sharpe Ratio

The population value of the Sharpe [11, 12] performance measure for portfolio *i* is defined as $Sr_i = \frac{\mu_i - z}{\sigma_i} = \frac{\mu_i - R_f}{\sigma_i}$, or the excess of the expected return of a portfolio above the risk-free rate of return divided by the standard deviation of the excess returns for the portfolio. If solely considering a financial portfolio i, a second investment portfolio may also be defined with a set proportion of funds allocated to it. If this second portfolio is defined as a risk-free rate of return, R_f, the proportion of funds allocated will be (1 – γ) if the proportion allocated to portfolio i is defined as γ. Thus, the expected return of the new combined portfolio j is: $\mu_j = \gamma\mu_i + (1-\gamma)R_f$.

As the risk free return has zero variance by definition, the standard deviation of the new portfolio j is $\sigma_j = \gamma\sigma_i$. Recognizing that the aforementioned standard deviations may also be presented as $\gamma = \frac{\sigma_j}{\sigma_i}$, substituting this into the equation representing the expected return of the new portfolio j yields:

$$\mu_j = \frac{\sigma_j}{\sigma_i}\mu_i + (1 - \frac{\sigma_j}{\sigma_i})R_f = R_f + \frac{\sigma_j}{\sigma_i}(\mu_i - R_f)$$

Therefore, it may be shown that $\mu_j = R_f + \lambda\sigma_j$, wherein: $\lambda = \frac{\mu_i - R_f}{\sigma_i} = Sr_i$.

For a portfolio population mean μ, fixed risk-free rate of return z or R_f with zero variance, and portfolio standard deviation σ, the sample counterparts used to estimate these population parameters via the sample mean and sample standard deviation are calculated from a random sample (X_1, X_2, X_3… X_n) generally as a sample mean $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ and a sample standard deviation, $s = \sqrt{s^2}$ where $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$. Therefore, the applied point estimate for the Sharpe ratio may essentially be represented as $S\tilde{r}_i = \frac{E(R_i) - R_f}{s_i} = \frac{\bar{x} - R_f}{s_i} = E(Sr_i)$

While Vinod and Morey [13, 14] did not use this aforementioned sampling approach (i.e., X_1, X_2, X_3,…,X_n) directly in their development of the Double Sharpe ratio, these authors proposed the use of the bootstrap methodology to generate a large number of resamples from the original sample (X_1, X_2, X_3,…, X_n). Some 999 bootstrap resamples were used to calculate the estimate of the Sharpe ratio, coupled with an estimation of the standard deviation of these ratios by using bootstrapped means as $\sigma_{S\tilde{r}}$. Therefore, the improvements proposed by Vinod and Morey [13, 14] consisted of the estimation of both the numerators and the denominators of the Sharpe ratio using the bootstrap-resamples, separately by their bootstrap-mean estimates of the numerator and of the denominator.

As discussed previously, the applied point estimate for the Sharpe ratio may be represented as $S\tilde{r} = \frac{\bar{x} - R_f}{s_i} = E(Sr)$. The novel estimators of the Sharpe ratios constructed and proposed in the current research endeavor consist of improved estimation in both the numerator and denominators of the ratio using 999 bootstrap resamples as presented by Vinod and Morey [13, 14]. In this context, the first undertaking is to establish an efficient estimator of the inverse of the normal standard deviation via Lemma 1, which minimizes the statistical estimation error under the common assumption that the loss function is the squared error.

Lemma 1. For a random sample (X1, X2, X3,…,Xn) from a normal population N(μ, σ), the minimum mean square error (MMSE) of the inverse of the normal standard deviation may be designated as $C^*(1/s)$, wherein:

$$C^* = \frac{\sqrt{\frac{2}{n-1}} \cdot \Gamma(\frac{n}{2}-1)}{\Gamma(\frac{n-3}{2})}$$

Proof: C* is the MMSE of given that:

$$C^* = \frac{E(\frac{1}{s}) \cdot (\frac{1}{\sigma})}{E(\frac{1}{s^2})}$$

Furthermore, it is well-established that (n-1)(s2) ~ χ2n-1. □

The second lemma of the current investigation corrects for overestimation by identifying that the negative covariance between two variables representing the estimate of the Sharpe ratio and the standard deviation can be used for corrective purposes. In developing this, it is again established that the Sharpe ratio is defined as $Sr_i = \frac{\mu_i - z}{\sigma_i} = \frac{\mu_i - R_f}{\sigma_i}$ which is the excess expected return of portfolio i relative to a risk-free return divided by the standard deviation of the asset return distribution, estimated as $S\tilde{r}_i = \frac{E(R_i) - R_f}{s_i} = \frac{\bar{x} - R_f}{s_i} = E(Sr_i)$ and recalling again that the risk-free rate of return in this ratio has zero variance. Extending this definition of the Sharpe ratio, a bootstrap resample may be defined as $E_B(Sr_i) = E_B(\frac{E(R_i) - R_f}{s_i})$. The aforementioned must be noted in the context of formalizing Lemma 2, as follows.

Lemma 2. For two variables defined as $E(R_i) - R_f$ and s_i within the Sharpe ratio, representing the excess expected return of portfolio i relative to a risk-free return with zero variance and the standard deviation of the asset return distribution, respectively, the bootstrap resample E_B for the given characteristics is as follows:

$$E_B\left(\frac{E(R_i) - R_f}{s_i}\right) > \frac{E_B(E(R_i) - R_f)}{E_B(s_i)}$$

Proof: The aforementioned may be established given that:

$$E_B\left(\frac{E(R_i) - R_f}{s_i}\right) E_B(s_i) > E_B(s_i)$$

under plausible conditions of either $E_B(s_i) > 0$ or $\mathrm{Cov}\left(\frac{E(R_i) - R_f}{s_i}, s_i\right) < 0$, which holds as $\frac{E(R_i) - R_f}{s_i}$ may either increase or decrease as s_i increases or decreases, respectively. □

The Double Sharpe Ratio was defined by Vinod and Morey [13, 14] as $DS\tilde{r} = S\tilde{r} / \sigma_{S\tilde{r}}$, noting the importance of the standard deviation of the Sharpe ratio estimates which was estimated using 999 bootstrap resamples with replacements from the original sample. In the current investigation, the efficient estimation of the Sharpe ratio's numerator is adopted through similar approach, though adding more explicit control through Lemma 1 and Lemma 2. To illustrate the current study's findings in a comprehensible fashion and for comparative reference, the case-study employed by Vinod and Morey [13, 14] is also used, wherein Sharpe measures were calculated from original excess return series from 30 largest historical growth funds. These authors justified the 999 measures calculated because the rank-ordered 25^{th} and 975^{th} values estimated yielded useful 95 percent confidence intervals. Results of this previous investigation reported the excess monthly mean return, the standard deviation of the excess monthly returns, the Sharpe ratios, and the mean and standard deviations of the bootstrapped Sharpe ratios. Furthermore, the upper and lower bounds of the confidence intervals of the bootstrapped Sharpe ratios were reported, as were their widths. Complementary analyses were conducted for their proposed Double Sharpe ratio. It is important to recognize, both within Vinod and Morey [13, 14] and within the current investigation, that the mean values of bootstrapped Sharpe ratios will always be observed to be higher than those values computed that ignore estimation error (i.e., point estimates of the Sharpe ratio); this observation occurs because the sampling distribution that represents the statistical estimation error for the Sharpe ratio has been reported to be markedly non-normal with positive skew.

In applying Lemma 1 and Lemma 2 more formally to the Sharpe ratio or Double Sharpe ratio, and as a prelude to the forthcoming, it should be emphasized that the stochastic variation of the denominator remains the key challenge that requires estimation error minimization when the Sharpe ratio is calculated. To reach this investigative goal, again, an empirical simulation study was used that parallels the Double Sharpe ratio of Vinod and Morey [13, 14], wherein the bootstrap resamples of both their numerator and denominator should be determined prior to that of the ratio itself. Recalling, the Double Sharpe ratio defines the numerator as the estimated Sharpe ratio and the denominator as the standard error of this estimated Sharpe ratio (i.e., the standard deviation of the sample estimates of Sharpe ratio from various bootstrap resamples),

$DS\tilde{r} = {S\tilde{r}}/{\sigma_{S\tilde{r}}}$. In the current study, the 'estimation error corrected bootstrap point estimate of the Sharpe ratio' is designated as $E_{B,EEC}(Sr)$ relative to the Vinod and Morey [13, 14] bootstrap point estimate of the Sharpe ratio as $E_{B,VM}(Sr)$. Based upon Lemma 1 and Lemma 2, it may be expected that $E_{B,VM}(Sr)$ provides an overestimate due to the estimation error present. Building upon $E_{B,EEC}(Sr)$, computational intelligence described by Engelbrecht [3] was also applied, which incorporates information from the simulation to yield additional improvements, designated as the 'estimation error corrected bootstrap point estimate of the Sharpe ratio via computational intelligence', or $E_{B,EEC,CI}(Sr)$. The additional analyses that provided this computational intelligence indicated that an optimal choice of the design parameter m was a value of 3, to yield:

$$E_{B,EEC,CI}(Sr) = E_{B,EEC}(Sr) \cdot m$$

where *m* is defined as a positive integer. Overall, the final numerator after incorporating computation intelligence is:

$$E_{B,EEC,CI}(Sr) = E_{B,EEC}(Sr) - 3\left(E_{B,VM}(Sr) - E_{B,EEC}(Sr)\right)$$

Capturing the concepts relating to Lemma 1, Lemma 2, and computational intelligence $E_{B,EEC,CI}(Sr)$, three final efficient point estimators were ultimately analyzed in the current study via the empirical simulation study:

1) Applying Lemma 1, the first proposed efficient point estimator of the Sharpe ratio estimator $E_{B,EEC,1}(Sr)$ using all resamples is:

$$E_{B,EEC,1}(Sr) = C^* \cdot \left(E_{B,VM}(Sr)\right) = \left(\frac{\sqrt{\frac{2}{n-1}} \cdot \Gamma(\frac{n}{2} - 1)}{\Gamma(\frac{n-3}{2})} \right) \cdot \left(E_{B,VM}(Sr)\right)$$

2) Applying Lemma 1 in conjunction with Lemma 2, the second proposed efficient point estimator of the Sharpe ratio $E_{B,EEC,1+2}(Sr)$ using all resamples is:

$$E_{B,EEC,1+2}(Sr) = C^* \cdot \left(\frac{\bar{x}}{\bar{s}}\right) = \left(\frac{\sqrt{\frac{2}{n-1}} \cdot \Gamma(\frac{n}{2}-1)}{\Gamma(\frac{n-3}{2})}\right) \cdot \left(\frac{\bar{x}}{\bar{s}}\right)$$

and

3) Applying Lemma 1 with Lemma 2 and adding the results of computational intelligence, the third proposed efficient point estimator of the Sharpe ratio $E_{B,EEC,1+2+CI}(Sr)$ is:

$$E_{B,EEC,1+2+CI}(Sr) = 4\left(E_{B,EEC,1+2}(Sr)\right) - 3\left(E_{B,EEC,1}(Sr)\right)$$

To assess the relative performance of these three final point estimators $E_{B,EEC,1}(Sr)$, $E_{B,EEC,1+2}(Sr)$ and $E_{B,EEC,1+2+CI}(Sr)$, the Double Sharpe ratio $DS\tilde{r}$ from Vinod and Morey [13, 14] was defined as referent computation through which relative efficiencies were measured.

In addition to the three point estimates presented above, the current investigation also sought to develop a more efficient estimation error corrected 95% confidence interval for the Sharpe ratio, $E_{B,EEC,95\%\ CI}(Sr)$. In articulating this confidence interval and recalling that the Sharpe ratio may be described as $S\tilde{r}_i = \frac{E(R_i) - R_f}{s_i} = \frac{\bar{x} - R_f}{s_i} = E(Sr_i)$, a series of 999 values of $S\tilde{r}_i$ was determined for each of 999 bootstrap resamples. If arranged in ascending order of these values, the resultant row vector array will be of the order (1, 999) and can be designated as $E_{B,EEC\ Array}(Sr_i)$. Consequently, the 25th and 975th elements of this array yield a 95% confidence interval at $E_{B,EEC\ Array}(Sr_i)(25)$ and $E_{B,EEC\ Array}(Sr_i)(975)$. After applying Lemma 2, the current study proposes a more efficient 95% confidence interval of the Sharpe ratio, $E_{B,EEC,95\%\ CI}(Sr)$ defined as:

$$E_{B,EEC,95\%\ CI}(Sr) = [C^* \cdot E_{B,EEC,Array}(Sr_i)(25), C^* \cdot E_{B,EEC,Array}(Sr_i)(975)] =$$

$$= \left[\left(\frac{\sqrt{\frac{2}{n-1}} \cdot \Gamma(\frac{n}{2}-1}{\Gamma(\frac{n-3}{2})} \right) E_{B,EEC,Array}(Sr_i)(25), \left(\frac{\sqrt{\frac{2}{n-1}} \cdot \Gamma(\frac{n}{2}-1}{\Gamma(\frac{n-3}{2})} \right) E_{B,EEC,Array}(Sr_i)(975) \right]$$

For consistency, the 95% confidence intervals for the Double Sharpe ratio from Vinod and Morey [13, 14] were established as the referent computation through which relative efficiencies were measured, designated as:

$$E_{B,VM,95\%\ CI}(Sr) = [E_{B,VM,Array}(Sr)(25), E_{B,VM,\ Array}(Sr)(975)] .$$

3 Main Results: Empirical Simulation Study

The empirical simulation study for the current investigation was conducted using Matlab 2010b (The Mathworks Inc., Natick, Massachusetts, USA) for various illustrative values of the sample sizes N = 11, 21, 31, 41, 51, 71, and 101. Using parameters established from Vinod and Morey [13, 14], the parent population was taken to be normal with the population Sharpe ratio of 0.20. Also from Vinod and Morey [13, 14], the various values of population standard deviation were defined as σ = 3.25, 3.75, 4.25, 4.75, 5.25, 5.75, 6.25, 6.75, and 7.25. Some 1111 replications were conducted in the current simulation for the 999 bootstrap resamples utilizing bootstrap methodology outlined by Davison and Hinkley [2]. The actual mean squared error (MSE) of the Vinod and Morey [13, 14] bootstrap point estimate of the Sharpe ratio, $E_{B,VM}(Sr)$ and of the three final point estimators $E_{B,EEC,1}(Sr)$, $E_{B,EEC,1+2}(Sr)$ and $E_{B,EEC,1+2+CI}(Sr)$ were calculated by averaging the squared deviation of the estimator's value from the population Sharpe ratio (i.e., 0.20) using 999 resamples for each of the original

1111 samples. Furthermore, the relative efficiency of the three proposed estimators relative to that of Vinod and Morey [13, 14] were calculated via the following formula:

$$\mathrm{Re}\,lEff\left(\mathrm{E}_{\mathrm{B,EEC,1}}(\mathrm{Sr}),\mathrm{E}_{\mathrm{B,EEC,1+2}}(\mathrm{Sr}),\text{ or }\mathrm{E}_{\mathrm{B,EEC,1+2+CI}}(\mathrm{Sr})\right)$$
$$=\left[\frac{\mathrm{MSE}\left(\mathrm{E}_{\mathrm{B,VM}}(\mathrm{Sr})\right)\cdot 100}{\mathrm{MSE}(\mathrm{E}_{\mathrm{B,EEC,1}}(\mathrm{Sr}),\mathrm{E}_{\mathrm{B,EEC,1+2}}(\mathrm{Sr}),\text{or }\mathrm{E}_{\mathrm{B,EEC,1+2+CI}}(\mathrm{Sr}))}\right]\%$$

Results of the relative efficiencies of the three proposed estimation error corrected point estimators are presented in Table 1. Across the 999 bootstrap resamples replicated 1111 times, the relative efficiencies for all three proposed estimation error corrected (EEC) approaches were comprehensively improved versus Vinod and Morey [13, 14]. Furthermore, within these three proposed EEC approaches, the method that employed Lemma 1 and Lemma 2 with computational intelligence, $\mathrm{E}_{\mathrm{B,EEC,1+2+CI}}(\mathrm{Sr})$, yielded the highest relative efficiencies with values ranging from 105.210 percent (N = 101, σ = 6.25) to 198.959 percent (N = 11, σ = 3.25). Not unexpectedly, improvements were also more pronounced at smaller sample sizes.

Table 2 presents the relative performance of the proposed estimation error corrected confidence interval within the current study, versus the Vinod and Morey [13, 14] approach, $\mathrm{E}_{\mathrm{B,EEC,95\%\ CI}}(\mathrm{Sr})$. For the 1111 replication samples and respective 999 bootstrap resamples, the 95% confidence interval performance characteristics calculated included: coverage probability; coverage error; length; left bias; right bias; and relative bias. For each of these performance characteristics, results of the current empirical simulation study indicated that $\mathrm{E}_{\mathrm{B,EEC,95\%\ CI}}(\mathrm{Sr})$ offered substantial improvements versus $\mathrm{E}_{\mathrm{B,VM,95\%\ CI}}(\mathrm{Sr})$. Across all simulated combinations of sample sizes and standard deviations, the coverage probability for the EEC confidence interval fared equally to or better than the Vinod and Morey [13, 14] approach in each of the 20 empirical scenarios, with all estimation error corrected values more closely approaching 0.95.

Importantly as well, the coverage error for $E_{B,EEC,95\%\ CI}(Sr)$ was equal or improved across all simulations, and the confidence interval length was uniformly shorter. Finally, unlike $E_{B,VM,95\%\ CI}(Sr)$, the $E_{B,EEC,95\%\ CI}(Sr)$ also offered a more balanced left and right bias, which ultimately resulted in a markedly improved relative bias throughout the empirical investigation.

4 Conclusion

The Sharpe ratio is a commonly-used method that is used to assess financial portfolio performance based upon risk versus return. Despite its popularity, only limited research has been conducted to improve the statistical properties of this metric. The current study sought to improve the statistical estimation error and overestimation associated with calculations of the Sharpe ratio, offering a methodology for both point estimates and confidence intervals that utilize implicit bootstrap resampling and computational intelligence while adding explicit analytic control via two central lemmas. Results of the empirical simulation study indicated improved estimation error correction of the Sharpe ratio based upon the relative efficiency of point estimates and the coverage probability, coverage error, length, and relative bias of confidence intervals.

Table 1. Simulation Results of Relative Efficiencies for Three Proposed Estimation Error Corrected (EEC) Point Estimates of the Sharpe Ratio Relative to Vinod and Morey [13, 14]

		Relative Efficiency (%)								
		σ = 3.25	**σ = 3.75**	**σ = 4.25**	**σ = 4.75**	**σ = 5.25**	**σ = 5.75**	**σ = 6.25**	**σ = 6.75**	**σ = 7.25**
N = 11	$E_{B,EEC,1}(Sr)$	134.093	134.699	134.313	134.327	133.611	134.172	134.750	135.234	135.548
	$E_{B,EEC,1+2}(Sr)$	154.803	154.415	150.784	152.999	149.760	150.718	154.864	153.519	155.145
	$E_{B,EEC,1+2+CI}(Sr)$	198.959	183.912	181.423	182.684	175.459	179.455	194.275	188.390	190.247
N = 21	$E_{B,EEC,1}(Sr)$	115.357	115.223	114.121	114.921	114.807	115.562	115.004	114.846	114.826
	$E_{B,EEC,1+2}(Sr)$	122.107	121.179	119.336	121.304	121.198	121.697	121.129	121.032	120.561
	$E_{B,EEC,1+2+CI}(Sr)$	138.928	135.208	130.911	136.208	136.483	137.629	135.771	136.610	134.465
N = 31	$E_{B,EEC,1}(Sr)$	109.201	109.152	110.108	109.649	110.008	110.499	109.638	109.884	109.241
	$E_{B,EEC,1+2}(Sr)$	112.943	112.957	114.103	113.973	114.587	114.398	113.080	114.468	113.171
	$E_{B,EEC,1+2+CI}(Sr)$	122.643	122.730	124.407	125.043	127.025	124.804	121.235	127.279	123.522
N = 41	$E_{B,EEC,1}(Sr)$	107.384	106.722	107.272	107.147	107.232	106.620	106.839	106.932	106.956
	$E_{B,EEC,1+2}(Sr)$	110.405	109.335	109.976	109.974	110.207	109.197	109.758	109.537	109.747
	$E_{B,EEC,1+2+CI}(Sr)$	118.628	116.247	117.083	117.465	118.255	115.806	117.720	116.346	117.326
N = 51	$E_{B,EEC,1}(Sr)$	105.365	105.779	105.589	105.299	105.240	105.657	105.752	105.190	105.301
	$E_{B,EEC,1+2}(Sr)$	107.380	108.105	108.085	107.525	107.505	107.775	107.955	107.223	107.768
	$E_{B,EEC,1+2+CI}(Sr)$	112.852	114.491	115.027	113.708	113.723	113.400	113.988	112.612	114.610
N = 71	$E_{B,EEC,1}(Sr)$	103.755	103.713	104.190	104.140	103.921	103.814	103.984	103.879	103.816
	$E_{B,EEC,1+2}(Sr)$	105.181	105.128	105.862	105.746	105.274	105.451	105.406	105.336	105.138
	$E_{B,EEC,1+2+CI}(Sr)$	109.056	108.984	110.539	110.206	108.998	110.065	109.278	109.294	108.667
N = 101	$E_{B,EEC,1}(Sr)$	102.622	102.749	102.572	102.600	102.766	102.439	102.355	102.606	102.897
	$E_{B,EEC,1+2}(Sr)$	103.783	103.834	103.619	103.662	103.820	103.337	103.131	103.716	103.996
	$E_{B,EEC,1+2+CI}(Sr)$	107.118	106.893	106.540	106.660	106.770	105.781	105.210	106.806	107.093

$E_{B,EEC,1+2+CI}(Sr)$: Bootstrap expected value of the point estimate for the estimated error corrected Sharpe ratio applying Lemma 1 and 2, and Computational Intelligence, respectively; Comparator for relative efficiency defined as the Vinod and Morey [13, 14] bootstrap expected value of the Sharpe ratio

Table 2. Simulation Results of Performance for Proposed Estimation Error Corrected (EEC) Sharpe ratio 95% Confidence Interval Relative to Vinod and Morey [13, 14]

		95% Confidence Interval Performance Characteristics					
		Coverage Probability	Coverage Error	Length	Left Bias	Right Bias	Relative Bias
N = 31, σ = 3.25	$E_{B,VM,95\%CI}(SR)$	0.927093	0.022907	0.756226	0.030603	0.042304	0.160494
	$E_{B,EEC,95\%CI}(SR)$	0.930693	0.019307	0.724091	0.032403	0.036904	0.064935
N = 31, σ = 4.25	$E_{B,VM,95\%CI}(SR)$	0.932493	0.017507	0.754222	0.027903	0.037804	0.150685
	$E_{B,EEC,95\%CI}(SR)$	0.934293	0.015707	0.722172	0.032403	0.035104	0.040000
N = 31, σ = 5.25	$E_{B,VM,95\%CI}(SR)$	0.942394	0.007606	0.755497	0.020702	0.036904	0.281250
	$E_{B,EEC,95\%CI}(SR)$	0.943294	0.006706	0.723393	0.023402	0.033303	0.174603
N = 31, σ = 6.25	$E_{B,VM,95\%CI}(SR)$	0.946895	0.003105	0.750657	0.025203	0.027903	0.050847
	$E_{B,EEC,95\%CI}(SR)$	0.947795	0.002205	0.718758	0.027003	0.025203	0.034483
N = 31, σ = 7.25	$E_{B,VM,95\%CI}(SR)$	0.920792	0.029208	0.754303	0.029703	0.049505	0.250000
	$E_{B,EEC,95\%CI}(SR)$	0.920792	0.029208	0.722249	0.031503	0.047705	0.204545
N = 51, σ = 3.25	$E_{B,VM,95\%CI}(SR)$	0.940594	0.009406	0.574231	0.023402	0.036004	0.212121
	$E_{B,EEC,95\%CI}(SR)$	0.941494	0.008506	0.559706	0.024302	0.034203	0.169231
N = 51, σ = 4.25	$E_{B,VM,95\%CI}(SR)$	0.947795	0.002205	0.573691	0.027903	0.024302	0.068966
	$E_{B,EEC,95\%CI}(SR)$	0.950495	0.000495	0.559180	0.023402	0.026103	0.054545
N = 51, σ = 5.25	$E_{B,VM,95\%CI}(SR)$	0.945995	0.004005	0.575705	0.019802	0.034203	0.266667
	$E_{B,EEC,95\%CI}(SR)$	0.945995	0.004005	0.561143	0.022502	0.031503	0.166667
N = 51, σ = 6.25	$E_{B,VM,95\%CI}(SR)$	0.938794	0.011206	0.574478	0.017102	0.044104	0.441176
	$E_{B,EEC,95\%CI}(SR)$	0.941494	0.008506	0.559948	0.018002	0.040504	0.384615
N = 51, σ = 7.25	$E_{B,VM,95\%CI}(SR)$	0.937894	0.012106	0.574725	0.027903	0.034203	0.101449

	$E_{B,EEC,95\%CI}(SR)$	0.937894	0.012106	0.560188	0.028803	0.033303	0.072464
N = 71, σ = 3.25	$E_{B,VM,95\%CI}(SR)$	0.939694	0.010306	0.481578	0.022502	0.037804	0.253731
	$E_{B,EEC,95\%CI}(SR)$	0.940594	0.009406	0.472907	0.023402	0.036004	0.212121
N = 71, σ = 4.25	$E_{B,VM,95\%CI}(SR)$	0.944194	0.005806	0.481914	0.026103	0.029703	0.064516
	$E_{B,EEC,95\%CI}(SR)$	0.948695	0.001305	0.473237	0.026103	0.025203	0.017544
N = 71, σ = 5.25	$E_{B,VM,95\%CI}(SR)$	0.957696	0.007696	0.481648	0.016202	0.027903	0.265306
	$E_{B,EEC,95\%CI}(SR)$	0.955896	0.005896	0.472976	0.016202	0.026103	0.234043
N = 71, σ = 6.25	$E_{B,VM,95\%CI}(SR)$	0.954095	0.004095	0.482228	0.018002	0.028803	0.230769
	$E_{B,EEC,95\%CI}(SR)$	0.953195	0.003195	0.473545	0.018902	0.027003	0.176471
N = 71, σ = 7.25	$E_{B,VM,95\%CI}(SR)$	0.943294	0.006706	0.481991	0.019802	0.036904	0.301587
	$E_{B,EEC,95\%CI}(SR)$	0.948695	0.001305	0.473312	0.019802	0.031503	0.228070
N = 101, σ = 3.25	$E_{B,VM,95\%CI}(SR)$	0.936094	0.013906	0.400797	0.035104	0.028803	0.098592
	$E_{B,EEC,95\%CI}(SR)$	0.937894	0.012106	0.395757	0.033303	0.028803	0.072464
N = 101, σ = 4.25	$E_{B,VM,95\%CI}(SR)$	0.947795	0.002205	0.402040	0.023402	0.027903	0.087719
	$E_{B,EEC,95\%CI}(SR)$	0.947795	0.002205	0.396985	0.026103	0.026103	0.000000
N = 101, σ = 5.25	$E_{B,VM,95\%CI}(SR)$	0.944194	0.005806	0.401403	0.019802	0.036004	0.290323
	$E_{B,EEC,95\%CI}(SR)$	0.945995	0.004005	0.396356	0.021602	0.032403	0.200000
N = 101, σ = 6.25	$E_{B,VM,95\%CI}(SR)$	0.942394	0.007606	0.401476	0.032403	0.024302	0.142857
	$E_{B,EEC,95\%CI}(SR)$	0.943294	0.006706	0.396428	0.029703	0.027903	0.031250
N = 101, σ = 7.25	$E_{B,VM,95\%CI}(SR)$	0.937894	0.012106	0.401857	0.016202	0.045905	0.478261
	$E_{B,EEC,95\%CI}(SR)$	0.938794	0.011206	0.396805	0.017102	0.044104	0.441176

$E_{B,VM,95\%CI}(Sr)$: Bootstrap expected value of the 95% confidence interval for Vinod and Morey [13, 14] Sharpe ratio

$E_{B,EEC,95\%CI}(Sr)$: Bootstrap expected value of the 95% confidence interval for the estimated error corrected Sharpe ratio.

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