The economic cost of procrastination A statistical model

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Abstract

This paper proposes a statistical model for estimating the economic cost of procrastination. The proposed model made used of given random samples on length of time of delay or procrastination by a random sample of individuals with respect to a given enterprise to calculate sample mean which was used to estimate μ , which in turn was used to estimate the parameter of any hypothesized distribution of unknown. The estimated parameters were therefore used to calculate the require probabilities, expected numbers and costs of procrastination. The model was illustrated with an example. The results showed that a salary earner of eighteen thousand naira (N18, 000.00) given the various delay time periods has an average cost of procrastination of three thousand two hundred and seventy naira twenty eight kobo (N3, 270.28) per day.

Keywords: Enterprise, probability, delay, sample mean, estimate, and parameters

1 Introduction

Procrastination is defined as the avoidance of doing a task which needs to be accomplished. According to Wikipedia is refers to as the counterproductive deferment of action or tasks to a later time. This can lead to feelings of quilt, stress, inadequacy, depression and self-doubt, as well as a societal disapproval for

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not meeting responsibility or commitment among people. Procrastination has a high potential for painful consequences. Thus, it is putting off till tomorrow what one ought to have done today. It is sometimes also euphemistically termed the 'thief of time'. The so call African time, the tendency to regard scheduled commencement time of an event as a mere suggestion and not the time the event is actually expected to commence, for example delays in acquiring further education, starting an industry or enterprise, building, getting married, etc, then one is procrastinating.

However, time cannot wait, one needs to keep up with time. In a moneydriven economy procrastination could be very costly. Cost of goods and services tend to increase with time. Appropriate human and material resources may be depleted or lost with time. There are also invisible costs such as cost levels of satisfaction and appreciation damaged inter-personal relationship, social fabrics and rapport.

Quantifying these costs separately is rather difficult. However, a researcher may use indices of cost to represent both the visible and invisible costs in an exploratory model to study the economic cost of procrastination. Researchers that have worked on procrastination include [1], [2], [3], [4], [5], [6], and [7].

2 The proposed model

Let n(0) be the number or group of individual in a given year who delay starting an event or enterprise for different time periods. Let X be a random variable representing the length of time it takes a randomly selected individual in the group to start an enterprise that should have been started immediately other things being equal, than, X may be interpreted as the delay time, lag-time or 'procrastination time period' between the expected and actual commencement time of the enterprise by the randomly selected individual. We may assume that X has a continuous probability distribution with pdf f(x), for $x \ge 0$.

We further assume that both the costs of delay for x time unit are respectively $c_1(x)$ and $c_2(x)$ for $x \ge 0$.

Then the probability that a randomly selected individual from the group delays less than x time units (measured in, for example hours, days months, years etc) before starting the enterprise is

$$q(X) = p(X < x) = \int_{0}^{x} f(y)d_{y} = F(x)$$

$$\tag{1}$$

The number of individuals who delay less than x time units before starting the enterprise are

$$d(x) = n(0)q(x) = n(0)f(x)$$
(2)

Hence, the number of individuals in the group who delay more than x time unit before starting the enterprise is

$$n(x) = n(0) - d(x) = n(0)(1 - f(x))$$
(3)

The probability that a randomly selected individual takes between x and z time units to start an enterprise, that is, start the enterprise in the time interval (x, z) is

$$q(z-x) = P(x < X < z) = \int_{x}^{z} f(y)d_{y} = F(z) - F(x)$$
(4)

The corresponding number of individual who start the enterprise in the time interval (x, z) is

$$d(z-x) = n(x)q(z-x) = n(x)(F(z) - f(x))$$
(5)

Hence, the number of individual who start the enterprise after the time interval (x, z) that is at the end of this time interval is

$$n(z) = n(x) - d(z - x) = n(x)(1 - q(z - x))$$
(6)

Note that the number of person-time unit interval (x, z) by individuals who delay between x and z time units before starting the enterprise is

$$L(z-x) = \int_{x}^{z} n(y)d_{y}$$
⁽⁷⁾

And the total number of person-time units of delay or procrastination by the entire group is

$$T_0 = \int_0^\infty n(y) d_y \tag{8}$$

Hence, the total cost of delay or procrastination is

$$W_0 = \int_0^\infty n(y) (c_1(y) + c_2(y)) d_y$$
(9)

Therefore, the average or expected cost of delay or procrastination for the group is

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$$\overline{C} = \frac{W_0}{\left(\int_0^\infty n(y)d_y\right)} = \frac{\left[\int_0^\infty n(y)(c_1(y) + c_2(y))d_y\right]}{\left(\int_0^\infty n(y)d_y\right)}$$
(10)

The probability that a randomly selected individual delays starting the enterprise beyond time x but not later than time z(z > x) is

$$q(z/x) = P\left(x < \frac{z}{X} > x\right) = \frac{P(x < X < z)}{P(X > x)}$$

That is,

$$q\left(\frac{z}{x}\right) = \frac{F(z) - F(x)}{1 - f(x)} \tag{11}$$

The number of individuals in the category is

$$d\left(\frac{z}{x}\right) = n(x)q\left(\frac{z}{x}\right) \tag{12}$$

Hence, the number of individuals who start the enterprise neither before time x nor before time z(z > x) is

$$n(z) = n(x) - d\left(z / x\right) = n(xd\left(\frac{z}{x}\right) = n(x)q\left(\frac{z}{x}\right))\frac{\left(1 - F(z)\right)}{\left(1 - F(x)\right)}$$
(13)

The probability that the randomly selected individual does not start the enterprise before time x is

$$q(>x) = P(X > x) = \int_{0}^{\infty} f(y)d_{y}$$
(14)

The expected cost of delay or procrastination in this situation is estimated as in equation (10) except that equation (13) now replaces equation (6). That is, n(y) in the form of equation (6) in equation (10). Finally, the expected or mean length of delay or procrastination is

$$\mu = \int_{0}^{\infty} y f(y) d_{y}$$
(15)

Whose unbiased estimator is the sample mean (\overline{X}) .

Hence, if an appropriate random sample on length of time of delay or procrastination by a random sample of individuals with respect to some enterprise is available, the calculated sample mean may be used to estimate μ , which would in this be used to estimate the parameter of any hypothesized distribution of unknown. The estimated parameter may now be used in equation 1-13 to calculate the require probabilities, expected numbers and costs of procrastination.

3 A Hypothesized Probability Distribution for X

Suppose X has the pdf

$$f(X) = \frac{xe^{-\left(\frac{x}{\beta}\right)}}{\beta^2}, \quad x \ge 0, \ \beta > 0$$
(16)

Now, from equation (1), we have that the probability that a randomly selected individual from the group delays starting an enterprise less than X time unit is

$$q(X) = \frac{1}{\beta^2} \int_0^x y e^{-\left(\frac{y}{\beta}\right)} dy$$

That is

$$q(X) = 1 - \left(1 + \frac{x}{\beta}\right)e^{-\left(\frac{y}{\beta}\right)}$$
(17)

The number of individuals who delay starting the enterprise less than time X is from equation (12).

$$d(X) = n(0) \left[1 - \left(1 + \frac{x}{\beta}\right) e^{-\left(\frac{x}{\beta}\right)} \right]$$
(18)

From equation (3), we have that the number of individuals failing to start the enterprise before time X is

$$n(0)\left(1+\frac{x}{\beta}\right)e^{-\left(\frac{x}{\beta}\right)}$$
(19)

Also, from equation (4), we have that the probability that a randomly selected individual starts the enterprise in the time interval (x, z) is

$$q(z-x) = \left(1 + \frac{x}{\beta}\right)e^{-\left(\frac{x}{\beta}\right)} - \left(1 + \frac{z}{\beta}\right)e^{-\left(\frac{z}{\beta}\right)}$$
(20)

Hence, the number of individuals who start the enterprise in the time period (x, z) from equation (5) is

$$d(z-x) = n(x) = \left(1 + \frac{x}{\beta}\right)e^{-\left(\frac{x}{\beta}\right)} - \left(1 + \frac{z}{\beta}\right)e^{-\left(\frac{z}{\beta}\right)}$$
(21)

From equation (6), we have that the number of individuals failing to start an enterprise before time z, that is, before the end of the time interval (x, z) is

$$n(z) = n(x) - d(z - x)$$

where d(z-x) is given by equation (21). The probability that a randomly selected individual who has not started an enterprise before time x starts before time z is given by equation (11) as

$$q(z/x) = \frac{\left(1 + \frac{x}{\beta}\right)e^{-\left(\frac{x}{\beta}\right)} - \left(1 + \frac{z}{\beta}\right)e^{-\left(\frac{z}{\beta}\right)}}{\left(1 + \frac{x}{\beta}\right)e^{-\left(\frac{x}{\beta}\right)}}$$
(22)

The number of individuals in the category (equation (12) is

$$d(z/x) = \frac{n(x)\left\{\left(1+\frac{x}{\beta}\right)e^{-\left(\frac{x}{\beta}\right)} - \left(1+\frac{z}{\beta}\right)e^{-\left(\frac{z}{\beta}\right)}\right\}}{\left(1+\frac{x}{\beta}\right)e^{-\left(\frac{x}{\beta}\right)}}$$
(23)

The number of individuals who start an enterprise neither before time x nor before time z is from equation (13)

$$n(z) = n(x) - d\left(z / x\right) = \frac{n(x)\left(1 + \frac{z}{\beta}\right)e^{-(z-x)}}{\left(1 + \frac{x}{\beta}\right)}$$
(24)

From equation (14), we have that the probability that the randomly selected individual does not start the enterprise before time x is

$$q(>x) = \left(1 + \frac{x}{\beta}\right)e^{-\left(\frac{x}{\beta}\right)}$$
(25)

The corresponding number of individuals is

$$d(>x) = n(x)\left(1 + \frac{x}{\beta}\right)e^{-\left(\frac{x}{\beta}\right)}$$
(26)

The n(z) values are used in equation (10) to obtain an estimate of procrastination as shown in the illustrative example.

Finally, the expected value of x, the mean delay or procrastination time period is obtained by using equations (15) and (16) as

$$\mu = \mathrm{E}(X) = 2\beta$$

so that an unbiased sample of β is

$$\hat{\beta} = \frac{\overline{X}}{2} \tag{27}$$

where \overline{X} is the sample mean. In evaluating results of Equations 17-24, we may replace β with its estimate $\hat{\beta}$ of equation (27).

4 Illustrative Example and Results

Table1: Distribution of frequency of delay time periods at various intervals for an employee at N18,000 Salary Level

Delay time	0-	1-	2-	3-	4-	5-	6-	7-	8-	9-	10-	11-	total
period	1	2	3	4	5	6	7	8	9	10	11	12	
Frequency	6	8	6	5	4	2	2	2	2	1	1	1	40

Table2: Summary of results of computations for various parameters using table 1

Dela y Tim e peri od	x	F	<i>q</i> (<i>x</i>)	nq(x)	(0 _i -e _i)	$\frac{O_i - E_i}{E_i}$	$W(x)=L(x)c_1(x)$	$\mathbf{n}(\mathbf{z}) = \mathbf{n}(\mathbf{x}) - \mathbf{d}(\mathbf{x})$	d(x)=q(x)n(z)	$L(x) = \frac{n(x) + n(z)}{2}$
0-1	0.5	6	0.0972	3.888	2.112	1.1473	3.8322	39.426	3.8322	37.5099
1-2	1.5	8	0.1838	7.352	0.648	0.0571	7.2465	35.5938	6.5421	31.9706
2-3	2.5	6	0.1838	7.352	-1.352	0.2486	7.2465	28.3473	5.2102	24.7241
3-4	3.5	5	0.1533	6.132	-1.132	0.20897	6.044	21.1008	3.2348	18.0788
4-5	4.5	4	0.1167	4.668	-0.668	0.0956	4.601	15.0568	1.7571	12.7563
5-6	5.5	2	0.085	3.4	-1.4	0.5765	3.3512	10.4558	0.887	8.7802
6-7	6.5	2	0.0597	2.388	-0.368	0.063	2.3537	7.1046	0.4241	5.9278
7-8	7.5	2	0.0408	1.632	0.368	0.08298	1.6086	4.7509	0.1938	3.9466
8-9	8.5	2	0.0274	1.096	0.904	0.7456	1.0803	3.1423	0.0861	2.6022
9-10	9.5	1	0.0182	0.728	0.272	0.1016	0.7176	2.062	0.0375	1.7032
10- 11	10.5	1	0.012	0.478	0.522	0.5701	0.4711	1.3444	0.01613	1.1089
11- 12	11.5	1	0.0078	0.312	0.688	1.5171	0.3075	0.8733	0.0068	0.7196
Total		40	0.9857	39.426		5.4145		0.5658		

Hence, C= 489979.98/149.8282 = 3270.28. This implies that the average of procrastination of a salary earner of **N18**, 000 naira is **N3**, 270.28 naira per day.

5 Summary and Conclusion

We have here presented a statistical model which can be used to estimate the economic cost of procrastination at any given time delay. The proposed model made used of given random samples on length of time of delay or procrastination by a random sample of individuals with respect to a given enterprise to calculate sample mean which was used to estimate μ , which in turn was used to estimate the parameter of any hypothesized distribution of unknown. The estimated parameters were therefore used to calculate the require probabilities, expected numbers and costs of procrastination. The model was illustrated with an example. The results showed that a salary earner of eighteen thousand naira (N18,000.00) given the various delay time periods has an average cost of procrastination of three thousand two hundred and seventy naira twenty eight kobo (N3,270.28) per day.

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