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An Analysis of 6-th order Moment for the Case of One Sided Affiliated Impact Vibration

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Abstract

Among many dimensional and dimensionless amplitude parameters, kurtosis (4th normalized moment of probability density function) is recognized to be a sensitive good parameter for machine diagnosis. In this paper, one sided affiliated impact vibration is approximated by half triangle and a simplified calculation method is introduced. Various models are examined for this model and it is shown that simplified calculation method of 6-th normalized moment is much more sensitive than those of kurtosis. For the further detection, the concept of an absolute deterioration factor of n-th order moment in a broad sense is introduced and the analysis is executed. From the result of comparison, an absolute deterioration factor of 6-th order moment in a broad sense is better than other factors in the

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viewpoint of sensitivity and practical use. Thus, n-th order moment in a broad sense was examined and evaluated, and we have obtained practical good results.

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Keywords: impact vibration, probability density function, kurtosis, 6-th normalized moment, deterioration

1 Introduction

Machine diagnosis techniques play important roles on manufacturing. So far, many signal processing methods for machine diagnosis have been proposed [1]. As for sensitive parameters, Kurtosis, Bicoherence, Impact Deterioration Factor (ID Factor) were examined [2],[4]-[7],[10]. In this paper, we focus our attention to the index parameters of vibration. Kurtosis is one of the sophisticated inspection parameters which calculates normalized 4th moment of Probability Density Function (PDF). Kurtosis has a value of 3.0 under normal condition and the value generally goes up as the deterioration proceeds. But there were cases that kurtosis values went up and then went down when damages increased as time passed which were observed in our experiment in the past [6],[7].

Formerly, we have introduced a simplified calculation method of kurtosis and 6-th normalized moment for the analysis of impact vibration including affiliated impact vibration [11]. In this paper, one sided affiliated impact vibration is approximated by half triangle and a simplified calculation method is introduced. Furthermore an absolute deterioration factor is introduced. Various models are examined for this model. Comparing them, we show that the absolute deterioration factor of 6-th normalized moment is much more sensitive than those of Kurtosis.

For further detection, the concept of an absolute deterioration factor of n-th order moment in a broad sense is introduced and the analysis is executed. In this paper, we consider the case such that the impact vibration occurs on the gear when the failure arises. Higher moments would be more sensitive compared with 4-th moment. Kurtosis value is 3.0 under the normal condition and when the failure increases, the value grows big. Therefore, it is a relative index. On the other hand, Bicoherence is an absolute index which is close to 1.0 under the normal condition and tends to be 0 when the failure increases.

In this paper, we deal with the generalized n-th moment. When the theoretical value of n-th moment is divided by the calculated value of n-th moment, it would behave as an absolute index. The new index shows that it is 1.0 under the normal condition and tends to be 0 when the failure increases. As Bicoherence can be considered to be a kind of 6-th order moment, several factors concerning absolute deterioration factor of 6-th order moment are compared and evaluated. Trying several n, we search n which shows the most similar effect to the behavior of Bicoherence. From the result of comparison, an absolute deterioration factor of 6-th order moment in a broad sense is better than other factors in the viewpoint of sensitivity and practical use.

The rest of this study is organized as follows. We survey each index of deterioration in section 2. A simplified calculation method of Kurtosis including one sided affiliated impact vibration is stated in section 3. A simplified calculation method of 6-th normalized moment including one sided affiliated impact vibration is introduced in section 4. Numerical example is exhibited in section 5 and the corresponding method is summarized in Section 6. Remarks are made in section 6. Section 7 is a summary.

2 Factors for vibration calculation

In cyclic movements such as those of bearings and gears, the vibration grows larger whenever the deterioration becomes bigger. Also, it is well known that the vibration grows large when the setting equipment to the ground is unsuitable (Yamazaki, 1977). Assume the vibration signal is the function of time as $x(t)$. And also assume that it is a stationary time series with mean 0. Denote the probability density function of these time series as $p(x)$. Indices for vibration amplitude are as follows.

$$X_{root} = \left[\int_{-\infty}^{\infty} |x|^{\frac{1}{2}} p(x) dx\right]^2 \tag{1}$$

$$X_{rms} = \left[\int_{-\infty}^{\infty} x^2 p(x) dx\right]^{\frac{1}{2}} \tag{2}$$

$$X_{abs} = \int_{-\infty}^{\infty} |x| p(x) dx \tag{3}$$

$$X_{peak} = \lim_{n\to\infty}\left[\int_{-\infty}^{\infty} x^n p(x) dx\right]^{\frac{1}{n}} \tag{4}$$

These are dimensional indices which are not normalized. They differ by machine sizes or rotation frequencies. Therefore, normalized dimensionless indices are required. There are four big categories for this purpose.

A. Normalized root mean square value

B. Normalized peak value

C. Normalized moment

D. Normalized correlation among frequency domain

A. Normalized root mean square value

a.Shape Factor : SF

$$SF = \frac{X_{rms}}{\overline{\overline{X}}_{abs}} \tag{5}$$

($\overline{X}_{abs}$: mean of the absolute value of vibration)

B. Normalized peak value

b.Crest Factor : CrF

$$CrF = \frac{X_{peak}}{X_{rms}} \tag{6}$$

(X_{peak} : peak value of vibration)

c.Clearance Factor : ClF

$$ClF = \frac{X_{peak}}{X_{root}} \tag{7}$$

d.Impulse Factor : IF

$$IF = \frac{X_{peak}}{\overline{\overline{X}}_{abs}} \tag{8}$$

e.Impact Deterioration Factor : ID Factor

$$ID = \frac{X_{peak}}{X_c} \tag{9}$$

(X_c : vibration amplitude where the curvature of PDF becomes maximum)

C. Normalized moment

f.Skewness : SK

$$SK = \frac{\int_{-\infty}^{\infty} x^3 p(x)dx}{\left[\int_{-\infty}^{\infty} x^2 p(x)dx\right]^{\frac{3}{2}}} \tag{10}$$

g.Kurtosis : KT

$$KT = \frac{\int_{-\infty}^{\infty} x^4 p(x)dx}{\left[\int_{-\infty}^{\infty} x^2 p(x)dx\right]^{2}} \tag{11}$$

D. Normalized correlation in the frequency domain

h.Bicoherence

Bicoherence means the relationship of a function at different points in the frequency domain and is expressed as

$$Bic,_{xxx}(f_1, f_2) = \frac{B_{xxx}(f_1, f_2)}{\sqrt{S_{xx}(f_1) \cdot S_{xx}(f_2) \cdot S_{xx}(f_1 + f_2)}} \tag{12}$$

Here

$$B_{xxx}(f_1, f_2) = \frac{X_T(f_1) \cdot X_T(f_2) \cdot X_T^*(f_1 + f_2)}{T^{\frac{3}{2}}} \tag{13}$$

means Bispectrum and

$$X_T(t) = \begin{cases} x(t) & (0 < t < T) \\ 0 & (else) \end{cases} \tag{14}$$

T : Basic Frequency Interval

$$X_T(f) = \int_{-\infty}^{\infty} X_T(t) e^{-j2\pi ft} dt \tag{14}$$

$$S_{xx}(f) = \frac{1}{T} X_T(f) X_T^*(f) \tag{15}$$

Range of Bicoherence satisfies

$$0 < Bic,_{xxx}(f_1, f_2) < 1 \tag{16}$$

When there exists a significant relationship between frequencies f_1 and f_2, Bicoherence is near 1 and otherwise comes close to 0.

These indices are generally used in combination and machine condition is judged totally. Among them, Kurtosis is said to be superior index [3] and many researches on this have been made [2],[4],[5]. Judging from the experiment we have made in the past, we may conclude that Bicoherence is also a sensitive good index [6],[7].

Eq. (15) is a power spectrum. Power spectrum is a Fourier Transform of Autocorrelation function. Therefore it is a kind of second order moment in a broad sense. Watching at the denominator of Eq. (12), a square root is taken for the triple products of power spectrum. Normalization is executed by this item. That is, Bicoherence is equivalent to the square root of normalized 6-th order moment in a broad sense. Therefore, Bicoherence can considered to be a kind of an absolute deterioration factor of normalized 6-th order moment in a broad sense.

In [2], ID Factor is proposed as a good index. In this paper, we focusing on the indices of vibration amplitude, a simplified calculation method of Kurtosis and the 6-th normalized moment including one sided affiliated impact vibration is introduced. Furthermore an absolute deterioration factor is introduced. Varying the shape of triangle, various models are examined. An absolute index of n-th moment and Bicoherence are compared with this index and analysis is carried out.

3 Simplified calculation method of Kurtosis

3.1 Absolute index of *n*-th moment

Mean value $\bar{x}$ of $x(t)$ is calculated as:

$$\bar{x} = \int_{-\infty}^{\infty} xp(x)dx$$

Discrete time series are stated as follows.

$$xk = x(k\Delta t)$$

where Δt is a sampling time interval. $\bar{x}$ is stated as follows under discrete time series.

$$\bar{x} = \lim_{M\to\infty} \frac{1}{M}\sum_{i=1}^{M} x_i$$

Under the following Gaussian distribution:

$$\phi(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2} \tag{17}$$

its moment is described as follows which is well known(Hino,1977).

$$\overline{x^{(2n-1)}} = 0 \tag{18}$$

$$\overline{x^{(2n)}} = \prod_{k=1}^{n} (2k-1)\sigma^{2n} \tag{19}$$

If we divide Eq. (19) by σ^{2n}, we can obtain the normalized moment. In general, the normalized n-th moment is stated as follows.

$$Q(n)=\frac{\int_{-\infty}^{\infty}(x-\bar{x})^n p(x)dx}{\left[\int_{-\infty}^{\infty}(x-\bar{x})^2 p(x)dx\right]^{\frac{n}{2}}} \tag{20}$$

In discrete time system, it is described as:

$$Q(n)=\lim_{M\to\infty}\frac{\frac{1}{M}\sum_{i=1}^{M}(x_i-\bar{x})^n}{\left\{\frac{1}{M}\sum_{i=1}^{M}(x_i-\bar{x})^2\right\}^{\frac{n}{2}}} \tag{21}$$

We describe $Q(n)$ as $Q_N(n)$ if it is calculated by using N amount of data.

$$Q_N(n)=\frac{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^n}{\left\{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2\right\}^{\frac{n}{2}}} \tag{22}$$

The absolute index of n-th moment is described as follows.

$$Z_N(n)=\frac{\prod_{k=1}^{n/2}(2k-1)}{\frac{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^n}{\left\{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2\right\}^{\frac{n}{2}}}} \tag{23}$$

Under the normal condition, $Z_N(n)\to 1$ ($N\to\infty$), and if failure becomes larger, $Z_N(n)\to 0$.

3.2 Several facts on Kurtosis

Kurtosis (*KT*) is a normalized 4-th moment stated as follows.

$$KT = \frac{\int_{-\infty}^{\infty} x^4 p(x)\,dx}{\left[\int_{-\infty}^{\infty} x^2 p(x)\,dx\right]^2} = \lim_{N\to\infty} \frac{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^4}{\left\{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2\right\}^2} \quad (24)$$

Let KT of N amount of data is stated as KT_N.

3.3 Simplified calculation method of Kurtosis

When there arise failures on bearings or gears, peak values arise cyclically. In the early stage of the defect, this peak signal usually appears clearly. Generally, defects will injure other bearing or gears by contacting the inner covering surface as time passes. When defects grow up, one sided affiliated impact vibration arises. Impact vibration including one sided affiliated impact vibration occurs in the case that there is a failure of such as bearings' outer race. See Chart 1.

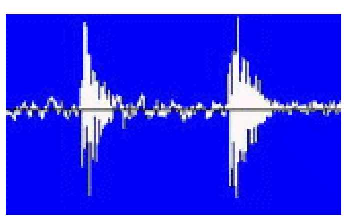

Chart 1: Example of one sided affiliated impact vibration

These signals can be approximated by the half triangle model. Hereafter, we analyze these cases by utilizing the simplified model.

Assume that the peak signal which has *p* times magnitude from normal signals arises during *m* times measurement of samplings. As for determining the sampling interval, the sampling theorem is well known. But in this paper, we do not pay much attention on this point in order to focus on our proposal theme.

Suppose that one sided affiliated vibration can be approximated by the half triangle and set the sampling count as *d*, then we can assume the following half triangle model (Figure 1).

When $d = 1$, the peak signal which has p times magnitude from normal signals arises.

When $d = i$, the peak signal which has $p - (i-1)\frac{p-1}{q}$ times magnitude from normal signals arises ($i = 1, \cdots, q$).

When $d \geq q+1$, normal signal.

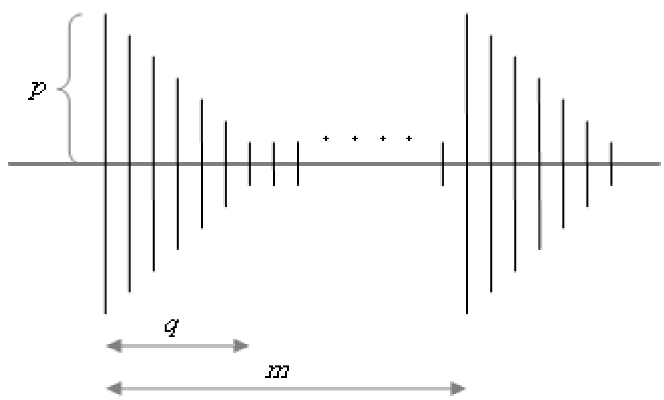

Figure 1: Impact vibration and one sided affiliated vibration

Let $\sigma_N{}^2$ state as $\overline{\sigma_N}^2$ when impact vibration occurs.

$\overline{\sigma_N}^2$ can be calculated as follows.

$$\overline{\sigma_N}^2 = \frac{1}{m-1}\sum_{i=1}^{m}\left(x_i - \bar{x}\right)^2$$
$$= \left[\sum_{i=1}^{q}\left\{p-(i-1)\frac{(p-1)}{q}\right\}^2\right]\frac{\sigma_N^{\ 2}}{m-1} + (m-1-q)\frac{\sigma_N^{\ 2}}{m-1} \tag{25}$$
$$= \sigma_N^{\ 2} + \frac{\sigma_N^{\ 2}}{m-1}(q+1)(p-1)\left\{1+\frac{(p-1)(2q+1)}{6q}\right\}$$

As for $\overline{MT_N}(4)$, utilizing:

$$\sum_{i=1}^{n} i^3 = \left\{\frac{n(n+1)}{2}\right\}^2$$
$$\sum_{i=1}^{n} i^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1)$$

$\overline{MT_N}(4)$ can be calculated as follows.

$$\overline{MT_N(4)} = \frac{1}{m-1}\sum_{i=1}^{m}\left(x_i - \bar{x}\right)^4$$
$$= \frac{1}{m-1}[\sum_{i=1}^{q}\left\{p-(i-1)\frac{(p-1)}{q}\right\}^4]MT_N(4) + \frac{m-1-q}{m-1}MT_N(4) \tag{26}$$
$$= [1+\frac{1}{m-1}(q+1)(p-1)\{\frac{1}{30}(p-1)^3\frac{1}{q^3}(2q+1)(3q^2+3q-1)$$
$$+(p-1)^2\frac{1}{q}(q+1)+(p-1)\frac{1}{q}(2q+1)+2\}]MT_N(4)$$

Then we get $\overline{KT_N}$ as:

$$\overline{KT_N} = \frac{1+\frac{1}{m-1}(q+1)(p-1)\{\frac{1}{30}(p-1)^3\frac{1}{q^3}(2q+1)(3q^2+3q-1)+(p-1)^2\frac{1}{q}(q+1)+(p-1)\frac{1}{q}(2q+1)+2\}KT_N}{[1+\frac{1}{m-1}(q+1)(p-1)\{\frac{2q+1}{6q}(p-1)+2\}]^2} \tag{27}$$

If the system is under normal condition, we may suppose p(x) becomes a normal distribution function. Under this condition, KT is always:

$$KT = 3.0$$

Therefore,

$$KT_N \cong 3.0 \tag{28}$$

As failure increases, $\overline{KT_N}$ value grows up. The absolute deterioration factor such as Bicoherence is easy to handle because it takes the value of 1.0 under the normal condition and tends to be 0 when damages increase. Therefore inverse number of the ($\overline{KT_N}$ −2) would make an absolute deterioration factor.

$$\overline{Z_N} = \frac{1}{\overline{KT_N} - 2} \tag{29}$$

Under the normal condition, $\overline{Z_N}$ is 1 and tends to be 0 when damages increase. Another method for an absolute deterioration factor is considered to be as follows.

$$\overline{V_N} = \frac{3}{\overline{KT_N}} \tag{30}$$

4 Simplified calculation method of 6-th normalized moment

4.1 Several Facts on 6-th Normalized Moment

6-th normalized moment is transformed into the one for the continuous time system as:

$$Q = \frac{\int_{-\infty}^{\infty} (x - \overline{x})^6 p(x)dx}{\left[\int_{-\infty}^{\infty} (x - \overline{x})^2 p(x)dx\right]^3} \tag{31}$$

And it is transformed into the one for discrete time system as:

$$Q = \lim_{M \to \infty} \frac{\frac{1}{M-1}\sum_{i=1}^{M}(x_i - \bar{x})^6}{\left\{\frac{1}{M-1}\sum_{i=1}^{M}(x_i - \bar{x})^2\right\}^3} \tag{32}$$

4.2 Simplified Calculation Method of 6-th Normalized Moment

As for 6-th moment and 6-th normalized moment, let $MT_N(6)$ and Q_N state as $\overline{MT_N(6)}$, $\overline{Q_N}$ when impact vibration occurs.

As for $\overline{MT_N(6)}$, utilizing:

$$\sum_{i=1}^{n} i^3 = \left\{\frac{n(n+1)}{2}\right\}^2 \tag{33}$$

$$\sum_{i=1}^{n} i^4 = \frac{n}{30}(n+1)(2n+1)(3n^2+3n-1) \tag{34}$$

$$\sum_{i=1}^{n} i^5 = \frac{1}{12}n^2(n+1)^2(2n^2+2n-1) \tag{35}$$

$$\sum_{i=1}^{n} i^6 = \frac{1}{42}n(n+1)(2n+1)(3n^4+6n^3-3n+1) \tag{36}$$

$\overline{MT_N(6)}$ can be calculated as follows.

$$\begin{aligned}\overline{MT_N(6)} &= \frac{1}{m-1}\sum_{i=1}^{m}(x_i - \bar{x})^6 \\ &= \frac{1}{m-1}\left[\sum_{i=1}^{q}\left\{p-(i-1)\frac{(p-1)}{q}\right\}^6\right]MT_N(6) + \frac{m-1-q}{m-1}MT_N(6) \\ &= R \cdot MT_N(6)\end{aligned} \tag{37}$$

Here,

$$R = 1 + \frac{1}{m-1}[(p-1)(q-1)[\frac{(p-1)^5}{q^5}\frac{(2q-1)}{42}\left\{3q(q-1)(q^2-q-1)+1\right\} - \\ -\frac{1}{2}\frac{(p-1)^4 p}{q^3}(q-1)(2q^2-2q-1)+ \\ +\frac{(p-1)^3 p^2}{2q^3}(2q-1)(3q^2-3q-1)- \\ -5\cdot\frac{(p-1)^2 p^3}{q}(q-1)+\frac{5}{2}\frac{(p-1)p^4}{q}(2q-1)-3p^5] \\ +(p^6-1)q] \tag{38}$$

$\overline{Q_N}$ is stated as follows.

$$\overline{Q_N} = \frac{R}{S}Q_N \tag{39}$$

Here,

$$S = \left[1 + \frac{1}{m-1}(q+1)(p-1)\left\{\frac{2q+1}{6q}(p-1)+1\right\}\right]^3 \tag{40}$$

Under the normal condition, Q is always:

$$Q = 15.0$$

[10]. Therefore

$$\overline{Q_N} \cong 15.0$$

As failure increases, $\overline{Q_N}$ value grows up.

As stated before in 3.3., absolute deterioration factor for $\overline{Q_N}$ is introduced in the same way,

$$\overline{W_N} = \frac{1}{\overline{Q_N} - 14} \tag{41}$$

Another method for an absolute deterioration factor is considered to be as follows.

$$\overline{U_N} = \frac{15}{\overline{Q_N}} \tag{42}$$

5 Numerical example

5.1 Transition of $\overline{KT_N}$

Under the assumption of 3.3., let $m = 12$. Considering the case $p = 2, 3, \ldots, 6$ and $q = 1, 2, 3, 4$, we obtain Table 1 from the calculation of Eq.(27).

Table 1: $\overline{KT_N}$ for each case

		p					
		1	**2**	**3**	**4**	**5**	**6**
q	**1**	3.0	4.377	8.319	12.984	17.106	20.367
	2	3.0	4.266	7.227	10.275	12.762	14.649
	3	3.0	4.194	6.555	8.703	10.338	11.535
	4	3.0	3.603	6.018	7.593	8.736	9.552

As p increases, $\overline{KT_N}$ increases. On the other hand, $\overline{KT_N}$ decreases as q increases when p is the same.

When damages increase or transfer to other places, peak level grows up and one sided affiliated impact vibrations spread. This means that $\overline{KT_N}$ value shift from the left-hand side upwards to the right-hand side downwards in Table 1. For example, the following transition of $\overline{KT_N}$ can be supposed.

When $q=1$, $p=1$, $\overline{KT_N}=3.0$

When $q=2$, $p=2$, $\overline{KT_N}=4.266$

When $q=4$, $p=4$, $\overline{KT_N}=7.593$

When $q=4$, $p=6$, $\overline{KT_N}=9.552$

Calculation $\overline{Z_N}$ and $\overline{V_N}$ in the same way, we obtain Table 2 and Table 3 respectively.

Table 2: Transition of $\overline{Z_N}$

		p					
		1	**2**	**3**	**4**	**5**	**6**
q	**1**	1.0	0.421	0.158	0.091	0.066	0.054
	2	1.0	0.441	0.191	0.121	0.093	0.079
	3	1.0	0.456	0.220	0.149	0.120	0.105
	4	1.0	0.624	0.249	0.179	0.148	0.132

Table 3: Transition of $\overline{V_N}$

		p					
		1	**2**	**3**	**4**	**5**	**6**
q	**1**	1.0	0.685	0.361	0.231	0.175	0.147
	2	1.0	0.703	0.415	0.292	0.235	0.205
	3	1.0	0.715	0.458	0.345	0.290	0.260
	4	1.0	0.833	0.499	0.395	0.343	0.314

5.2 Transition of $\overline{Q_N}$

Under the assumption of 3.3., let $m = 12$. Considering the case $p = 2, 3, ..., 6$ and $q = 1, 2, 3, 4$, we obtain Table 4 from the calculation of Eq.(39).

Table 4: $\overline{Q_N}$ for each case

		p					
		1	**2**	**3**	**4**	**5**	**6**
q	**1**	15.000	48.947	195.551	424.009	661.866	870.169
	2	15.000	43.188	136.705	258.660	373.537	469.160
	3	15.000	39.620	104.047	175.469	236.761	285.220
	4	15.000	36.523	82.759	127.906	164.091	191.691

As p increases, $\overline{Q_N}$ increase. On the other hand, $\overline{Q_N}$ decrease as q increases when p is the same.

When damages increase or transfer to another place, peak level grows up and one sided affiliated impact vibrations spread. This means that $\overline{Q_N}$ value shift from the left-hand side upwards to the right-hand side downwards in Table 4. For example, following transition of $\overline{Q_N}$ can be supposed.

When $q = 1$, $p = 1$, $\overline{Q_N} = 15.000$

When $q = 2$, $p = 2$, $\overline{Q_N} = 43.188$

When $q = 4$, $p = 4$, $\overline{Q_N} = 127.906$

When $q = 4$, $p = 6$, $\overline{Q_N} = 191.691$

Calculating $\overline{W_N}$ and $\overline{U_N}$ in the same way, we obtain Table 5 and Table 6 respectively.

Table 5: Transition of $\overline{W_N}$

		p					
		1	**2**	**3**	**4**	**5**	**6**
q	**1**	1.000	0.029	0.006	0.002	0.002	0.001
	2	1.000	0.034	0.008	0.004	0.003	0.002
	3	1.000	0.039	0.011	0.006	0.004	0.004
	4	1.000	0.0444	0.015	0.009	0.007	0.006

Table 6: Transition of $\overline{U_N}$

		p					
		1	**2**	**3**	**4**	**5**	**6**
q	**1**	1.000	0.306	0.077	0.035	0.023	0.017
	2	1.000	0.347	0.120	0.058	0.040	0.032
	3	1.000	0.379	0.144	0.085	0.063	0.053
	4	1.000	0.411	0.181	0.117	0.091	0.078

From Table 5 and Table 6, following transition of $\overline{W_N}$ and $\overline{U_N}$ can be supposed respectively.

When $q = 1$, $p = 1$, $\overline{W_N} = 1.000$ $\overline{U_N} = 1.000$

When $q = 2$, $p = 2$, $\overline{W_N} = 0.034$ $\overline{U_N} = 0.347$

When $q = 4$, $p = 4$, $\overline{W_N} = 0.009$ $\overline{U_N} = 0.117$

When $q = 4$, $p = 6$, $\overline{W_N} = 0.006$ $\overline{U_N} = 0.078$

Comparing the case of $\overline{KT_N}$ with $\overline{Q_N}$, we can see that $\overline{Q_N}$ is much more sensitive and machine troubles can be detected easily. Though the mathematical formulation of $\overline{Q_N}$ is little more complex, both methods enable us to calculate the new indices even on a pocketsize calculator quickly and easily in the industry.

6 Simplified absolute index of n-th moment

To compare n-th moment and Bicoherence with newly introduced index stated above, we show simplified absolute index of n-th moment [8].

6.1 Simplified absolute of *n*-th moment

When the number of failures on bearings or gears arise, the peak value arise cyclically. In the early stage of the defect, this peak signal usually appears clearly. Generally, defects will injure other bearings or gears by contacting the inner covering surface as time passes.

Assume that we get N amount of data and then newly get L amount of data. Assume that mean, variance and moment are same with 1~*N* data and *N*+1~*N*+*L* data except for the case where a special peak signals arises.

Let mean ,variance and n-th moment calculated by using 1~N data state as:

$$\overline{x}_N,\ \ \sigma^2{}_N,\ \ M_N(n)$$

And as for *N*+1~*N*+*L*, let them state as:

$$\overline{x}_{N/l},\ \ \sigma^2{}_{N/l},\ \ M_{N/l}(n)$$

Where

$$M_N(n)=\frac{1}{N}\sum_{i=1}^{N}\left(x_i-\overline{x}\right)^n \tag{43}$$

$$M_{N/l}(n)=\frac{1}{l}\sum_{i=N+1}^{N+l}\left(x_i-\overline{x}\right)^n \tag{44}$$

Therefore, $Q_{N+l}(n)$ is stated as:

$$Q_N(n)=\frac{M_N(n)}{\sigma^n{}_N} \tag{45}$$

Assume that the peak signal which has S times impact from normal signals arises in each *m* times samplings. As for determining the sampling interval, the sampling theorem which is well known can be used. But in this paper, we do not pay much attention on this point in order to focus on the proposed theme.

Let $\sigma^2{}_{N/l}$ and $M_{N/l}$ of this case, of $N + 1$~$N + l$ be $\overline{\sigma}^2{}_{N/l}$, $\overline{M}_{N/l}$, then we get:

$$\begin{aligned}\overline{\sigma}^2_{N/l} &=\frac{1}{l}\sum_{i=N1}^{N+l}\left(x_i-\overline{x}\right)^2\\ &\cong\frac{l-\frac{l}{m}}{l}\sigma^2{}_N+\frac{\frac{l}{m}}{l}S^2\sigma^2{}_N=\sigma^2{}_N\left(1+\frac{S^2-1}{m}\right)\end{aligned} \tag{46}$$

$$\begin{aligned}\overline{M}_{N/l}(n) &=\frac{1}{l}\sum_{i=N+1}^{N+l}\left(x_i-\overline{x}\right)^n\\ &\cong\frac{l-\frac{l}{m}}{l}M_{N/l}(n)+\frac{\frac{l}{m}}{l}S^nM_{N/l}=\left(1+\frac{S^n-1}{m}\right)M_{N/l}(n)\end{aligned} \tag{47}$$

From these equations, we obtain $\overline{Q}_{N+l}(n)$ as $Q_{N+l}(n)$ of the above case:

$$\bar{Q}_{N+l}(n) \cong \frac{\frac{N}{N+l} M_N(n) + \frac{l}{N+l}\left(1+\frac{S^n-1}{m}\right) M_N(n)}{\left\{\frac{N}{N+l}\sigma^2{}_N + \frac{l}{N+l}\sigma^2{}_N\left(1+\frac{S^2-1}{m}\right)\right\}^{\frac{n}{2}}}$$
$$= \frac{1+\frac{l}{N+l}\cdot\frac{S^n-1}{m}}{\left(1+\frac{l}{N+l}\cdot\frac{S^2-1}{m}\right)^{\frac{n}{2}}}\cdot\frac{M_N(n)}{\sigma^2{}_N} = \frac{1+\frac{l}{N+l}\cdot\frac{S^n-1}{m}}{\left(1+\frac{l}{N+l}\cdot\frac{S^2-1}{m}\right)^{\frac{n}{2}}} Q_N(n) \tag{48}$$

While $Q_{N+l}(n)$ is Kurtosis when $n=4$,

$$Q_N(4) = KT$$

We assume that time series are stationary as is stated before in 2. Therefore, even if sample pass may differ, mean and variance are naturally supposed to be the same when the signal is obtained from the same data occurrence point of the same machine.

We consider such case when the impact vibration occurs. Except for the impact vibration, other signals are assumed to be stationary and have the same means and variances. Under this assumption, we can derive the simplified calculation method for machine diagnosis which is a very practical one.

From the above equation, we obtain $\overline{KT_{N+l}}$ in the following way.

$$\overline{KT} \cong \frac{1+\frac{l}{N+l}\cdot\frac{S^4-1}{m}}{\left(1+\frac{l}{N+l}\cdot\frac{S^2-1}{m}\right)^2}\times 3.0 \tag{49}$$

Consequently, we obtain $\bar{Z}_{N+l}(n)$ as:

$$\bar{Z}_{N+l}(n) \cong \frac{\prod_{k=1}^{n/2}(2k-1)}{Q_{N+l}(n)} = \frac{\prod_{k=1}^{n/2}(2k-1)}{\dfrac{1+\dfrac{l}{N+l}\cdot\dfrac{S^n-1}{m}}{\left(1+\dfrac{l}{N+l}\cdot\dfrac{S^2-1}{m}\right)^{\frac{n}{2}}} Q_N(n)} \tag{50}$$

Under the normal condition,

$$Q_N(n) \cong \prod_{k=1}^{n/2}(2k-1) \tag{51}$$

Therefore, we get:

$$\bar{Z}_{N+l}(n) \cong \frac{\left(1+\dfrac{l}{N+l}\cdot\dfrac{S^2-1}{m}\right)^{\frac{n}{2}}}{1+\dfrac{l}{N+l}\cdot\dfrac{S^n-1}{m}} \tag{52}$$

6.2 Numerical Examples

If the system is under normal condition, we may suppose $p(x)$ becomes a normal distribution function. Under this condition, $Q(n)$ is as follows theoretically when $n = 4, 6, 8$

$$Q(4) = 3.0, \quad Q(6) = 15.0, \quad Q(8) = 105.0$$

Under the assumption of 3.3, let $m = 12$. Considering the case $S = 2, 4, 6$ for 3.3, and setting $N \to 0,\ l \to N$, we obtain Table 7.

Table 7: $\overline{Q}, \overline{Z}$ by the variation of S

		S	1	2	4	6
n	4	$\overline{Q}$	3.0	4.32	13.2	21.3
		$\overline{Z}$	1.0	0.69	0.23	0.14
	6	$\overline{Q}$	15.0	48.65	450.71	970.89
		$\overline{Z}$	1.0	0.31	0.03	0.02(0.016)
	8	$\overline{Q}$	105.0	956.93	22378.44	62453.16
		$\overline{Z}$	1.0	0.11	0.0047	0.0017

7 Remarks

We introduced the two absolute deterioration factors for each model respectively. We compare them with Bicoherence and evaluate them accordingly. In [2], the waveform is simulated in three cases as (a) normal condition, (b) small defect condition (maximum vibration is two times compared with (a)), (c) big defect condition (maximum vibration is six times compared with (a)). They showed the result of Kurtosis in these cases. We showed the relations between those results and $\overline{Q_N}$ (in detail see [8]). Subsequently, we examine

Bicoherence. We made experiment in the past [6], [7]. Summary of the experiment is as follows. Pitching defects are pressed on the gears of small testing machine.

Small defect condition	Pitching defects pressed on 1/3 gears of the total gear.
Middle defect condition	Pitching defects pressed on 2/3 gears of the total gear.
Big defect condition	Pitching defects pressed on whole gears of the total gear.

We examined several cases for the f_1, f_2 in Eq.(12). We got best-fit result in the following case.

$$\begin{cases} f_1 & : \text{peak frequency of power spectrum} \\ f_2 & : 2\, f_1 \end{cases}$$

We obtained the following Bicoherence values in this case (Table 8).

Table 8: Transition of $\overline{U_N}$

Condition	Normal	Small defect	Middle defect	Big defect
Bicoherence	0.99	0.38	0.09	0.02

Thus, Bicoherence proved to be a very sensitive good index. These results can be taken into account, though the definition of defect size does not necessarily coincide. Bicoherence is an absolute index of which range is 1 to 0. Therefore it can be said that it is a universal index.

Now, we compare this index with proposed simplified absolute indices. The proposed methods are the absolute indices of which range are from 1 to 0 similarly as Bicoherence. As for sensitivity, $\overline{Z_N}$ is better than $\overline{V_N}$ for the group of $\overline{KT_N}$. $\overline{W_N}$ is better than $\overline{V_N}$ in sensitivity but $\overline{W_N}$ falls too fast. Even when $p = 2$, it has the value of less than 0.09. While $\overline{U_N}$ decreases smoothly, which may be appropriate for the practical use. As a whole, indices of $\overline{Q_N}$ group ($\overline{W_N}$, $\overline{U_N}$) are sensitive than those of $\overline{KT_N}$ group ($\overline{Z_N}$, $\overline{V_N}$). Comparing $\overline{W_N}$ with $\overline{U_N}$, $\overline{U_N}$ is better than $\overline{W_N}$ for the practical use.

Thus, we can get a sensitive index for machine diagnosis in a simple way. These deterioration factors are said to a kind of 6-th order moment in a broad sense. They have the value around Bicoherence. Someone is much more sensitive than Bicoherence and someone is moderate compared with Bicoherence. Therefore proposed one is a practical good index with simple calculation method.

Next, we compare Bicoherence with proposed simplified absolute index of n-th moment. The proposed method is an absolute index of which range is from 1 to 0 similarly as Bicoherence. As for sensitivity, the case of $n = 6$ is quite similar to Bicoherence, but the proposed one is slightly much more sensitive. The value is already 0.31 at small defect condition and 0.03 at middle defect condition which show quite sensitive behavior. It is suitable for especially early stage failure detection.

Sensitivity is better in $n = 6$ than that of $n = 4$. Sensitivity is much better in $n = 8$, but the value falls too fast therefore the judgment becomes hard. Therefore the case $n = 6$ is good for the practical use. As Bicoherence is one of the kind of 6-th order moment in a broad sense, these deterioration factors of 6-th order moment in a broad sense found to be sensitive and practical indicies.

This calculation method is simple enough to execute even on a pocketsize calculator as is shown in Eq. (52). Compared with Bicoherence which has to be calculated by Eq. (12)~(15), proposed method is by far a simple one and easy to handle on the field defection. Comparing with these indices, simplified calculation method of 6-th normalized moment includes index with much more sensitive than others. Judging from the machine status, suitable method should be selected. For example, sensitive index should be selected for the machine which requires severe control for machine failure.

8 Conclusion

We proposed a simplified calculation method of Kurtosis and 6-th normalized moment for the analysis of impact vibration including affiliated impact vibration. One sided affiliated impact vibration was approximated by half triangle and simplified calculation method was introduced. Furthermore an absolute deterioration factor was introduced.

Varying the shape of triangle, various models were examined and it was shown that absolute deterioration factor of 6-th normalized moment was much more sensitive than those of kurtosis. Utilizing this method, the behavior of 6-th normalized moment would be forecasted and analyzed while watching machine condition and exquisite diagnosis would be performed.

For further detection, an absolute index of n-th moment and Bicoherence were compared with this index and analysis was executed. Focusing that Bicoherence was considered to be a kind of absolute deterioration factor of normalized 6-th order moment in a broad sense, analysis was conducted. From the result of comparison, absolute factor of 6-th order moment in a broad sense was better than other factors in the viewpoint of sensitivity and practical use. The effectiveness of this method should be examined in various cases.

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